Linking the Regions in the International Comparisons Program at Basic Heading Level and at Higher Levels of Aggregation

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Abstract:
Both the 2005 and 2011 rounds of the International Comparisons Program (ICP) are broken up into six regions. Global results are then obtained by linking these regions together. A requirement in both ICP rounds is that the global results at basic heading level and at higher levels of aggregation satisfy within-region fixity (i.e., the relative parities of a pair of countries in the same region are the same in the global comparison as in the within-region comparison). Standard multilateral methods (such as CPD at basic heading level and GEKS above basic heading level) violate this within-region fixity requirement and hence cannot be used to construct the global results. A new method is proposed here that resolves this problem in an optimal way by altering the multilateral price indexes (calculated say using CPD or GEKS) by the minimum least-squares amount necessary to ensure that within-region fixity is satisfied. The method’s underlying rationale is similar to that of the GEKS method. In fact it can be viewed as an extension of GEKS for achieving within-region fixity. The resulting global price indexes are base country-invariant, and treat all countries within each region and all regions symmetrically. The method can be applied equally well at basic heading level or at higher levels of aggregation, and to either the ring country framework used in ICP 2005 or the core product framework that will be used in ICP 2011. (JEL: C43, O53)
1 Introduction

Both ICP 2005 and 2011 are broken up into six separate regional comparisons, each of which has its own list of products for each basic heading.\(^1\) It is then necessary to link the results across regions both at basic heading level and at the aggregate level. In ICP 2005, the regions were linked both at basic heading level and at the aggregate level using an across-region variant proposed by Diewert (2008a) on the country-product-dummy (CPD) method (see Summers 1973, Rao 2004, Diewert 2008b, and Hill and Syed 2010). However, it later emerged that Diewert’s method of linking, as applied at the aggregate level, was not invariant to the choice of within-region base countries (see Sergeev 2009 and Diewert 2010b). Hence, at least at the aggregate level, a new approach for linking the regions will be required in ICP 2011.

An important difference between ICP 2005 and 2011 is that in 2005 to facilitate the linking of the regions an additional so-called ‘ring’ comparison was made between 18 countries drawn from the six regions. This ring comparison had its own product list for each basic heading. In 2011, the ring comparison has been dropped. The regions will be linked through ‘core’ products rather than ring countries. The core products are products that are included in the product lists of every region. Every basic heading will include some core products.

An important complication that arises in the region-linking process is that the regions require that the global results are consistent with the within-region results. In other words, the relative parities of a pair of countries in the same region must be the same in the global comparison as in the within-region comparison. Within-region fixity is required both at basic heading level and at all higher levels of aggregation up to GDP.

In this paper a new and flexible method is proposed for linking the regions while maintaining within-region fixity. The method, which can be applied either at basic heading level or the aggregate level, is optimal in the sense that it alters the multilateral

\(^1\)A basic heading is the lowest level of aggregation at which expenditure weights are available. A basic heading consists of a group of similar products defined within a general product classification. Food and non-alcoholic beverages account for 29 headings, alcoholic beverages, tobacco and narcotics for 5 headings, clothing and footwear for 5 headings, etc. (see Blades 2007).
price indexes (e.g., CPD at basic heading level or GEKS at the aggregate level) by the
minimum least-squares amount necessary to ensure that within-region fixity is satisfied.\(^2\)
The method’s underlying rationale is similar to that of GEKS, which by comparison
alters Fisher price indexes by the minimum least-squares amount necessary to ensure
transitivity. Also, like GEKS, the method takes geometric means of a number of chained
comparisons between a pair of reference countries. In both these senses, therefore, the
method can be viewed as an extended version of GEKS that achieves within-region fixity.
The resulting global price indexes are base country-invariant, and treat all countries
within each region and all regions symmetrically. The method can be applied equally
well at basic heading level or at the aggregate level, and to either the ring country
framework used in ICP 2005 or the core product framework that will be used in ICP
2011.

