International Liquidity Rents

Maya Eden *
World Bank
June 26, 2012

Abstract

When producers are liquidity constrained, there are opportunities for liquidity suppliers to extract rents. In this paper I explore the implications of this in a global equilibrium context, in which producers in emerging economies are liquidity constrained, and developed economies have a comparative advantage in creating liquidity. From the perspective of emerging economies, liquidity inflows from developed economies bid up wages to a point at which domestic producers can no longer afford the domestic wage bill, and must rely on foreign liquidity. Rents to liquidity supply imply a net flow of goods from emerging economies to developed economies. Globally, financial integration increases the returns to creating liquid claims, which may be a socially wasteful activity. This model suggests that unrestricted international liquidity flows are (a) welfare reducing for emerging economies and (b) Pareto inefficient.

JEL Classification: E40, E50, F30, G15
Keywords: Liquidity constraints, the welfare effects of financial integration, international liquidity flows, liquidity creation

*I thank Alessandro Barattieri, Ricardo Caballero, Arnaud Costinot, John Duca, Ben Eden, Anton Korinek, Ha Nguyen, Luis Serven and Christoph Steffen for helpful comments. I have also benefited from comments made by seminar participants at MIT’s international breakfast and conference participants at the 14th Money and Finance Conference at IGIDR and the INFINITI conference, 2012. I thank Tomoyuki Yagi for excellent research assistance. This paper reflects my own views and not necessarily those of the World Bank, its Executive Directors or the countries they represent. Please send comments to meden@worldbank.org.
1 Introduction

The international economics literature has traditionally focused on two types of flows: international trade in goods and services, and international borrowing and lending for the purpose of risk sharing or consumption smoothing (often through the accumulation of physical capital). In this paper, I focus on a different type of flow, which is international trade in liquidity. Liquid claims are claims that are easily transferable, and can be used for transaction purposes. An international liquidity market allows producers to borrow liquidity in order to pay labor, and repay with interest after production revenues are generated. Given a relatively inelastic labor supply, liquidity affects wages more than employment (and output). Unlike other forms of international integration, integrating liquidity markets may reduce Pareto efficiency, and may be welfare reducing for liquidity-scarce economies.

The relevance of studying liquidity flows stems primarily from their potential roles in recent crisis episodes, both in emerging and in developed economies. In emerging economies, shocks to foreign liquidity supply have been shown to cause large fluctuations in employment and output, suggesting a role for foreign liquidity in facilitating domestic production. In developed economies, the heightened demand for liquidity from emerging economies seems to have lead to over-accumulation of physical capital and excessive private creation of liquid claims. During the recent crisis, such privately-created liquid claims became illiquid, exacerbating the crisis.

In light of their fragility, several emerging economies have considered restricting short-term flows by introducing some form of capital controls. While there is broad agreement that such measures are likely to reduce fragility, little is known about how these restrictions will affect the levels of consumption, output, and capital accumulation. In this paper, I explore the implications of restricting liquidity flows in a riskless environment, and show that, even absent any risk, restricting international liquidity flows may improve welfare.

---

1 This idea has been advocated, for example, by Neumeyer and Perri [2005], who argue that firms in emerging economies depend on working capital financing from developed economies.
3 See recent IMF publications, including “Recent Experiences in Managing Capital Inflows - Cross-Cutting Themes and Possible Policy Framework”, prepared by a team led by R. Baqir and V. Chensavasdijai. For a theoretical discussion, see Korinek [2011].
The quantitative importance of international liquidity flows is difficult to assess directly. However, there is some evidence suggesting that the magnitude of these flows may be substantial. In emerging economies, short term debt constitutes over a fifth of the debt stock. While there might be other reasons for emerging economies to borrow short term, these flows may capture borrowing and lending for liquidity purposes. The magnitudes of short term flows are potentially larger when considering other forms of financing. For example, prior to introducing capital controls, short term capital accounted for over 90% of capital inflows to Chile. In terms of outcomes, capital inflows to emerging economies are typically associated with rapid appreciations consistent with the price effects of liquidity inflows.

The starting point of this model is the assumption that, compared to developed economies, emerging economies are inferior in their ability to create liquid claims. This assumption is consistent with figure that shows that the average level of broad money (M3) as a fraction of nominal GDP is substantially higher in developed economies. In this paper, liquid claims are modeled as single period IOU notes, issued at the beginning of the period and redeemed at the end of the period. To be “liquid”, a claim must be easy to enforce; thus, liquid claims must be backed either by the government or by collateral. Firms use liquid claims to pay workers, who supply their labor inputs before revenues are generated.

In emerging economies, there is not enough liquidity to finance the unconstrained wage bill. Consequently, the autarkic wage is depressed relative to the marginal product of labor. This wedge creates an incentive for firms to borrow liquidity from abroad. In the integrated equilibrium, “imported” liquidity bids up wages to a point at which domestic firms can no longer afford the domestic wage.

---

4 According to World Bank data, short term debt (with maturity of less than a year) accounts for 20% of the debt stock for developing countries in Latin America and the Caribbean, 18% in emerging Europe and Central Asia, and 46% in East Asia and the Pacific.

5 See, for example, Broner et al. [Forthcoming], that focuses on the price benefits of short term debt.

6 See Edwards [1999].

7 See the IMF publication titled “Recent Experiences in Managing Capital Inflows: Cross-Cutting Themes and Possible Policy Framework”, prepared by a team led by R. Baqir and V. Chensavasdi jai.

8 This result is broadly consistent with the empirical findings in Chari et al. [2012], who show that capital market integration in emerging economies is associated with an increase in wages.

---

3
bill and must rely on foreign liquidity. Aggregate domestic income is lower, as rents must be paid to foreign liquidity suppliers.

In developed economies, the demand for liquidity from emerging economies increases the incentives to create liquid claims. This is socially inefficient, as it leads to excessive accumulation of physical capital (as capital can be used as collateral to back liquid claims). While financial integration may increase consumption in developed economies, it is Pareto inefficient: both emerging and developed economies would benefit from restricting liquidity flows, with a transfer from emerging to developed economies.

The general principle underlying the mechanism in this paper is that agents are willing to undertake costly measures to relax binding constraints, even when doing so is socially inefficient. For emerging economies, this takes the form of borrowing liquidity from abroad. Privately, each firm thinks it can increase its profits by borrowing from abroad and increasing its labor inputs. In general equilibrium, borrowed liquidity merely bids up the wage rate, leaving firms worse off as they face higher wages and must make interest payments to foreign liquidity suppliers. For developed economies, the inefficiency takes the form of accumulating collateral. Privately, each agent stands to profit from creating liquid claims, backed by physical capital; however, in general equilibrium, this merely reduces the value of liquidity and wastes valuable resources.

2 Related literature

The global equilibrium view expressed in this paper is closely related to the “asset shortage” view, summarized in Caballero [2006] and Caballero et al. [2008]. The “asset shortage” view attributes the current account deficit in the United States, as well as recent bubble episodes, to the insufficient ability of emerging economies to create quality stores of value. In this model, liquid claims constitute such a quality store of value, and the underlying motivations (as well as many of the equilibrium implications) are similar. The key difference is in the use of the asset: while Caballero et al. [2008] emphasize the shortage in long term (safe) saving facilities, my focus here is on short term liquidity. The difference is important because, to some extent, changes in liquidity supply have the potential to affect
See Appendix A for a description of the data.

Figure 1: Real balances as a fraction of GDP ($M_3 / P_Y$) in a sample of emerging and developed economies, averages for 2002-2011.

prices more than allocations. This aspect of liquidity is central to the welfare implications explored here.

