1. Introduction

Suppose we are concerned with the inequalities that exist in a society with respect to the distribution of some desirable good or advantage – wealth, life expectancy, literacy, or wage-earning capacity. The causes of inequality in that distribution can be partitioned into two categories: those for which individuals should not be held responsible, and those for which they should be. We need not here be concerned with the problem of free will, and the possibility that people are not responsible for anything if they lack free will, because every society has a conception of responsibility, and we may take that as the politically salient conception. Thus, in many societies, it is thought wrong that an individual’s income be strongly correlated with her parent’s education or social position, for, assuming that correlation reflects causality, these family characteristics seem to be things for which the child should not be held responsible. On the other hand, most societies believe that adults should be held responsible for various choices that they make, assuming that they possess adequate information about the alternatives. Let us call the social and biological aspects of a person’s environment for which society believes he should not be responsible his circumstances, and those choices for which he should be held responsible, his effort. Call the desirable good whose distribution we are concerned with the objective.

When we have a data set that permits us to measure the inequality in the distribution of the objective, and its correlation with circumstances and effort, it is usually necessary (because data sets are finite) to choose a fairly small number of circumstances, each of which can take on a fairly small number of values. Thus, one circumstance might be parental education, which we could partition into three values; another might be race, partitioned into three categories, and so on. Call a vector of circumstances a type. Thus, we will have partitioned the population of the data set into a
finite number of types, where a type is the set of individuals with the same vector of circumstances. Denote the types by \( t = 1, \ldots, T \). Denote the level of the objective with which we are concerned (income, wage-earning capacity, or life expectancy) by \( u \), which is a function of circumstances, policy, and effort (with an error term). Thus, \( u'(e, \varphi) \) is the average level of the objective for individuals of type \( t \) whose effort choices are summarized by the vector \( e \) if the policy is \( \varphi \). Denote the policy space by \( \Phi \). In this formulation, any characteristic of the individual is either a component of circumstance, or of effort.

Effort, here is measured so that so that increasing effort produces an increasing value of the objective. In this way, effort’s role in the functions \( u' \) differs from its relationship to utility in economics. For example, if the objective is health status, then refraining from smoking constitutes positive effort – although that abstinence may lower utility in the usual sense, where the utility function is a representation of subjective preferences.

If the population faces a policy \( \varphi \in \Phi \), there will ensue a distribution of effort in each type; denote the distribution functions of these probability distributions by \( G'_{\varphi}(\cdot) \).

These distribution functions will, of course, have characteristics that reflect type – that is, circumstances. For instance, we will find different distributions of smoking behavior in different socio-economic types. Because the goal of equal-opportunity policy is to compensate persons for their circumstances, we should compensate them as well for the effect of their circumstances on their effort. How can we decide when two persons, of different types, have expended comparable degrees of effort? I propose to measure the degree of a person’s effort by his rank in the distribution \( G'_{\varphi} \). Rank sterilizes out of the distribution aspects of it which reflect circumstances. Thus, for example, if we view years of education chosen as effort, and persons in two different types both rank at the 80th centile of the distributions of education in their respective types, we will declare that they have expended equal degrees of effort (although their actual years of education may be quite different).

We may thus define the function

\[
v'(\pi, \varphi) = u'((G'_{\varphi})^{-1}(\pi), \varphi), \quad (1)
\]
which is the (average) value of the objective, when the policy is \( \phi \), of the individuals at the \( \pi^{th} \) quantile of the distribution of effort of their type. If effort is unidimensional, the function \( v' \) is well-defined. If \( e \) is multi-dimensional, then in general it is not, and we should then replace vectors of effort with, for example, the linear combination of its components which best explains the value of the objective. For practical purposes, however, in many applications, one need never measure effort: one can simply define the values \( v'(\pi, \phi) \) directly as the level of the objective in type \( t \) at the \( \pi^{th} \) quantile of the distribution of the objective in that type. Implicitly, this approach assumes that effort is declared to be that constellation of choices that enhance the value of the objective.

In figure 1, we see the distribution function of post-fisc income of three types of men in Austria, in 2005, where the unique circumstance defining type is the level of education of the individual’s more educated parent. The yellow graph is the distribution function of those men whose parent had at least some tertiary education; the red graph, of those men whose parent had 12 years of education, and the blue graph, of those men whose parent had less than 12 years of education.

Figure 1. CDFs of income for three types of Austrian male workers
The inverses of the functions in the graph are the functions \( v'(\pi) \). So to see the graphs of the three functions \( v' \), simply reflect figure 1 over the vertical axis and then rotate it 90 degrees clockwise.

Holding persons responsible for their effort means that if two individuals in the same type (who are exposed to identical policy treatments by hypothesis) sustain different values of the objective, there is no inequity, because by hypothesis, this is due to differential effort, something for which they are responsible. However, differences between the functions \( v' \) are ethically undesirable — a reflection of unequal opportunities — because individuals are not responsible for their type/circumstances. Therefore, the goal of policy should be so render the functions \( \{v'\} \) as similar as possible. Since we identify individuals at the same rank of their distributions as having expended equal degrees of effort, the goal is to render the distribution functions (in Figure 1) as close together as they can be (that is, to minimize the horizontal distance between the functions).

But we do not want equality of distribution functions at a low level: we therefore want some kind of maxi-minimization. Suppose we fixed a particular value of \( \pi \); inequality in the numbers \( \{v'(\pi, \varphi) | t \in T\} \) is due not to differential effort but to differential circumstances. Thus, we should choose the policy to

\[
\max_{\varphi \in \Phi} \min_{t \in T} v'(\pi, \varphi).
\]

But we are concerned with every level of effort: a reasonable way of addressing all effort levels is to take the average of the numbers being maximized in (2), that is, to choose policy to

\[
\max_{\varphi \in \Phi} \frac{1}{t} \int_0^1 \min_{t \in T} v'(\pi, \varphi) d\pi.
\]

I call the solution to (3) the equal opportunity policy. It must be emphasized that this policy is conditional upon the definition of circumstances, and the choice of policy space.

Define \( W^{EO}(\varphi) = \frac{1}{t} \int_0^1 \min_{t \in T} v'(\pi, \varphi) d\pi \). \( W^{EO} \) defines an ordering on \( \Phi \): that is we say that:
\[ \varphi \succeq_{EO} \varphi' \iff W^{EO}(\varphi) \geq W^{EO}(\varphi') \]

In words, \( W^{EO}(\varphi) \) is the average value of the lower envelope of the objective functions, across types.