2 A GEKS-Type Method for Linking the Regions
at Basic-Heading Level while Retaining Within-
Region Fixity

2.1 Linking at Basic Heading Level Through Core Products

The regions in the comparison are indexed here by \(A, B, C,\) etc. There are \(N_A\) countries
in region \(A, N_B\) in region \(B, N_C\) in region \(C,\) etc.

In ICP 2011, the products within each basic heading are divided into core and
noncore groups. The core products are priced by all regions. The noncore products are
region specific.

\(P_{Aa}^{\text{region}}\) denotes a within-region \(A\) price index for country \(Aa,\) with country \(A1\) as
the base. The ‘region’ superscript denotes the fact that the price index is calculated
over the full product list of region \(A\) (i.e., both core and noncore). Similarly, \(P_{Bb}^{\text{region}}\)
denotes a within-region \(B\) price index for country \(Bb\) with country \(B1\) as the base,
calculated over the full product list of region \(B,\) etc.

\(^2\)GEKS is named after Gini (1931), Eltető and Köves (1964) and Szulc (1964).
$P_{Aa}^{\text{core}}$ and $P_{Bb}^{\text{core}}$ denote price indexes for countries $Aa$ and $Bb$, obtained from a global comparison with country $A1$ as the base country. These indexes differ from the within-region indexes described above in two important respects. First, they are only calculated over the core products within the basic heading. Second, they are calculated over all countries in the world (that are participating in ICP), and violate within-region fixity.

Both the within-region and core price indexes can be calculated using standard formula such as CPD or Eurostat Jevons-S (see Sergeev 2003 and Hill and Hill 2009).

The method outlined below uses these within-region and core price indexes to construct global price indexes at the basic heading level that satisfy within-region fixity, but which at the same time treat all countries within each region symmetrically, and all regions symmetrically (to the extent this is possible given that each region contains a different number of countries).

The global fixed (i.e., satisfying within-region fixity) price indexes are calculated for an arbitrary reference pair of countries $Aj$ and $Bk$ as follows:

$$
\frac{P_{Bk}^{\text{global}}}{P_{Aj}^{\text{global}}} = \left[ \prod_{a=1}^{N_A} \prod_{b=1}^{N_B} \left( \frac{P_{Aa}^{\text{region}}}{P_{Aj}^{\text{region}}} \times \frac{P_{Bb}^{\text{core}}}{P_{Bb}^{\text{region}}} \right) \right]^{1/(N_A \times N_B)}
$$

$$
= \left( \frac{P_{Bk}^{\text{region}}}{P_{Aj}^{\text{region}}} \right)^{\frac{N_B}{N_A}} \left[ \prod_{a=1}^{N_A} \left( \frac{P_{Aa}^{\text{region}}}{P_{Bk}^{\text{region}}} \right)^{1/1} \right]^{1/N_B}.
$$

(1)

It is important to remember that in (1) each region has its own base country (i.e., $P_{A1}^{\text{region}} = P_{B1}^{\text{region}} = 1$), while there is only one base country in the core comparison (i.e., $P_{A1}^{\text{core}} = 1$ while in general $P_{B1}^{\text{core}} \neq 1$).

Each term $(P_{Aa}^{\text{region}}/P_{Aj}^{\text{region}}) \times (P_{Bb}^{\text{core}}/P_{Aa}^{\text{core}}) \times (P_{Bb}^{\text{region}}/P_{Bb}^{\text{region}})$ in (1) can be thought of as a chained price index that compares countries $Aj$ and $Bk$ via countries $Aa$ and $Bb$ (i.e., the chaining path is $Aj - Aa - Bb - Bk$). The overall global fixed price index $P_{Bk}^{\text{global}}/P_{Aj}^{\text{global}}$ is the geometric average of the chained price indexes obtained by using all possible chain paths from $Aj$ to $Bk$. For example, suppose there are three countries in region $A$, denoted by $A1$, $A2$, and $A3$, and two countries in region $B$, denoted by $B1$ and $B2$. There are then a total of six possible paths from $A1$ to $B1$. These and
their corresponding chained price indexes are listed below:

\[ A_1 - B_1 : \left( \frac{p_{\text{core}}^{B_1}}{p_{\text{core}}^{A_1}} \right) \]

