Financial Distortions and the Distribution of Global Volatility

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Abstract

Decreasing returns on the macro level are an outcome of productive efficiency on the micro level: when inputs are scarce, an efficient economy carries out only the most productive projects. When inputs become more abundant, the economy runs out of high-return activities and starts implementing less productive projects as well. This link between productive efficiency on the micro level and decreasing returns on the macro level suggests that misallocation can reduce the extent of aggregate decreasing returns. I show that this is generically the case when the underlying distortion decreases the relative likelihood that better projects are assigned higher priorities. Under this assumption, I consider a model in which fluctuations in inputs are driven by shocks to funding and show that (a) distortions increase the sensitivity of aggregate output to funding shocks; and (b) financial integration amplifies shocks in relatively distorted economies, but mitigates shocks in less distorted economies.

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1 Introduction

The foundations of the law of decreasing returns build heavily on concepts of productive efficiency. As Adam Smith proposed in “The Wealth of Nations”:

“As the colony increases, the profits of stock gradually diminish. When the most fertile and best situated lands have been all occupied, less profit can be made by the cultivation of what is inferior both in soil and situation, and less interest can be afforded for the stock which is so employed.”


As Smith explains, declining marginal returns on the macro level are an outcome of an efficient allocation of inputs on the micro level: when inputs are scarce, only the most productive projects are carried out. When inputs become more abundant, the economy runs out of high-return activities, and starts implementing projects with lower returns.

This tight link between decreasing returns and productive efficiency suggests that economies in which misallocation is prevalent may be less subject to decreasing returns. When the economy is imperfect in its ability to channel inputs to their most productive uses, some unproductive projects may be implemented when inputs are scarce, while more productive projects may be implemented only when resources become more abundant. In Smith’s colony, this type of misallocation could occur, for example, when colonists have incomplete information regarding which lands are the most fertile. Compared to an economy that is perfectly able to tell apart good lands from bad lands, the marginal return to inputs would decline less steeply.

Starting from this basic insight, this paper studies the implications of input misallocation on the shape of the aggregate production function. I consider a simple environment in which there are heterogeneous projects, and a single input of production. Buying the input requires funding, so the amount of funds determines the level of production. There are a variety of distortions that prevent the efficient allocation of funds. The equilibrium is characterized by a stochastic matching between projects and funding priorities. Misallocation is measured by the relative likelihood

\footnote{For more on the origin of the law of diminishing returns, see Cannan 1892.}
that better projects receive lower funding priorities. This form of complementarity at the micro level implies that, at the macro level, the marginal return to inputs is log-supermodular in inputs and the level of distortion.

This property has far-reaching implications with respect to the sensitivity of aggregate output to funding shocks. I embed this representation in a simple general equilibrium model, in which shocks to the supply of funding lead to fluctuations in production. I first consider the closed economy equilibrium and show that economies with greater misallocation are more sensitive to funding shocks. Intuitively, decreasing returns play an important role in mitigating fluctuations in inputs: in decreasing-return economies, the projects implemented at the margin are the least productive among all implemented projects. Whether or not they are implemented has a relatively small effect on total output. In less-steeply-declining marginal return economies, the expected productivities of the marginal projects are relatively higher; discontinuation of these projects will have a relatively larger impact on aggregate output.

Next, I consider the implications of financial integration between countries with heterogeneous levels of misallocation. In the integrated regime, funding can move freely across countries - shocks to the domestic supply of funds need not adjust through changes in domestic production, as adjustments can also be made through changes in the supply of funding to other countries. The analysis of the integrated environment suggest a new view regarding the relationship between misallocation and volatility in the context of financial integration. Whether financial integration amplifies or reduces volatility depends on the relative (rather than the absolute) severity of the distortion. Relatively more distorted economies become more volatile upon financial integration, but by the same token, relatively less distorted economies become less volatile in the financially integrated environment. In essence, financial integration shifts the margin of adjustment to fluctuations in the global supply of funds disproportionately to more distorted economies. In equilibrium, distorted economies essentially serve as a “buffer zone”, insulating less distorted economies from fluctuations in their own supply of funds.

I illustrate how these theoretical results may be useful for thinking about the effects of financial distortions on the global distribution of volatility. I discuss several potential applications:

- **The excess volatility of emerging markets.** There is increasing evidence that misallocation of inputs is more prevalent in economies that are in ear-
lier stages of development (see, for example, Hsieh and Klenow [2009]). The mechanism in this paper suggests that this may partly explain why output fluctuations in emerging economies are larger than output fluctuations in developed economies.

- **The comovements of emerging economies.** The theory implies that countries with high levels of misallocation should be similarly affected by shocks to the global supply of funds. This is consistent with observed comovements in output among emerging economies (e.g., the Asian crisis), and suggests that the comovements may be driven by a common sensitivity to shocks to external funding.

- **The “Great Moderation” trend in the US.** Up until the recent crisis, the US has been experiencing a steep and steady decline in the volatility of output and employment (see Gali and Gambetti [2009]). This model suggests that this trend may be an equilibrium outcome of financial integration between the US and emerging economies. Fluctuations in the supply of funding in the US (e.g., monetary shocks) have a smaller effect on the domestic economy, because they adjust primarily through changes in funding supplied to emerging markets.

- **The financial crisis in the US.** Many attribute the amplification of the recent financial crisis in the US to excessive use of “trenching”, or the practice of creating diversified assets out of a collection of more specialized loans (such as mortgage backed securities backed by sub-prime mortgage loans). The mechanism in this paper suggests that by obscuring the differences between different projects, this practice reduces the aggregate extent of decreasing returns, making it more difficult to discontinue projects differentially when a shock hits. In an extension, I illustrate that the adoption of such “pooling” arrangements may be an equilibrium outcome of financial integration, that lowers the expected cost of funds from the perspective of developed economies.

This paper is related to several strands of literature. The idea that financial distortions exacerbate output volatility appears prominently in the context of collateral constraints. Kiyotaki and Moore [1997], Fostel and Geanakoplos [2008], and Caballero and Krishnamurthy [2001] are important examples, the latter two with specific applications to open economies. The general formulation in this paper suggests that the
link between financial distortions and volatility is not unique to collateral constraints, and is common to many forms of misallocation, such as incomplete information, market segmentation or search frictions. Furthermore, it provides further predictions regarding the divergence in volatility following financial integration, in particular with respect to the mitigation of output volatility in less-distorted economies.

Methodologically, this paper is related to the literature emphasizing the role of supermodularity and log supermodularity conditions in various economic fields. Prominent examples include, among others, Milgrom and Weber [1982] in auction theory; Bulow et al. [1985] in industrial organization; Jewitt [1987] and Athey [2002] in monotone comparative statics under uncertainty; Costinot [2009] in international trade; and, more recently, Acemoglu and Jensen [2012] in dynamic general equilibrium models. Shimer and Smith [2000] consider a model in which heterogeneous agents search for partners for joint production, where the output produced by the match depends positively on each type. They use log supermodularity conditions on the joint production function to generate sufficient conditions for positive assortative matching. Broadly, the model in this paper is the inverse exercise: I characterize conditions on the matching process that deliver log supermodularity of the aggregate production function.

2 An aggregate representation of misallocation

There is a single final good. For simplicity, I will assume that there is only one input of production, which I will refer to as “labor”. In the general equilibrium model (presented in the next section), fluctuations in funding will translate into fluctuations in employment - therefore labor should be interpreted more generally as production inputs which need to be purchased with funds.

There are a unit measure of projects indexed \( x \in [0, 1] \). Each project requires one unit of labor to implement. Projects are of heterogeneous productivity: the projects indexed \( x \in [0, \alpha] \) (where \( \alpha \leq 1 \)) produce a return of \( A \) if implemented; the rest of the projects are unproductive and produce a return of 0. Section 6 extends this setup to environments with arbitrary productivity distributions.

Denote the aggregate amount of (employed) labor by \( L \). Whether or not a specific project is carried out depends on the whether \( L \) is high enough. For each project \( x \), there is a threshold \( p(x) \), such that the project is implemented if and only if \( L \geq p(x) \).
Of course, the efficient allocation is to implement all productive projects before any unproductive projects; however, the assumption is that various technological and economic frictions make this allocation unfeasible. In principle, the threshold \( p(x) \) is an equilibrium object that is determined by a wide variety of factors, including the project’s perceived productivity, the pledgeability of its returns, its connections to people in power, luck, etc. Importantly for our purposes, this equilibrium object can be summarized as a priority: when the aggregate amount of labor is \( L \), all projects with priorities \( p \in [0, L] \) are implemented.