There is a special case of interest. Typically, the policy space \( \Phi \) is fairly small – it may contain only a small, finite number of possible policy choices. In this case, it may well be that there is one type – denote it type 1 -- that for all policies \( \varphi \in \Phi \) is unambiguously the most disadvantaged one, in the sense that its distribution function is dominated (first-order stochastically) by the others at every policy. This is the case in figure 1, where the distribution functions are stacked unambiguously – it is obvious that the most disadvantaged type is the one of men whose more educated parent had less than twelve years of education, as its income distribution function is FOSD by the distributions of the other two types. In this case, the left-hand envelope of the distribution functions is simply the distribution function of a single type, and we have:

\[
W^{EO}(\varphi) = \int_{0}^{1} v_{1}(\pi, \varphi) d\pi = \bar{v}_{1}(\varphi) \quad (4)
\]

where \( \bar{v}_{1}(\varphi) \) is the average value of the objective\(^1\) in the most disadvantaged type under policy \( \varphi \). In this case, the equal-opportunity ethic directs us to choose the policy to maximize the average value of the objective in the most disadvantaged type – assuming that this type is unambiguously the most disadvantaged, for any feasible policy.

It is worthwhile contrasting the equal-opportunity ethic with one of its main competitors, the utilitarian ethic. Denote by \( f^{t} \) the fraction of the population in type \( t \). The utilitarian policy maximizes the ordering given by:

\[
W^{U}(\varphi) = \sum_{t=1}^{T} f^{t} \int_{0}^{1} v^{t}(\pi, \varphi) d\pi \quad (5)
\]

i.e., the average value of the objective in the population.

A third ordering, associated with John Rawls, is the ordering which maximizes the minimum value of the objective in the population; I will write:

\(^1\) Recall that the area above a distribution function and bounded by the line at ordinate value one is the mean of the distribution.
We see that the equal-opportunity ethic lies ‘between’ utilitarianism and the Rawlsian difference principle; it is less extreme than the Rawlsian formulation, in that it maximizes an average of minima across effort levels. Actually, my naming of (6) as the Rawlsian view is not quite fair, for Rawls wrote that the difference principle should apply to ‘social groups,’ not individuals. If we take the different types to be the relevant social groups, then, at least in the special case where (4) holds, the equal-opportunity ethic maximizes the minimum objective value over social groups, and hence possesses a Rawlsian ancestry.

In the general case, however, if the distribution functions cross, the solution of (3) does not entail maximizing the average value of the most disadvantaged type, but rather, maximizing the area above the left-hand envelope of the distribution functions of the types.

To summarize, we have provided an ordering of policies with respect to the equal-opportunity ethic. That ordering takes as data a particular social view of personal responsibility, summarized in a set of circumstances and an implied typology – a partition of the population – and a policy space. The objective for which opportunities are to be equalized is typically some measurable and interpersonally comparable kind of advantage, the kind of thing a ministry in a government might be concerned with, such as income, health, life expectancy, or educational achievement. For this reason – and this should be emphasized— effort is conceived of as those choices which increase the value of the objective, which is often not the case when we think of effort, as in conventional decision theory, as a source of disutility to the decision maker.

2. Economic development

I believe economic development should be measured by the extent to which a society has achieved a desirable distribution of advantage. Desirability should include considerations of both efficiency and justice or fairness. Indeed, the most common measure of economic development, GDP per capita, is based upon the utilitarian ethic. Taking the objective to be income, GDP per capita is exactly the utilitarian’s measure of the justness of the society in question.
Even the human development index (HDI) is utilitarian, but here the objective is a linear combination of a person’s income (or consumption), degree of literacy, and longevity. The human development index is just an average of this objective over the whole society. So the HDI does not part from conventional practice in changing the ethic, but rather in changing the argument of the objective, from consisting of just income, to consisting of a linear combination of income and other kinds of advantage. To be precise, this description is valid for the HDI as defined up to the 2009 Human Development Report of the UNDP. But in 2010, the Human Development Report introduced the ‘inequality adjusted human development index,’ which is not a utilitarian measure.

Utilitarianism -- in either of the two forms I have just mentioned -- is insensitive to the distinction between circumstances and effort. It is what is called a welfarist measure, in that the data it requires to assess the desirability of a particular allocation are simply the values of the objective for members of the population. The equal-opportunity view, however, focuses upon the distinction between circumstances and effort. Thus, to assess the desirability of an allocation, it requires not only the data just mentioned, but also the type of each individual. It is therefore a non-welfarist measure. Utilitarianism condemns inequalities if their elimination would increase total welfare (however it is measured); opportunity egalitarianism condemns them to the extent they are due to circumstances beyond the control of the individuals concerned. The views are quite orthogonal.

I believe that opportunity egalitarianism is not only a superior ethic to utilitarianism, it is the one implicitly endorsed by members of many societies. Suppose you ask the proverbial man on the street, “Do you think that the inequality between the rich Mr. A and poor Ms. B is unjustified?,” it is unlikely that he will answer, “Only if a redistribution from A to B would increase their total welfare.” But he might well answer, “It depends upon how hard they each tried.” In other words, my claim is that popular views of justice are not welfarist, they are based upon notions of desert, and desert is based upon measurements of effort\(^2\). Our man on the street must know more than the

\(^2\)In a welfarist view, diagnoses of justice or injustice can be made knowing only the distribution of welfare. In the equal-opportunity view, one must also have additional
aggregate distribution of the objective to assess whether that distribution is fair – he must know the (disaggregated) distributions of the objective by type.

We cannot maintain that the most common measures of economic development are value free: they are derived from a utilitarian ethic. To this claim one might object that the measure of GDP per capita has nothing to do with utilitarianism, it is simply a proxy for technological development. But I respond that economists are not interested in technological prowess per se: we are interested in human welfare. We would not consider a society highly developed which possessed a fine technology run by slaves, whose product all went, but for the slaves’ subsistence, to the prince. So an attempt to justify the GDP per capita measure of development as a value-free measure of technological accomplishment has the undesirable consequence of obliterating the distinction between economics and engineering – namely, that economics must always focus upon human welfare.

Therefore, we should use the best conception of justice or social welfare to measure economic development. I am not sure that the extent to which opportunities have been equalized is the best such conception, but I do advocate it over utilitarian measures. I therefore believe it is better to measure the level of economic development by some statistic which reflects equality of opportunity than by a utilitarian measure.

But what version of equality of opportunity should we use? What should be the circumstances and the objective, if we want a measure that can be used to evaluate economic development globally? To be most similar to GDP per capita, the objective should be income – I would choose post-fisc income including the per capita value of public goods. I would start with circumstances that include the educational level and occupations of the parents of the individuals, and the ethnicity and gender of the individual\(^3\). One could also construct an opportunity measure to compare to the (pre-2010) human development index. In this case, one would take as the objective the same linear combination of income, literacy, and longevity (or health status) as the HDI uses for the nation, but measure it for individuals. Either of these choices will give individual information: what the circumstances of individuals are, which determines the partition into types.

\(^3\) How one treats gender depends upon whether one uses households or the individual as the unit.
values $\{y'(\pi)\}$ as the indices of welfare for individuals, indexed by type and degree of effort. One then can compute the level of human development measure as the average level of $y$ of the most disadvantaged type, reflecting the opportunity-egalitarian, rather than the utilitarian ethic as the current HDI does.

3. **The degree of opportunity equality**

I have proposed to measure the level of development of a society as the value of the equal-opportunity social welfare function. Of course, we are highly restricted in our ability to measure economic development when we must use a single number to represent it. In particular, applying the measure defined in (4) to income does not allow us to distinguish the wealth of the society from the degree to which the society has succeeded in eliminating injustice – that being the influence of circumstances upon inequality. To do this, I will propose a second measure, which I call the *degree of opportunity equality*.