\[ A_1 - B_2 - B_1 : \left( \frac{p_{\text{core}}^{B_2}}{p_{\text{core}}^{A_1}} \times \frac{p_{\text{region}}^{B_1}}{p_{\text{region}}^{B_2}} \right) \]

\[ A_1 - A_2 - B_1 : \left( \frac{p_{\text{region}}^{B_2}}{p_{\text{region}}^{A_1}} \times \frac{p_{\text{core}}^{B_1}}{p_{\text{core}}^{A_2}} \right) \]

\[ A_1 - A_2 - B_2 - B_1 : \left( \frac{p_{\text{region}}^{A_2}}{p_{\text{region}}^{A_1}} \times \frac{p_{\text{core}}^{B_1}}{p_{\text{core}}^{A_2}} \times \frac{p_{\text{region}}^{B_2}}{p_{\text{region}}^{B_2}} \right) \]

\[ A_1 - A_3 - B_1 : \left( \frac{p_{\text{region}}^{A_3}}{p_{\text{region}}^{A_1}} \times \frac{p_{\text{core}}^{B_1}}{p_{\text{core}}^{A_3}} \right) \]

\[ A_1 - A_3 - B_2 - B_1 : \left( \frac{p_{\text{region}}^{A_3}}{p_{\text{region}}^{A_1}} \times \frac{p_{\text{core}}^{B_2}}{p_{\text{core}}^{A_3}} \times \frac{p_{\text{region}}^{B_2}}{p_{\text{region}}^{B_2}} \right) \]

The global price index between \( A_1 \) and \( B_1 \) in this case is obtained by taking the geometric mean of these six chained price indexes.

Countries from third regions, such as \( C \), drop out if they are included in the chain path. This is because the core price indexes are transitive. For example, as shown below the path \( A_j - C_l - B_k \) reduces to \( A_j - B_k \):

\[
\frac{p_{\text{region}}^{A_j}}{p_{\text{region}}^{A_j}} \times \frac{p_{\text{core}}^{C_l}}{p_{\text{core}}^{A_j}} \times \frac{p_{\text{core}}^{B_k}}{p_{\text{core}}^{C_l}} \times \frac{p_{\text{region}}^{B_k}}{p_{\text{region}}^{B_k}} = \frac{p_{\text{region}}^{A_j}}{p_{\text{region}}^{A_j}} \times \frac{p_{\text{core}}^{B_k}}{p_{\text{core}}^{A_j}} \times \frac{p_{\text{region}}^{B_k}}{p_{\text{region}}^{B_k}} .
\]

Similarly, additional countries from either regions \( A \) or \( B \) drop out since the within-region price indexes are also transitive. For example, as shown below, the path \( A_j - A_l - A_a - B_b - B_k \) reduces to \( A_j - A_a - B_b - B_k \):

\[
\frac{p_{\text{region}}^{A_l}}{p_{\text{region}}^{A_j}} \times \frac{p_{\text{region}}^{A_a}}{p_{\text{region}}^{A_l}} \times \frac{p_{\text{core}}^{B_b}}{p_{\text{core}}^{A_a}} \times \frac{p_{\text{region}}^{B_k}}{p_{\text{region}}^{B_b}} \times \frac{p_{\text{region}}^{B_k}}{p_{\text{region}}^{B_b}} = \frac{p_{\text{region}}^{A_a}}{p_{\text{region}}^{A_j}} \times \frac{p_{\text{core}}^{B_b}}{p_{\text{core}}^{A_a}} \times \frac{p_{\text{region}}^{B_k}}{p_{\text{region}}^{B_b}} .
\]

An analogy can be drawn here with the GEKS formula, which transitivizes Fisher price indexes (denoted here by \( P_{j,k}^F \) where \( j \) and \( k \) denote countries) in a similar way:

\[
\frac{P_{2,1}^{\text{GEKS}}}{P_{1,1}^{\text{GEKS}}} = \prod_{k=1}^{K} \left( \frac{P_{k,2}^F}{P_{k,1}^F} \right)^{1/K} .
\]

