Three meanings of intergenerational mobility: a follow-up with an application to educational mobility in Mexico

Gastón Yalonetzky

Leeds University Business School, OPHI

June 2012
Table of contents

Introduction

Methodology

Empirical application: Educational mobility in Mexico
  Data
  Methods
  Results

Concluding remarks
Introduction: three meanings of intergenerational mobility
Introduction: three meanings of intergenerational mobility

- In 2001 van de Gaer, Schokkaert and Martinez published their seminal article "Three meanings of intergenerational mobility".
Introduction: three meanings of intergenerational mobility

- In 2001 van de Gaer, Schokkaert and Martinez published their seminal article "Three meanings of intergenerational mobility".
- The article explains, and clarifies, three meanings of mobility captured by indices based on quantile transition matrices.
Introduction: three meanings of intergenerational mobility

- In 2001 van de Gaer, Schokkaert and Martinez published their seminal article "Three meanings of intergenerational mobility".
- The article explains, and clarifies, three meanings of mobility captured by indices based on quantile transition matrices.
- They also showed incompatibilities between some mobility axioms, including axioms of meaning.
Introduction: three meanings of intergenerational mobility

- In 2001 van de Gaer, Schokkaert and Martinez published their seminal article "Three meanings of intergenerational mobility".
- The article explains, and clarifies, three meanings of mobility captured by indices based on quantile transition matrices.
- They also showed incompatibilities between some mobility axioms, including axioms of meaning.
- They also proposed ways of measuring mobility as opportunity, and mobility as equalization of life chances.
Introduction: three meanings of intergenerational mobility

- In 2001 van de Gaer, Schokkaert and Martinez published their seminal article "Three meanings of intergenerational mobility".
- The article explains, and clarifies, three meanings of mobility captured by indices based on quantile transition matrices.
- They also showed incompatibilities between some mobility axioms, including axioms of meaning.
- They also proposed ways of measuring mobility as opportunity, and mobility as equalization of life chances.
Introduction: three meanings of intergenerational mobility

The three meanings are:

1. Mobility as movement: the reduction in the likelihood that the offspring replicates the wellbeing achievements of parents.
2. Mobility as equality of opportunity: more proximity between the conditional cumulative distributions of the offspring variables, i.e. lower opportunity dominance (Fleurbaey, 2008).
3. Mobility as equalization of life chances: more proximity between the conditional probability distributions of the offspring variables, i.e. the categories of the variable do not have relative desirability.
Introduction: three meanings of intergenerational mobility

The three meanings are:

1. Mobility as *movement*: the reduction in the likelihood that the offspring replicates the wellbeing achievements of parents.
Introduction: three meanings of intergenerational mobility

The three meanings are:

1. Mobility as *movement*: the reduction in the likelihood that the offspring replicates the wellbeing achievements of parents.

2. Mobility as *equality of opportunity*: more proximity between the conditional cumulative distributions of the offspring variables, i.e. lower *opportunity dominance* (Fleurbaey, 2008).
Introduction: three meanings of intergenerational mobility

The three meanings are:

1. Mobility as *movement*: the reduction in the likelihood that the offspring replicates the wellbeing achievements of parents.

2. Mobility as *equality of opportunity*: more proximity between the conditional cumulative distributions of the offspring variables, i.e. lower *opportunity dominance* (Fleurbaey, 2008).

3. Mobility as *equalization of life chances*: more proximity between the conditional probability distributions of the offspring variables, i.e. the categories of the variable do not have relative desirability.
Introduction: This paper’s methodological contributions

- A discussion of situations in which the incompatibilities detected by van de Gaer et al. (2001) can be surmounted.
Introduction: This paper’s methodological contributions

- A discussion of situations in which the incompatibilities detected by van de Gaer et al. (2001) can be surmounted.
- The proposal of new indices of mobility as equality of opportunity, based on Silber and Yalonetzky (2011) and suitable for ordinal variables and size matrices.
Introduction: This paper’s methodological contributions

- A discussion of situations in which the incompatibilities detected by van de Gaer et al. (2001) can be surmounted.
- The proposal of new indices of mobility as equality of opportunity, based on Silber and Yalonetzky (2011) and suitable for ordinal variables and size matrices.
- The proposal of new indices of mobility as equalization of life chances, based on Reardon and Firebaugh (2002) and suitable for ordinal variables and size matrices.
Introduction: This paper’s empirical contributions

- Study of intergenerational mobility of education in Mexico using the ESRU-EMOVI 2011 dataset.
Introduction: This paper’s empirical contributions

- Study of intergenerational mobility of education in Mexico using the ESRU-EMOVI 2011 dataset.
- First question: how different are the transition matrices by gender (i.e. father-son versus mother-daughter), by cohorts.
Introduction: This paper’s empirical contributions

- Study of intergenerational mobility of education in Mexico using the ESRU-EMOVI 2011 dataset.
- First question: how different are the transition matrices by gender (i.e. father-son versus mother-daughter), by cohorts.
- Second question: documenting mobility trends across cohorts using indices that capture different meanings of mobility (including the new proposals).
Introduction: The menu for the rest of the presentation

- I will introduce the axioms of van de Gaer et al. (2001) and their incompatibility theorems.
Introduction: The menu for the rest of the presentation