The model in this paper implies a net flow of goods from constrained economies to liquidity supplying regions. The mechanism is loosely related is to international seigniorage (as in Matsuyama et al. 1993 and Eden 2009)). In the monetary literature, the fact that foreigners use Dollars allows the US central bank to collect seigniorage payments from foreigners. Printing new Dollars not only taxes Americans, but also foreigners who hold Dollars for transaction purposes. An implication of this is that the foreigners would be better off if using Dollars were illegal, as the only seigniorage payments would be collected by the domestic government and consumed domestically. The results here are of similar flavor: under autarky, there are rents to liquidity supply, but they are consumed domestically. Under financial integration, foreigners extract some liquidity rents, which lowers domestic income.

The literature on working capital flows has mostly focused on volatility. In Neumeyer and Perri 2005, there is an exogenous working capital constraint that

9See, among others, Feldstein 1999 and Tong and Wei 2011.
forces producers to finance part of the wage bill with short-term loans from abroad. It is shown that this foreign dependence creates non-fundamental volatility in employment. The mechanism in this paper endogenizes the equilibrium working capital requirement, and highlights an additional problematic aspect of working capital inflows: in addition to creating volatility, they may also lower the level of income.

The mechanics of this model are close to Eden [2012]. In Eden [2012], I consider a closed economy with liquidity constraints in which the price of production inputs in terms of liquidity is determined in equilibrium. The equilibrium determination of this price is the key ingredient in both models. The main difference between the papers is in their focus. Here, I focus on the welfare implications of international liquidity flows and on the distribution of surplus between emerging and developed economies. In Eden [2012] I focus on the general equilibrium costs of financial intermediation in a closed economy context.

This paper contributes to the theoretical literature on the welfare implications of financial integration in emerging economies, including (among others) Gourinchas and Jeanne [2006] and Levine [2001]. Most closely related to this paper is Mendoza et al. [2007], that considers the welfare implications of financial integration for a financially underdeveloped economy, in which agents face borrowing constraints. Similarly, the conclusion is that financial integration is welfare reducing. However, the mechanism is quite distinct: in Mendoza et al. [2007], financial integration increases to cost of borrowing, which reduces equilibrium welfare. In contrast, in this paper, financial integration makes borrowing (for liquidity purposes) cheaper. However, this does not increase welfare, as liquid claims are not a real input in production; rather, welfare is reduced because rents to liquidity are sent abroad rather than consumed domestically.

Finally, this paper is broadly related to the literature on the distributional implications of opening to trade and the welfare implications of liberalization. Bhagwati and Brecher [1980] present the idea that in the presence of foreign-owned inputs of production, trade liberalization can decrease domestic welfare if it increases the relative wage of the inputs that are disproportionately owned by foreigners. In this model, the distributional implications of liberalization go in the opposite direction: financial integration increases the return to domestic labor,
while it lowers the return to liquidity that can be supplied by foreigners. However, welfare decreases as the economy imports liquidity in equilibrium, despite the fact that the economy’s liquidity needs can be satisfied in autarky at no aggregate cost. The distributional implications of financial integration are in the spirit of Antras and Caballero [2009]: the returns to liquidity decline, as liquidity can be imported more cheaply from abroad, allowing labor to absorb a higher share of output.

3 A dynamic general equilibrium model

I consider a discrete time infinite horizon model, where time periods are indexed by $t = 0, 1, 2, \ldots$.

**Technology.** There is a unit measure of identical firms indexed $i \in [0, 1]$. The production technology is time invariant and given by:

$$F(k_{i,t}, l_{i,t}) = k_{i,t}^{\alpha} l_{i,t}^{1-\alpha}$$

(1)

Where $l_{i,t}$ is the labor employed by firm $i$ and $k_{i,t}$ is firm $i$’s physical capital.

**Physical capital.** Physical capital is owned by firms. The final good can be invested and turned into physical capital, to be used in the next period. Capital depreciates at the rate $\delta$.

**Timing of production.** The timing of production is as follows:

- Firms enter the period with capital.
- Firms purchase labor inputs.
- Production takes place.

**Paying labor.** Firms need to hire labor with promises on post-production output. However, workers are limited in their ability to enforce labor contracts ex-post. Thus, they accept only promises that are backed, either by the government or by collateral.
Backed promises will be referred to as *liquid claims*. This simplified modeling of liquid claims captures two important aspects of liquidity. First, it can easily be used for transaction purposes (here, purchasing labor). Second, liquid claims can be enforced without any expertise or enforcement power, a property that is necessary for liquid claims to be widely acceptable.\(^{10}\)

**Publicly backed claims.** The government can guarantee a firm’s promises up to \(m\) units of output. Publicly guaranteed claims are promises that are issued by the firm, and enforced by the government. This guarantee can take various forms. For example, the government can commit to enforcing contracts in case of a dispute, or repay workers in case of a default. Alternatively, the government can issue transferable claims on output, that can be used by firms to pay workers (such as “money” in Caballero and Krishnamurthy [2005]).

I assume that the limit on publicly backed claims is proportional to aggregate output:

\[
m_t \leq \theta Y_t
\]

The proportion coefficient \(\theta\) captures the economy’s aptitude for creating publicly backed liquid claims.

**Collateral backed claims.** In addition, firms can privately back their promises against part of their capital stock.\(^{11}\) Firm \(i\) can issue private promises to labor against a fraction \(\gamma\) of its capital stock. The parameter \(\gamma\) is the fraction of capital that can be used as collateral.

Collateral backed claims are essentially ownership rights on capital, to be delivered at the end of the period. The fraction \(\gamma\) represents the transferable portion of the value of the capital stock.

To summarize, the amount of liquid claims that a firm can issue is \(m_t + \gamma k_{i,t}\).

---

\(^{10}\) See Holmstrom and Tirole [2011], Holmstrom [2008], and Kurlat [2010] for a more rigorous discussion of the necessary attributes of liquid claims.

\(^{11}\) The distinction between “publicly backed” and “privately backed” liquid claims is in the spirit of Farhi and Tirole [2011], who similarly distinguish between “inside” and “outside” liquidity.
Liquidity markets. There is a liquidity market, in which firms can borrow and lend liquidity. Firms are able to enforce promises made by other firms. Thus, they are willing to accept unbacked promises, or illiquid claims (they are not accepted as means of payment to labor). In the liquidity market, firms exchange liquid claims for illiquid claims. The price of liquid claims in terms of illiquid claims is $1 + r_t$: a backed claim for one unit of output is traded for an unbacked claim of $1 + r_t$ units of output. The return to liquidity, $r_t$, is determined in equilibrium.

It is convenient to think of the liquidity market as a market for borrowing and lending backed IOU notes between firms. The market rate of return on backed IOU notes is $r_t$; a firm borrowing one backed IOU note at the beginning of the period must repay $1 + r_t$ goods after production takes place.

Denote by $b_{i,t}$ the amount of liquid claims that firm $i$ borrows at time $t$. I will also refer to $b_{i,t}$ as the firm’s liquidity demand. A positive $b_{i,t}$ means that the firm is a net borrower of liquid claims: part of its labor purchases is financed by liquid claims issued by other firms. A negative value of $b_{i,t}$ means that the firm is a net lender of liquid claims, and issues more liquid claims than it uses for its own labor purchases.

Table 1: Liquid and illiquid claims issued by firms

<table>
<thead>
<tr>
<th>Backed (by collateral or public guarantees)</th>
<th>Liquid claims</th>
<th>Illiquid claims</th>
</tr>
</thead>
<tbody>
<tr>
<td>Accepted by labor</td>
<td>Yes</td>
<td>No</td>
</tr>
<tr>
<td>Accepted by firms</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Value in terms of illiquid claims</td>
<td>$1 + r_t$</td>
<td>1</td>
</tr>
<tr>
<td>Issuing limit</td>
<td>$m_t + \gamma k_{i,t}$</td>
<td>Value of profits</td>
</tr>
</tbody>
</table>

It might be useful to clarify that, while firms are constrained in their ability to create liquid claims, they are not borrowing constrained: they can issue illiquid claims up to the full value of their profits, and borrow liquidity against these claims.\footnote{In this model, there is an assumption that labor is able to accept only claims that are backed, while other firms - both foreign and domestic - can accept uncollateralized obligations. This division is different from Caballero and Krishnamurthy [2001], who assume that domestic agents are willing to accept a wider variety of collateral relative to foreigners.}
**Households.** There is a unit measure of households indexed $h \in [0, 1]$. For simplicity, I assume that each household owns one firm (but cannot supply its own labor to its firm). Households derive utility from consumption. Household $h$’s preferences over consumption sequences $\{c_{h,t}\}_{t=0}^{\infty}$ are represented by:

$$U(\{c_{h,t}\}_{t=0}^{\infty}) = \sum_{t=0}^{\infty} \beta^t u(c_{h,t})$$

Where $u(\cdot)$ satisfies the standard assumptions: $u'(c) > 0$, $u''(c) < 0$.