A society will have achieved equality of opportunity to the extent that the contribution of differential circumstances to total inequality in the distribution of the objective is small. Let the distribution function of the objective in a given society be $H$, and the distribution functions of the objectives in the types be $H';$ then

$$H = \sum f'H'. \quad (7)$$

Suppose we measure inequality in a distribution by the coefficient of variation squared ($CV^2$), that is:

$$C(H) = \frac{\text{var} H}{\mu^2}. \quad (8)$$

where the mean of $H$ is $\mu$. Denote the mean of $H'$ by $\mu'$. Define the distribution:

$$\Phi^T(x) = \sum_{i=0}^{k} f'i \text{ on the interval } \mu_i \leq x \leq \mu_{i+1}, \quad (9)$$

where $k = 0, ..., n$ and $\mu_0 = 0$ and $\mu_n = \infty$. Clearly the mean of $\Phi^T$ is $\mu$. If $\Phi^T$ were the actual distribution of the objective in society, then everybody in a given type would have exactly the same value of the objective, equal to the mean of the objective in that type. (The distribution function $\Phi^T$ is a step function with the same mean as $H$. ) Were this the case, then the contribution of effort to inequality would be nil, as no variation of
accomplishment would exist within any type. Now it is well-known (see the appendix to this paper) that we can decompose $C(H)$ as follows:

$$C(H) = C(\Phi^T) + \sum f'(\rho')^2 C(H') ,$$  \hspace{1cm} (10)

where $\rho' = \frac{\mu'}{\mu}$. Since both contributions in this decomposition are positive, it is natural to interpret $C(\Phi^T)$ as a lower bound of the amount of inequality due to circumstances, and $\sum f'(\rho')^2 C(H')$ as an upper bound on the amount of inequality due to effort. I therefore propose, as a measure of an upper bound on the degree of opportunity equalization, the index:

$$\eta^{UP} = 1 - \frac{C(\Phi^T)}{C(H)} .$$  \hspace{1cm} (11)

The reason that the measure $\eta^{UP}$ is only an upper bound on the fraction of inequality due to effort is that circumstances continue to influence the second term in the decomposition (10).

The next step is to compute a lower bound on the fraction of inequality due to effort. This is done by constructing a ‘dual’ decomposition to (10). Recall our notation – that $v'(\pi)$ is the value of the objective (under the status-quo policy) at the $\pi^{th}$ quantile of the objective distribution in type $t$. Now construct the distribution functions $E^x$ defined as follows (I illustrate the procedure assuming there are exactly three types):

$$\forall \pi \in [0,1] \quad E^\pi(y) = \begin{cases} 
0, & \text{if } 0 \leq y \leq v'(\pi) \\
 f_1, & \text{if } v'(\pi) \leq y < v^2(\pi) \\
f_1 + f_2, & \text{if } v^2(\pi) \leq y < v^3(\pi) \\
1, & \text{if } y \geq v^3(\pi) 
\end{cases}$$

Next, construct the distribution function $E$ as follows:

$$E(y) = \pi \text{ if } \sum f_iv'(\pi) = y.$$  

The distribution $E$ is the counterfactual distribution under which everyone who expended effort degree $\pi$ had the same value of the objective. The distribution $E^x$ is the
distribution of the objective in the \( \pi^{th} \) tranche of the population. (I use ‘type’ to denote the set of individuals with the same circumstances, and ‘tranche’ to denote the set of individuals who expend the same degree of effort.)

Now the application of formula (10) to the society partitioned into tranches is:

\[
C(F) = C(E) + \int_{0}^{1} (\rho^\pi)^2 C(E^\pi) d\pi \tag{12},
\]

where \( \rho^\pi = \sum f_i y_i^\pi(\pi) / \mu \). By exactly the same reasoning as before, we can interpret

\[
\eta^{lo} = \frac{C(E)}{C(F)}
\]

as a lower bound on the fraction of total inequality due to effort.

I therefore propose as our estimate of the degree of inequality due to effort the average of the lower and upper bounds, that is:

\[
\eta = \frac{1}{2} (1 - \frac{C(\Phi^T)}{C(F)} + \frac{C(E)}{C(F)}) \tag{12}.
\]

My suggestion is that we measure economic development by the ordered pair \( d = (W^{EO}, \eta) \). \( W^{EO} \) replaces GDP per capita: it is the average income across all effort levels of those who, at those effort levels, belong to the disadvantaged type\(^4\). In what follows, I compute these pairs for a set of European countries.

For the social-choice theorist, note that neither component of \( d \) is a welfarist measure. One cannot recover either \( W^{EO} \) or \( \eta \) from knowledge of the distribution of income (more generally, the objective) alone. One must know, as well, the circumstances of individuals, which capture the concept of responsibility salient for the society in question.

4. **Country calculations of the level and degree of development**

\(^4\) Or, more generally, as I explained above, it is the average value of the objective of those in the population who comprise the left-hand envelope of the type distributions of the objective.
The data upon which these calculations are based are taken from EU-SILC 2005. The sample which we use consists of male workers, who are partitioned into three types, based upon the maximum of the worker’s parents’ educational levels:

**Type 1:** the worker’s more educated parent had at most lower secondary education

**Type 2:** the worker’s more educated parent had at least upper secondary education but not tertiary education

**Type 3:** the worker’s more educated parent had at least some tertiary education.

The net income for each respondent is recorded, which includes earnings, self-employment income, after taxes and transfers. The single characteristic of type in these calculations is parental education.

The fact that income does not include the value of public goods is a weakness of the measure. If a country has a high rate of taxation, and a substantial fraction of tax revenues finance public goods (as opposed to transfer payments), this will not be reflected in the income data. Transfer payments are included in the definition of income.

In figure 1, I presented the income-distribution functions for Austria, by type, which is in many ways typical. Since the left-hand envelope of the three CDFs is, for all practical purposes, the CDF of type 1, the level of development is simply the mean of type 1’s income. For Austria, the level and degree of development, as defined in the previous section, are:

\[(W^{EO}, \eta) = (20975, 0.945).\]

(Incomes are measured in Euros.) It may surprise the reader that only about 5% of income inequality is attributed to circumstances, but this is quite typical for advanced European countries. For Latin American countries, this number will be considerably

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5 I am grateful to Daniele Checchi and Francesco Scervini for providing me with the data set. For an exact description of the data set, see Checchi, Peragine, and Scerlenga (2010). The computation of the degrees of development and the type-distributions of income were performed by the author using Mathematica, and the I will supply the code upon request.
larger. This suggests that other circumstances should be included in judging
development of advanced countries: more on this below.

In Figure 2, I present the income CDFs of the three types for one of the least
developed countries in
sample, Hungary.