- I will introduce the axioms of van de Gaer et al. (2001) and their incompatibility theorems. These will be re-interpreted for size matrices.
Introduction: The menu for the rest of the presentation

- I will introduce the axioms of van de Gaer et al. (2001) and their incompatibility theorems. These will be re-interpreted for size matrices.
- Then I will briefly discuss the situations under which the incompatibilities can be surmounted.
Introduction: The menu for the rest of the presentation

- I will introduce the axioms of van de Gaer et al. (2001) and their incompatibility theorems. These will be re-interpreted for size matrices.
- Then I will briefly discuss the situations under which the incompatibilities can be surmounted.
- Then I present the new indices of mobility as equality of opportunity.
Introduction: The menu for the rest of the presentation

- I will introduce the axioms of van de Gaer et al. (2001) and their incompatibility theorems. These will be re-interpreted for size matrices.
- Then I will briefly discuss the situations under which the incompatibilities can be surmounted.
- Then I present the new indices of mobility as equality of opportunity.
- Then I present the new indices of mobility as equalization of life chances.
Introduction: The menu for the rest of the presentation

- I will introduce the axioms of van de Gaer et al. (2001) and their incompatibility theorems. These will be re-interpreted for size matrices.
- Then I will briefly discuss the situations under which the incompatibilities can be surmounted.
- Then I present the new indices of mobility as equality of opportunity.
- Then I present the new indices of mobility as equalization of life chances.
- Then the empirical application to educational mobility in Mexico.
Methodology

The axioms of meaning: Mobility as movement (MOV)

\[ M(B) > M(A) \]
The axioms of meaning: Mobility as equality of opportunity (EOP)

\[ \mathcal{M}(B) > \mathcal{M}(A) \]

\[ \mathcal{F}_{i|1}^B \geq \mathcal{F}_{i|3}^B \quad \forall i \in [1,4] \]

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.7</td>
<td>0.2</td>
</tr>
<tr>
<td>0.9</td>
<td>0.7</td>
</tr>
<tr>
<td>1</td>
<td>0.9</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>0.7-\varepsilon</td>
<td>0.2</td>
</tr>
<tr>
<td>0.9-\varepsilon</td>
<td>0.7</td>
</tr>
<tr>
<td>1</td>
<td>0.9</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>
The axioms of meaning: Mobility as equalization of life chances (ELC)

\[ M(B) > M(A) \]

\[ \tilde{p}_{1|2}^B \geq \tilde{p}_{1|4}^B \land \tilde{p}_{4|2}^B \leq \tilde{p}_{4|4}^B \]

<table>
<thead>
<tr>
<th>A</th>
<th>0.7</th>
<th>0.2</th>
<th>0.1</th>
<th>0</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.2</td>
<td>0.5</td>
<td>0.2</td>
<td>0.1</td>
<td></td>
</tr>
<tr>
<td>0.1</td>
<td>0.2</td>
<td>0.5</td>
<td>0.2</td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>0.1</td>
<td>0.2</td>
<td>0.7</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>B</th>
<th>0.7</th>
<th>0.2 - \varepsilon</th>
<th>0.1</th>
<th>0 + \varepsilon</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.2</td>
<td>0.5</td>
<td>0.2</td>
<td>0.1</td>
<td></td>
</tr>
<tr>
<td>0.1</td>
<td>0.2</td>
<td>0.5</td>
<td>0.2</td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>0.1 + \varepsilon</td>
<td>0.2</td>
<td>0.7 - \varepsilon</td>
<td></td>
</tr>
</tbody>
</table>
Methodology

The axioms of permutation: Anonymity (AN)

\[ \mathcal{M}(B) = \mathcal{M}(A) \]

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.7</td>
<td>0.2</td>
</tr>
<tr>
<td>0.2</td>
<td>0.5</td>
</tr>
<tr>
<td>0.1</td>
<td>0.2</td>
</tr>
<tr>
<td>0</td>
<td>0.1</td>
</tr>
</tbody>
</table>

\[ \begin{array}{cccc}
0 & 0.2 & 0.1 & 0.7 \\
0.1 & 0.5 & 0.2 & 0.2 \\
0.2 & 0.2 & 0.5 & 0.1 \\
0.7 & 0.1 & 0.2 & 0.0 \\
\end{array} \]
The axioms of permutation: Focus on Probability (FP)

\[ M(B) = M(A) \]

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.7</td>
<td>0.2</td>
<td>0.1</td>
</tr>
<tr>
<td>0.2</td>
<td>0.5</td>
<td>0.2</td>
</tr>
<tr>
<td>0.1</td>
<td>0.2</td>
<td>0.5</td>
</tr>
<tr>
<td>0</td>
<td>0.1</td>
<td>0.2</td>
</tr>
<tr>
<td>0</td>
<td>0.1</td>
<td>0.2</td>
</tr>
<tr>
<td>0.7</td>
<td>0.2</td>
<td>0.1</td>
</tr>
<tr>
<td>0.1</td>
<td>0.2</td>
<td>0.5</td>
</tr>
<tr>
<td>0</td>
<td>0.1</td>
<td>0.2</td>
</tr>
</tbody>
</table>
The axioms of minimum mobility: Immobility (IM)
\[ M(B) \leq M(A) \]

