Estimating Rates of Return on Capital

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The World Bank ~ Kyiv, Ukraine
Rate of Return on Capital (RoR K)

• ... provides a basis for assessing the contribution of capital investment to the growth process

• ... is a key guide for estimating the opportunity cost of capital
RoR K: Important for applied growth economic work

- Measures the contribution of investment to economic growth (Solow, 1956)
- Presents a significant part of country-level diagnostics analysis of impediments to growth:

  Key Question: Is investment in physical capital low because: (i) the return of capital is low, or (ii) the cost of financing is too high?
RoR K: Definition

RoR K \( (\rho_t) \) is the ratio of the value of national income accrued to capital \( (Y_t^K) \) during the year to the value of the national capital sock at the beginning of the year \( (K_t) \).

\[
\rho_t = \frac{Y_t^K}{K_t}
\]

\[
Y_t^K = \alpha Y_t
\]

where \( \alpha \) is the share of capital in income \( Y_t \)

Therefore:

\[
\rho_t = \alpha_t \frac{Y_t}{K_t} = \frac{Y_t^K}{K_t}
\]
Estimation procedure: Problems

- Data in National Accounts are usually not in directly applicable form!
- The marginal productivity of labor in the public sector may differ from wages and salaries paid
- National accounts frequently identify employee compensation with income from labor, which implies that the income of unincorporated enterprises is entirely attributed to capital
Income Accruing to Capital

Total Capital Income = (National Income - Total Labor Income)

\[ Y^K_t = Y_t - Y^L_t \]

Total Labor Income = (Total Wages&Salaries + Imputed Wages&Salaries)

\[ Y^L_t = \sum_{j=1}^{h} \left( p_j X_j + s^L_{UE} Y_j \right) \]

Imputed Wages&Salaries: a fraction of average income generated by a typical employee in each sector to account for proprietors, partners, self-employed, and unpaid family members
Income Accruing to Capital

Or

Obtaining estimates of $\alpha$:

- dynamic capital shares of income as reported in nat. accounts
- 'rule of thumb' assumes a constant capital share of income of 0.4
- regress capital shares reported by Bernanke and Gurkaynak (2001) on the value added share of industry and region dummies
- regress capital shares reported by Gollin (2002) on region dummies

In the case of Ukraine:

- we obtain four estimates of $\alpha$ and use them to calculate four sets of Income Accruing to Capital
Income Accruing to Capital

Share of National Income Accruing to Capital

\[ \alpha_t = \frac{Y_t - \left( \sum_{j=1}^{h} p_j X_j + s_{UE}^L Y_t \right)}{Y_t} = \frac{Y_t - Y_t^L}{Y_t} = \frac{Y_t^K}{Y_t} \]

Income accruing to capital

\[ Y_t^K = Y_t - \sum_{j=1}^{h} (p_j X_j + s_{UE}^L Y_j) = \alpha Y_t \]
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<th>Year</th>
<th>Rule of thumb</th>
<th>Capital Share (B&amp;G Region Dummies)</th>
<th>Capital Share (Gollin Region Dummies)</th>
<th>Capital Share (dynamic)</th>
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Estimating Income Accruing to Capital: Ukraine

Income Accruing to Capital
constant 2001 LCU, mln

Rule of thumb  
Capital Share (B&G Region Dummies)  
Capital Share (dynamic)
Estimating the Stock of Capital

Total capital stock comprises of different categories of assets, usually:

Fixed Assets
- Machinery and equipment
- Buildings and construction

Inventory
Methodology

Perpetual Inventory Method:

Net capital stock at the beginning of the period \( (K_{t+1}) \) is a function of the net capital stock \( (K_t) \) plus yearly Gross Investment \( (I_t) \) minus depreciation \( (D_t) \)

\[
K_{t+1} = K_t + I_t - D_t
\]  

(1)

Assumptions:

- depreciation method has to be chosen
- level & time profile of depreciation rate has to be made
- initial capital stock has to be constructed
Perpetual Inventory Method: Assumptions