![Figure 2: Type distributions of income in Hungary](image)

We see that the inter-type dispersion is considerably more dramatic than in Austria. The level and degree of development for Hungary are:

$$(W^{EQ}, \eta) = (3257, 0.775).$$

In Hungary, parental education explains 23% of total income inequality. The graphs of the three CDFs for the other countries in the sample are presented in the appendix.
Figure 3  Ordered pairs \((W_j^{EO}, \eta_j)\) for European countries

In figure 3, I plot the ordered pairs \((W_j^{EO}, \eta_j)\) for all 22 countries in the sample\(^6\).

Some comments:

1. The eastern European countries are the worst off with respect to the index \(W_j^{EO}\): these comprise Lithuania (LT), Estonia (EE), the Czech Republic (CZ), Poland (PL), Latvia (LV), and Hungary (HU). (Slovenia (SI) does somewhat better.) But Spain (ES) is also very low on this measure. However, w.r.t. to the degree of development, \(\eta\), the eastern European countries span a wide range from about 75\% to 94\%.

2. Greece appears to do very well on the degree of development: I do not believe the data.

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\(^6\) EU-SILC also contains data for Cyprus, but there are so few observations that I do not consider the CDFs to be meaningful. I excluded as well Ireland from the sample, because I believe the data have been miscoded: according to the data, the middle type in Ireland is worse off than the most disadvantaged type.
3. We may define a partial order with respect to development; a country \( j \) dominates a country \( k \) if 

\[
(W_j^{EO}, \eta_j) > (W_k^{EO}, \eta_k).
\]

With regard to this partial order, no country in the sample dominates all others. Thus, we can say \textit{there exists no most developed European country}. Conversely, however, there are two countries that are undominated by any other: Denmark (DK) and Iceland (IS). These data are from 2005, and doubtless Iceland, post-crash, no longer enjoys this status.

We see that Denmark, Finland (FI), the Netherlands (NL), Sweden (SE), Austria (AT), and France (FR) dominate many other countries. Although Germany (DE), the UK, Luxembourg (LU) and Belgium (BE) do very well on the first component, they fare relatively poorly on the degree of inequality due to effort.

Table 1 presents the upper and lower bounds \( \eta^{up}, \eta^{lo} \) for the set of countries.
I now turn to another question. One may well ask: Does the partial ordering of countries with respect to the ordered pairs \((W^E_k, \eta_k)\) add anything to the ordering of countries given by GDP per capita? One way to ask this question more precisely is as follows. Let us create a complete ordering of countries, based on the opportunity ordered pair, using an aggregation of the form \((W^{EO}_k)^\theta \eta^{1-\theta}\), for some value \(\theta \in [0,1]\). The advantage of using this Cobb-Douglas aggregation is that the ordering of countries induced will be independent of scale, that is, of the currency unit used to measure income. (Thus the fact that our income measure is in the thousands and our ‘degree’
measure is in the interval $[0,1]$ will not affect the Cobb-Douglas rankings.) We ask: what is the rank correlation of country order induced by various choices of $\theta \in [0,1]$, with the country order induced by GDP per capita? I calculated this for the European data set, using mean income in the country samples as the proxy for GDP per capita: it turns out that for $\theta \geq 0.3$, the rank correlation is always over 0.95! The rankings given by the two measures are very similar. This occurs because of two features of this sample of countries: first, the values of $\eta_k$ are all larger than 0.75 and second, the ranking countries by $W_k^{EO}$ is virtually identical to their ranking by mean income. So I cannot say that there is significant value added in measuring development for highly developed countries by the measure I am proposing, versus using the traditional measure. This might appear to take the wind out of the sails of the equal-opportunity approach as a measure of development.

To study the usefulness of my proposed measure, we must apply it to a large set of countries, including ones at many levels of development. If economic development measure by the $\theta$—weighted opportunity indices $(W_k^{EO})^\theta \eta_k^{1-\theta}$ turns out to be essentially the same as economic development measured by GDP per capita, that would be an important observation. My conjecture is that there will be significant value added by using the two-dimensional measure I have proposed. Testing the conjecture requires having comparable data sets to EU-SILC for a large set of countries.

5. The degree of opportunity equalization in Sweden

It is perhaps surprising that in the panel of European countries examined here, the circumstance of parental education accounts for a relatively small fraction of total inequality. One would like to include other circumstances, but in cross-sectional data sets, this is not easy to do so. In Sweden, however, there is a longitudinal data set, comprising observations on about one-third of Swedish males, which contains sufficiently detailed information to enable a much finer typology to be constructed. Bjorklund, Jantti, and Roemer (in press), hereafter BJR, do so; they include as the set of circumstances parental education (3 levels), parental income (4 levels), the IQ of the individual during adolescence (3 levels), number of siblings (3 levels), body mass index
during adolescence (4 levels), and family structure (2 levels), yielding a total of 1152 types. Call the listed characteristics the *direct* circumstances, while the *indirect* circumstance is the effect of the direct circumstances on the distribution of ‘effort’ in a type.

BJR decompose the overall level of income inequality (here, it is total market income before taxes) into the effects of each of the six direct circumstances, the indirect circumstance, and effort. This is done using the Shapley value, as proposed by Shorrocks (1999). Take, for example, the Gini coefficient as the measurement of inequality. One computes the inequality that a given ‘coalition’ of circumstances produces by seeing what the Gini would have been had the whole sample had a common value (the average) for all the circumstances not in the ‘coalition.’ This defines a cooperative game among coalitions of circumstances, whose Shapley value gives the contribution of each circumstance, and effort, to the total Gini.

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<td>67.4</td>
</tr>
</tbody>
</table>

Table 2. Shapley-value decomposition of inequality in Sweden (BJR)

Table 2 reports the decomposition of four measures of inequality into the contributions of the six direct circumstances, the indirect effect of these circumstances on the distribution
of effort, and the effort, taken to be the error term sterilized of the indirect effect.
(GE(0) and GE(1) are two measures from the generalized entropy class, and CV$^2$ is the square of the coefficient of variation.) We see there is quite a variation in the degree of inequality attributed to circumstances, as we vary the inequality measure – from almost 33% for CV$^2$ to only 11.3% with GE(0). To compare the results here with the ones reported earlier, where only parental education was taken as a circumstance, we should use CV$^2$. Doing so, we see that the much finer typology adopted for the Swedish data in BJR increases the degree to which circumstances account for inequality from 1.6% to 32.6%, showing the importance of including a larger set of circumstances. Note, however, that the Swedish income data upon which table 1 is based report market income – pre-fisc. Were the effect of the fisc taken into account, the share of inequality due to circumstances would in all likelihood be reduced. (The income data of EU-SILC include transfer payments in income.)

From table 2, we see that three circumstances stand out as important: parental income, IQ, and the effect of the direct circumstances upon effort (the indirect effect). It is noteworthy that parental education is of very little importance once parental income is included. These results underscore the importance of taking account of the effect of a person’s circumstances on her effort, and in particular, not holding the person responsible for this effect in measuring inequality of opportunity. Importantly, as a special case, it is clear that more disadvantaged types will almost always acquire less education than more advantaged ones, and the amount of education a person acquires is, at the most micro level, a choice. The fact that that choice is heavily influenced by circumstances must be taken into account in measuring the effort expended by the individual – which is why I have proposed to measure the degree of a person’s effort by her rank in the effort (our outcome) distribution of her type, rather than by an absolute measure (like years of education acquired).