<table>
<thead>
<tr>
<th>A</th>
<th>B=I</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.7</td>
<td>0</td>
</tr>
<tr>
<td>0.2</td>
<td>0.5</td>
</tr>
<tr>
<td>0.1</td>
<td>0.2</td>
</tr>
<tr>
<td>0</td>
<td>0.1</td>
</tr>
</tbody>
</table>
The axioms of minimum mobility: Perfect Predictability (PP)

\[ M(B) = M(A) \]
The axioms of maximum mobility: Perfect mobility (PM) 
\( M(B) > M(A) \)

<p>| | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>0.7</td>
<td>0.2</td>
<td>0.1</td>
<td>0</td>
</tr>
<tr>
<td>0.2</td>
<td>0.5</td>
<td>0.2</td>
<td>0.1</td>
</tr>
<tr>
<td>0.1</td>
<td>0.2</td>
<td>0.5</td>
<td>0.2</td>
</tr>
<tr>
<td>0</td>
<td>0.1</td>
<td>0.2</td>
<td>0.7</td>
</tr>
</tbody>
</table>

<p>| | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>P1</td>
<td>P1</td>
<td>P1</td>
<td>P1</td>
</tr>
<tr>
<td>P2</td>
<td>P2</td>
<td>P2</td>
<td>P2</td>
</tr>
<tr>
<td>P3</td>
<td>P3</td>
<td>P3</td>
<td>P3</td>
</tr>
<tr>
<td>P4</td>
<td>P4</td>
<td>P4</td>
<td>P4</td>
</tr>
</tbody>
</table>
The axioms of maximum mobility: Perfect unpredictability (PU)

\[ M(B) > M(A) \]

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.7</td>
<td>0.2</td>
<td>0.1</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>0.2</td>
<td>0.5</td>
<td>0.2</td>
<td>0.1</td>
</tr>
<tr>
<td></td>
<td>0.1</td>
<td>0.2</td>
<td>0.5</td>
<td>0.2</td>
</tr>
<tr>
<td></td>
<td>0</td>
<td>0.1</td>
<td>0.2</td>
<td>0.7</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>B</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.25</td>
<td>0.25</td>
<td>0.25</td>
<td>0.25</td>
</tr>
<tr>
<td></td>
<td>0.25</td>
<td>0.25</td>
<td>0.25</td>
<td>0.25</td>
</tr>
<tr>
<td></td>
<td>0.25</td>
<td>0.25</td>
<td>0.25</td>
<td>0.25</td>
</tr>
<tr>
<td></td>
<td>0.25</td>
<td>0.25</td>
<td>0.25</td>
<td>0.25</td>
</tr>
</tbody>
</table>
Incompatibility theorems by van de Gaer et al. (2001)

1. MOV and PM are incompatible.
Incompatibility theorems by van de Gaer et al. (2001)

1. MOV and PM are incompatible.
2. MOV and AN are incompatible.
Incompatibility theorems by van de Gaer et al. (2001)

1. MOV and PM are incompatible.
2. MOV and AN are incompatible.
3. MOV and PP are incompatible (corollary).
Incompatibility theorems by van de Gaer et al. (2001)

1. MOV and PM are incompatible.
2. MOV and AN are incompatible.
3. MOV and PP are incompatible (corollary).
4. MOV and FP are incompatible.
Incompatibility theorems by van de Gaer et al. (2001)

1. MOV and PM are incompatible.
2. MOV and AN are incompatible.
3. MOV and PP are incompatible (corollary).
4. MOV and FP are incompatible.
5. EOP and FP are incompatible.
A matrix with quasi-maximal diagonal is characterized by:

\[ \mu_j p_{j|j} \geq \mu_i p_{i|j}; \mu_j, \mu_i > 0 \forall j, i, \text{ which implies: } p_{j|j} > 0 \forall j \in [1, E_{\text{top}}]. \]
Situations in which incompatibilities are surmounted: matrices with quasi-maximal diagonals

A matrix with quasi-maximal diagonal is characterized by:
\[ \mu_j p_{j\mid j} \geq \mu_i p_{i\mid j}; \mu_j, \mu_i > 0 \forall j, i, \text{ which implies: } p_{j\mid j} > 0 \forall j \in [1, E_{top}]. \]

- In matrices with quasi-maximal diagonals theorem 1 (on MOV and PM) does not hold. (Shorrocks, 1978).
A matrix with quasi-maximal diagonal is characterized by:
\[ \mu_j \rho_{j\mid j} \geq \mu_i \rho_{i\mid j}; \mu_j, \mu_i > 0 \forall j, i, \] which implies:
\[ p_{j\mid j} > 0 \forall j \in [1, E_{top}]. \]

- In matrices with quasi-maximal diagonals theorem 1 (on MOV and PM) does not hold. (Shorrocks, 1978).
- Likewise, with those matrices, corollary 1 (on MOV and PP) does not hold (because permutations of columns of the identity matrix are not allowed under the quasi-maximal diagonality restriction).
Situations in which incompatibilities are surmounted: monotone matrices