Using the fact that Fisher price indexes satisfy the country reversal test, this formula can be rewritten as follows:

\[
\frac{P_{2,1}^{\text{GEKS}}}{P_{1,1}^{\text{GEKS}}} = \left( P_{1,2}^F \right)^2 \prod_{k=3}^{K} \left( P_{1,k}^F \times P_{k,2}^F \right)^{1/K} .
\]
Written in this way, it can be seen that GEKS can itself be interpreted as taking the geometric mean of a direct price index (given double the weight), i.e., $P_{1,2}^F$, and $K - 2$ chained price indexes obtained by chaining through all possible third countries, i.e., $P_{1,k}^F \times P_{k,2}^F$. In this sense, the method in (1) is analogous to GEKS (except that it does not give twice the weight to the direct comparison). In section 2.2 this analogy is further extended. It is shown that both methods can be derived as the solution to least-squares minimization problems.

From (1) it can be seen that the global price indexes are transitive. Hence the numerators and denominators in (1) can be separated into region specific components as follows:

$$P_{\text{global}}^{A_j} = P_{\text{region}}^{A_j} \left[ \prod_{a=1}^{N_A} \left( \frac{P_{\text{core}}^{A_a}}{P_{\text{region}}^{A_a}} \right)^{1/N_A} \right]$$  \hspace{1cm} (3)

$$P_{\text{global}}^{B_k} = P_{\text{region}}^{B_k} \left[ \prod_{b=1}^{N_B} \left( \frac{P_{\text{core}}^{B_b}}{P_{\text{region}}^{B_b}} \right)^{1/N_B} \right]$$  \hspace{1cm} (4)

$$P_{\text{global}}^{C_l} = P_{\text{region}}^{C_l} \left[ \prod_{\gamma=1}^{N_C} \left( \frac{P_{\text{core}}^{C_{\gamma}}}{P_{\text{region}}^{C_{\gamma}}} \right)^{1/N_C} \right]$$ etc.  \hspace{1cm} (5)

None of these indexes as it stands are normalized to 1. However, they can easily be rescaled to achieve whatever normalization is desired.

The overall global results are invariant (after rescaling) to the choice of the regional base countries $A_1, B_1, C_1$, etc., and the base country in the core comparison $A_1$.

From (3), (4) and (5) it can be seen that the global price indexes satisfy within-region fixity. For example, the ratio of the global price indexes of countries $B_2$ and $B_3$ is the same as that of the corresponding within-region ratio, i.e.:

$$\frac{P_{\text{global}}^{B_3}}{P_{\text{global}}^{B_2}} = \frac{P_{\text{region}}^{B_3}}{P_{\text{region}}^{B_2}} \left[ \prod_{b=1}^{N_B} \left( \frac{P_{\text{core}}^{B_b}}{P_{\text{region}}^{B_b}} \right)^{1/N_B} \right] / P_{\text{region}}^{B_2} \left[ \prod_{b=1}^{N_B} \left( \frac{P_{\text{core}}^{B_b}}{P_{\text{region}}^{B_b}} \right)^{1/N_B} \right] = \frac{P_{\text{region}}^{B_3}}{P_{\text{region}}^{B_2}}.$$

### 2.2 Least-Squares Optimality of the Method

The GEKS method alters intransitive Fisher price indexes by the logarithmic least squares amount required to obtain transitivity (see Eltető and Köves 1964 and Szulc 1964). More precisely, GEKS solves the following problem:

$$\text{Min}_{ln(P_k/P_j)} \left\{ \sum_{j=1}^{K} \sum_{k=1}^{K} \left[ \ln \left( \frac{P_k}{P_j} \right) - \ln P_{j,k}^F \right]^2 \right\}.$$  \hspace{1cm} (6)
That is, the solution for $P_k/P_j$ obtained from (6) is the GEKS formula stated in (2).