Households supply $l_{h,t} = l$ units of labor inelastically. The results build heavily on this assumption, and introducing an elastic labor supply may alter some of the conclusions. However, as the focus is primarily on permanent changes in the wage rate, assuming an inelastic labor supply is not unrealistic. Long run labor supply is typically thought of as being inelastic, as permanent changes in the wage rate have both an income and substitution effect that roughly cancel out.

Household $h$ that owns firm $i$ faces the following optimization problem:

$$\max_{\{c_{h,t},k_{i,t+1},l_{i,t},b_{i,t}\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t u(c_{h,t})$$

s.t.

$$c_{h,t} + k_{i,t+1} = (F(k_{i,t}, l_{i,t}) - w_t l_{i,t} - r_t b_{i,t} + (1 - \delta)k_{i,t}) + w_t l$$

$$k_{i,t+1} \geq 0$$

$$w_t l_{i,t} \leq m_t + \gamma_t k_{i,t} + b_{i,t}$$

The initial capital levels are given by $k_{i,0} = k_0$ (identical across firms).

At time $t$, households have four choice variables: household consumption ($c_{h,t}$), next-period capital levels ($k_{i,t+1}$), labor demand ($l_{i,t}$) and liquidity demand ($b_{i,t}$).

Equation (5) is the household’s budget constraint. The household’s income is composed of dividend income and labor income. This income is divided between consumption and the accumulation of physical capital. Equation (6) states that capital cannot be negative (this will not be binding).

---

See, for example, Basu and Kimball [2002].
Equation 7 is the household’s liquidity constraint. It states that the firm’s labor purchases have to be financed with guaranteed claims: $m_t$ are claims guaranteed by the government, $\gamma k_{i,t}$ are claims backed by collateral, and $b_{i,t}$ are guaranteed claims issued by other firms.

**Timing.** To summarize, the within-period timing of the model is as follows:

1. Firms enter period $t$ with $k_{i,t}$ units of physical capital. The amount of liquid claims that the firm can issue is $m_t + \gamma k_{i,t}$.

2. Firms decide how many additional liquid claims to borrow (or lend), $b_{i,t}$.

3. Firms use their liquid claims ($m_t + \gamma k_{i,t} + b_{i,t}$) to hire labor ($l_{i,t}$). The equilibrium wage ($w_t$) is set so that the labor market clears.

4. After production, firms repay liquidity suppliers (with interest). Profits are distributed as dividend payments to households.

5. Households (who are shareholders) decide how much to invest in next period’s physical capital ($k_{i,t+1}$). They consume their income (dividends + labor income) minus their investment in capital.

It is important to clarify that the interest rate, $r_t$, denotes the within period rate of return on liquid claims. There is no borrowing and lending across periods.

### 3.1 Global environment

There are two economies: an emerging economy ($em$) and a developed economy ($d$). The economies differ both in their ability to create publicly backed liquid claims and in their ability to create collateral backed liquid claims.

The emerging economy is assumed to be unable to create collateral backed liquid claims:

$$\gamma^{em} = 0$$ (8)

The emerging economy is also not particularly good at creating publicly backed liquid claims: the fraction of output that the government can guarantee, $\theta$, is such
that the liquidity constraint is binding in equilibrium. Recall that given the Cobb-Douglas production technology, the wage bill in an unconstrained economy is a fraction $1 - \alpha$ of output. Thus, the following restriction guarantees that in the autarkic emerging economy, the liquidity constraint is binding in equilibrium:

$$\theta_{em} < 1 - \alpha$$

The developed economy is superior to the emerging economy both in its ability to create publicly backed liquid claims and in its ability to create privately backed liquid claims. I assume that in the developed economy, a positive fraction of capital can be used as collateral:

$$\gamma^d = \gamma > 0$$

However, the government is able to back enough claims to finance the autarkic wage bill:

$$\theta^d \geq 1 - \alpha$$

**Initial capital levels.** To abstract from other sources of heterogeneity between emerging and developed economies, I assume that initial capital levels are identical in both types of economies:

$$k_{d,0} = k_{em,0} = k_0$$

The results qualitatively generalize to more realistic settings in which $k_{em,0} < k_{d,0}$.

### 3.2 Planner’s problem

It is useful to consider the social planner’s problem. Consider a social planner, facing the following problem:

---

\[14\] In the following sections, it will be shown that the autarkic equilibrium is Pareto efficient, and the integrated equilibrium is not. When $k_{em,0} < k_{d,0}$, neither the autarkic nor the integrated equilibria are Pareto efficient; however, it is possible to show that there is a Pareto improvement when moving from the integrated equilibrium to the autarkic equilibrium, with a transfer from emerging to developed economies (under the assumption that there is no borrowing and lending across periods, just within-period liquidity markets).
\[
\max_{c_{em,t}, c_{d,t}, k_{em,t+1}, k_{d,t+1}, w_{em,t}, w_{d,t}} \sum_{t=0}^{\infty} \beta^t u(c_{d,t}) + \Psi \sum_{t=0}^{\infty} \beta^t u(c_{em,t})
\]

s.t.
\[
c_{d,t} + c_{em,t} + k_{d,t+1} + k_{em,t+1} = F(k_{em,t}, l) + F(k_{d,t}, l) + (1 - \delta)(k_{em,t} + k_{d,t})
\]

\[
w_{em,t} l + w_{d,t} l \leq \theta_{em} F(k_{em,t}, l) + \theta_{d} F(k_{d,t}, l) + \gamma k_{d,t}
\]

Where \( k_{em,0} = k_{d,0} \) is given.

The social planner maximizes a weighted sum of developed and emerging market utilities, given an aggregate budget constraint and an aggregate liquidity constraint. \( \Psi \) is the Pareto weight that the planner puts on emerging economies. Note that the standard equivalence applies here: any Pareto efficient allocation corresponds to the solution to the planner’s problem, for some value of \( \Psi \). In other words, if an allocation does not correspond to the planner’s solution for some value of \( \Psi \), both emerging and developed economies can be made better off by switching to an allocation that does.

Though the planner faces an aggregate liquidity constraint, his optimization problem is the same as a problem that ignores that constraint. This is because the planner can circumvent the aggregate liquidity constraint by choosing \( w_{em,t} = w_{d,t} = 0 \). As labor is supplied inelastically, there is no real cost to expropriating labor, and offering a wage sufficiently low so that the liquidity constraint does not bind. Thus, the above problem is equivalent to that of a planner facing only the first constraint.

**Lemma 1** The solution to the planner’s problem satisfies the following equations:

\[
k_{em,t} = k_{d,t}
\]

\[
u'(c_{d,t}) = \Psi u'(c_{em,t})
\]

The proof of this lemma, together with other omitted proofs, is in the appendix.
3.3 Closed economy equilibrium

Definition 1 An equilibrium of the closed economy is a sequence of interest rates \( \{r_t\}_{t=0}^{\infty} \), a sequence of wages \( \{w_t\}_{t=0}^{\infty} \), a sequence of labor demands \( \{l_{i,t}\}_{t=0}^{\infty} \), a sequence of physical capital stocks \( \{k_{i,t}\}_{t=0}^{\infty} \), a sequence of liquidity demands \( \{b_{i,t}\}_{t=0}^{\infty} \), and a set of consumption paths \( \{c_{h,t}\}_{t=0}^{\infty} \) that jointly satisfy the following conditions:

1. Given the sequence of interest rates and the wage sequence, the consumption sequences, the labor demand sequences, the sequence of liquidity demands, and the capital sequences solve the optimization problem of household \( h \) (owner of firm \( i \)), given by equations 4-7.