6. Equity ‘versus’ development

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7 A number of the points in this section were made my review of the 2006 World Development Report, Roemer (2006).
It is often said that equity and efficiency are competing goals, that equity is purchased at the expense of efficiency. There are two senses in which this phrase is uttered. The first is that redistributive taxation may be purchased only at the cost of Pareto inefficiency, due to workers’ and firms’ facing different effective wages. The second sense is that redistribution may lower total output. These two claims are in principle independent. There may be policies which re-allocate income in a more equitable manner, lower total output, but are not Pareto inefficient. (Think, for example, of re-allocating educational funds from tertiary education to secondary education in a poor country. This might have a purely redistributive effect, without significant consequences for Pareto efficiency.)

I wish to criticize the second usage of the phrase. Saying that there may be a trade-off between equity and efficiency where efficiency is measured as total output is equivalent to saying there is a trade-off between equity and the utilitarian measure of development, which (in its simplest form) is given by output per person. To show I am not erecting a straw man, consider the following quotations from the otherwise fine report of the World Bank report WDR 2006, entitled *Equity and Development*. In these quotations, equity and development are counter-posed:

Greater equity is thus doubly good for poverty reduction: through potential beneficial effects on aggregate long-run development and through greater opportunities for poorer groups within any society (p.2)

If the opportunities faced by children like N. are so much more limited than those faced by children like P. or S., and if this hurts development progress in the aggregate, then public action has a legitimate role in seeking to broaden opportunities…(p.3)

Third, the dichotomy between policies for growth and policies specifically aimed at equity is false (p.10)

In the first quotation, saying that equity is ‘doubly good,’ in that it is good for the poor and also good for long-run development, only makes sense if one assumes that equity and
long-run development are different goals. In my view, long-run development means approaching equity – that is, equality of opportunity. I submit that the authors of this sentence had in mind GNP per capita as the measure of long-run development, and so what is being said is that equalizing opportunities will increase GDP per capita. This is peculiar in a report that is devoted to advocating the view that economic development requires the achievement of equal opportunity. In the second quotation, the assumption is that redressing the inequality of opportunity among the children is justifiable because that inequality hurts development: but my view is that that inequality comprises underdevelopment, and so the sentence is tautological. I submit that the authors have a utilitarian concept in mind as the measure of economic development. Finally, the third quotation would likewise be a tautology for me: but in the context, the authors are saying that policies which increase equality of opportunity also lead to an increase in total income. (That is, the third quotation is offered as an empirical claim, while for me, it is a tautology as a theoretical statement.) Again, there is an ambivalence in the conceptualization of economic development: does it mean equalizing opportunities, or increasing per capita output?

It will often be the case that policies that redress inequality of opportunity will also increase total output, because improving opportunities for the disadvantaged releases talents that were, before, unused. But this need not be the case, and I maintain, our justification for redressing inequality of opportunity should not depend on its being the case. There may be groups in society that are so disadvantaged that it is very costly to compensate them: the return in output per funds invested may be small. Equity may be advanced only by shifting investment from uses where it generates high output to ones where it generates lower output. (This may be so, particularly in the short-run.) But if this is the case, it does not mean that the policy in question should not be undertaken, nor does it mean that development is thereby reduced if it is.

The ambivalence in Equity and Development is a reflection of the competing conceptions of justice represented by utilitarianism and opportunity-equalization.

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8 To say that development 'requires' equalizing opportunities is weaker than saying that it is synonymous with equalizing opportunities: I have been advocating the latter position in this paper.
Utilitarianism has a strong hold on economists; it is the virtually ubiquitous ethic used in macroeconomics. This is a hold-over from an earlier period where utilitarianism was the only game in town – let us say, the first half of the 20th century. Economists and mathematicians developed optimization techniques (think of the Bellman equation) which are suited to solving problems where utilities are added up across persons, but not to solving problems where the minimum is maximized. And so it is comfortable to work with utilitarian formulations. I submit, however, that this is a bad habit that we need not continue practicing.

If my view of economic development is adopted, there may be a significant change in policy evaluation. One would not have to justify investment in very disadvantaged social groups by showing that such investment increased total output. As I indicated, in the long run, such a conflict might not exist: but often, policy makers must evaluate the consequences of their policy choices in the short run. If a country is evaluated on the basis of its ordered-pair statistic \((W^{EO}, \eta)\) rather than on GDP per capita, policies could be quite different.

6. Varia

In this section, I will discuss briefly several special points.

a. The treatment of children

In some cases, the objective that we are interested in involves an accomplishment of children – for example, proficiency in academic achievement at a given age, or number of years of school attended. Should we attempt to decompose the inequality in the distribution of these objectives into circumstances and effort?

The issue is this: At what age should children be held responsible for their choices? Again, we can attempt to take an age for which there is a social consensus. Suppose we say that age is 15 years. Then any differences which exist in the value of the objective among children up to age 15 should be treated as due entirely to circumstances. In particular, if we are measuring inequality in wages at age 30, we should include measures of accomplishment at age 15 as a circumstance.

We do not have to settle the nature-nurture debate here: indeed, both nature or nurture are circumstances for which the child should not be held responsible.
b. Are people responsible for their preferences?

There is a long tradition in economics that takes preferences as given, and implicitly, holds individuals responsible for them. But this view does not lie behind the present analysis. The differential choices individuals make, that lead to differential effort and accomplishment, would be traditionally modeled as due to their having different preferences: but as I have said, we should not hold persons entirely responsible for those choices – we should attempt to sterilize those choices of the impact of circumstances upon them, in measuring the extent to which individuals are responsible for them. This practice may strike some as paternalistic, and perhaps it is.

This issue is somewhat contentious. In particular, the philosopher and legal scholar Ronald Dworkin, who did much to introduce the issue of responsibility into egalitarian political philosophy, takes the position that if a person identifies with her preferences, then, even if those preferences were formed under conditions of disadvantage, she should be held responsible for the consequences of choices that those preferences induce (see Dworkin (1981a, 1981b)). Dworkin’s defense of this move is that not to do so would be insulting to the person’s concept of herself – since she identifies, by hypothesis, with these preferences.