An analogous result exists for the method proposed here. The objective now is to alter the core price indexes by the minimum logarithmic least squares amount necessary to ensure that within-region fixity is satisfied. Mathematically, this logarithmic least squares problem is slightly more complicated than the standard GEKS problem. It can be written as follows:

$$
\min_{\ln(P_{Bb}/P_{Aa})} \left\{ \sum_{a=1}^{N_A} \sum_{b=1}^{N_B} \left[ \ln\left(\frac{P_{Bb}}{P_{Aa}}\right) - \ln\left(\frac{P_{core}}{P_{core}}\right) \right]^2 \right\}, \tag{7}
$$

subject to

$$
\ln\left(\frac{P_{Aa}}{P_{Aj}}\right) = \ln\left(\frac{P_{region}}{P_{region}}\right), \quad \text{for } a = 1, \ldots, N_A \tag{8}
$$

and

$$
\ln\left(\frac{P_{Bb}}{P_{Bk}}\right) = \ln\left(\frac{P_{region}}{P_{region}}\right), \quad \text{for } b = 1, \ldots, N_B, \tag{9}
$$

where $Aj$ and $Bk$ denote arbitrary reference countries from regions $A$ and $B$ respectively.

The two sets of constraints in (8) and (9) can be combined as follows:

$$
\ln\left(\frac{P_{Aa}}{P_{Aj}}\right) = \ln\left(\frac{P_{region}}{P_{region}}\right), \quad \text{for } a = 1, \ldots, N_A, b = 1, \ldots, N_B, \tag{10}
$$

or equivalently:

$$
\ln\left(\frac{P_{Bb}}{P_{Aa}}\right) = \ln\left(\frac{P_{Bk}}{P_{Aj}}\right) + \ln\left(\frac{P_{Bk}}{P_{region}}\right), \quad \text{for } a = 1, \ldots, N_A, b = 1, \ldots, N_B. \tag{10}
$$

The advantage of the constraint in (10) is that it can be substituted directly into (7). This simplifies the optimization problem to the following:

$$
\min_{\ln(P_{Bb}/P_{Aa})} \left\{ \sum_{a=1}^{N_A} \sum_{b=1}^{N_B} \left[ \ln\left(\frac{P_{Bb}}{P_{Aj}}\right) - \ln\left(\frac{P_{region}}{P_{region}}\right) \right]^2 \right\}. \tag{11}
$$

That is, instead of having to solve for $N_A \times N_B$ unknowns of the form $\ln(P_{Bb}/P_{Aa})$ in (7), subject to $N_A + N_B$ constraints in (8) and (9), there is now in (11) only one unknown $\ln(P_{Bb}/P_{Aa})$ and no constraints. Differentiating the argument of (11)
with respect to \( \ln(P_{Bk}^{global}/P_{Aj}^{global}) \), yields the following first order condition:\(^3\)

\[
2N_AN_B \ln \left( \frac{P_{Bk}^{global}}{P_{Aj}^{global}} \right) - 2 \sum_{a=1}^{N_A} \sum_{b=1}^{N_B} \ln \left( \frac{P_{region}^{Bk}}{P_{region}^{Bb}} \frac{P_{region}^{Aj}}{P_{region}^{Aa}} \right) = 0. \tag{12}
\]

Finally, rearranging (12) yields the following solution for \( P_{Bk}^{global}/P_{Aj}^{global} \):

\[
\frac{P_{Bk}^{global}}{P_{Aj}^{global}} = \left( \frac{P_{region}^{Bk}}{P_{region}^{Bb}} \frac{P_{region}^{Aj}}{P_{region}^{Aa}} \right)^{1/(N_A \times N_B)}, \tag{13}
\]

which on inspection can be seen to be equivalent to the global price indexes already presented in (1).

### 2.3 Linking at the Basic Heading Level Through Ring Countries

With slight modifications the method can be applied to an ICP 2005 context in which ring countries were used instead of core products to link the regions together. The ring comparison in ICP 2005 had its own product list and involved 18 countries drawn from the regions.