The matching between projects and priorities is stochastic. The marginal probability that priority \( p \) is assigned to project \( x \) is given by \( \theta(x, p) \). I restrict attention to allocations that weakly prefer productive projects over unproductive projects. Formally, for any \( p' \leq p \), it will be required that the probability that a productive project is assigned priority \( p' \) is higher than the the probability that a productive project is assigned priority \( p \):

\[
\int_0^{p'} \theta(x, p')dx \geq \int_0^{p} \theta(x, p)dx \tag{1}
\]

The matching between projects and priorities takes place simultaneously in many different sub-locations. Given \( A, L \) and \( \theta \), aggregate output is deterministic and equal to the expected output produced by all implemented projects, that is, all projects with priorities \( p \leq L \):

\[
Y(A, L, \theta) = A \int_0^{L} \int_0^{\alpha} \theta(x, p)dx dp \tag{2}
\]

The expression above can be understood as follows. The expected output of the project implemented with the \( p \)-th priority is \( A \) times the probability that it is assigned to a productive project, which is \( A \int_0^{\alpha} \theta(x, p)dx \). Output is equal to the total amount produced by all projects who received priorities \( p \leq L \), yielding the expression above.

The aggregate marginal return to labor, denoted \( y \), is given by the expected

\[\text{[6]}\]

\[\text{\footnotesize{Acemoglu and Zilibotti 1997 present a model in which the key ingredient is that idiosyncratic risk affects aggregate variables when only few projects are carried out. The theory suggests an alternative channel through which financial development may reduce volatility. In my model, this channel is essentially shut down by the application of the law of large numbers (furthermore, countries will not systematically differ in the number of projects that are carried out, but only in the efficiency of the implemented set).}}\]
productivity of the project with the \(L\)-th priority:

\[
y(A, L, \theta) = \frac{\partial Y(A, L, \theta)}{\partial L} = A \int_0^\alpha \theta(x, L) dx
\]  

Note that the condition in equation (1) guarantees that the marginal return to labor is weakly decreasing. It also implies that the extent of decreasing returns depends on the relative “advantage” that is given to productive projects over unproductive projects.

A model of misallocation has three components: first, there is a parameter space \(\Omega\). Second, there is a partial ordering of \(\Omega\) according to a “more distorted than” relation. For example, in a model of incomplete information, \(\omega \in \Omega\) might be interpreted as the observability of productivity - if the information in \(\omega'\) is, in some sense, less noisy than \(\omega\), we might say that an economy whose information structure is given by \(\omega'\) is less distorted than an economy whose information structure is \(\omega\). The third component of the model is a function \(\theta_{\omega}(\cdot, \cdot)\), that determines the stochastic mapping between projects and priorities given the parameter \(\omega\). In the example of incomplete information, \(\theta_{\omega}(x, p)\) may depend not only on the quality of the project \(x\), but also on the its perceived productivity, which is determined by the information structure.

**Definition 1** A model of misallocation is a triple \(\{\Omega, \{\theta_{\omega}\}_{\omega \in \Omega}, \preceq\}\), where \(\Omega\) is a parameter space, \(\theta_{\omega}\) is a stochastic matching between projects and priorities given \(\omega\), and \(\preceq\) is a partial ordering on \(\Omega\) such that \(\omega' \preceq \omega\) is interpreted as “an economy with parameters \(\omega\) is weakly more distorted than an economy with parameters \(\omega'\)”.

A minimal requirement on the “\(\preceq\)” relation is that \(Y(A, L, \theta_{\omega})\) is monotone decreasing in \(\omega\); that is, with the same technology level \((A)\) and the same labor inputs \((L)\), a more distorted economy produces less output. Since \(Y(A, 1, \theta_{\omega}) = \alpha A\) regardless of \(\omega\), monotonicity implies that for any \(\omega' \preceq \omega\), \(Y(A, 1, \theta_{\omega}) - Y(A, L, \theta_{\omega}) \geq Y(A, 1, \theta_{\omega'}) - Y(A, L, \theta_{\omega'})\) and thus:

\[
\frac{Y(A, 1, \theta_{\omega}) - Y(A, L, \theta_{\omega})}{Y(A, L, \theta_{\omega})} \geq \frac{Y(A, 1, \theta_{\omega'}) - Y(A, L, \theta_{\omega'})}{Y(A, L, \theta_{\omega'})}
\]  

This minimal requirement therefore implies that distortions reduce the “global” extent of decreasing returns: increasing labor from any \(L \leq 1\) to \(L = 1\) leads to higher output growth in more distorted economies.

\(^3\)When \(L = 1\), all projects are implemented so the implementation order doesn’t matter.
It is useful to observe that, on the micro level, equation 4 is equivalent to assuming that for every \( p \), the average productivity of projects with priorities \( p' \leq p \) relative to the average productivity of projects with priorities \( p' \geq p \) is higher in more distorted economies:

\[
\int_0^1 \frac{1}{1-p} \int_0^x \theta(x, p') dx dp' \geq \int_0^1 \frac{1}{1-p} \int_0^x \theta(x, p') dx dp',
\]

(5)

I will restrict attention to models of misallocation in which this property is satisfied locally as well: at any range of priorities, distortions reduce the relative likelihood that higher priorities are assigned to productive projects. Notice that this condition is a complementarity condition between three variables: projects \( (x) \), priorities \( (p) \), and parameters \( (\omega) \).

**Assumption 1** For any two parameters \( \omega' \preceq \omega \) and for any two priorities \( p' \leq p \):

\[
\frac{\int_0^x \theta(x, p) dx}{\int_0^x \theta(x, p') dx} \geq \frac{\int_0^x \theta(x, p) dx}{\int_0^x \theta(x, p') dx}
\]

(6)

This condition compares the probability that a lower priority is assigned to a productive project with the probability that a higher priority is assigned to a productive project. In more distorted economies these probabilities are more similar, reflecting a matching that is less correlated with productivity.

**Random allocation example.** Consider the following example (that will also be used to illustrate the results in the following sections). Let \( \Omega \) be a set of parameters that includes an element \( \omega_{\text{max}} \). The partial ordering is defined as follows: for every \( \omega \in \Omega \), \( \omega \) is less than \( \omega_{\text{max}} \) (\( \omega \preceq \omega_{\text{max}} \)). Thus \( \omega_{\text{max}} \) is a maximal element. Let \( \theta_{\omega_{\text{max}}} \) be given by \( \theta_{\omega_{\text{max}}}(x, p) = 1 \); in other words, an economy with the parameter \( \omega_{\text{max}} \) assigns priorities to projects completely at random, and does not strictly prefer productive projects (a productive project is just as likely to be implemented as an unproductive project, regardless of \( L \)). By equation 3, the marginal return to labor is constant and equal to \( \alpha A \).

This model of misallocation is consistent with Assumption 1 because, by equation 3, \( \int_0^x \theta(x, p) \) is weakly decreasing in \( p \). Thus, for any \( p' \leq p \) and \( \omega \preceq \omega_{\text{max}} \):
\[ \frac{\int_0^\alpha \theta_{\omega_{\text{max}}}(x,p)dx}{\int_0^\alpha \theta_{\omega_{\text{max}}}(x,p')dx} = \frac{\alpha}{\alpha} = 1 \geq \frac{\int_0^\alpha \theta_\omega(x,p)dx}{\int_0^\alpha \theta_\omega(x,p')dx} \]  

(7)

In this extreme case, the marginal return to labor is completely flat given the maximal level of distortion \((\omega = \omega_{\text{max}})\), and decreasing otherwise.

Section 6 extends this setup to environments with arbitrary productivity distributions, and presents several additional examples of models of misallocation that are consistent with Assumption 1, including models of incomplete information, collateral constraints, search frictions and market segmentation.

### 2.1 A reduced form representation of misallocation

The property central to all of the results that follow is that the marginal return to labor is log supermodular\(^4\) in the level of employment and the level of distortion:

**Property 1** For any \(\omega' \preceq \omega\) and for any \(L' \leq L\),

\[ \frac{y(A,L,\theta_\omega)}{y(A,L',\theta_\omega)} \geq \frac{y(A,L,\theta_{\omega'})}{y(A,L',\theta_{\omega'})} \]  

(8)

Property 1 can be understood as follows. Fix any two levels of employment, \(L'\) and \(L\), such that \(L' \leq L\). Property 1 requires that the ratio of the marginal returns to labor at \(L\) and \(L'\) is relatively lower in more efficient economies; loosely stated, the marginal return to labor declines faster in more efficient economies.