A monotone matrix is characterized by:

\[ F_{i|j} \geq F_{i|j+1} \quad \forall i \in [1, E_{top}] \land j \in [1, E_{top} - 1]. \]
Situations in which incompatibilities are surmounted: monotone matrices

- In monotone matrices, all theorems (and corollary) are overcome.
Situations in which incompatibilities are surmounted: monotone matrices

- In monotone matrices, all theorems (and corollary) are overcome.
- The (restricted) domain of monotone matrices forbids any permutation of columns or rows yielding a non-monotone matrix, whereas AN, PP and FP contemplate any permutation.
Situations in which incompatibilities are surmounted: monotone matrices

- In monotone matrices, all theorems (and corollary) are overcome.
- The (restricted) domain of monotone matrices forbids any permutation of columns or rows yielding a non-monotone matrix, whereas AN, PP and FP contemplate any permutation.
- When matrices are monotone:
Situations in which incompatibilities are surmounted: monotone matrices

- In monotone matrices, all theorems (and corollary) are overcome.
- The (restricted) domain of monotone matrices forbids any permutation of columns or rows yielding a non-monotone matrix, whereas AN, PP and FP contemplate any permutation.
- When matrices are monotone:
  1. Minimum mobility is attained only in the case of an identity matrix.
Situations in which incompatibilities are surmounted: monotone matrices

- In monotone matrices, all theorems (and corollary) are overcome.
- The (restricted) domain of monotone matrices forbids any permutation of columns or rows yielding a non-monotone matrix, whereas AN, PP and FP contemplate any permutation.
- When matrices are monotone:
  1. Minimum mobility is attained only in the case of an identity matrix.
  2. Maximum mobility is attained only when matrices have the identical columns.
Methodology

Situations in which incompatibilities are surmounted: monotone matrices

▶ In monotone matrices, all theorems (and corollary) are overcome.

▶ The (restricted) domain of monotone matrices forbids any permutation of columns or rows yielding a non-monotone matrix, whereas AN, PP and FP contemplate any permutation.

▶ When matrices are monotone:
  1. Minimum mobility is attained only in the case of an identity matrix.
  2. Maximum mobility is attained only when matrices have the identical columns.
  3. Any probability mass transfer generating mobility as movement also generates mobility as equality of opportunity. (But not necessarily the other way around).
New indices of mobility as equality of opportunity

▶ There are not many indices fulfilling EOP for transition matrices.

The family proposed by van de Gaer et al (2001) also uses cardinal information from the variable, which is sensible for continuous variables.

▶ However, in the case of ordinal variables (e.g. educational levels), scaling is arbitrary.

▶ In that context, I propose indices that only use information from a size transition matrix for ordinal variables and fulfill EOP.

The mobility indices are based on the inequality-of-opportunity indices by Silber and Yalonetzky (2011).

▶ The original indices have been re-normalized to be suitable for transition matrices and their fulfillment of mobility axioms (van de Gaer et al. (2001)) is studied.
New indices of mobility as equality of opportunity

- There are not many indices fulfilling EOP for transition matrices.
- The family proposed by van de Gaer et al (2001) also uses cardinal information from the variable, which is sensible for continuous variables.
New indices of mobility as equality of opportunity

- There are not many indices fulfilling EOP for transition matrices.
- The family proposed by van de Gaer et al (2001) also uses cardinal information from the variable, which is sensible for continuous variables.
- However, in the case of ordinal variables (e.g. educational levels), scaling is arbitrary.
New indices of mobility as equality of opportunity

- There are not many indices fulfilling EOP for transition matrices.
- The family proposed by van de Gaer et al (2001) also uses cardinal information from the variable, which is sensible for continuous variables.
- However, in the case of ordinal variables (e.g. educational levels), scaling is arbitrary.
- In that context, I propose indices that only use information from a size transition matrix for ordinal variables and fulfill EOP.
New indices of mobility as equality of opportunity

- There are not many indices fulfilling EOP for transition matrices.
- The family proposed by van de Gaer et al (2001) also uses cardinal information from the variable, which is sensible for continuous variables.
- However, in the case of ordinal variables (e.g. educational levels), scaling is arbitrary.
- In that context, I propose indices that only use information from a size transition matrix for ordinal variables and fulfill EOP.
- The mobility indices are based on the inequality-of-opportunity indices by Silber and Yalonetzky (2011).
Methodology

New indices of mobility as equality of opportunity

- There are not many indices fulfilling EOP for transition matrices.
- The family proposed by van de Gaer et al (2001) also uses cardinal information from the variable, which is sensible for continuous variables.
- However, in the case of ordinal variables (e.g. educational levels), scaling is arbitrary.
- In that context, I propose indices that only use information from a size transition matrix for ordinal variables and fulfill EOP.
- The mobility indices are based on the inequality-of-opportunity indices by Silber and Yalonetzky (2011).
- The original indices have been re-normalized to be suitable for transition matrices and their fulfillment of mobility axioms (van de Gaer et al. (2001)) is studied.
New indices of mobility as equality of opportunity

\[ I_\alpha = \frac{E_{top}^\alpha}{\sum_{k=1}^{E_{top}} [k(E_{top} - k)^\alpha + k^\alpha (E_{top} - k)]} \sum_{j=1}^{E_{top}} \sum_{i=1}^{E_{top}} |F_{i,j} - F_{i,prom}^\alpha| \quad \forall \alpha \geq 1 \]
New indices of mobility as equality of opportunity