But I agree with G.A. Cohen (1989) that Dworkin’s view is unappealing. Consider a person who grows up in a community where very few acquire higher education, and indeed, preferences have evolved in this community to disvalue higher education, perhaps viewing it as an unmanly thing to acquire. Our person has these preferences himself, and chooses not to work hard in school, having no intention of acquiring higher education. Would it be wrong for the state to undertake a policy of exposing children in this community to the benefits of higher education? Would it be wrong to offer financial incentives to tip the scales, causing a fraction of such children to choose higher education? Perhaps – it could be argued – that children in this community are suffering from lack of information, and hence such policies would not be paternalistic. But surely, to some extent at least, the choice not to acquire higher education is based upon preferences: it is not fully explained by lack of information, and therefore, honesty requires us to say that such policies are to a degree paternalistic. They are also, however, correct policies in my view, and claiming that they show disrespect for
individuals on the grounds that they attempt to circumvent choices that people would make using preferences with which they identify is, I say, misguided. It is a liberal fallacy, based upon not fully taking on board the endogeneity of preferences.

c. Meritocracy versus equal opportunity

A meritocracy is a society in which persons are assigned to occupations, or are allocated scarce educational resources, on the basis of attributes that they possess which are relevant for success in pursuing these occupations or education. It is contrasted with non-meritocratic assignment, in which irrelevant attributes are use to allocate positions – such as the race or tribe one belongs to, or the social connections one’s parent has. But meritocracy is also in conflict with equality of opportunity: for an equal-opportunity policy may assign resources to persons in order to redress disadvantage, even if those resources might be more effectively transformed into social output by more advantaged individuals.

Affirmative action policies, for example, may be non-meritocratic but opportunity-enhancing. This immediately raises the question: How far should equal-opportunity policies go in attempting to compensate persons for disadvantages for which they are not responsible, when these policies come into conflict with assigning resources according to merit – where merit is defined in the traditional sense?

The debates over affirmative action policy in the United States are instructive for thinking about this question. These debates occurred with regard to two different loci of affirmative action: universities and jobs. The nature of the critique of affirmative-action policy was quite different for these loci. With respect to university admissions, the critique was that race is not a good measure of disadvantage, and that instead, the socio-economic circumstances of (the parents of the) individuals should be taken as the appropriate measure. Consequently, the large public universities (California, Texas, Florida) amended their affirmative action policies to be predicated upon measures of socio-economic disadvantage, rather than race. But the *principle* of affirmative action was not -- at least by many of the most vocal critics – challenged. However, with respect to jobs and assignment of contracts, the critique was against the principle – rather, jobs and contracts should be assigned according to merit, the critics said.
This distinction is important. Justice requires that we take account of fairness to those who are competing for social positions – university slots or occupations— but it requires as well that we take account of the welfare of the entire society, and therefore of the effect of assigning social positions to those who may not be as efficient in producing outputs that society in general consumes. What the debate in the US indicates is that many people are quite willing to undertake compensatory policies when the formation of skills is at issue, but not when the direct provision of outputs that others consume is at issue. For example, we might be willing to admit a certain percentage of high-effort but disadvantaged students to medical schools, but not be willing to give handicaps to disadvantaged medical-school graduates who are attempting to pass the board certification exam to become surgeons.

To take a somewhat absurd example, consider the case of professional basketball teams, which discriminate in hiring against players who are short – short stature is, let us assume, something beyond the individual’s control. We would find it ridiculous to require professional basketball teams to recruit a certain percentage of short players. This is because, I maintain, there are only a few basketball players, but there are millions of fans who derive pleasure from consuming their output, basketball games, and, let’s assume for the purposes of argument that the game would be less interesting if a fraction of players were required to be short. Here, those who consume the output are much more numerous than those who produce it, and so merit becomes the right principle for assigning positions. Note, however, that we might well outlaw discriminating against the recruitment of black players; in this case, the discrimination goes against the merit principle, while in the case of short players, it does not. Indeed, when the discrimination against certain attributes does not go against the merit principle, we do not call its exercise ‘discrimination.’

Now the anti-egalitarian will argue that we should not apply affirmative action even in the case of admission to universities and medical schools: after all, if we admit some disadvantaged students to medical schools, where disadvantage does conflict with merit, we will be reducing the quality of the pool of those who compete at the surgery boards and will be forced to lower the bar for board certification in order to recruit the necessary number of surgeons. So, the anti-egalitarian will argue, the equal-opportunity
policies will come home to roost in eventually lowering the quality of consumable goods (in this example, appendectomies) for the society as a whole.

I do not have a fully worked out view on the proper scope of opportunity-enhancing policies. But my rule of thumb is taken from the American discussion: I advocate equality of opportunity policies where they involve the education and training of people for social positions, but the merit principle in assigning persons to occupations.

d. Is IQ special?

A person’s IQ is a consequence of both nature and nurture. As such, it is a circumstance in the sense that a person cannot be reasonably held responsible for her IQ, at least in adolescence. Nevertheless, compensating persons for low IQ in an equal-opportunity policy can come into conflict with the merit principle in an obvious way. The same considerations therefore apply to engaging in such policies that were brought to bear in the discussion in point c.

That is: society may decide not to count IQ as a circumstance when admitting students to universities, because the conflict with merit is too great. On the other hand, it may at the same time decide to allocate additional resources to the elementary education of low-IQ children. Indeed, all advanced countries do so for the extreme case of children who are considered to be mentally handicapped. This is again a case where the proximity to a person’s occupying an occupational position in society is taken into account in deciding the extent to which equal-opportunity is followed.

In my advocacy of a new statistic for economic development, I did not propose that IQ be a circumstance of which account is taken in measuring the extent to which a society is developed. Perhaps, at some point in the future, it will be appropriate to include IQ as a circumstance in this measure. But at present, the grossest form of unequal opportunity, in most countries, is due to the effect of social background on a person’s life chances. To the first order, I believe that we should reorient development policy to be sensitive to the inequalities that are caused by socio-economic disadvantage, but that the inclusion of IQ as a circumstance in its own right would be more contentious.

7. A World Bank proposal for measuring equal opportunity
The World Bank has been an important innovator in bringing considerations of equal opportunity into economic development. Its two important publications, to date, have been the 2006 World Development Report, *Equity and Development*, and a monograph, *Measuring inequality of opportunities in Latin America and the Caribbean* (Paes de Barros et al., 2009). The more recent publication contains a wealth of information on the effects of social circumstances on various measures of achievement and output. Indeed, it should be possible to compute the development indices \((W^{EO}, \eta)\) that I have advocated from the data base used for this report for the Latin American countries.

Paes de Barros et al. (2009) proposes a measure of equality of opportunity. Consider a particular kind of opportunity, such as ‘attending the sixth grade in elementary school.’ Let the total sixth-grade attendance in a country be \(H\), and the total number of children of sixth-grade age be \(N\), and define \(\bar{p} = \frac{H}{N}\) to be the access on average of children to the opportunity of a sixth-grade education. \(\bar{p}\) measures the level of this opportunity in the country, but not the extent to which access is unequal to different children, based upon their social circumstances. Now using a logit model, estimate the probability that each child, \(j\), in the country has of attending the sixth grade, where that probability is a function of a vector of circumstances; denote this estimated probability by \(\hat{p}_j\). Define \(D = \frac{1}{2 \bar{p} N} \sum |\hat{p}_j - \bar{p}|\). \(D\) measures the variation in access to the opportunity in question across children in the country. The normalization guarantees that \(0 \leq D \leq 1\). Now define the human opportunity index as

\[
O = \bar{p}(1 - D);
\]

note that \(0 \leq O \leq \bar{p}\).