In a ring country context, the formula in (1) is modified slightly as follows:

\[
\frac{P_{Bk}^{global}}{P_{Aj}^{global}} = \left[ \prod_{a=1}^{N_A} \prod_{b=1}^{N_B} \left( \frac{P_{region}^{Bk}}{P_{region}^{Bb}} \times \frac{P_{ring}^{Bb}}{P_{region}^{Bb}} \times \frac{P_{region}^{Aj}}{P_{region}^{Aa}} \right) \right]^{1/(N_A \times N_B)} \\
= \left( \frac{P_{region}^{Bk}}{P_{region}^{Bb}} \frac{P_{region}^{Aj}}{P_{region}^{Aa}} \right)^{1/N_B} \left[ \prod_{b=1}^{N_B} \left( \frac{P_{ring}^{Bb}}{P_{region}^{Bb}} \right) \right]^{1/N_A}. \tag{14}
\]

Again, the numerators and denominators in (14) can be separated into region specific components as follows:

\[
P_{Aj}^{global} = P_{Aj}^{region} \left[ \prod_{a=1}^{N_A} \left( \frac{P_{ring}^{Aa}}{P_{region}^{Aa}} \right)^{1/N_A} \right]. \tag{15}
\]

\(^3\)The second derivative of the argument in (11) with respect to \( \ln(P_{Bk}^{global}/P_{Aj}^{global}) \) equals \( 2N_AN_B \) which is greater than zero, thus ensuring that the solution found is a minimum.
There are two changes here as compared with (1). First, the core price indexes $P_{Aa}^{core}$ have been replaced by ring price indexes $P_{Aa}^{ring}$. Second, while $a = 1, \ldots, N_A$ in (1) indexes all the countries in region $A$, in (14) it indexes only the ring countries in region $A$. The global price index for a non-ring country (say $Az$) is then obtained by linking it to the global price index of one of the ring countries from its region (say $Aj$) as follows:

$$P_{Az}^{global} = \left( \frac{P_{region}^{Az}}{P_{region}^{Aj}} \right) \times P_{Aj}^{global} = P_{Az}^{region} \left[ \prod_{a=1}^{N_A} \left( \frac{P_{Aa}^{core}}{P_{region}^{Aa}} \right)^{1/N_A} \right].$$

It can be seen from (18) that for a region containing more than one ring country (as all regions did in ICP 2005), it does not matter which is used as the link for the non-ring countries in that region. The linking ring country (here $Ak$) drops out of the $P_{Az}^{global}$ formula.

More generally, moving beyond ICP 2005, there is no reason why in ICP 2011 every country from every region must be included in the core comparison. Once the core products have been decided, it is still possible to select only a sample of countries from each region to participate in the calculation of the core price indexes that are used in (1), such as those with a more complete coverage of the products in the core list (while ensuring that there is enough representation from each region). Furthermore, the list of countries used in the core comparison could vary from one basic heading to the next. The countries excluded from the core comparison can then be linked back in using (18). Such an approach might prove useful if some countries are unable to price any of the core products in a few basic headings.
3 A GEKS-Type Method for Linking the Regions at the Aggregate Level while Retaining Within-Region Fixity

ICP 2005 and 2011 require within-region fixity to be satisfied at all levels of aggregation from basic heading level up to GDP. This means that, once the basic heading price indexes have been linked across regions, it is not enough to simply use a standard multilateral method to construct global price indexes at the aggregate level since this would lead to a violation of within-region fixity.\(^4\)

With slight modifications to (1), the method outlined in section 2.1 can be used to construct aggregate price indexes at the global level that satisfy within-region fixity.

\[
\frac{P_{Bk}^{\text{global}}}{P_{A_j}^{\text{global}}} = \left[ \prod_{a=1}^{N_A} \prod_{b=1}^{N_B} \left( \frac{P_{A_a}^{\text{region}}}{P_{A_j}^{\text{region}}} \times \frac{P_{B_b}^{\text{unfixed}}}{P_{A_a}^{\text{unfixed}}} \times \frac{P_{B_k}^{\text{region}}}{P_{B_b}^{\text{region}}} \right) \right]^{1/(N_A \times N_B)}
\]