\[ I_\alpha = \frac{E_{\text{top}}^\alpha}{\sum_{k=1}^{E_{\text{top}}} [k(E_{\text{top}} - k)^\alpha + k^\alpha(E_{\text{top}} - k) \sum_{j=1}^{E_{\text{top}}} \sum_{i=1}^{E_{\text{top}}} |F_{i|j} - F_i^{\text{prom}}|\alpha \forall \alpha \geq 1} \]

\[ I_1 = \frac{3}{E_{\text{top}}^2 - 1} \sum_{j=1}^{E_{\text{top}}} \sum_{i=1}^{E_{\text{top}}} |F_{i|j} - F_i^{\text{prom}}| \]
New indices of mobility as equality of opportunity

\[ I_\alpha = \frac{E_{\text{top}}^\alpha}{\sum_{k=1}^{E_{\text{top}}} [k(E_{\text{top}} - k)^\alpha + k^{\alpha}(E_{\text{top}} - k)]} \sum_{j=1}^{E_{\text{top}}} \sum_{i=1}^{E_{\text{top}}} |F_{ij} - F_{i}^{\text{prom}}|^{\alpha} \forall \alpha \geq 1 \]

\[ l_1 = \frac{3}{E_{\text{top}}^2 - 1} \sum_{j=1}^{E_{\text{top}}} \sum_{i=1}^{E_{\text{top}}} |F_{ij} - F_{i}^{\text{prom}}| \]

The indices satisfy EOP, but not MOV or ELC. They fulfill AN but not FP. They also satisfy PM, IM and PP.
New indices of mobility as equalization of life chances

- There are not many indices fulfilling ELC for transition matrices either.
New indices of mobility as equalization of life chances

- There are not many indices fulfilling ELC for transition matrices either.
- The family proposed by van de Gaer et al (2001) is most suitable for quantile matrices.
New indices of mobility as equalization of life chances

- There are not many indices fulfilling ELC for transition matrices either.
- The family proposed by van de Gaer et al (2001) is most suitable for quantile matrices.
- Hence, I propose indices that are suitable for a size transition matrix and fulfill ELC.
New indices of mobility as equalization of life chances

- There are not many indices fulfilling ELC for transition matrices either.
- The family proposed by van de Gaer et al (2001) is most suitable for quantile matrices.
- Hence, I propose indices that are suitable for a size transition matrix and fulfill ELC.
- The mobility indices are based on the segregation indices by Reardon and Firebaugh (2002).
New indices of mobility as equalization of life chances

- There are not many indices fulfilling ELC for transition matrices either.
- The family proposed by van de Gaer et al (2001) is most suitable for quantile matrices.
- Hence, I propose indices that are suitable for a size transition matrix and fulfill ELC.
- The mobility indices are based on the segregation indices by Reardon and Firebaugh (2002).
- The original indices have been re-normalized to be suitable for transition matrices and their fulfillment of mobility axioms (van de Gaer et al. (2001)) is studied.
New indices of mobility as equalization of life chances

- There are not many indices fulfilling ELC for transition matrices either.
- The family proposed by van de Gaer et al (2001) is most suitable for quantile matrices.
- Hence, I propose indices that are suitable for a size transition matrix and fulfill ELC.
- The mobility indices are based on the segregation indices by Reardon and Firebaugh (2002).
- The original indices have been re-normalized to be suitable for transition matrices and their fulfillment of mobility axioms (van de Gaer et al. (2001)) is studied.
- Parker and Rougier (2001) also have an index of *mobility as unpredictability*, that fulfills ELC, but its benchmark of maximum mobility (i.e. complete unpredictability) is at odds with PM.
Methodology

New indices of mobility as equalization of life chances

\[ P_C = \frac{1}{E_{top}(E_{top} - 1)} \sum_{j=1}^{E_{top}} \sum_{i=1}^{E_{top}} \left( \frac{p_{i|j} - p_{i}^{prom}}{p_{i}^{prom}} \right)^2 \]
New indices of mobility as equalization of life chances

\[ PC = \frac{1}{E_{top}(E_{top} - 1)} \sum_{j=1}^{E_{top}} \sum_{i=1}^{E_{top}} \frac{(p_{i|j} - p_{i}^{prom})^2}{p_{i}^{prom}} \]

\[ R = \frac{1}{(E_{top}) - 1} \sum_{j=1}^{E_{top}} \sum_{i=1}^{E_{top}} (p_{i|j} - p_{i}^{prom})^2 \]
New indices of mobility as equalization of life chances

\[
PC = \frac{1}{E_{top}(E_{top} - 1)} \sum_{j=1}^{E_{top}} \sum_{i=1}^{E_{top}} \frac{(p_{i|j} - p_i^{prom})^2}{p_i^{prom}}
\]

\[
R = \frac{1}{(E_{top}) - 1} \sum_{j=1}^{E_{top}} \sum_{i=1}^{E_{top}} (p_{i|j} - p_i^{prom})^2
\]

\[
P = \frac{1}{E_{top}} \sum_{j=1}^{E_{top}} \sum_{i=1}^{E_{top}} \frac{(p_{i|j} - p_i^{prom})^2}{1 - p_i^{prom}}
\]
New indices of mobility as equalization of life chances

\[ D = \frac{1}{2E_{top} - 1} \sum_{j=1}^{E_{top}} \sum_{i=1}^{E_{top}} |p_{ij} - p_{i}^{prom}| \]
New indices of mobility as equalization of life chances

\[ D = \frac{1}{2E_{top} - 1} \sum_{j=1}^{E_{top}} \sum_{i=1}^{E_{top}} |p_{i,j} - p_{i,j}^{prom}| \]