2. Given the wage \( w_t \), the labor market clears:

\[
\int_0^1 l_{i,t} \, di = l \tag{18}
\]

3. Given the interest rate \( r_t \), the liquidity market clears:

\[
\int_0^1 b_{i,t} \, di = 0 \tag{19}
\]

The last condition states that the liquidity market clears domestically; there is no international borrowing and lending for liquidity purposes, so the domestic \( r \) may be different across countries. This implies that in an autarkic equilibrium, the entire wage bill is financed by domestically issued claims.

The following lemma characterizes the closed economy equilibrium in the developed economy:

Lemma 2 1. The equilibrium of the developed economy is characterized by the following equations:

\[
u'(c_t) = \beta u'(c_{t+1}) \left( \frac{\partial F(k_{t+1}, l)}{\partial k} + 1 - \delta \right) \tag{20}\]

\[
\frac{\partial F(k_t, l)}{\partial l} = w_t \tag{21}\]

14
2. The autarkic developed economy converges to a steady state, in which \( r, w, k \) and \( c \) are constant. The steady state is characterized by the following equations:

\[
\frac{1}{\beta} = \frac{\partial F(k^{ss}, l)}{\partial k} + 1 - \delta \tag{24}
\]

\[
\frac{\partial F(k^{ss}, l)}{\partial l} = w^{ss} \tag{25}
\]

\[
r^{ss} = 0 \tag{26}
\]

\[
c^{ss} = F(k^{ss}, l) - \delta k^{ss} \tag{27}
\]

As the developed economy generates sufficient liquidity, the autarkic equilibrium is essentially the equilibrium of the unconstrained economy. The first equilibrium condition (equation 20) is the Euler equation with respect to the accumulation of physical capital. As the liquidity constraint is not binding, the fact that capital can be used as collateral is inconsequential. The within period return to liquidity is 0, so the fact that capital can be used to back liquid claims does not affect the incentives for capital accumulation.

Equation 21 states that, in the autarkic developed economy, the wage is equated with the marginal product of labor. In other words, producers are indifferent with respect to hiring an additional unit of labor: the marginal productivity of labor exactly offsets its cost. This indifference is evidence that the liquidity constraint is not binding: a producer endowed with another unit of liquidity would be indifferent between using it or not. The market return on liquidity is therefore 0 (equation 22).
The closed economy equilibrium in the emerging economy is different in nature. In the emerging economy, liquidity is insufficient to account for the entire real return to labor. Thus, in equilibrium, the liquidity constraint is binding.

However, importantly, while the binding liquidity constraint changes the equilibrium wage and the distribution of surplus between firms’ profits and labor income, it does not change the equilibrium levels of consumption and capital. In other words, consumption and capital are the same as in the unconstrained economy; the binding liquidity constraint does not create any inefficiency. The following lemma summarizes this result:

**Lemma 3** 1. The equilibrium of the emerging economy is characterized by the following equations:

\[ u'(c_t) = \beta u'(c_{t+1}) \left( \frac{\partial F(k_{t+1}, l)}{\partial k} + 1 - \delta \right) \]  
\[ \frac{1}{w_t} \frac{\partial F(k_t, l)}{\partial l} = 1 + r_t \]  
\[ w_t l = \theta^{em} F(k_t, l) \]  
\[ c_t + k_{t+1} = F(k_t, l) + (1 - \delta)k_t \]

2. The autarkic emerging economy converges to a steady state, in which \( r, w, k \) and \( c \) are constant. The steady state is characterized by the following equations:

\[ \frac{1}{\beta} = \frac{\partial F(k^{ss}, l)}{\partial k} + 1 - \delta \]  
\[ \frac{1}{w^{ss}} \frac{\partial F(k^{ss}, l)}{\partial l} = 1 + r^{ss} \]  
\[ w^{ss} l = \theta^{em} F(k^{ss}, l) \]
Note that equations (28) and (31) fully characterize the consumption and capital accumulation sequences, and that these equations are identical to the equilibrium of the unconstrained developed economy. Thus, while the shortage of liquidity depresses the equilibrium wage and increases the equilibrium return to liquidity, it does not affect equilibrium welfare.

In the autarkic emerging economy, the liquidity constraint is binding for every firm. From the firm’s perspective, an additional backed claim on output could purchase $\frac{1}{w}$ units of labor; at the margin, each unit of labor produces $\frac{\partial F(k,l)}{\partial l} > w$ units of output. The return to liquidity is greater than 0, reflected in a positive equilibrium $r$.

On the aggregate level, even though each producer would like to employ more labor at the equilibrium wage, there is no more labor to employ. All labor is employed in equilibrium, and an increase in the liquidity supply would merely lead to an appreciation of the wage.

**Corollary 1** The closed economy equilibrium is Pareto efficient.

To see this, note that the autarkic equilibrium corresponds to the solution to the planner’s problem with $\Psi = 1$. Under autarky, consumption is identical across regions in every period, and capital accumulation is efficient.

### 3.4 Integrated equilibrium

In the open economy, it is no longer required that the market for liquidity clears domestically: foreigners and domestic firms can borrow and lend liquidity in a global liquidity market.

Within-period borrowing and lending for liquidity purposes is the only type of flow considered. There is no borrowing and lending for consumption smoothing purposes, or for capital accumulation.\(^\text{15}\)

\(^{15}\)This assumption is made for simplicity. It will be shown that, in the integrated equilibrium, the developed economy accumulates capital faster than the emerging economy following financial integration. Allowing for borrowing and lending across periods is therefore likely to result in
Definition 2 An equilibrium of the integrated economy is a sequence of interest rates \( \{r_t\}_{t=0}^{\infty} \), a sequence of emerging market wages and developed market wages \( \{w_{\text{em},t}, w_{\text{d},t}\}_{t=0}^{\infty} \), a sequence of labor demands \( \{\{l^d_{i,t}, l^\text{em}_{i,t}\}_{t=0}^{\infty}\}_{i\in[0,1]} \), a sequence of physical capital stocks \( \{\{k^d_{i,t}, k^\text{em}_{i,t}\}_{t=0}^{\infty}\}_{i\in[0,1]} \), a sequence of liquidity demands \( \{\{b^d_{i,t}, b^\text{em}_{i,t}\}_{t=0}^{\infty}\}_{i\in[0,1]} \), and a set of consumption paths \( \{\{c^d_{h,t}, c^\text{em}_{h,t}\}_{t=0}^{\infty}\}_{h\in[0,1]} \) that jointly satisfy the following conditions:

1. Given the sequence of interest rates and the wage sequences, the consumption sequences, the labor demand sequences, the sequences of liquidity demands, and the capital sequences solve the optimization problem of household \( h \) (owner of firm \( i \)), given by equations \[2\] to \[7\].

2. Given the wages \( w_{\text{em},t} \) and \( w_{\text{d},t} \), the domestic labor markets clear:

\[
\int_0^1 l^\text{em}_{i,t} \, di = \int_0^1 l^d_{i,t} \, di = l \tag{36}
\]

3. The interest rate \( r_t \) is such that the global liquidity market clears:

\[
\int_0^1 b^\text{em}_{i,t} \, di + \int_0^1 b^d_{i,t} \, di = 0 \tag{37}
\]

The definition of an equilibrium in the integrated economy departs from the definition of the autarkic equilibrium only in the last condition: rather than requiring that the market for liquidity clears domestically, it is required that the rate of return on liquidity is equated across countries.