\[
= \left( \frac{P_{B_k}^{\text{region}}}{P_{A_j}^{\text{region}}} \right) \left[ \prod_{b=1}^{N_B} \left( \frac{P_{B_b}^{\text{unfixed}}}{P_{B_b}^{\text{region}}} \right)^{1/N_B} \right] \left[ \prod_{a=1}^{N_A} \left( \frac{P_{A_a}^{\text{unfixed}}}{P_{A_a}^{\text{region}}} \right)^{1/N_A} \right], \tag{19}
\]

where \(P_{A_a}^{\text{region}}\) is now an aggregate within-region price index for country \(Aa\), with \(A1\) as the base, while \(P_{B_b}^{\text{region}}\) is an equivalent within-region price index for region \(B\) with \(B1\) as the base.\(^5\) \(P_{A_a}^{\text{unfixed}}\) and \(P_{B_b}^{\text{unfixed}}\) in (19) are global multilateral price indexes, that violate within-region fixity. This is why they have ‘unfixed’ superscripts. In principle, any multilateral method, such as GEKS, Geary-Khamis, IDB or a minimum-spanning-tree, can be used to compute the unfixed price indexes.\(^6\)

Global fixed price indexes (i.e., that satisfy within region fixity) at the aggregate

\(^4\)At a conceptual level, such an approach might also be undesirable since while the basic headings may match from one region to the next, the underlying products from which the basic heading price indexes are constructed do not.

\(^5\)In ICP 2005, the within-region price indexes at the aggregate level were calculated using GEKS for all regions except Africa which used IDB. IDB was proposed by Iklé (1972), Dikhanov (1997), and Balk (1996) (see also Dievert 2010a for a detailed analysis of the properties of IDB).

\(^6\)Geary-Khamis was developed by Geary (1958) and Khamis (1972), while minimum-spanning-tree methods were developed by Hill (1999) and Dievert (2010b).
level, denoted here by \( P_{Aa}^{\text{global}} \), \( P_{Bb}^{\text{global}} \), \( P_{Cc}^{\text{global}} \), etc., are derived from (19) as follows:

\[
P_{Aj}^{\text{global}} = P_{Aj}^{\text{region}} \prod_{a=1}^{N_A} \left( \frac{P_{Aa}^{\text{unfixed}}}{P_{Aa}^{\text{region}}} \right)^{1/N_A}
\]

(20)

\[
P_{Bk}^{\text{global}} = P_{Bk}^{\text{region}} \prod_{b=1}^{N_B} \left( \frac{P_{Bb}^{\text{unfixed}}}{P_{Bb}^{\text{region}}} \right)^{1/N_B}
\]

(21)

\[
P_{Cl}^{\text{global}} = P_{Cl}^{\text{region}} \prod_{c=1}^{N_C} \left( \frac{P_{Cc}^{\text{unfixed}}}{P_{Cc}^{\text{region}}} \right)^{1/N_C}
\]

(22)

This method applied at the aggregate level has the same least-squares optimality property as when applied at basic heading level, and it is also invariant to the choice of base country in each region and the choice of base country in the global unfixed comparison.

While symmetric treatment of countries may be desirable, it is not actually necessary to include all countries in the global unfixed comparison. It might be preferable to exclude a few countries if their basic heading prices and/or expenditure shares are deemed to be of dubious quality. Their exclusion would prevent their data problems contaminating the between-region links. These countries could then be linked back in at the end using a formula of the type outlined in (18), but with the ‘ring’ superscripts replaced by ‘unfixed’ superscripts.

Dikhanov (2007) has proposed an alternative method for linking regions at the aggregate level, referred to by Dievert (2010b) as the global comparison GEKS method. Dikhanov proposes making a global unfixed GEKS comparison, and using this to determine the quantity shares of each region. The within-region results from each region are then linked in such a way that region shares exactly match those obtained from the global unfixed GEKS comparison. One similarity between the global comparison GEKS method and the method proposed here is that other multilateral methods can be substituted for GEKS if desired in the global unfixed comparisons. In fact, Kravis, Heston and Summers (1982) used essentially the same method with Geary-Khamis replacing GEKS. The global comparison GEKS method and its variants, however, lack the algebraic simplicity and least-squares optimality property of my method.
4 An Illustrative Example

This example focuses on the case of linking at the basic heading level using core products. If the word ‘core’ is replaced by ‘ring’ or ‘unfixed’ in the discussion that follows, the example applies equally well to the cases of linking at basic heading level using ring countries or linking at higher levels of aggregation.