The indices satisfy ELC, but not MOV or EOP. They also fulfill AN, FP; PM, IM and PP.
The data and the variables of the illustration

- Study of educational mobility in Mexico using the ESRU-EMOVI 2011 dataset (Centro de Estudios Espinosa Yglesias).
The data and the variables of the illustration

- Study of educational mobility in Mexico using the ESRU-EMOVI 2011 dataset (Centro de Estudios Espinosa Yglesias).
- Education measured in four levels: Less than complete primary, Complete primary, Complete secondary, Complete tertiary.
Empirical application: Educational mobility in Mexico

Data

The data and the variables of the illustration

- Study of educational mobility in Mexico using the ESRU-EMOVI 2011 dataset (Centro de Estudios Espinosa Yglesias).
- Education measured in four levels: Less than complete primary, Complete primary, Complete secondary, Complete tertiary.
- Male (father-son) and female (mother-daughter) matrices compared.
The data and the variables of the illustration

- Study of educational mobility in Mexico using the ESRU-EMOVI 2011 dataset (Centro de Estudios Espinosa Yglesias).
- Education measured in four levels: Less than complete primary, Complete primary, Complete secondary, Complete tertiary.
- Male (father-son) and female (mother-daughter) matrices compared.
- Mobility trends computed for four age cohorts.
The data and the variables of the illustration

<table>
<thead>
<tr>
<th>Cohorts</th>
<th>Men</th>
<th>Women</th>
</tr>
</thead>
<tbody>
<tr>
<td>1: 25-34</td>
<td>3,184</td>
<td>2,118</td>
</tr>
<tr>
<td>2: 35-44</td>
<td>1,178</td>
<td>1,177</td>
</tr>
<tr>
<td>3: 45-54</td>
<td>862</td>
<td>969</td>
</tr>
<tr>
<td>4: 55-64</td>
<td>898</td>
<td>833</td>
</tr>
<tr>
<td>Total</td>
<td>6,122</td>
<td>5,097</td>
</tr>
</tbody>
</table>
Empirical application: Educational mobility in Mexico

Methods

Homogeneity tests for the empirical application

I use the test by Anderson and Goodman (1957), with Ho: \( M(\text{Men}) = M(\text{Women}) \) and Ha: \( M(\text{Men}) \neq M(\text{Women}) \).
Homogeneity tests for the empirical application

I use the test by Anderson and Goodman (1957), with Ho: \( M(\text{Men}) = M(\text{Women}) \) and Ha: \( M(\text{Men}) \neq M(\text{Women}) \). They use the Pearson statistic for contingency tables:

\[ \chi^2 = \sum_{g=1}^{G} \sum_{j=1}^{E_{top}} \sum_{i=1}^{N_{g,j}} \frac{(p_{i|j} - p_{i}^*)^2}{p_{i}^*} \]

The statistic has a limiting distribution of chi-square with \((G-1)E_{top}(E_{top}-1)\) degrees of freedom, under the null.
Homogeneity tests for the empirical application

I use the test by Anderson and Goodman (1957), with Ho: \( M(Men) = M(Women) \) and Ha: \( M(Men) \neq M(Women) \). They use the Pearson statistic for contingency tables:

\[
\chi = \sum_{g=1}^{G} \sum_{j=1}^{E_{top}} \sum_{i=1}^{E_{top}} \frac{N_{g,j} (p_{i|j} - p_{i}^{*})^2}{p_{i}^{*}}
\]
Homogeneity tests for the empirical application

I use the test by Anderson and Goodman (1957), with Ho: $M(\text{Men}) = M(\text{Women})$ and Ha: $M(\text{Men}) \neq M(\text{Women})$. They use the Pearson statistic for contingency tables:

$$
\chi = \sum_{g=1}^{G} \sum_{j=1}^{E_{\text{top}}} \sum_{i=1}^{E_{\text{top}}} \frac{N_{g}^{j}(p_{i|j} - p_{i}^{*})^{2}}{p_{i}^{*}}
$$

The statistic has a limiting distribution of chi-square with $(G - 1)E_{\text{top}}(E_{\text{top}} - 1)$ degrees of freedom, under the null.
Homogeneity tests for the empirical application

To measure the degree of matrix heterogeneity I also compute a Pearson-Cramer index that results from normalizing the Pearson statistic by its maximum:
Homogeneity tests for the empirical application

To measure the degree of matrix heterogeneity I also compute a Pearson-Cramer index that results from normalizing the Pearson statistic by its maximum:

\[ PC = \frac{\chi}{Nmin(G - 1, E_{top} - 1)E_{top}} \]
Empirical application: Educational mobility in Mexico