The human opportunity index is a non-welfarist measure of development, because the probabilities \(\hat{p}_j\) can only be computed knowing the circumstances of the children. The measure combines a concern with the level of provision of opportunities and the inequality of the distribution of them. This is to be contrasted with my ordered pair
(\(W^{EO}, \eta\)) \), which separates these two concerns into two measures. Obviously, some information is lost in using a single measure rather than two measures.

The concern of the 2009 report is in large part with children. As I have said, where children are concerned, all inequality should be counted as due to circumstances, and none to effort, and so the fact that the human opportunity index does not explicitly make the distinction between effort and circumstances is unobjectionable. However, if the measure is used for addressing inequality of opportunity for adults, this may be a defect.

To study this, let us take an opportunity for adults – earning an income above \(M\), measured in PPP exchange rates. Suppose there are three types of worker, according to the level of education of their more educated parent. Denote the distribution of income in type \(t\) as \(F^t\); let the fraction of type \(t\) be \(f^t\) and let \(F\) be the distribution of income in the society as a whole. Then \(\bar{p} = 1 - F(M)\) is the average access to the opportunity in question in the country. Now for all members \(j\) of a given type, \(t\), we will compute that \(\hat{p}_j = 1 - F^t(M)\): this is because the probabilities \(\hat{p}_j\) are computed by taking the independent variables in the logit regression as the circumstances. Hence, the human opportunity measure is:

\[
O = \bar{p} \left( 1 - \frac{1}{2\bar{p}} \sum f^t |1 - F^t(M) - (1 - F(M))| \right) = (1 - F(M)) - \frac{1}{2} \sum f^t |F(M) - F^t(M)|
\]

(13)

Despite the fact that effort is not explicitly mentioned in defining the index, effort is reflected in measure, because the distributions \(F^t\) appear in the calculation. Indeed, the first term \(1 - F(M)\) measures the level of opportunity in the country, while the second term is a penalty for the degree to which this opportunity is mal-distributed with respect to circumstances (e.g., if there were no inequality of opportunity, then \(F^t(M) = F(M)\) for all \(t\), and the penalty is zero).

In expression (13), the first term on the right-hand side, \(1 - F(M)\), plays the role that \(W^{EO}\) plays in my measure: it measures the level of development. But while \(W^{EO}\) focuses upon how well off the most disadvantaged type is doing, \(1 - F(M)\) is a level for the society at large. The second component of my measure, \(\eta\), is explicitly derived to
show the degree to which inequality is due to circumstances, while the second term on the right-hand side of (13) is a form of a variance. Certainly these two measures are getting at the same phenomenon. I have a slight preference for my proposal, as it is more carefully justified as measuring what we are concerned with. But these are minor criticisms; certainly, the measure \( O \) is in the spirit of thinking of economic development as opportunity equalization.

8. **Conclusion**

Inequality has become a more important focus in development economics in recent years, and this is an important step forward, from the days when only GDP per capita was considered to be salient as far as development was concerned. But an important weakness in the entry of inequality into the field has been treating all inequality as having the same ethical status. This is reflected in the very large literature on how to measure inequality in the most salient way; in this literature, the focus is upon whether the statistical properties of various inequality measures conform to our intuitions concerning when equality is *large* or *small*. These discussions ignore the issue of whether inequality is *innocuous* or *undesirable* – that is, the ethical status of the inequality. The equal-opportunity literature introduced this distinction in economic theory, and it built on the introduction of the issue of responsibility into egalitarian political philosophy, through the writings of Dworkin (1981a,b), G.A. Cohen (1989) and Richard Arneson (1989). For a discussion of this literature, from an economist’s viewpoint, see Roemer (1996) and the recent treatment of Fleurbaey (2010).

It is useful to compare the equal-opportunity approach to inequality with the approach represented by the human development index, based upon the work of Amartya Sen, on functionings and capability. As is well known, Sen’s (1980) major point, made initially in 1980, was that there are objective measures of human functioning that important for any conception of welfare, and the set of vectors of functionings, available to a person, which Sen defined as her *capability*, is a measure of the opportunities that she has. Sen’s intervention was post-Rawls and pre-Dworkin: his main foil was Rawl’s choice of primary goods as the equalisandum, which he proposed to replace by capabilities; and his conception of responsibility was implicit in the idea that, if
capabilities, so defined, were ‘equal’ (whatever that should mean) across persons, then if individuals chose different vectors of functioning from these sets, that was of no ethical consequence. The treatment of responsibility, in Dworkin (1981,1982), was significantly more explicit, and led to the equal-opportunity literature.

Indeed, the human development index, at least until the most recent publication of the Human Development Report, was a ‘utilitarian’ measure: it was an aggregate index for a country, computed as an average of three functionings across the population—literacy/education, life expectancy/health, and income/consumption. The ‘freedom’ or ‘responsibility’ part of Sen’s proposal was not reflected in the index.

The proposal I have advanced in this paper, and the human development index (HDI), are complementary. The HDI broadens the objective of concern from income (GDP) to a set of functionings, but continues to average over the population as a whole. The equal-opportunity approach—as I have advocated applying it to a set countries—retains income as the objective, but disaggregates the population into types based upon circumstances which are beyond the control of individuals. In other words, the HDI approach says that human accomplishment along dimensions other than income is important, and the equal-opportunity approach says that inequality is bad, but only inequality of a certain kind.

Of course, it is possible to unite the two approaches. Instead of using income as the measure in my proposal, one could measure the human development index disaggregated by types, where type continues to be defined according to a set of circumstances, and then my two-dimensional index would allow us to assess levels and degrees of development with regard to the various Sen-inspired functionings. Indeed, it would be ideal to have data sets that permitted us to do this. The reason I proposed using only income in this paper is that I think, at this point, we do not have the data to compute levels and degrees of human development by type for a large set of countries.

Note that the issue of the ethical status of inequality is quite different from another way that inequality can be good or bad, and that is, with regard to incentives.

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9 In the most recent publication of the Human Development Report, a measure of inequality in human development within a country is introduced: but that inequality is not parsed into that due to circumstances and that due to effort.
Bad inequality in this sense – inequality that is bad for incentives – will be condemned by the utilitarian measure of GDP per capita, because its elimination will increase social output. This is to be distinguished from inequality that is bad because it reflects disadvantage due to circumstances: as I have emphasized, eliminating this kind of inequality is not -- at least in the short run – synonymous with increasing total output. Whether inequality is good or bad for incentives is not the same thing as whether it is ethically good or bad. This is not to say that the two kinds of diagnosis are orthogonal: sometimes, inequalities are bad for incentives because they reflect social discrimination. But the two kinds of diagnosis of bad inequality are different.

The equal-opportunity approach, which focuses upon eliminating inequalities that are due to circumstances for which persons should not be held responsible, is both good ethics, and also good policy – by which I mean it is policy supported by the great majority of people in many countries. For we know from survey data that, globally, people believe injustice occurs when low incomes are due to bad luck as opposed to low effort. What differs across countries is the extent to which citizens attribute low incomes to bad luck, as opposed to low effort: in Brazil, a much larger fraction believe poverty is due to bad luck than in the United States (and perhaps this reflects reality). Indeed, the popular moniker associated with equality of opportunity – it levels the playing field -- can be interpreted as a way of saying that disadvantages that some face due to circumstances beyond their control should be eliminated before the competition for economic goods begins.