More accurately, Property 1 requires that \(\ln(y)\) (rather than simply \(y\)) declines faster in more distorted economies. Imposing the condition on \(\ln(y)\) rather than on \(y\) means that relative degrees of misallocation are invariant to scaling by \(A\). Note that the slope of \(y\) is proportional to \(A\); a lower \(A\) naturally implies that the slope of \(y\) is lower, but this reflects a lower level of aggregate technology and has nothing to do with misallocation. A less-steeply-declining \(\ln(y)\) means that the return to the next unit of labor is relatively more similar to the return to the last unit of labor. Since the condition relates to relative rather than absolute returns, comparing two

\(^4\)Recall that a function \(f(x_1,x_2)\) is log supermodular in \(x_1\) and \(x_2\) if for any \(x'_1 < x_1\) and \(x'_2 < x_2\), we have that \(f(x_1,x_2)f(x'_1,x'_2) \geq f(x_1',x_2)f(x_1,x_2')\). If \(f\) is strictly positive, this condition implies that \(\frac{f(x_1,x_2)}{f(x'_1,x'_2)} \leq \frac{f(x_1',x_2)}{f(x'_1,x'_2)}\). If \(f\) is strictly positive and differentiable, this corresponds to \(\frac{\partial^2 \ln f(x_1,x_2)}{\partial x_1\partial x_2} \geq 0\).
economies in terms of their misallocation is independent from any differences in labor productivity.

Lemma 1  A model of misallocation satisfies Property \( P \) if and only if it is consistent with Assumption \( I \).

This lemma follows trivially from equation \( 3 \). Property \( P \) can therefore be used as an aggregate reduced form representation of misallocation, under Assumption \( I \).

3 Closed economy equilibrium

In this section I characterize the closed economy equilibrium. To close the model, consider the following simple model of aggregate employment. The amount of potential labor is fixed at 1. However, funding is required in order to hire labor; consequently, the amount of employment is determined by the supply of funds. The supply of funds is stochastic and denoted by \( F \). The wage rate (in terms of funds) is determined prior to the realization of \( F \), and is denoted \( w \). Thus, the equilibrium level of employment is given by \( L = F w \).

The stochastic process of \( F \) is given by \( F = \epsilon \bar{F} \), where \( \epsilon \) is a random variable. As a background story, it is convenient to think of an economy in which \( \epsilon = 1 \) is a high probability event, and \( \bar{F} \) is the “long-run” (or steady state) level of funding. The wage \( w \) is set so that \( L = \bar{L} \) whenever \( \epsilon = 1 \) (so \( w = \frac{\bar{F}}{\bar{L}} \)). The realization \( \epsilon = 1 \) can be thought of as the economy’s “benchmark” or “steady state”, and \( \epsilon \neq 1 \) can be thought of as a shock to funding supply, that has a proportional effect on employment. Note that given \( \epsilon \), labor is given by \( \epsilon \bar{L} \).

\[
\epsilon \bar{F} = F = wL \Rightarrow L = \frac{\epsilon \bar{F}}{w} = \frac{\epsilon w \bar{L}}{w} = \epsilon \bar{L} \tag{9}
\]

It is important to note that in this framework, the slope of the marginal return to employed labor reflects the efficiency in which the economy allocates funding to projects, rather than its ability to allocate scarce labor. Broadly, misallocation of labor in this context can be thought of as an outcome of financial distortions, or distortions that prevent funding from being allocated to the right projects.

\(^5\) Assuming that \( \epsilon \) is bounded from above by \( \frac{1}{\bar{L}} \) guarantees that given the wage \( w \), the supply of labor does not constrain employment.
Figure 1: The autarkic equilibrium in the random allocation example.

The figure on the left depicts the closed economy equilibrium in an economy with $\omega \lesssim \omega_{\text{max}}$, and the figure on the right depicts the closed economy equilibrium in an economy with $\omega_{\text{max}}$. Wages are normalized to $w = 1$ (so $F = L$). In both economies there is a positive shock to funding supply. The average productivity of implemented projects declines in the $\omega$ economy but remains constant in the $\omega_{\text{max}}$ economy. In the $\omega_{\text{max}}$ economy, the shock leads to a higher percent increase in output (in the sense that $\Delta Y$ is larger).

The first result is that output in more distorted economies is more sensitive to shocks to the supply of funding:

**Proposition 1** Output is more sensitive to $\epsilon$ in more distorted economies:

$$\frac{\partial \ln Y(A, \epsilon L, \theta_\omega)}{\partial \epsilon} \text{ is increasing in } \omega$$

The average productivity of funded projects is more sensitive to funding supply shocks in less distorted economies:

$$\frac{\partial}{\partial \epsilon} \left( \frac{Y(A, \epsilon L, \theta_\omega)}{\epsilon L} \right) \text{ is increasing in } \omega$$

The proof of this proposition, together with other omitted proofs, is in the appendix. This result illustrates the role of decreasing returns in the mitigation of shocks. In decreasing-return economies, an expansion in the supply of funds leads to the implementation of projects with lower funding priorities that are relatively less productive. Consequently, there is a decline in average productivity, which dampens the expansionary effect. In more distorted economies, the decline in average pro-
ductivity is less steep, as the marginal projects are more similar in quality to the projects implemented inframarginally. This could be either because there are relatively more productive projects matched to marginal priorities, or because there are more unproductive projects that are being implemented inframarginally, lowering the average productivity of funded projects. These two channels imply that the ratio of the marginal return to funding and the average return to funding is higher in more distorted economies. Thus, as the underlying distortion increases, output becomes more sensitive (in percentage terms) to fluctuations in the supply of funds. Using the random allocation example, figure 1 illustrates this result.

4 Integrated equilibrium

In this section I characterize the implications of financial integration between countries with heterogeneous levels of misallocation. Consider a global environment with $n$ countries of equal size, with $\omega_i \leq \omega_{i+1}$ for all $1 \leq i \leq n - 1$. Funds can move freely across countries (but labor is immobile). The value of $\bar{L}$ is identical across countries, but $A_i$ and $\bar{F}_i$ may be country-specific. In order to isolate the effects of financial heterogeneity on the global equilibrium environment, it is convenient to assume that shocks to funding supply are perfectly correlated across countries ($\epsilon_i = \epsilon_j$).\footnote{Assuming that funding supplies are perfectly correlated isolates the effects of financial heterogeneity because, under this assumption, financial integration between identical economies would have no effect on output volatility. In contrast, independent funding supplies would imply that financial integration between identical economies has a moderating effect on output, as shocks to funding supply are shared across countries. Replacing the assumption that funding supplies are perfectly correlated with the assumption that they are independent would therefore decrease volatility in all countries; however, the result that volatility induced by financial integration would be relatively higher for more distorted economies would still hold.}

Let $L_i(\epsilon)$ denote the level of employment in country $i$ given $\epsilon$, and let $F_w(\epsilon)$ and $r(\epsilon)$ denote the global supply of funds given $\epsilon$ and the global return to funding given $\epsilon$ respectively. The global equilibrium is characterized by three equations:

$$\sum_{i=1}^{n} w_i L_i(\epsilon) = \epsilon \sum_{i=1}^{n} \bar{F}_i = F_w(\epsilon)$$  \hspace{1cm} (12)

$$\frac{1}{w_i} y(A_i, L_i(\epsilon), \theta_{\omega_i}) = r(\epsilon)$$  \hspace{1cm} (13)
\( \bar{L} = L_i(1) \) \hspace{1cm} (14)

The first equation is a market clearing condition, stating that the sum of wage bills in all countries must be equated with the global supply of funds. The second condition is the optimality condition of the financial system, requiring that there are no gains from reallocating funds from one country to another \(^7\) (to see this, note that a marginal unit of funding in country \( i \) can purchase \( \frac{1}{w_i} \) units of labor, that produce a marginal return of \( y(A_i, L_i, \theta_{\omega_i}) \)). The third condition is the labor market clearing condition, requiring that wages are set so that absent any shocks (when \( \epsilon = 1 \)) employment is \( \bar{L} \) in every country.

Recall that (by assumption) the autarkic volatility of employment is the same in all economies, regardless of the level of distortion. This is no longer the case in the integrated equilibrium. Under financial integration, countries with higher levels of distortion experience larger fluctuations in employment, as shocks to the global supply of funds adjust disproportionately through changes in employment in more distorted regions. The following proposition states this result:

**Proposition 2** In the financially integrated equilibrium, for any \( 1 \leq i \leq j \leq n \), and for any \( \epsilon \), \( |L_j(\epsilon) - \bar{L}| \geq |L_i(\epsilon) - \bar{L}| \) and \( |\ln Y(A, L_j(\epsilon), \theta_{\omega_j}) - \ln Y(A, \bar{L}, \theta_{\omega_j})| \geq |\ln Y(A, L_i(\epsilon), \theta_{\omega_i}) - \ln Y(A, \bar{L}, \theta_{\omega_i})| \).