The following parametric restriction guarantees that in the integrated equilibrium, there is insufficient liquidity to finance the aggregate returns to labor in an unconstrained economy:

Assumption 1 Let \( k^{ss} \) denote the autarkic steady state capital level (defined by equation \[32\]). Assume the following parametric restrictions:

\[
k_0 \leq k^{ss} \tag{38}
\]

developed economies borrowing from emerging economies upon integration of liquidity markets.

Of course, the direction of these flows may change under the more realistic assumption \( k_{\text{em},0} < k_{\text{d},0} \).
\[(\theta^{d} + \theta^{m})(k^{ss})^{\alpha l^{1-\alpha}} + \gamma k^{ss} < 2(1 - \alpha)(k^{ss})^{\alpha l^{1-\alpha}} \]  \hspace{1cm} (39)

The first part of the assumption states that both economies start off with capital levels that are less than or equal to their autarkic steady state levels (otherwise, if there is a lot of capital, there may be enough collateral backed claims to make the liquidity constraints not binding). The second condition guarantees that at the autarkic steady state, the aggregate amount of liquidity is insufficient to finance the unconstrained wage bill. The left hand side is the aggregate amount of liquidity in the autarkic steady state. The right hand side is the aggregate unconstrained wage bill at the autarkic steady state (which is a fraction \(1 - \alpha\) of steady state output in each region).

The following proposition summarizes the main results of this paper:

**Proposition 1**

1. Opening to international liquidity flows reduces equilibrium welfare in the emerging economy. Compared to autarky, consumption is lower in every period.

2. In the financially integrated environment, developed economies supply liquidity to emerging economies. In every period, there is a net transfer of goods from emerging to developed economies.

3. The financially integrated equilibrium is not Pareto efficient. The incentive to create collateral backed claims leads to an over accumulation of physical capital in the developed economy.

The first result is that financial integration reduces equilibrium welfare in emerging economies. Privately, each constrained firm finds it optimal to borrow from abroad to increase the amount of labor that it can employ. However, in general equilibrium, this merely bids up the wage rate. Domestic income is lower in equilibrium, as firms must pay interest on foreign liquidity.

The second result - related to the first - is that, in every period, there is a net flow of goods from emerging to developed economies. This provides a potential explanation to the “global imbalances” phenomenon, and suggests that the net exports from emerging to developed economies may be a steady state equilibrium outcome.
In equilibrium, producers in emerging economies rely on foreign liquidity to finance the wage bill. Though not modeled here explicitly, this creates unnecessary fragility with respect to foreign liquidity supply shocks (as in Neumeyer and Perri [2005]). The “working capital” requirement, now standard in the literature, emerges here as an equilibrium outcome.

Finally, the third result states that financial integration not only reduces welfare in emerging economies, but does so in a Pareto inefficient fashion. This is because households in developed economies inefficiently accumulate physical capital for the purpose of creating backed claims.

### 3.5 Numerical example

I illustrate the quantitative implications of the model with a numerical example. I assume the following functional form for the household’s consumption utility:

$$u(c) = \ln(c)$$ (40)

<table>
<thead>
<tr>
<th>Notation</th>
<th>Variable</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>Discount factor</td>
<td>0.97</td>
</tr>
<tr>
<td>$\delta$</td>
<td>Depreciation rate</td>
<td>0.1</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Capital share in production</td>
<td>0.33</td>
</tr>
<tr>
<td>$l$</td>
<td>Labor supply</td>
<td>1</td>
</tr>
<tr>
<td>$\theta^d$</td>
<td>Public backing as a share of output: developed economies</td>
<td>0.67</td>
</tr>
<tr>
<td>$\theta^{em}$</td>
<td>Public backing as a share of output: emerging economies</td>
<td>0.345</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>Collateral as a share of capital: developed economies</td>
<td>0.1</td>
</tr>
</tbody>
</table>

The choice of parameters is summarized in table 2. The first three parameters, $\beta$, $\delta$, and $\alpha$, are standard. The choice of $l = 1$ is a convenient normalization. The value of $\theta^d$ was chosen such that in developed economies, there are exactly enough publicly backed claims to finance the unconstrained wage bill. The value of $\theta^{em}$ was chosen as half of $\theta^d$, roughly corresponding to the relative values of $\frac{M_3}{PY}$ (see figure 1).

The choice of $\theta^d$ restricts the values of $\gamma$ that comply with Assumption 1.
value $\gamma = 0.1$ was chosen to generate a steady state interest rate in the integrated economy of roughly 5%.

**Steady state analysis.** I begin by computing the steady states under autarky and under financial integration. The following lemma characterizes the steady state solutions:

**Lemma 4** Let $l = 1$.

1. Both in emerging and developed economies, the autarkic steady state values of capital and consumption are characterized by the following equations:

   $$k_{ss} = \left(\frac{\alpha}{\frac{1}{\beta} - 1 + \delta}\right)^{\frac{1}{\alpha}}$$  \hspace{1cm} (41)

   $$c_{ss} = (k_{ss})^\alpha - \delta k_{ss}$$  \hspace{1cm} (42)

   For developed economies, the wage and the interest rate are given by:

   $$w_{ds}^{ss} = (1 - \alpha)(k_{ss})^\alpha$$  \hspace{1cm} (43)

   $$r_{ds}^{ss} = 0$$  \hspace{1cm} (44)

   For emerging economies, the wage and the interest rate are given by:

   $$w_{ems}^{ss} = \theta_{em}(k_{ss})^\alpha$$  \hspace{1cm} (45)

   $$r_{ss} = \frac{(1 - \alpha)(k_{ss})^\alpha}{w_{ss}} - 1$$  \hspace{1cm} (46)

2. In the open economy, the steady state solves the following system of equations:

   $$k_{ds}^{ss} = \left(\frac{\alpha}{\frac{1}{\beta} - 1 + \delta - r_{ss}^{ss} \gamma}\right)^{\frac{1}{\alpha}}$$  \hspace{1cm} (47)

   $$k_{ems}^{ss} = \left(\frac{\alpha}{\frac{1}{\beta} - 1 + \delta}\right)^{\frac{1}{\alpha}}$$  \hspace{1cm} (48)

   $$\frac{(1 - \alpha)k_{dss}^{ss}}{w_{dss}^{ss}} = \frac{(1 - \alpha)k_{dss}^{ss}}{w_{dss}^{ss}} = 1 + r_{ss}$$  \hspace{1cm} (49)
The autarkic steady state can be solved analytically. However, the integrated steady state has to be solved numerically. I consider a range of $r$ between 0 and the emerging market’s autarkic steady state level of $r$, and minimize the error terms of the liquidity market clearing condition (equation $50$).

The steady state values of the autarkic and open equilibria, given the calibrated parameters in table 2, are summarized in table 3.

Table 3: Steady state comparison

<table>
<thead>
<tr>
<th>Variable</th>
<th>Autarky</th>
<th>Open</th>
<th>Ratio (autarky divided by open)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$c_{em}$</td>
<td>1.1793</td>
<td>1.1545</td>
<td>1.0215</td>
</tr>
<tr>
<td>$c_d$</td>
<td>1.1793</td>
<td>1.2111</td>
<td>0.9737</td>
</tr>
<tr>
<td>$k_{em}$</td>
<td>3.974</td>
<td>3.974</td>
<td>1</td>
</tr>
<tr>
<td>$k_d$</td>
<td>3.974</td>
<td>4.222</td>
<td>0.9411</td>
</tr>
<tr>
<td>$w_{em}$</td>
<td>0.5282</td>
<td>1.0041</td>
<td>0.526</td>
</tr>
<tr>
<td>$w_d$</td>
<td>1.0564</td>
<td>1.0244</td>
<td>1.0312</td>
</tr>
<tr>
<td>$r_{em}$</td>
<td>1</td>
<td>0.0521</td>
<td>19.1939</td>
</tr>
<tr>
<td>$r_d$</td>
<td>0</td>
<td>0.0521</td>
<td>0</td>
</tr>
<tr>
<td>$b$</td>
<td>0</td>
<td>0.4759</td>
<td>0</td>
</tr>
<tr>
<td>$(\frac{n_x}{y})_{em}$</td>
<td>0</td>
<td>0.0158</td>
<td>0</td>
</tr>
<tr>
<td>$(\frac{n_x}{y})_d$</td>
<td>0</td>
<td>-0.0154</td>
<td>0</td>
</tr>
</tbody>
</table>

In this numerical example, steady state consumption in emerging economies is about 2.2% lower in the financially integrated equilibrium, while steady state consumption in developed economies is about 2.7% higher.