## Methods

### Mobility indices for the empirical application

<table>
<thead>
<tr>
<th>Index</th>
<th>Axioms fulfilled (those fulfilled under restrictions are in parentheses)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( ST = \frac{E_{top} - \sum_{i=1}^{E_{top}} p_{i</td>
<td>i}}{E_{top} - 1} )</td>
</tr>
<tr>
<td>( E2 = 1 -</td>
<td>\lambda_2</td>
</tr>
<tr>
<td>( B2 = \frac{1}{E_{top}(E_{top} - 1)} \sum_{j=1}^{E_{top}} \sum_{i=1}^{E_{top}} p_{i</td>
<td>j}</td>
</tr>
<tr>
<td>( PR = \frac{1}{E_{top} - 1} \left( \sum_{j=1}^{E_{top}} \sum_{i=1}^{E_{top}} p_{i</td>
<td>j}^2 - 1 \right) )</td>
</tr>
<tr>
<td>( V2 = 1 - \frac{1}{2(E_{top} - 1)} \sum_{j=1}^{E_{top}} \sum_{i=1}^{E_{top}}</td>
<td>p_{i</td>
</tr>
<tr>
<td>( I^1 = \frac{3}{E_{top}^2 - 1} \sum_{j=1}^{E_{top}} \sum_{i=1}^{E_{top}}</td>
<td>F_{i</td>
</tr>
<tr>
<td>( PC = \frac{1}{E_{top}(E_{top} - 1)} \sum_{j=1}^{E_{top}} \sum_{i=1}^{E_{top}} \frac{(p_{i</td>
<td>j} - p_i^{prom})^2}{p_i^{prom}} )</td>
</tr>
<tr>
<td>( R = \frac{1}{(E_{top} - 1)} \sum_{j=1}^{E_{top}} \sum_{i=1}^{E_{top}} (p_{i</td>
<td>j} - p_i^{prom})^2 )</td>
</tr>
</tbody>
</table>
Homogeneity test results

<table>
<thead>
<tr>
<th>Cohort</th>
<th>Pearson statistic</th>
<th>P-value</th>
<th>“Homogeneous” columns at 1% significance</th>
<th>Pearson-Cramer heterogeneity index</th>
</tr>
</thead>
<tbody>
<tr>
<td>1: 25-34</td>
<td>34.50374</td>
<td>0.000562***</td>
<td>2,3,4</td>
<td>0.083303</td>
</tr>
<tr>
<td>2: 35-44</td>
<td>18.62266</td>
<td>0.098051*</td>
<td>2,3,4</td>
<td>0.097266</td>
</tr>
<tr>
<td>3: 45-54</td>
<td>39.41353</td>
<td>8.99E-05***</td>
<td>3,4</td>
<td>0.18656</td>
</tr>
<tr>
<td>4: 55-64</td>
<td>35.74536</td>
<td>0.000356***</td>
<td>2,3,4</td>
<td>0.287895</td>
</tr>
</tbody>
</table>

*Null hypothesis rejected at 10% significance level.
** Null hypothesis rejected at 5% significance level.
*** Null hypothesis rejected at 1% significance level.
Mobility results

[Graphs showing mobility results for Index ST and Index E2, with data points for different age groups (25-34, 35-44, 45-54, 55-64) and gender (Hombres, Mujeres).]
Empirical application: Educational mobility in Mexico

Results

Mobility results

- Graphs showing mobility results for different age groups (25-34, 35-44, 45-54, 55-64) for males (Hombres) and females (Mujeres).
Empirical application: Educational mobility in Mexico

Results

Mobility results

![Graphs showing mobility results for different age groups and gender.](image)
Mobility results
Concluding remarks

- Paper is meant to be a follow-up to the seminal contribution by van de Gaer et al. (2001).

Situations in which incompatibilities between certain axioms can be overcome were discussed. These require restricting the domain of admissible matrices. The paper explores two restrictions: monotone matrices and matrices with quasi-maximal diagonals. The former restriction implies the latter.

New indices of mobility as equality of opportunity based on Silber and Yalonetzky (2011). These are suitable for ordinal variables and size matrices.

New indices of mobility as equalization of life chances based on Reardon and Firebaugh (2002). These are suitable for ordinal variables and size matrices.
Concluding remarks

- Paper is meant to be a follow-up to the seminal contribution by van de Gaer et al. (2001).

- Situations in which incompatibilities between certain axioms can be overcome were discussed. These require restricting the domain of admissible matrices. The paper explores two restrictions: monotone matrices and matrices with quasi-maximal diagonals.
Concluding remarks

- Paper is meant to be a follow-up to the seminal contribution by van de Gaer et al. (2001).

- Situations in which incompatibilities between certain axioms can be overcome were discussed. These require restricting the domain of admissible matrices. The paper explores two restrictions: monotone matrices and matrices with quasi-maximal diagonals. The former restriction implies the latter.
Concluding remarks

- Paper is meant to be a follow-up to the seminal contribution by van de Gaer et al. (2001).

- Situations in which incompatibilities between certain axioms can be overcome were discussed. These require restricting the domain of admissible matrices. The paper explores two restrictions: monotone matrices and matrices with quasi-maximal diagonals. The former restriction implies the latter.