Geometric depreciation

\[ K_t = (1 - \delta)K_{t-1} + I_t \]

Different depreciation rates \( \delta \) across the types of investment considered

Initial Stock of Capital

\[ K_{t+1} = \sum (1 - \delta) I_{t-i} \]

\[ K_{t+1} = (1 - \delta)^t K_1 + \sum (1 - \delta)^i I_{t-i} \]

\[ I_t = (\delta + \gamma) K_t \]
Estimating Initial Stock of Fixed Assets

1. Take *Gross Investment* for fixed assets (buildings and construction; and machinery and equipment) to generate capital-stock series (source: National Income Accounts)

2. Assume appropriate rates of depreciation ($\delta$) and of economic growth ($\gamma$)

3. Estimate the initial stock of each type of reproducible capital:

   \[ K_1 = \frac{I_t}{\delta + \gamma} \]

   Where:
   - $I$ is gross investment,
   - $\delta$ is the rate of depreciation,
   - $\gamma$ is the annual rate of growth of capital stock, and
   - $K$ is the capital stock at the beginning of the year
Estimating Capital Stock in Transition Economies

- Upward bias in recorded values of capital stocks prior to 1990, because part of old capital does not contribute any longer to the production process and is still in the records
- Estimate “old” Soviet capital stock:

\[ K_{t, old} = (1 - \mu) K_0 (1 - \delta_{old})^t \]

\( \mu \) – share of unusable capital and
\( \delta_{old} \) – depreciation rate of old capital

- Izyumov and Vahaly (2008) estimate (i) 1989 capital stock using data from Goskomstat; (ii) 1991 capital stock using PIM \( \delta_{old} = 0.05; \mu = 0.3 \)
- Estimate “new” capital stock (from 1992 until year \( t \))

\[ K_{t, new} = \sum_{n=0}^{t} I_{t-n} (1 + \delta_{new})^n \]
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<th>Depreciation of old capital</th>
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<th>Depreciation of capital stock</th>
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Note: SSCU series Capital stock deflated into constant 2001 LCU; GCF - SSCU series Investment in fixed capital deflated into constant 2001 LCU; depreciation 5%
Source: SSCU
Estimating Capital Stock: Ukraine

Capital Stock
constant 2001 LCU, mln

Note: SSCU series Capital stock deflated into constant 2001 LCU; GCF - SSCU series Investment in fixed capital deflated into constant 2001 LCU; depreciation 5%
Source: SSCU
Estimating Stock of Inventories

\[ Inv_0 = \left( \sum_{n=0}^{t} \frac{Inv_{t-n}}{(GDP_t - GDP_n)} \right)^n \times GDP \]

\[ Inv_t = Inv_{t-1} + Inv_t \]

Retrieve

Net Inventory Accumulation from National Accounts

Calculate

Marginal Inventory/GDP = Net Inventory Accumulation / Change in GDP

[Ukraine = 10.35%]

Apply ratio [10.35%] to initial year GDP and estimate Initial Inventory Stock

Derive Inventory Stock by adding Net Inventory Changes to Initial Stock of Inventory for each year after
Estimating Inventory Stock: Ukraine

Inventory Stock
constant 2001 LCU, mln

- net inventory change
- Inventory stock at beginning of year
The Rate of Return on Capital

1. Estimate Total Capital Stock
   \[ K_t = \Sigma [Fixed \ Capital]_t; \ (Inventories)_t \]

2. National Income Accruing to Capital
   \[ Y^K_t = Y_t - \sum_{j=1}^{n} (p_j X_j + s_{UE}^L Y_j) \]
   or
   \[ Y^K_t = \alpha Y_t \]

3. Estimate the rate of return \((\rho_t)\)
   \[ \rho_t = \alpha \frac{Y^K_t}{K_t} = \frac{Y^K_t}{K_t} \]
Rates of Return on Capital: Ukraine

- Rule of thumb
- Capital Share (B&G Region Dummies)
- Capital Share (dynamic)
- Average RoRK (over dynamic shares, rule of thumb and B&G)


Selected References