The last two rows are net exports as a fraction of GDP. These parameters suggest that, in the steady state, the emerging market exports roughly 1.5% of its GDP to developed economies, as payment for liquidity services.
Transitional dynamics. Starting from the autarkic steady state, I calibrate the transitional dynamics to the financially integrated equilibrium.

At $t = 0$, the economies are closed to international liquidity flows. At $t = 1$, the economies open (unexpectedly). I solve for the equilibrium using dynare. Figure 2 describes the transitional dynamics.

Consistent with recent trends, the return to liquidity in global markets ($r$) declines over time. From the perspective of developed economies, financial integration is associated with an initial spike in $r$ (from its steady state value of 0), followed by a gradual decline. The decline in $r$ is consistent with the “Greenspan conundrum” (see Caballero et al. [2008]), the declining trend in interest rates, despite efforts of the US central bank to increase it. This model suggests that part of the decline in $r$ may be accounted for by the increase in the supply of liquid claims, in response to heightened demand for liquidity from emerging markets. As the supply of liquid claims increases, the return to liquidity declines.

The incentive to create liquid claims leads to an increase in capital in the developed world. In the autarkic equilibrium, the return to liquidity in developed markets was 0; agents did not have any incentives to accumulate capital for the purpose of creating collateralized obligations. However, in the globally integrated environment, the heightened demand for liquid claims distorts the incentives to accumulate capital. The return to capital faced by private agents includes both the return in production, and the return to the liquid claims that can be backed by that capital.

The heightened incentives to accumulate capital lead to an initial decline in consumption in the developed economies. Faced with a higher return to capital, households in developed economies initially sacrifice consumption in favor of capital accumulation. However, their consumption quickly surpasses its autarkic level, as they consume rents from liquidity supplied to emerging economies.

For emerging economies, consumption and capital quickly converge to their open-economy steady state levels. The steady state capital level is the same as in the autarkic economy, while consumption is about 2.2% lower. After an initial large drop in consumption, there is a mild gradual increase in consumption, as the economy converges to its new steady state. The increase in consumption reflects the buildup of liquidity in developed economies, which lowers the price of “imported”
Figure 2: The transitional dynamics from the autarkic steady state to the integrated equilibrium.
liquidity.

On impact, wages in emerging and developed economies converge. For emerging economies, this implies an appreciation of the wage, whereas for developed economies, wages decline. As the developed economy accumulates capital, wages slightly diverge, reflecting the higher marginal product of labor in the capital-abundant developed economy.

Finally, this model implies that, on impact, the net exports of emerging markets increase to roughly 2% of GDP. This increase is mirrored by a net import of the same magnitude in developed economies. As the returns to liquidity decline, net exports in emerging economies decline as well, but never to 0: they converge to their steady state level of roughly 1.5% of GDP.

3.6 Discussion

This model suggests that, while the integration of liquidity markets leads to a Pareto inefficient equilibrium, it increases welfare in developed economies. In principle, this would suggest that developed economies (if not appropriately compensated) should oppose restrictions on liquidity flows. However, there is a sense that this model does not fully capture the welfare implications of the creation of privately backed liquid claims. In this model, the inefficiency of private claims is merely that it leads to excessive capital accumulation. When thinking about the US economy, this is clearly part of the problem: the ability to create mortgage backed securities (which could be used for liquidity purposes, specifically as collateral in the REPO market), led to inefficient accumulation of houses, and perhaps also to an inefficiently large financial industry. But it has become apparent that this activity has consequences beyond the inefficient use of resources, leading (in one way or another) to a deep and prolonged recession. A richer model that accounts for the financial fragility induced by privately backed claims seems necessary for evaluating the welfare implications of liquidity flows for developed economies.

Financial fragility is likely to increase the welfare costs of international liquidity flows. However, there are other forces that might work in the opposite direction. In the presence of an elastic labor supply, the autarkic wedge between the marginal
product of labor and the wage creates a real inefficiency in emerging economies. International liquidity flows would equalize this wedge across countries, which is likely to be associated with global efficiency gains. While it is not unreasonable that long-run labor supply is inelastic, a broader interpretation would allow for other inputs of production that need to be purchased with liquid claims, some of which are internationally tradable. International liquidity flows may allow for a more efficient allocation of these inputs across countries.

These additional channels - financial fragility and elastic input supply - are potentially important for assessing the "bottom line" welfare implications of international liquidity flows. A richer quantitative model that incorporates these aspects in addition to the channels highlighted here is a natural next step, which I reserve for future research.

4 Conclusion

The presence of binding liquidity constraints implies a transfer of surplus to liquidity suppliers. In the closed economy, liquidity is supplied domestically so the "liquidity rents" (if any) are consumed domestically. However, in the integrated equilibrium, binding liquidity constraints in emerging economies imply a transfer of surplus to foreign liquidity suppliers, thereby increasing consumption in developed economies at the expense of emerging markets.

From a policy perspective, this model suggests that emerging economies have an incentive to discourage foreign liquidity supply, even if domestic firms are heavily constrained. Opening to liquidity flows will increase the supply of liquidity, but this will only bid up the domestic input prices such as labor and land. This, in turn, will make the domestic firms less liquidity constrained but more heavily reliant on foreign liquidity.

For developed economies, international liquidity flows increase the returns to liquidity, and increase the incentives to create liquid assets. Responding to these incentives may be Pareto inefficient, as in equilibrium prices may adjust so that the supply of liquidity is sufficient to satisfy aggregate liquidity needs.

It is interesting to note that in this model, "global imbalances" are an equilibrium artifact, and in some sense, an accounting error. If liquid and illiquid
claims were valued at their equilibrium relative prices, the current account would be balanced. The trade deficit in developed economies is accounted for by rents to liquidity supply.

While this paper focuses on liquidity flows, the general mechanism may apply to other types of flows as well. For example, foreign direct investment (FDI) may be subject to a similar problem: if foreign owners are less liquidity constrained, they are in a better position to exploit the wedge between the marginal product of labor and the wage. The presence of FDI may therefore divert “liquidity rents” from domestic producers to foreigners.

References


A Money aggregates for selected emerging and developed economies

Table 4 presents the figures used to create figure 1 (the “advanced” column corresponds to the average of developed economies, and the “emerging” column corresponds to the average of emerging economies).