- New indices of mobility as equality of opportunity based on Silber and Yalonetzky (2011).
Concluding remarks

- Paper is meant to be a follow-up to the seminal contribution by van de Gaer et al. (2001).

- Situations in which incompatibilities between certain axioms can be overcome were discussed. These require restricting the domain of admissible matrices. The paper explores two restrictions: monotone matrices and matrices with quasi-maximal diagonals. The former restriction implies the latter.

- New indices of mobility as equality of opportunity based on Silber and Yalonetzky (2011). These are suitable for ordinal variables and their size matrices.
Concluding remarks

- Paper is meant to be a follow-up to the seminal contribution by van de Gaer et al. (2001).

- Situations in which incompatibilities between certain axioms can be overcome were discussed. These require restricting the domain of admissible matrices. The paper explores two restrictions: monotone matrices and matrices with quasi-maximal diagonals. The former restriction implies the latter.

- New indices of mobility as equality of opportunity based on Silber and Yalonetzky (2011). These are suitable for ordinal variables and their size matrices.

- New indices of mobility as equalization of life chances based on Reardon and Firebaugh (2002).
Concluding remarks

- Paper is meant to be a follow-up to the seminal contribution by van de Gaer et al. (2001).

- Situations in which incompatibilities between certain axioms can be overcome were discussed. These require restricting the domain of admissible matrices. The paper explores two restrictions: monotone matrices and matrices with quasi-maximal diagonals. The former restriction implies the latter.

- New indices of mobility as equality of opportunity based on Silber and Yalonetzky (2011). These are suitable for ordinal variables and their size matrices.

- New indices of mobility as equalization of life chances based on Reardon and Firebaugh (2002). These are suitable for ordinal variables and size matrices.
Concluding remarks

Mexico case study:

▶ Increase in the similarity between father-son and mother-daughter matrices among younger cohorts.

▶ Monotonic increase in mobility, in its three meanings, when moving from older to younger cohorts.

▶ More evidence necessary to corroborate these trends.

Previous research (e.g. Binder and Woodruff, 2002) find a decrease in educational mobility in the youngest cohort. But differences in results may differ by variables, indices/methods, samples. It is worth pinpointing the reasons.

▶ Larger sample sizes would enable a finer cohort partition and the ability to control also for parental cohorts and discriminate between cohort and life-cycle effects.
Concluding remarks

Mexico case study:

- Increase in the similarity between father-son and mother-daughter matrices among younger cohorts.

Previous research (e.g. Binder and Woodruff, 2002) find a decrease in educational mobility in the youngest cohort. But differences in results may differ by variables, indices/methods, samples. It is worth pinpointing the reasons.

- Larger sample sizes would enable a finer cohort partition and the ability to control also for parental cohorts and discriminate between cohort and life-cycle effects.
Concluding remarks

Mexico case study:

- Increase in the similarity between father-son and mother-daughter matrices among younger cohorts.
- Monotonic increase in mobility, in its three meanings, when moving from older to younger cohorts.

Previous research (e.g. Binder and Woodruff, 2002) find a decrease in educational mobility in the youngest cohort. But differences in results may differ by variables, indices/methods, samples. It is worth pinpointing the reasons.

Larger sample sizes would enable a finer cohort partition and the ability to control also for parental cohorts and discriminate between cohort and life-cycle effects.
Concluding remarks

Mexico case study:

- Increase in the similarity between father-son and mother-daughter matrices among younger cohorts.
- Monotonic increase in mobility, in its three meanings, when moving from older to younger cohorts.
- More evidence necessary to corroborate these trends.
Concluding remarks

Mexico case study:

- Increase in the similarity between father-son and mother-daughter matrices among younger cohorts.
- Monotonic increase in mobility, in its three meanings, when moving from older to younger cohorts.
- More evidence necessary to corroborate these trends. Previous research (e.g. Binder and Woodruff, 2002) find a decrease in educational mobility in the youngest cohort.

But differences in results may differ by variables, indices/methods, samples. It is worth pinpointing the reasons.

Larger sample sizes would enable a finer cohort partition and the ability to control also for parental cohorts and discriminate between cohort and life-cycle effects.
Concluding remarks

Mexico case study:

- Increase in the similarity between father-son and mother-daughter matrices among younger cohorts.
- Monotonic increase in mobility, in its three meanings, when moving from older to younger cohorts.
- More evidence necessary to corroborate these trends. Previous research (e.g. Binder and Woodruff, 2002) find a decrease in educational mobility in the youngest cohort. But differences in results may differ by variables, indices/methods, samples. It is worth pinpointing the reasons.
Concluding remarks

Mexico case study:

- Increase in the similarity between father-son and mother-daughter matrices among younger cohorts.
- Monotonic increase in mobility, in its three meanings, when moving from older to younger cohorts.
- More evidence necessary to corroborate these trends. Previous research (e.g. Binder and Woodruff, 2002) find a decrease in educational mobility in the youngest cohort. But differences in results may differ by variables, indices/methods, samples. It is worth pinpointing the reasons.
- Larger sample sizes would enable a finer cohort partition and the ability to control also for parental cohorts and discriminate between cohort and life-cycle effects.