**Proof:** By the third equilibrium condition, \( w_i \) is set such that \( L(1) = \bar{L} \). By the second equilibrium condition, for any \( i \) and for any \( \epsilon > 1 \):

\[
\frac{y(A_i, L_i(\epsilon), \theta_{\omega_i})}{y(A_i, \bar{L}, \theta_{\omega_i})} = \frac{r(\epsilon)}{r(1)} \hspace{1cm} (15)
\]

By Property \( \square \) holding \( L_i(\epsilon) \) constant, the LHS is increasing in \( \omega_i \). It follows that if \( \omega_i \leq \omega_j \), \( L_i(\epsilon) \leq L_j(\epsilon) \). A similar argument shows that for \( \epsilon < 1 \), \( L_i(\epsilon) \geq L_j(\epsilon) \). Thus, more distorted economies experience relatively larger fluctuations in employment in the integrated equilibrium as a result of \( \epsilon \) shocks.

\(^7\)The results easily generalize if there is a constant wedge in marginal returns, that is, the second equilibrium condition is replaced with a condition of the form \( \frac{1}{w_i}y_i(A_i, L_i(\epsilon), \theta_{\omega_i}) = \lambda_i r(\epsilon) \).

\(^8\)It is easy to see that this proposition holds true for any \( L \) only if Property \( \square \) is satisfied. The proposition can thus be rephrased as an "if and only if" statement.
To conclude the proof, it is left to show that, in the integrated equilibrium, output is more sensitive to $\epsilon$ shocks in more distorted economies. This follows from Proposition 1 for the same shock to employment, output responds more strongly in more distorted economies. Larger fluctuations in employment in more distorted economies further amplify the differences in output response:

$$\left| \ln Y(A_j, L_j(\epsilon), \theta_{\omega_j}) - \ln Y(A_j, \bar{L}, \theta_{\omega_j}) \right| \geq \left| \ln Y(A_i, L_i(\epsilon), \theta_{\omega_j}) - \ln Y(A_i, \bar{L}, \theta_{\omega_j}) \right|$$

$$\geq \left| \ln Y(A_i, L_i(\epsilon), \theta_{\omega_i}) - \ln Y(A_i, \bar{L}, \theta_{\omega_i}) \right|$$

The proposition above illustrates that, similar to the closed economy environment, output volatility in the integrated equilibrium is positively related to the level of distortion. Unlike the closed economy environment, in the integrated equilibrium the result extends to the volatility of employment.

The key to this result is that shocks to the global supply of funds adjust disproportionately through changes in the supply of funding to relatively more distorted economies. This equilibrium property is closely related to Property 1. In less distorted economies, projects are implemented in an order that is more reflective of their productivity: the expected productivity of projects with high funding priorities well exceeds $r$, while most unimplemented projects generate returns which are well below $r$. Small fluctuations in $r$ therefore have a relatively small impact on the amount of implemented projects. In contrast, in more distorted economies, the order in which projects are implemented is more arbitrary; this implies that the return generated by the next unit of funding is more similar to the return generated by the previous unit. The same fluctuations in $r$ therefore induce larger fluctuations in the amount of implemented projects. Figure 2 illustrates this equilibrium property in the random allocation example.

The analysis of the integrated equilibrium allows for further results regarding the effect of financial integration of output volatility. A key insight of this model is that financial integration leads to an increase in volatility in relatively distorted economies, but a moderation in volatility in relatively less distorted economies. The following

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9 The first inequality follows from the fact that the output response ($\Delta \ln Y$) is increasing in $L(\epsilon)$ (for $\epsilon > 1$) and homogeneous in $A$.

10 In the random allocation example, the equilibrium level of $r$ does not change following a shock to global funding supply. Rather, the shock is adjusted entirely through changes in the quantity of funding supplied to the random economy.
proposition summarizes this result:

**Proposition 3** For every \( \epsilon \), there exists \( 1 < m \leq n \) such that (compared to autarky) financial integration amplifies the effects of an \( \epsilon \) shock for every \( j \geq m \) but mitigates the effects of the shock for every \( i < m \):

\[
|L_i(\epsilon) - \bar{L}| \leq |\epsilon - 1|\bar{L} \leq |L_j(\epsilon) - \bar{L}|
\]

\[
|\ln Y(A_i, L_i(\epsilon), \theta_{\omega_i}) - \ln Y(A_i, \bar{L}, \theta_{\omega_i})| \leq |\ln Y(A_i, \epsilon \bar{L}, \theta_{\omega_i}) - \ln Y(A_i, \bar{L}, \theta_{\omega_i})| \tag{16}
\]

\[
|\ln Y(A_j, L_j(\epsilon), \theta_{\omega_j}) - \ln Y(A_j, \bar{L}, \theta_{\omega_j})| \geq |\ln Y(A_j, \epsilon \bar{L}, \theta_{\omega_j}) - \ln Y(A_j, \bar{L}, \theta_{\omega_j})| \tag{17}
\]

\[
|\ln Y(A_i, L_i(\epsilon), \theta_{\omega_i}) - \ln Y(A_i, \bar{L}, \theta_{\omega_i})| \geq |\ln Y(A_i, \epsilon \bar{L}, \theta_{\omega_i}) - \ln Y(A_i, \bar{L}, \theta_{\omega_i})| \tag{18}
\]

**Proof:** Using the third equilibrium condition and the first equilibrium condition for \( \epsilon = 1 \),

\[
\sum_{i=1}^{n} w_i \bar{L} = \sum_{i=1}^{n} \bar{F}_i
\]

(19)

The first equilibrium condition can therefore be rewritten as:

\[
\sum_{i=1}^{n} w_i (L_i(\epsilon) - \epsilon \bar{L}) = 0
\]

(20)

Consider the case of \( \epsilon > 1 \) (the case \( \epsilon < 1 \) is analogous). By Proposition 2, given a positive shock to funding supply, labor is increasing the level of distortion:

\[
L_1(\epsilon) \leq L_2(\epsilon) \cdots \leq L_n(\epsilon)
\]

(21)

If there is some \( L_i \) for which \( L_i(\epsilon) > \epsilon \bar{L} \) it must be the case that for every \( j \geq i \), \( L_j(\epsilon) > \epsilon \bar{L} \). Thus, there exists a minimal \( m \) for which \( L_m(\epsilon) > \epsilon \bar{L} \). Since the LHS of equation 20 must be 0, it follows that \( 1 < m \) (if \( m = 1 \), the LHS is a sum of strictly positive numbers), and that for every \( i \leq m \), \( L_i(\epsilon) \leq \epsilon \bar{L} \). This proves the first part of the proposition. As \( \ln Y \) is increasing in \( L \) at a faster rate in more distorted economies (Proposition 1), the proposition follows.

The divergence result is an outcome of simple accounting: a shock to the supply of funds must adjust somewhere. If, compared to autarky, some countries are less affected by the shock, it is necessarily the case that other countries are more affected by the shock. Figure 2 illustrates this result in the random allocation example (in a two-country world).
This figure illustrates the integrated equilibrium in a two-country world in which \( \omega_1 \lesssim \omega_2 = \omega_{\text{max}} \). Wages are normalized to 1 (\( w_1 = w_2 = 1 \)), so \( F_1 = L_1 \). The origin of country 1 is on the left corner, and the origin of country 2 is on the right corner. The market clearing condition requires that the distance between the two corners is equal to \( F_w \). The intersection of the marginal returns to funding determines \( r \), and hence the division of funding between the two countries. Shocks to the global supply of funds adjust entirely through changes in employment in country 2. A positive shock to \( F_w \) is illustrated as a shift of country 2’s origin to the right.

5 Applications

These results naturally lend themselves to several applications concerning the global distribution of volatility. This section illustrates how the model can be used to reconcile several (seemingly-unrelated) stylized facts, and proposes a couple of simple extensions.

5.1 The excess volatility of emerging economies

It is well known that output volatility in emerging economies is higher than output volatility in developed economies.\(^{11}\) At the same time, there is increasing evidence that the allocation of inputs across firms in emerging economies is less efficient than in developed economies (see Hsieh and Klenow [2009]).

Under the assumption that emerging economies are more distorted than developed economies, Propositions 1 and 2 would suggest that misallocation may be part of the explanation for their excess volatility. Specifically, Proposition 1 suggests that

\(^{11}\)See, for example, Aguiar and Gopinath [2007].
part of the problem may be suboptimal adjustment to funding shocks: some highly productive projects must contract when financing requirements increase, while less productive projects remain relatively unaffected. Some evidence for this type of amplification was found in the study of the Argentinian 2001-2002 crisis. Sandleris and Wright [2011] and Neumeyer and Sandleris [2010] find that during the crisis, misallocation of inputs increased dramatically. This is consistent with the view that the “wrong” projects contracted, amplifying the crisis.