<table>
<thead>
<tr>
<th>Country</th>
<th>Average $\frac{M_3}{Y}$</th>
<th>Years</th>
</tr>
</thead>
<tbody>
<tr>
<td>Australia</td>
<td>0.83</td>
<td>2002-2011</td>
</tr>
<tr>
<td>Denmark</td>
<td>0.58</td>
<td>2002-2011</td>
</tr>
<tr>
<td>Euro Area</td>
<td>0.94</td>
<td>2002-2011</td>
</tr>
<tr>
<td>Japan</td>
<td>2.12</td>
<td>2003-2010</td>
</tr>
<tr>
<td>Singapore</td>
<td>1.22</td>
<td>2002-2011</td>
</tr>
<tr>
<td>Sweden</td>
<td>0.61</td>
<td>2002-2011</td>
</tr>
<tr>
<td>Switzerland</td>
<td>1.25</td>
<td>2002-2010</td>
</tr>
<tr>
<td>United States</td>
<td>0.8</td>
<td>2002-2005</td>
</tr>
<tr>
<td>Argentina</td>
<td>0.32</td>
<td>2002-2011</td>
</tr>
<tr>
<td>Brazil</td>
<td>0.59</td>
<td>2002-2011</td>
</tr>
<tr>
<td>Chile</td>
<td>0.88</td>
<td>2002-2011</td>
</tr>
<tr>
<td>Colombia</td>
<td>0.36</td>
<td>2003-2010</td>
</tr>
<tr>
<td>Costa Rica</td>
<td>0.48</td>
<td>2002-2010</td>
</tr>
<tr>
<td>Czech Republic</td>
<td>0.66</td>
<td>2002-2011</td>
</tr>
<tr>
<td>Hungary</td>
<td>0.55</td>
<td>2002-2011</td>
</tr>
<tr>
<td>India</td>
<td>0.78</td>
<td>2006-2011</td>
</tr>
<tr>
<td>Malaysia</td>
<td>1.34</td>
<td>2002-2011</td>
</tr>
<tr>
<td>Mexico</td>
<td>0.53</td>
<td>2002-2011</td>
</tr>
<tr>
<td>Morocco</td>
<td>1.03</td>
<td>2002-2010</td>
</tr>
<tr>
<td>Philippines</td>
<td>0.38</td>
<td>2002-2007</td>
</tr>
<tr>
<td>Poland</td>
<td>0.48</td>
<td>2002-2011</td>
</tr>
<tr>
<td>Romania</td>
<td>0.37</td>
<td>2007-2011</td>
</tr>
<tr>
<td>Slovenia</td>
<td>0.53</td>
<td>2002-2006</td>
</tr>
<tr>
<td>South Africa</td>
<td>0.74</td>
<td>2002-2011</td>
</tr>
<tr>
<td>Turkey</td>
<td>0.38</td>
<td>2002-2010</td>
</tr>
<tr>
<td>Ukraine</td>
<td>0.46</td>
<td>2002-2011</td>
</tr>
</tbody>
</table>

I use data on M3 and nominal GDP (in domestic currency) taken from the International Monetary Fund (IMF) International Financial Statistics (IFS). The
choice of countries is based on data availability. Comparisons for M2 or the “Broad Money” category yield qualitatively similar results. To calculate averages, I first calculate the average for each country, and use those averages to calculate the group average.

B Proofs

B.1 Proof of Lemma 1

First, note that the aggregate liquidity constraint (equation 15) is never binding, as the planner can choose \( w_{em,t} = w_{d,t} = 0 \). Let \( \lambda_t \) be the Lagrange multiplier on the aggregate budget constraint of time \( t \) (equation 14). The Lagrangian is given by:

\[
\max_{c_{em,t}, c_{d,t}, l_{em,t}, l_{d,t}} \sum_{t=0}^{\infty} \beta^t u(c_{d,t}) + \Psi \sum_{t=0}^{\infty} \beta^t u(c_{d,t})
- \sum_{t=0}^{\infty} \lambda_t (c_{d,t} + c_{em,t} + k_{d,t+1} + k_{em,t+1} - F(k_{em,t}, l) - F(k_{d,t}, l) - (1 - \delta)(k_{em,t} + k_{d,t}))
\]

The FOC with respect to \( k_{d,t+1} \) is:

\[
\lambda_t - \lambda_{t+1} \left( \frac{\partial F(k_{d,t+1}, l)}{\partial k} + 1 - \delta \right) = 0
\]

Replacing \( k_{d,t+1} \) with \( k_{em,t+1} \) yields the first order condition with respect to \( k_{em,t+1} \). Thus,

\[
\frac{\partial F(k_{d,t+1}, l)}{\partial k} = \frac{\partial F(k_{em,t+1}, l)}{\partial k} \Rightarrow k_{d,t+1} = k_{em,t+1}
\]

The equality for \( t = 0 \) is by assumption.

To prove the second part of the lemma, consider the FOC with respect to \( c_{d,t} \) and \( c_{em,t} \):

\[
\beta^t u'(c_{d,t}) = \lambda_t
\]

\[
\Psi \beta^t u'(c_{d,t}) = \lambda_t
\]
It follows that $\beta^t u'(c_{d,t}) = \Psi \beta^t u'(c_{d,t})$.

B.2 Proof of Lemma 2

Consider the following “unconstrained” problem, that ignores the liquidity constraint:

$$\max_{c_t} \sum_{t=0}^{\infty} u(c_t)$$

s.t.

$$c_t + k_{t+1} = F(k_t, l) + (1 - \delta)k_t$$

$$k_{t+1} \geq 0$$

Where $k_0$ is given.

When the liquidity constraint is not binding, households implement the welfare maximizing plan.

I continue by showing that the unconstrained solution is feasible in the decentralized economy: the government is able to create sufficient publicly backed claims so that the cash in advance constraint does not bind. I consider the firm’s unconstrained problem, and show that there is sufficient liquidity to finance the unconstrained wage bill.

The firm’s unconstrained problem can be reduced to:

$$\max_{l_{i,t}} k_{i,t}^{1-\alpha} l_{i,t}^{1-\alpha} - w_t l_{i,t}$$

The first order condition with respect to $l_{i,t}$ is:

$$(1 - \alpha)k_{i,t}^{\alpha} l_{i,t}^{\alpha} = w_t$$

The total wage bill is therefore given by a fraction $1 - \alpha$ of output:

$$(1 - \alpha)k_{i,t}^{\alpha} l_{i,t}^{1-\alpha} = w_t l_{i,t}$$

By symmetry, all firms choose the same value of $l_{i,t}$, and $k_{i,t}$ is identical across
firms. Thus, the unconstrained wage bill of firm $i$ is:

$$w_{t}l_{i,t} = (1 - \alpha)k_{i,t}^{\alpha}l_{i,t}^{1-\alpha} = (1 - \alpha)F(k_{t},l) \quad (65)$$

The assumption that $\theta^d > 1 - \alpha$ guarantees that the government can issue sufficient public guarantees to finance the unconstrained wage bill. Thus, in equilibrium, there is enough liquidity so that the liquidity constraint is not binding.

It follows that the firm’s optimization problem can be written as follows:

$$\max_{l_{i,t},b_{i,t}} F(k_{i,t},l_{i,t}) - w_{t}l_{i,t} - r_{t}b_{i,t} + (1 - \delta)k_{i,t} \quad (66)$$

When $r_{t} > 0$, the firm chooses $b_{i,t} \rightarrow -\infty$. When $r_{t} < 0$, the firm chooses $b_{i,t} \rightarrow \infty$. Market clearing therefore requires $r_{t} = 0$.

It is easy to show that the solution to the unconstrained problem satisfies the Euler equation and the aggregate budget constraint.

The standard neoclassical growth model that we are left with converges to a steady state, that can be computing by replacing time $t$ and $t + 1$ variables in the equilibrium equations with steady state values.

### B.3 Proof of Lemma 3

First, note that the liquidity constraint is binding. The proof of Lemma 2 illustrates that to finance the unconstrained wage bill, the liquidity supply must be at least a fraction $1 - \alpha$ of total output. In the emerging economy, there are only publicly backed claims, and publicly backed claims as a fraction of output are assumed to be less than $1 - \alpha$ ($\theta^{em} < 1 - \alpha$).