References


Appendix 1. Decomposing inequality into inequality of opportunity and within-type inequality

Let $F$ be a differentiable distribution function, and let it be decomposed into the distribution functions of $n$ types,

$$F = \sum f_i F_i$$  \hspace{1cm} (A.1)

where the $F_i$ are differentiable, and the frequency of type $i$ in the pop. is $f_i$. Denote the means of these distributions by $\mu$ and $\mu_i$ respectively.

Let $\text{var} (F)$ be the variance of a distribution $F$. We take as our measure of inequality the square of the coefficient of variation, that is:

$$C(F) = \frac{\text{var} F}{\mu^2}.$$  \hspace{1cm} (A.2)

Let $\rho_i \equiv \frac{\mu_i}{\mu}$.

Given a typology, we define the discrete distribution which has $n$ mass points, which are the means of the $n$ sub-populations $F_i$; the frequencies of these mass points are $f_i$. To be precise, if we denote by $T$ the given partition of the society into these $n$ types, then this discrete distribution function is given by:

$$\Phi^T (x) = \sum_{j=0}^{k} f_j \text{ on the interval } \mu_k \leq x < \mu_{k+1},$$  \hspace{1cm} (A.3)

where we define $f_0 = \mu_0 = 0$ and $\mu_{n+1} = \infty$. Clearly the mean of $\Phi^T$ is $\mu$.

Full equality of opportunity holds, for this typology, if and only if $C(\Phi^T) = 0$.

More generally, the size of $C(\Phi^T)$ indicates the degree of inequality of opportunity as it is a measure of inequality in the distribution of means of the $\{F_i\}$.

The strongest condition for equality of opportunity is that the distributions $F_i$ are identical. A weaker form, implied by the strong form, is that the means $\mu_i$ are identical – call this weaker condition EOP1. We now offer a decomposition of the inequality in the society as a whole, $C(F)$, into its parts.
Proposition

A. \( C(F) = \sum f_i \rho_i^2 C(F_i) + C(\Phi^T) \)

B. If \( EOp1 \) holds then \( C(F) = \sum f_i C(F_i) \).

C. Define \( I = \{ i \mid \mu_i \leq \mu \} \), \( I' = N \setminus I \). Suppose that \( \max_{i \in I} C(F_i) \leq \min_{i \in I'} C(F_i) \). Then

\[ C(F) = \sum f_i C(F_i) \Rightarrow EOp1. \]

D. If \( C(F_i) = C(F) \) for all \( i \), then \( EOp1 \) holds.

Proof:

1. We compute that

\[ C(\Phi^T) = \sum f_i \frac{\mu_i^2 - 2 \mu_i \mu + \mu^2}{\mu^2} = \sum f_i \rho_i^2 - 1. \]

2. Denote \( \varphi = F' = \sum f_i F_i' = \sum f_i \varphi_i \). Compute that

\[
\text{var} F = \int (x - \mu)^2 \sum f_i \varphi_i(x) dx = \\
\sum f_i \int ((x - \mu_i) + (\mu_i - \mu))^2 \varphi_i(x) dx = \\
\sum f_i (x - \mu)^2 \varphi_i(x) + \sum f_i (\mu_i - \mu)^2 + 0.
\]

Therefore:

\[
\frac{\text{var} F}{\mu^2} = \sum f_i \frac{\text{var} F_i}{\mu_i^2} + \sum f_i (\rho_i - 1)^2.
\]

But note that \( \sum f_i \rho_i = 1 \). Hence the above equation reduces to

\[ C(F) = \sum f_i \rho_i^2 C(F_i) + \sum f_i \rho_i^2 - 1 = \sum f_i \rho_i^2 C(F_i) + C(\Phi^T), \]

where the last step uses step 1 of this proof. This proves part A.

3. If \( \mu_i = \mu \) for all \( i \) then \( \rho_i = 1 \) and \( C(\Phi^T) = 0 \), and the formula of part A immediately gives us part B of the proposition.

4. If \( C(F) = C(F_i) \), all \( i \), then the formula of part A says that:

\[ C(F) = C(F) \sum f_i \rho_i^2 + C(\Phi) = C(F)(C(\Phi) + 1) + C(\Phi) \]

which implies that

\[ 0 = C(\Phi)(C(F) + 1). \]
Since the second term in this product is positive, we must have \( C(\Phi) = 0 \), and so EOp1 holds. This proves part D.

5. Finally, assume the premise of part C, and let \( r \) be a number such that

\[
i \in I, j \in I' \implies C(F_i) \leq r \leq C(F_j).
\]

Suppose the claim were false. Then \( C(\Phi) > 0 \) and so, from part A, we have

\[
\sum f_i \rho_i^2 C(F_i) < C(F) = \sum f_i C(F_i),
\]

or

\[
\sum f_i C(F_i) (\rho_i^2 - 1) < 0. \tag{A.4}
\]

But

\[
\sum f_i r (\rho_i^2 - 1) = r C(\Phi) > 0. \tag{A.5}
\]

Inequalities (A.4) and (A.5) contradict each other, because the negative terms in (A.4) have less weight than they do in (A.5), and the positive terms in (A.4) have more weight than they do in (A.5). So if the inequality (A.5) is positive, a fortiori, the inequality in (A.4) must be positive. The contradiction proves part C. □

The formula in part A can be viewed as decomposing inequality in the society into that part due to inequality of opportunity (of degree 1), namely \( C(\Phi) \), and residual inequality, due to within-distribution inequality of the various types. Parts B and C show that there is a close association between EOp1 and linearity of the functional \( C(\cdot) \).

EOp1 is a sufficient condition for that linearity, and if the functional is linear for a decomposition, and another condition holds, then EOp1 follows. Part D is initially surprising. Why not take two distributions with the same coefficient of variation but different means and combine them into one society? The reason this example does not contradict part D is that, although we will have \( C(F_i) = C(F_j) \), we will not have \( C(F_i) = C(F_j) = C(F) \). In particular, part D says that if its premise holds, then we have both EOp1 and EOp2.

Note that all the terms in the formula of part A are non-negative. So this is a real decomposition. We can thus view the ratio \( \eta(T) = \frac{C(\Phi^T)}{C(F)} \) as the extent to which inequality in society is due to inequality of opportunity.
Appendix 2. Country graphs of income distributions by parental-education typology

(see following pages)
Spain: Blue=1, Red=2, Yellow=3

Finland: Blue=1, Red=2, Yellow=3
France: Blue = 1, Red = 2, Yellow = 3

Greece: Blue = 1, Red = 2, Yellow = 3
Iceland: Blue = 1, Red = 2, Yellow = 3

Italy: Blue = 1, Red = 2, Yellow = 3
Latvia: Blue = 1, Red = 2, Yellow = 3

Netherlands: Blue = 1, Red = 2, Yellow = 3
United Kingdom: Blue=1, Red=2, Yellow=3