Proposition 2 further suggests that, in addition to amplifying the output response to a given shock to funding, misallocation increases the volatility of funding in a financially integrated environment, as the economy absorbs a larger share of shocks to global funds. Indeed, there are many studies illustrating the relative importance of shocks to external funding to volatility in emerging markets, such as Neumeyer and Perri [2005], Uribe and Yue [2006], Chang and Fernandez [2010], Broner and Ventura [2010], and Broner and Rigobon [2006].

Extension: productivity shocks in an integrated environment. It might be interesting to note that in this environment, misallocation not only increases the economy’s sensitivity to interest rate shocks, but also its sensitivity to idiosyncratic productivity shocks. Consider a simple extension, in which in addition to funding shocks, economies experience idiosyncratic productivity shocks. It turns out that funding responds similarly to idiosyncratic productivity shocks and to shocks to $r$. This perhaps suggests a link between the heightened sensitivity of emerging markets to interest rate shocks and the amplification of shocks to productivity (as in Caballero et al. [2005]). To see this link, note that in a small open economy, the level of funding is pinned down by a single indifference condition, equating the marginal product of funding with the world rate of return:

$$\frac{1}{w_i} y(A_i, L_i, \theta_{\omega_i}) = r \Rightarrow \int_0^\alpha \theta_{\omega_i}(x, L_i)dx = w_i \frac{r}{A_i}$$  \hspace{1cm} (22)

From the formulation above, it is easy to see that domestic employment is affected similarly by shocks to $r$ and shocks to $A_i$. Essentially, in this model, the respon-

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12Specifically, Sandleris and Wright [2011] find that more than half of the 10% decline in TFP is due to an increase in input misallocation; Neumeyer and Sandleris [2010] find that the gains from eliminating misallocation rise from around 50% during normal years to between 60-80% during the crisis.
siveness of funding to either type of shock captures the density of funding priorities for which the assigned project has an expected return equal roughly to \( r \). A small shock to aggregate productivity will shift their expected returns above or below the implementation threshold; similarly, small shocks to \( r \) will determine whether or not their expected returns are sufficient to justify implementation.

5.2 Comovements of emerging economies

The model naturally implies comovements between economies with similar levels of misallocation (even when the underlying frictions causing the misallocation are different). Economies that are relatively distorted will be similarly affected by shocks to the global supply of funding, while less-distorted economies will be less affected by these shocks. This simple insight perhaps sheds light on the comovement of emerging economies, and their common sensitivity to global interest rate shocks (as documented in [Uribe and Yue 2006]). The importance of shocks to the global supply of funds as a source of emerging market fluctuations naturally implies common movements in emerging market output levels. Similar to [Fostel and Geanakoplos 2008], comovements in emerging market economies result from a common sensitivity to an external supply of funds.

5.3 The “Great Moderation” trend in the US

Between the mid 80’s and the mid 2000’s, the US has experienced a large decline in output volatility, commonly referred to as “the Great Moderation”. This period was also characterized by the rapid financial integration of emerging economies. While, for emerging economies, financial integration was typically associated with episodes of extreme volatility, often related to fluctuations in the supply of external funding, for developed economies, fluctuations in the supply of funds did not seem to be an important source of volatility, at least not until the recent crisis.\(^{13}\) Further, evidence suggests that the relative importance of non-technology shocks in developed economies was declining over the period of financial globalization, as many attribute the “Great Moderation” trend in output volatility to a decline in non-fundamental

\(^{13}\)See Demirgüç-Kunt and Detragiache [1999], Kose et al. [2003] and Bekaert et al. [2006] for an empirical discussion of the differential effects of financial integration on emerging and developed economies.
Proposition 3 suggests a new explanation for the Great Moderation in the US: rather than adjusting domestically, fluctuations in the supply of funds in the US adjusted primarily through changes in funding supplied to emerging economies. As emerging economies became more financially open and more affected by funding shocks, the US became more insulated from these shocks and output volatility declined.

5.4 The financial crisis in the US

After a period of low and declining volatility, in 2007-2009 the US has experienced a financial crisis of nearly unprecedented scale. While the roots of the crisis are still not well understood, many have looked for explanations relating to the low interest rate environment in the pre-crisis era. On the micro level, many have focused on the consequences of “irresponsible” subprime lending, for the purpose of trenching them to create mortgage backed securities. In the literature, prominent examples include Brunnermeier [2009] and Gorton [2008], who discuss mechanisms through which a low interest rate environment may have led to a decline in lending standards and institutional changes which evidently amplified the subprime crisis.

A simple extension of the model in which the level of misallocation is determined endogenously can generate predictions that are consistent with these views:

**Extension: endogenous misallocation.** In each country there is a representative bank that chooses the extent of misallocation, prior to the realization of $\epsilon$. A more efficient allocation is associated with a higher cost. Banks are risk neutral and choose $\omega$ to maximize expected profits:

$$\max_{\omega} E(\pi_i(A_i, r(\epsilon), \omega_i))$$  \hspace{1cm} (23)

Where:

$$\pi_i(A_i, r(\epsilon), \omega_i) = \max_{L_i(\epsilon)} Y(A_i, L_i(\epsilon), \theta_{\omega_i}) - r(\epsilon)w_iL_i(\epsilon) - \lambda_i(\omega_i)$$  \hspace{1cm} (24)

\textsuperscript{14}See Gali and Gambetti [2009] for a structural VAR decomposition of the components of the “Great Moderation”.

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The function \( \lambda_i(\omega) \) is the country-specific cost of choosing a distortion level of \( \omega \). It is convenient to restrict attention to \( \Omega = [0, \infty) \), where \( \preceq \) is the ordering of real numbers, and assume that \( \lambda_i' \leq 0, \lambda_i'' \geq 0, \lim_{\omega \to 0} \lambda_i'(\omega) = -\infty \) and \( \lim_{\omega \to \infty} \lambda_i'(\omega) = 0 \).

Assume further that the process of \( \epsilon \) is characterized only by negative shocks, bounded by some \( \epsilon_0: \epsilon \in [\epsilon_0, 1] \).

**Lemma 2** Assume that \( Y(A, L, \theta_\omega) \) and \( y(A, L, \theta_\omega) \) are continuous and differentiable with respect to \( \omega \), and that \( \frac{\partial Y(A, L, \theta_\omega)}{\partial \omega} \) is decreasing in \( \omega \). For \( \epsilon_0 \bar{L} \) sufficiently large, the equilibrium level of \( \omega_1 \) (the distortion level in the most efficient economy) increases following financial integration.

The intuition is as follows. By Proposition 3, given the assumption that \( \epsilon \leq 1 \) for all \( \epsilon \), financial integration leads to an increase in employment. From the bank’s perspective, this is equivalent to a decline in the expected cost of funding projects (which, in principle, could reflect either a decline in \( r \) or a decline in \( w \)). In this model, a decline in the cost of funding (due to financial integration or otherwise) can lead to an endogenous decline in the level of scrutiny employed by banks. Intuitively, when most projects are likely to be implemented anyway, the return to distinguishing between projects declines.

This extension resonates with the experience of the recent crisis: as the interest rate declined, the US gravitated towards arrangements that effectively pooled productive and unproductive projects. A straightforward interpretation is the mortgage market itself. The creation of mortgage backed securities enabled the pooling of idiosyncratic risk of subprime loans. Once housing prices declined, issuing new subprime loans became difficult, perhaps in part because the process of issuing subprime loans did not allow for differentiation between relatively promising borrowers and relatively unpromising borrowers. A more subtle interpretation is in balance sheet effects (as in Brunnermeier [2009]). The heavy reliance of banks’ balance sheets on mortgage backed securities forced them to disengage from productive lending activities once the subprime crisis hit. This can be viewed as an additional mechanism that

\[ \text{This assumption means that a marginal improvement in allocation when the economy is more distorted has a greater effect on output.} \]

\[ \text{It might be interesting to note that this comparative static is true only for } \epsilon_0 \bar{L} \text{ sufficiently large. At smaller values of } \bar{L}, \text{ there may be a “scale effect” that dominates: when more projects are expected to be implemented, the returns to choosing a better allocation are higher.} \]
ties the implementation of productive projects to the implementation of unproductive projects.