Suppose there are three regions A, B and C, and that there are three countries in each of regions A and C, and two countries in region B. The within-region price indexes for the regional base countries $A_1$, $B_1$ and $C_1$ are normalized to 1 (i.e., $P_{region}^{A_1} = P_{region}^{B_1} = P_{region}^{C_1} = 1$). Also, the core price index for country $A_1$ is normalized to 1 (i.e., $P_{core}^{A_1} = 1$).

Suppose further that the within-region price indexes are:

- $P_{region}^{A_1} = 1$, $P_{region}^{A_2} = 1.2$, $P_{region}^{A_3} = 0.9$
- $P_{region}^{B_1} = 1$, $P_{region}^{B_2} = 1.1$
- $P_{region}^{C_1} = 1$, $P_{region}^{C_2} = 0.7$, $P_{region}^{C_3} = 1.1$

and that the core price indexes are:

- $P_{core}^{A_1} = 1$, $P_{core}^{A_2} = 1.4$, $P_{core}^{A_3} = 0.8$
- $P_{core}^{B_1} = 1.2$, $P_{core}^{B_2} = 1.4$
- $P_{core}^{C_1} = 0.7$, $P_{core}^{C_2} = 0.5$, $P_{core}^{C_3} = 0.9$

Applying the formulas in (3), (4) and (5) to these within-region and core price indexes the following global price indexes are obtained:

- $P_{global}^{A_1} = 1.0122$, $P_{global}^{A_2} = 1.2146$, $P_{global}^{A_3} = 0.9110$
- $P_{global}^{B_1} = 1.2358$, $P_{global}^{B_2} = 1.3594$
- $P_{global}^{C_1} = 0.7423$, $P_{global}^{C_2} = 0.5196$, $P_{global}^{C_3} = 0.8166$

The global price indexes can now be rescaled so that $P_{global}^{A_1} = 1$ by dividing through all of them by 1.0122 as follows:

- $P_{global}^{A_1} = 1.0000$, $P_{global}^{A_2} = 1.2000$, $P_{global}^{A_3} = 0.9000$
- $P_{global}^{B_1} = 1.2209$, $P_{global}^{B_2} = 1.3430$
- $P_{global}^{C_1} = 0.7334$, $P_{global}^{C_2} = 0.5134$, $P_{global}^{C_3} = 0.8067$. 

11
5 Conclusion

The method proposed here for linking regions in an ICP context is very flexible. It can be applied at either basic heading level or at the aggregate level, and can be combined with any multilateral method. It is algebraically quite simple and intuitive, and is optimal in a least-squares sense. It remains still to apply the method to real ICP data. It may be problematic for some basic headings to compute CPD price indexes over the core products for all participating countries (of which there will be about 150 in ICP 2011). If some countries fail to price any of the core products in a particular basic heading, then as mentioned earlier these countries can simply be excluded from the core comparison and then linked back in later using the formula in (18) (with ‘ring’ replaced by ‘core’). If, however, across all countries there are simply not enough core product price quotes in a particular basic heading to estimate the 150 country dummy variables with sufficient accuracy, then it may be preferable at least for this heading to stick with the regional variant on the CPD method proposed by Diewert and used in ICP 2005, and which although it lacks the least-squares optimality property of my method has the advantage that it requires the estimation of only region (of which there are only six) rather than country dummy variables. The practical relevance of this problem can only be determined empirically using real ICP data. At the aggregate level, however, a new method will definitely be required in ICP 2011 due to the failure of the Diewert region-CPD method in this context to satisfy base-country invariance. The aggregate level version of the method proposed here looks like a strong candidate for this role, particularly since at the aggregate level there is no need to make a global CPD comparison (and hence the criticisms above are no longer relevant). Again though the method needs testing on ICP data before a final decision can be made.

References


Blades, Derek (2007) GDP and Main Expenditure Aggregates. Chapter 3 in *ICP*