Thus, in equilibrium the liquidity constraint is binding for all firms:

$$w_{t}l_{i,t} = m_{t} + b_{i,t} \quad (67)$$

Aggregating across firms and imposing the market clearing conditions for labor and liquidity:

$$w_{t}l = \int_{0}^{1} w_{t}l_{i,t}di = \int_{0}^{1} (m_{t} + b_{i,t})di = m_{t} + \int_{0}^{1} b_{i,t}di = m_{t} \quad (68)$$
The constraint of the government’s ability to create liquid claims implies that:

\[ w_t l = m_t \leq \theta^m F(k_t, l) = \theta^m k^\alpha l^{1-\alpha} \tag{69} \]

Consider the firm’s problem, given that the liquidity constraint is binding. Since \( l_{i,t} = \frac{m_{i,t} + b_{i,t}}{w_t} \), the problem can be written as:

\[ \max_{l_{i,t}, b_{i,t}} F(k_{i,t}, \frac{m_{i,t} + b_{i,t}}{w_t}) - (m_{i,t} + b_{i,t}) - r_t b_{i,t} \tag{70} \]

The first order condition with respect to \( b_{i,t} \) is:

\[ \frac{1}{w_t} \frac{\partial F(k_{i,t}, l_{i,t})}{\partial l} = 1 + r_t \tag{71} \]

Note that for \( r_t = 0 \), the liquidity constraint is not binding (the marginal product of labor is equal to the wage). Thus, \( r_t > 0 \). As firms are identical, \( b_{i,t} = 0 \) for every \( i \).

Consider the household’s problem. Note that by replacing \( l_{i,t} \) with \( \frac{m_t}{w_t} \), the problem can be reduced to:

\[ \max_{c_t, k_t} \sum_{t=0}^{\infty} \beta^t u(c_t) \tag{72} \]

s.t.

\[ c_t + k_{t+1} = F(k_t, \frac{m_t}{w_t}) + (1 - \delta) k_t - w_t (\frac{m_t}{w_t}) + w_t l = \tag{73} \]

\[ = F(k_t, l) + (1 - \delta) k_t - w_t l + w_t l = F(k_t, l) + (1 - \delta) k_t \]

This is the standard neoclassical growth model. The equilibrium is described by the standard Euler equation, and the economy converges to the steady state.

### B.4 Proof of Proposition 1

I show that in the integrated equilibrium, \( r_t > 0 \) for every \( t \). Assume that \( r = 0 \). Then, the firm’s problem (both in emerging and developed economies) is given by:

\[ \max_{l_{i,t}, b_{i,t}} F(k_{i,t}, l_{i,t}) - w_t l_{i,t} \tag{74} \]
s.t. \[ w_t l_t \leq m_t + b_{i,t} \] (75)

The firm will choose \( b_{i,t} \) so that the liquidity constraint is not binding. In this case, the wage bill will be equal to a fraction \( 1 - \alpha \) of output:

\[ w_{d,t} l = (1 - \alpha) F(k_{d,t}, l) \] (76)

and:

\[ w_{em,t} l = (1 - \alpha) F(k_{em,t}, l) \] (77)

The aggregate liquidity supply is bounded from above by:

\[ \theta d F(k_{d,t}, l) + \theta em F(k_{em,t}, l) + \gamma k_{d,t} \] (78)

Subtracting the unconstrained wage bill from the (maximal) aggregate liquidity supply:

\[ (\theta d - (1 - \alpha)) F(k_{d,t}, l) + (\theta em - (1 - \alpha)) F(k_{em,t}, l) + \gamma k_{d,t} \] (79)

By Assumption 1, this expression is negative for \( k^d = k^{em} = k^{ss} \) (where here \( k^{ss} \) denotes the autarkic steady state level. It follows that, for \( k = k^{ss} \):

\[ (\theta^d + \theta^{em} - 2(1 - \alpha)) k^{1-\alpha} + \gamma k < 0 \Rightarrow (\theta^d + \theta^{em} - 2(1 - \alpha)) < 0 \] (80)

Assuming \( k^d = k_{em} = k \), consider the derivative of the above equation (aggregate liquidity minus the unconstrained wage bill) with respect to \( k \):

\[ \alpha(\theta^d + \theta^{em} - 2(1 - \alpha))(\frac{l}{k})^{1-\alpha} + \gamma \] (81)

Equating the above with 0 yields a unique solution for \( k > 0 \). It is easy to show that this is a minimum, as for \( k = 0 \) the expression is 0, for \( k \to \infty \) the expression goes to \( \infty \), and for \( k^{ss} \) the value of the expression is negative. Thus, the equation has a unique minimum which is less than 0.

It follows that if the expression is negative for \( k^{ss} \), it is negative for all \( k < k^{ss} \). Thus, at \( t = 0 \) (at which, by Assumption 1, both countries are assumed to have
identical capital stocks that satisfy $k_0 \leq k^{ss}$, there is insufficient aggregate liquidity to finance the unconstrained wage bill.

I show that the economy converges to a steady state in which liquidity constraints are binding, and show that the difference between the aggregate liquidity supply and the unconstrained wage bill is decreasing over time; it follows that the liquidity constraint is binding in every period.

To compute the steady state capital level, consider the Euler equation (with $c_t = c_{t+1}$). For emerging economies, this amounts to:

$$\frac{1}{\beta} = \frac{\partial F(k_{em}^{ss}, l)}{\partial k} + 1 - \delta \tag{82}$$

Thus, the steady state capital level in emerging economies is the same as in the autarkic equilibrium. For developed economies, the Euler equation is:

$$u(c_t) = \beta u(c_{t+1})(\frac{\partial F(k_{d,t}^{ss}, l)}{\partial k} + 1 - \delta + r_{t+1}\gamma) \tag{83}$$

There is an additional term: $r_{t+1}\gamma$. This is because a fraction $\gamma$ of each unit of capital can be used for issuing a collateral-backed claim, which yields a return of $r$.

The economy converges to a steady state with a constant $r$. The steady state capital level in developed economies is given by:

$$\frac{1}{\beta} = \frac{\partial F(k_d^{ss}, l)}{\partial k} + 1 - \delta + r^{ss}\gamma \tag{84}$$

If $r^{ss} = 0$, this is the same as the autarkic equilibrium. But this is not possible as Assumption 1 guarantees that given autarkic-steady-state capital levels, there is insufficient aggregate liquidity to finance the unconstrained wage bills. It follows that $r^{ss} > 0$.

To show that $r_t > 0$ for all $t$, it is left to show that the difference between the aggregate liquidity supply and the unconstrained wage bill is decreasing over time. This follows from the fact that capital accumulation in developed economies is always faster than capital accumulation in emerging economies (as there is a greater return to physical capital, and income is higher).

The fact that $r_t > 0$ for every $t$ implies that the emerging economy is an
importer of liquidity in every period (otherwise, there is sufficient liquidity to finance the unconstrained wage bill in developed economies). Since $b_{em,t} > 0$ for every $t$, welfare in the emerging economy is given as the solution to the following problem:

$$\max_{k_{em,t},c_{em,t}} \sum_{t=0}^{\infty} \beta^t u(c_t)$$

s.t.

$$c_t + k_{t+1} = F(k_t, l) + (1 - \delta)k_t - r_t b_{em,t}$$

The payments to foreign liquidity, $r_t b_{em,t}$, act as a lump sum tax on emerging economies, reducing their aggregate budget constraint. The solution to the above problem is clearly welfare inferior to the solution of the autarkic equilibrium (which was the same as the unconstrained economy).

So far, I have proved the first two parts of the lemma: welfare in emerging economies is lower compared to autarky, and there is a net transfer of goods from emerging to developed economies in every period. It is left to show that the integrated equilibrium is not Pareto efficient. This is easy to see as, by Lemma 1, any Pareto efficient allocation equates capital levels across regions. Here, steady state capital in the developed economy is higher than in the emerging economy.

### B.5 Proof of Lemma 4

The expressions for autarkic steady state capital and consumption follow directly from the steady state conditions in Lemmas 2 and 3. For the autarkic developed economy, the wage is equated with the marginal product of capital, and the interest rate is 0.

For the autarkic emerging economy, the wage bill is given by the aggregate liquidity supply:

$$w_{em} = \theta^e k^{\alpha} l^{1-\alpha} = \theta^e k^{\alpha}$$

By Lemma 3, the interest rate in the autarkic emerging economy is given by:

$$\frac{\partial F(k, l)}{\partial l} = 1 + r \Rightarrow r = \frac{(1 - \alpha)k^\alpha}{w} - 1$$

The expressions for the open economy similarly follow from replacing steady...
state values with chosen functional forms and $l = 1$. 