Under the assumption that financial institutions take time to adjust to the financially integrated environment, the analysis suggests the following dynamics associated with financial integration: on impact (before financial institutions adjust), there is a moderation in output volatility in developed countries. As financial institutions adjust to the low interest rate environment, there is an increase in misallocation in the developed world and a reversal in volatility trends, as developed economies become more similar to emerging economies in terms of their vulnerability to shocks to funding supply.\footnote{Lemma \ref{lem:1} would imply that relatively distorted economies would gravitate towards less misallocation - this would contribute to the reversal in volatility trends, as the relatively distorted economies absorb a smaller share of shocks.}

Of course, there is still work to be done towards understanding the quantitative relevance of this model with respect to the above mentioned stylized facts. The next section illustrates that the model’s predictions can be derived from a variety of microfoundations, and are not tied to a particular friction causing the misallocation. This suggests that the study of the macroeconomic implications of distortions can be somewhat separated from the study of their microeconomic sources.

6 Microfoundations

In this section, I extend the setup in section 2 to environments with arbitrary productivity distributions. I then present several examples, illustrating that Property 1 is consistent with many relevant forms of distortion.

6.1 The multiple-productivity environment

The setup in section 2 assumes a stark productivity distribution of projects - productivity $A$ for a fraction $\alpha$ of the projects, and 0 for the rest. At first glance, this may seem restrictive. However, this specification turns out to be quite general, as there is an intuitive mapping between this stark environment and models with multiple productivity types. The multi-type environment is equivalent to the two-type
environment, with different productivity types representing different lotteries over productive and unproductive projects.

Assume that the productivity of project \( x \) is given by \( A_g(x) \), where \( g(x) \geq 0 \) and (without loss of generality) \( g(\cdot) \) is weakly decreasing. Let \( \tilde{A} = A_g(0) \), and let \( \tilde{\alpha} = \frac{\int_0^1 A_g(x) dx}{\tilde{A}} \) (note that \( 0 \leq \tilde{\alpha} \leq 1 \)). The project \( x = 0 \) implements a productive project (with productivity \( \tilde{A} \)) with probability 1; the rest of the projects represent lotteries over productive and unproductive projects. Specifically, project \( x \) implements a productive project with probability \( g(x) g(0) \).

**Definition 2** The binary equivalent of \( \{\Omega, \{\theta_\omega\}_{\omega \in \Omega}, \succeq\} \) is a model of misallocation \( \{\Omega, \{\tilde{\theta}_\omega\}_{\omega \in \Omega}, \succeq\} \), where:

\[
\tilde{\theta}_\omega(x, p) = \begin{cases} 
\frac{1}{\tilde{\alpha}} \int_0^1 \theta_\omega(x', p) g(x') dx', & \text{if } x < \tilde{\alpha} \\
\frac{1}{1-\tilde{\alpha}} (1 - \int_0^1 \theta_\omega(x', p) g(x') dx'), & \text{otherwise.} 
\end{cases} \quad (25)
\]

**Lemma 3** In the multi-productivity environment, \( \{\Omega, \{\theta_\omega\}_{\omega \in \Omega}, \succeq\} \) satisfies property [7] if and only if its binary equivalent satisfies Assumption [7].

**Proof:** Note that:

\[
y(A, L, \omega) = A \int_0^1 \theta_\omega(x, L) g(x) dx = A g(0) \int_0^{\tilde{\alpha}} \tilde{\theta}_\omega(x, p) dx \quad (26)
\]

By Lemma [1], \( y(A, L, \theta_\omega) \) is log supermodular if and only if \( \{\Omega, \{\tilde{\theta}_\omega\}_{\omega \in \Omega}, \succeq\} \) satisfies Assumption [1] for \( \alpha = \tilde{\alpha} \).

### 6.2 Examples

**Incomplete information.** Consider an economy in which projects’ types are unobservable; instead, implementation decisions are made based on a signal that is correlated with the project’s quality, \( \omega(x) \). A fraction \( \gamma \) of the projects receive the signal 1 (a “good” signal) and the remaining \( 1 - \gamma \) of the projects receive the signal 0 (a “bad” signal). Let \( \Omega \) be the set of mappings \( \omega(\cdot) \) from \([0, 1]\) to the binary set \( \{0, 1\} \), such that the measure of \( x \) for which \( \omega(x) = 1 \) is exactly \( \gamma \). For 1 to be a “good” signal, it is required that the average quality of projects receiving the signal 1 is higher than the average quality of projects receiving the signal 0. For this to hold generically, it is
convenient to assume that for every \( x, \frac{1}{\gamma} \int_0^x \omega(x') dx' \geq \frac{1}{1 - \gamma} \int_0^x (1 - \omega(x')) dx' \). Under this condition, all projects with signal 1 will be implemented before all projects with signal 0 (among projects with the same signal, the implementation order is arbitrary).

The partial ordering of \( \Omega \) is defined by the stochastic dominance relation: \( \omega' \preceq \omega \) if for every \( x, \int_0^x \omega(x') dx' \leq \int_0^x \omega'(x') dx' \). To see that this model is consistent with Property \( \Box \) for any \( g(\cdot) \), note that for \( p', p \in [0, \gamma] \) or \( p', p \in (\gamma, 1] \), the condition holds trivially. For \( p' \in [0, \gamma] \) and \( p \in (\gamma, 1] \), note that:

\[
\frac{\int_0^1 \theta_{\omega} (x, p) dx}{\int_0^1 \theta_{\omega} (x, p') dx} = \frac{\int_0^1 (1 - \omega(x)) g(x) dx}{\int_0^1 \omega(x) g(x) dx} \tag{27}
\]

Thus, Property \( \Box \) is satisfied if:

\[
\frac{\int_0^1 (1 - \omega(x)) g(x) dx}{\int_0^1 \omega(x) g(x) dx} \geq \frac{\int_0^1 (1 - \omega'(x)) g(x) dx}{\int_0^1 \omega'(x) g(x) dx} \tag{28}
\]

Which follows trivially from stochastic dominance.

**Collateral constraints (in the spirit of Kiyotaki and Moore [1997]).** Let \( \Omega = [0, \infty) \), and let \( \preceq \) be the ordering of real numbers. Consider a model in which projects are characterized by their type, \( x \), and by their collateral type, \( b \). Assume that collateral types \( (b) \) are distributed independently from project types \( (x) \). Conditional on any project type, collateral is uniformly distributed on \([0, \omega]\). The collateral level of the project is given by the following decreasing function:

\[
\kappa(b) = g(\min\{b, 1\}) \tag{29}
\]

That is, collateral is decreasing in \( b \), and is bounded from below by the productivity of the least productive project.

A project with features \( (x, b) \) is implemented if and only if both its return and its collateral level exceed the price of funding:

\[
\min\{\kappa(b(x)), g(x)\} \geq r \tag{30}
\]

The mismatch between the quality of projects and their collateral changes the order in which projects are implemented. Note that a higher \( \omega \) implies that the ag-
aggregate collateral is lower. It is easy to see that $\omega \to 0$ corresponds to the efficient allocation, as all projects have sufficient collateral to be implemented when their return is high enough. The case $\omega \to \infty$ corresponds to an extremely distorted economy, in which essentially all projects are collateral constrained and can be implemented only when the market return is equal to the productivity of the least productive project; in this case, the aggregate marginal return to funding is close to constant, as projects are implemented essentially in a random order.

**Lemma 4** For $g(x) = \eta x^{-(1-\eta)}$, this model of misallocation satisfies Property [I].

**Search frictions (in the spirit of Duffie and Strulovici [2012])**. Let $\Omega = [\omega, 1]$ for some $0 < \omega < 1$, and let $\preceq$ be the ordering of real numbers. The distribution of productivities is $g(x) = x^{-(1-\omega)}$.\(^{18}\)

There is a unit measure of intermediaries. When $L$ is realized, the funds are divided equally among $L$ intermediaries, so that each intermediary is in charge of funding one project. Intermediaries choose how much effort to invest in searching for good projects. The expected return to their project, $g$, is increasing in search effort. The cost of search effort is given by $\frac{1}{2}g^2L^{1-\omega}$: it is increasing in $g$, but also in $L$, capturing the idea that when $L$ is higher there is more competition for productive projects. In this model, a higher $\omega$ implies higher search costs (as $L \leq 1$). More accurately, a higher $\omega$ implies that at any $L$, the competition for good projects is tougher.

Intermediaries are risk neutral and maximize the expected return to their project minus labor costs ($w$) and minus search costs:

$$\max_{g \in [0, \infty)} g - w - \frac{1}{2}g^2L^{1-\omega}$$

(31)

The first order condition yields:

$$1 = gL^{1-\omega} \Rightarrow g = \frac{1}{L^{1-\omega}}$$

(32)

Equilibrium aggregate output is given by:

$$Y = gL = L^\omega$$

(33)

\(^{18}\)Note that $g(0) \to \infty$, so, strictly speaking, the binary equivalent of the model is undefined. However, since $g(\cdot)$ is integrable, it can be arbitrarily-well approximated by bounded distributions.
The marginal return to labor is therefore \( y = \omega L^{-(1-\omega)} \). This model of misallocation satisfies Property 1 because:

\[
\frac{y(L)}{y(L')} = \left(\frac{L'}{L}\right)^{1-\omega}
\]

This expression is increasing in \( \omega \) (as \( \frac{L'}{L} \leq 1 \)).

**Market segmentation.** Let \( \Omega = \{1, 2, 3, \ldots\} \) and let \( \preceq \) be the ordering of natural numbers. Consider a model in which economies differ in the extent to which local banks can share risk. In each economy there are \( \omega \) local banks indexed \( i = 0, \ldots, \omega - 1 \). Project owners are unaware of their project’s type until right before production decisions must be made. Local banks are modeled as risk sharing arrangements among project owners. Each local bank shares risk among \( \frac{1}{\omega} \) project owners. The bank indexed \( i \) is a risk sharing arrangement between the owners of projects indexed \( x \in \left(\frac{i}{\omega}, \frac{i+1}{\omega}\right] \) (for \( i = 0 \), the segment of projects is the closed set \([0, \frac{1}{\omega}]\)). The process of allocating funds to projects follows two steps:

1. Funds are distributed randomly across projects owners.

2. Project owners handover their funds to their local bank. The local bank uses the funds to implement the best set of projects among those owned by the bank’s members.

The parameter \( \omega \) can be thought of as a measure inversely related to the integration of the domestic financial system. A high \( \omega \) captures a situation in which there are some banks with access to highly productive projects that lack funding to implement them, and some banks with a lot of funding but without access to good projects. The case \( \omega = 1 \) corresponds to the efficient allocation: there is only one bank in charge of allocating the entire supply of funds, and the bank has access to the entire set of projects. The optimal set of projects is therefore implemented. The case \( \omega \to \infty \) corresponds to the random allocation: project owners are essentially in autarky, as they must use their own funds to implement their own project.

**Lemma 5** For \( g(x) = e^{-x} \), this model of misallocation satisfies Property 1.
7 Conclusion

Decreasing returns on the macro level are implied by productive efficiency on the micro level. With this as a starting point, this paper derives conditions under which misallocation reduces the extent of decreasing returns. Intuitively, when productive projects become relatively less likely to be funded before unproductive projects, the marginal return to funding declines less steeply.

This view suggests a link between misallocation and output volatility. On the macro level, the extent of decreasing returns determines the sensitivity of output to contractions in inputs (or, more realistically, input employment). On the micro level, the extent of decreasing returns translates into the economy’s ability to adjust to shocks through the discontinuation of the least productive projects.

In the financially integrated environment, the extent of decreasing returns determines the economy’s sensitivity to changes in the global supply of funds. Changes in funding supply adjust disproportionately through countries that have less decreasing returns. Financial integration therefore leads to a divergence in volatility between relatively distorted and relatively undistorted economies, as distorted economies essentially serve as a “buffer zone”, insulating less distorted economies from shocks to global funding supply.

These results suggest natural explanations for several seemingly-unrelated stylized facts regarding the global distribution of volatility. Misallocation in emerging economies can perhaps help explain their excessive vulnerability to funding shocks. The superior ability of the US to efficiently allocate funds during the mid 80s up until the mid 2000s may have contributed to the “Great Moderation”, as shocks to funding in the US adjusted primarily through fluctuations in funding supplied to emerging economies. A low interest rate environment may have lead to an endogenous deterioration in lending standards in the US, which reduced its insulation from funding shocks in the run-up to the recent crisis.

The predictions of this model on the macro level are consistent with a variety of microfoundations, including models of incomplete information, collateral constraints, search frictions and market segmentation. The common element in all these microfoundations is that the underlying distortion makes it relatively less likely that higher priorities are assigned to better projects.
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## A Proofs

### A.1 Proof of Proposition 1

Let \( Y(F, \omega) = Y(A = 1, L = \frac{F}{w}, \theta_\omega) \) and \( y(n, \omega) = y(A = 1, L = \frac{n}{w}, \theta_\omega) \). To show that \( Y(F, \omega) \) is log supermodular, write \( Y(F, \omega) \) as:

\[
Y(F, \omega) = \int_{0}^{\infty} 1_{[0,F]}(n) y(n, \omega) dn \tag{35}
\]

Where \( 1_{[0,F]} \) denotes the indicator function which takes a value 1 over the interval \([0, F]\) (and 0 elsewhere).

Recall the definition of log supermodularity as it appears in Costinot [2009], which allows for 0 values:
Definition 3 For $X \subset \mathbb{R}^w$, a function $h : X \to \mathbb{R}^+$ is log supermodular if for all $z, z' \in X$,

$$h(\max(z_1, z'_1), \ldots, \max(z_m, z'_m))h(\min(z_1, z'_1), \ldots, \min(z_m, z'_m)) \geq h(z)h(z')$$  \hfill (36)

Claim 1 The function $h(F, n, \omega) = 1_{[0,F]}(n)$ is log supermodular in $F$, $n$, and $\omega$.

To see this, note that both sides of the inequality in (36) can be either 0 or 1, and consider the case in which the left hand side of the inequality is 0:

$$h(\max(F, F'), \max(n, n'), \max(\omega, \omega'))h(\min(F, F'), \min(n, n'), \min(\omega, \omega')) = 0$$  \hfill (37)

$$1_{[0,\max(F,F')]}(\max(n, n'))1_{[0,\min(F,F')]}(\min(n, n')) = 0$$  \hfill (38)

Assume without loss of generality that $\max(n, n') = n$. From the above equality, $1_{[0,\max(F,F')]}(\max(n, n')) = 0$ or $1_{[0,\min(F,F')]}(\min(n, n')) = 0$. Assume $1_{[0,\max(F,F')]}(\max(n, n')) = 0$. Thus,

$$n > \max(F, F') \Rightarrow n > F \Rightarrow 1_{[0,F]}(n) = 0 \Rightarrow 1_{[0,F]}(n)1_{[0,F']}(n') = 0$$  \hfill (39)

Assume instead that $1_{[0,\min(F,F')]}(\min(x, x')) = 0$. There are two cases: if $\min(F, F') = F'$,

$$n' > F' \Rightarrow 1_{[0,F']}(n') = 0 \Rightarrow 1_{[0,F]}(n)1_{[0,F']}(n') = 0$$  \hfill (40)

if, instead, $\min(F, F') = F$, since $\max(n, n') = n$,

$$n' > F \Rightarrow n > F \Rightarrow 1_{[0,F]}(n) = 0 \Rightarrow 1_{[0,F]}(n)1_{[0,F']}(n') = 0$$  \hfill (41)

Thus, log supermodularity is satisfied.

The assumption that $y(n, \omega)$ is log supermodular in $(n, \omega)$ implies trivially that it is log supermodular as a function of $(F, n, \omega)$.

Since the product of two log supermodular functions is log supermodular, and the integral of a log supermodular function is log supermodular\footnote{For proof see Karlin and Rinott \cite{1980}.}, it follows that $Y(F, \omega)$ is log supermodular. Thus, by log supermodularity, $\frac{\partial \ln Y(F, \omega)}{\partial F}$ is increasing in $\omega$.\footnote{For proof see Karlin and Rinott \cite{1980}.}
The second part of the proposition builds on the first part:

\[
\frac{Y(F, \omega)}{F} = Y(F, \omega)F^{-1} \tag{42}
\]

Since \(Y(F, \omega)\) is log supermodular, and \(F^{-1}\) is trivially log supermodular, it follows that average productivity is log supermodular as a product of two log supermodular functions. Thus, by log supermodularity, \(\frac{\partial \ln(Y(F, \omega))}{\partial F}\) is increasing in \(\omega\). Since the derivative is negative, this implies that the sensitivity of average productivity to the level of funding is higher in less distorted economies.

A.2 Proof of Lemma 2

It will be convenient to write the bank’s problem as follows:

\[
\max_{\omega_i, L_i(\epsilon)} E(Y(A_i, L_i(\epsilon), \theta) - r(\epsilon)w_iL_i(\epsilon) - \lambda_i(\omega_i)) \tag{43}
\]

The optimality condition with respect to \(\omega\) is:

\[
E\left(\frac{\partial}{\partial \omega_i} Y(A_i, L_i(\epsilon), \theta)\right) = \lambda_i'(\omega_i) \tag{44}
\]

To see that this has a solution, note first that \(\frac{\partial Y(A, L, \theta)}{\partial \omega} \leq 0\): by Proposition 1, \(Y\) is log supermodular in \(L\) and \(\omega\); thus, for any \(\omega' \leq \omega\),

\[
\ln Y(A, L, \theta) - \ln Y(A, L, \theta') \leq \ln Y(A, 1, \theta) - \ln Y(A, 1, \theta') = 0 \tag{45}
\]

It follows that \(Y(A, L, \theta) - Y(A, L, \theta') \leq 0\) for every \(L\), and thus \(\frac{\partial Y(A, L, \theta)}{\partial \omega} \leq 0\).

Second, note that since \(\lambda_i'(\cdot)\) is negative and increasing, \(\lim_{\omega \to 0} \lambda_i'(\omega) = -\infty\) and \(\lim_{\omega \to \infty} \lambda_i'(\omega) = 0\), and \(\frac{\partial Y(A, L, \theta)}{\partial \omega}\) is decreasing in \(\omega\), there is a unique interior solution.

By Proposition 3 upon financial integration, country 1 experiences an increase in \(L(\epsilon)\) for every \(\epsilon\). To show that this increases the LHS for every \(\omega\) (and hence the equilibrium choice of \(\omega_1\)), note that for every \(\omega\), there exists \(L^*(\omega) < 1\) such that for every \(L \geq L^*(\omega)\), \(\frac{\partial Y(A, L, \theta)}{\partial \omega L} \geq 0\). To see this, fix \(\omega\) and some \(\omega' \leq \omega\), and consider:

\[
\frac{\partial}{\partial L} (Y(A, L, \theta) - Y(A, L, \theta')) = y(A, L, \theta) - y(A, L, \theta') \tag{46}
\]

Since \(Y(A, 1, \theta) = Y(A, 1, \theta')\) but \(Y(A, L, \theta) \leq Y(A, L, \theta')\), there is a point
$L^*(\omega, \omega')$ such that for every $L \geq L^*(\omega, \omega')$ the above expression is positive. Since there is a continuous mapping from $\omega$ to $y(A, L, \theta_\omega)$, it follows that $\lim_{\omega' \to \omega} L^*(\omega, \omega')$ exists; for $L^*(\omega) = \lim_{\omega' \to \omega} L^*(\omega, \omega')$, the condition is satisfied.

Note that if $\epsilon_0 \bar{L} \geq L^*(\omega)$, then financial integration (which increases $L(\epsilon)$ for every $\epsilon$) increases $E(\frac{\partial}{\partial \omega} Y(A, L(\epsilon), \theta_\omega))$. Thus, for $\epsilon_0 \bar{L} \geq \sup_\omega L^*(\omega)$, financial integration leads to an increase in $E(\frac{\partial}{\partial \omega} Y(A, L(\epsilon), \theta_\omega))$ for every $\omega$; thus, the equilibrium $\omega_1$ increases.

### A.3 Proof of Lemma 4

It is convenient to normalize wages so that $w = 1$. Begin by considering the case $\omega \leq 1$. First, note that for every $F$, there is a threshold $\tilde{F}$ such that project $(x, b)$ is implemented if and only if $x \leq \tilde{F}$ and $b \leq \tilde{F}$. For $\tilde{F} < \omega$,

$$F = Pr(x \leq \tilde{F}, b \leq \tilde{F}) = \tilde{F} \cdot \frac{1}{\omega} \tilde{F} = \frac{1}{\omega} \tilde{F}^2 \Rightarrow \tilde{F} = \sqrt{\omega} \sqrt{\bar{F}}$$

(47)

Note that $\tilde{F} < \omega$ if and only if $F < \omega$. For $F > \omega$, it is easy to see that $\tilde{F} = F$.

For $F < \omega$, output is given by the following expression:

$$Y(\tilde{F}) = \int_0^{\tilde{F}} Pr(b \leq \tilde{F}) Ag(x) dx = \frac{1}{\omega} \tilde{F} \int_0^{\tilde{F}} Ag(x) dx = A \omega^{1+\alpha}$$

(48)

It follows that, for $F < \omega$:

$$Y(F, \omega) = A \omega^{\frac{\alpha-1}{2}} F^{\frac{1+\alpha}{2}}$$

(49)

The derivative of $Y$ with respect to $F$ is therefore given by:

$$y(F, \omega) = \frac{1 + \alpha}{2} A \omega^{\frac{\alpha-1}{2}} F^{\frac{\alpha-1}{2}}$$

(50)

$$\Rightarrow \ln y(F, \omega) = \ln(\frac{1 + \alpha}{2} A) - \frac{1 - \alpha}{2} \ln \omega - \frac{1 - \alpha}{2} \ln F$$

(51)

The derivative of above with respect to $F$ is:

$$\frac{\partial \ln y(F, \omega)}{\partial F} = -\frac{1 - \alpha}{2F}$$

(52)
The above does not depend on $\omega$ as long as $F < \omega$; the log supermodularity condition is trivially satisfied. However, Note that for a higher $\omega$, there are more values of $F$ such that $F < \omega$. Let there be $\omega$ and $\omega'$ such that $\omega' < F < \omega$. For $\omega'$, it is easy to see that the derivative of $\ln y(F, \omega')$ is given by the following expression:

$$y(F, \omega') = A\alpha F^{-(1-\alpha)} \Rightarrow \ln y(F, \omega') = \ln(A\alpha) - (1 - \alpha) \ln F$$  \hspace{1cm} (53)

$$\Rightarrow \frac{\partial \ln y(F, \omega')}{\partial F} = -\frac{1 - \alpha}{F} < -\frac{1 - \alpha}{2F} = \frac{\partial \ln y(F, \omega)}{\partial F}$$  \hspace{1cm} (54)

Thus, the derivative of $\ln y$ with respect to $F$ is higher in the more distorted economy $\omega$, consistent with Property 1.

Consider now the range $\omega \geq 1$. In this range, for $\tilde{F} < 1$, $F$ is given by equation 47, output is given by equation 48 and $\frac{\partial \ln y}{\partial F}$ is given by equation 52. In this range, $\frac{\partial \ln y}{\partial F}$ is constant with respect to $\omega$, so the log supermodularity condition is trivially satisfied.

Note that $\tilde{F} < 1$ if and only if $\sqrt{\omega F} < 1$, or $F < \frac{1}{\omega}$. This condition is violated for more values of $F$ if $\omega$ is larger. For $F > \frac{1}{\omega}$, the marginal product of funding is constant; the collateral constraint is binding for all implemented projects, so the productivity of the projects implemented with each unit of funding is the same. Thus, in the range $F > \frac{1}{\omega}$,

$$\frac{\partial \ln y(F, \omega)}{\partial F} = 0 > -\frac{1 - \alpha}{2F}$$  \hspace{1cm} (55)

The right hand side is equal to the derivative of $\ln y$ for the case $F < \frac{1}{\omega}$. It follows that the log supermodularity condition is satisfied for $\frac{1}{\omega} < F < \frac{1}{\omega'}$.

### A.4 Proof of Lemma 5

For simplicity, normalize $w = 1$. Output is given by the following expression:

$$Y(F, \omega) = \sum_{i=0}^{\omega} \int_{\frac{i}{\omega}}^{\frac{i+1}{\omega}} Ag(x)dx$$  \hspace{1cm} (56)

This is because each local bank has $\frac{F}{\omega}$ units of liquidity to allocate, and uses it to implement the first $\frac{F}{\omega}$ in the sample of projects available to it.
The marginal product of funding is given by:

\[
y(F, \omega) = \frac{1}{\omega} \sum_{i=0}^{\omega} Ag\left(\frac{i}{\omega} + \frac{F}{\omega}\right) = \frac{A}{\omega} \sum_{i=0}^{\omega} e^{-\left(\frac{i}{\omega} + \frac{F}{\omega}\right)}
\]  

(57)

\[
= \frac{Ae^{-\frac{F}{\omega}}}{\omega} \sum_{i=0}^{\omega} e^{-\frac{i}{\omega}}
\]  

(58)

It follows that:

\[
\ln y(F, \omega) = \ln\left(\frac{A}{\omega} \sum_{i=0}^{\omega} e^{-\frac{i}{\omega}}\right) + \ln e^{-\frac{F}{\omega}} = c - \frac{F}{\omega}
\]  

(59)

The derivative of above with respect to \( F \) is \(-\frac{1}{\omega}\), which is increasing in \( \omega \). The log supermodularity condition is satisfied, in accordance with Property 1.