

Theoretical Prerequisites for Fiscal Sustainability Analysis

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To define fiscal sustainability analysis it is useful to seek guidance from a dictionary. Webster's, for example, suggests using the adjective *sustainable* to describe something that can be kept up, prolonged, borne, etc. Or, it might be used to describe a method of harvesting a resource so that the resource is not depleted or permanently damaged in the process.

When we speak of fiscal sustainability we are typically referring to the fiscal policies of a government. Of course, we must make our definition of fiscal sustainability more precise than the dictionary definitions given above. On the other hand, they can guide our thinking.

The resource depletion analogy is not entirely appropriate, because a government's resources are not that comparable to mineral, or other, resources. On the other hand, it does suggest a concept of sustainability related to *solvency*. When we speak of *solvency* we refer to the government's ability to service its debt obligations without explicitly defaulting on them. One concept of fiscal sustainability relates to the government's ability to indefinitely maintain the same set of policies while remaining solvent. If a particular combination of fiscal and/or monetary policy would, if indefinitely maintained, lead to insolvency, then we refer to these policies as unsustainable. One role of fiscal sustainability analysis is to provide some indication as to whether a particular policy mix is sustainable or not.

Often governments will change their policies if it becomes clear that they are unsustainable. Thus, the focus of fiscal sustainability analysis is frequently not on default itself—which governments frequently avoid—rather it is on the consequences of the policy changes needed to avoid eventual default.

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Even when a government is solvent, and is likely to remain solvent, its fiscal policies may be costly. Sometimes fiscal sustainability analysis will refer to the ongoing costs associated with a particular combination of fiscal and monetary policies.

In the rest of this chapter we develop the simple theoretical framework within which fiscal sustainability analysis is usually conducted. We will introduce several concepts: the single-period government budget constraint, the lifetime budget constraint, the fiscal theory of the price level, the no-Ponzi scheme condition, and the transversality condition. In Chapter 2 we will show how these tools can be used in analyzing and interpreting data.

1. The Government Budget Constraint

The fundamental building block of fiscal sustainability analysis is the public sector or *government budget constraint*, which is an identity:

$$\text{net issuance of debt} = \text{interest payments} - \text{primary balance} - \text{seigniorage}. \quad (1.1)$$

The net issuance of debt is gross receipts from issuing new debt minus any amortization payments made in the period.¹

The identity, (1.1), can be expressed in mathematical notation as

$$B_t - B_{t-1} = I_t - X_t - (M_t - M_{t-1}). \quad (1.2)$$

Here the subscript t indexes time, which we will usually measure in years, B_t is quantity of public debt at the end of period t , I_t is interest payments, X_t is the primary balance (revenue minus noninterest expenditure), and M_t is the monetary base at the end of period t , all measured in units of local currency (LCUs). The only subtlety involved in (1.2) is in associating the net issuance of debt, a net cash flow, with the change in a stock, the quantity of debt, B_t . To be sure that these objects are equivalent, we should be more precise as to the definitions of the *quantity of debt* and *interest payments*. To define the former, we must decide how to value the government's outstanding debt obligations. To define the former we must divide debt service between amortization and interest. We will not explore precise definitions of debt and interest until Chapter 2.

Two clarifying statements should be made at this point. First, both the debt and interest payments concepts should be net; i.e. debt should be net of any marketable assets and

¹The government might also issue new debt in order to finance the repurchase of old debt. In this case, we would still be concerned with the net proceeds raised.

interest should be payments net of receipts. Second, the analysis must fix on a particular definition of the government or public sector since different measures of the variables in (1.2) would apply to different definitions of the public sector. For example, by including seigniorage revenue (the change in the monetary base) in our definition, we have implicitly defined the public sector to include, at least, the central bank in addition to the central government. It is probably most conventional to define the public sector as the consolidation of the central government, state and local governments, state-owned nonfinancial enterprises and the central bank. Sometimes state banks are also included in the definition of the public sector, but more commonly they are not.

As we will see in Chapter 2, (1.2) is the fundamental building block for studying the evolution of the government's debt over time, or, using a common phrase, the government's *debt dynamics*. But the flow budget constraint is also the first step in deriving the lifetime government budget constraint, which plays a crucial role in assessing a government's finances, interpreting its fiscal policies, and predicting the consequences of particular shocks to the economy for prices and exchange rates. To derive the lifetime budget constraint we need to first rewrite the flow budget constraint.

To begin, we will assume that time is discrete, that all debt has a maturity of one period, that debt is real, in the sense that its face value is indexed to the price level, and pays a constant real rate of interest, r . In this case (1.2) can be rewritten as

$$b_t = (1 + r)b_{t-1} - x_t - \sigma_t, \quad (1.3)$$

where $b_t = B_t/P_t$ is the end-of-period t stock of real debt, $x_t = X_t/P_t$ is the real primary surplus and $\sigma_t = (M_t - M_{t-1})/P_t$ is the real value of seigniorage revenue.²

Rearranging (1.3) we have

$$b_{t-1} = (1 + r)^{-1}b_t + (1 + r)^{-1}(x_t + \sigma_t). \quad (1.4)$$

Notice that the expression for b_{t-1} , on the right-hand side of (1.4), can be updated to period t , implying that

$$b_t = (1 + r)^{-1}b_{t+1} + (1 + r)^{-1}(x_{t+1} + \sigma_{t+1}). \quad (1.5)$$

²The correspondence between (1.2) and (1.3) can be verified if one assumes that the interest payment includes an indexation adjustment for both the interest and the principal on the loan. I.e.:

$$I_t = [(P_t/P_{t-1})(1 + r) - 1]B_{t-1}.$$

This can be used to substitute for b_t on the right-hand side of (1.4):

$$b_{t-1} = (1+r)^{-2}b_{t+1} + (1+r)^{-1}(x_t + \sigma_t) + (1+r)^{-2}(x_{t+1} + \sigma_{t+1}). \quad (1.6)$$

Clearly, the same procedure could be used to substitute for b_{t+1} on the right-hand side of (1.6), and then for b_{t+2} , etc., in a recursive fashion. Hence, after several iterations we would obtain

$$b_{t-1} = (1+r)^{-(j+1)}b_{t+j} + \sum_{i=0}^j (1+r)^{-(i+1)}(x_{t+i} + \sigma_{t+i}). \quad (1.7)$$

Equation (1.7) provides a link between the amounts of debt the government has at two dates: $t-1$ and $t+j$. In particular, the amount of debt the government has on date $t+j$ is a function of the debt it initially had at date $t-1$, as well as the primary surpluses it ran, and seigniorage it raised between these dates.

If we impose the condition

$$\lim_{j \rightarrow \infty} (1+r)^{-(j+1)}b_{t+j} = 0 \quad (1.8)$$

then we obtain what is frequently called the government's *lifetime budget constraint*:

$$b_{t-1} = \sum_{i=0}^{\infty} (1+r)^{-(i+1)}(x_{t+i} + \sigma_{t+i}). \quad (1.9)$$

Intuitively, the lifetime budget constraint states that the government finances its initial debt by raising seigniorage revenue and running primary surpluses in the future, whose present value is equal to its initial debt holdings.³

The lifetime budget constraint is a fundamental building block for a number of different tools and theoretical arguments developed in the literature and discussed here. In the next section we use the lifetime budget constraint as a theoretical tool to discuss the effects of government deficits on inflation. This discussion will be closely related to the theoretical arguments made in Sargent and Wallace's (1981) *unpleasant monetarist arithmetic* paper and Sargent's (1983, 1985) papers on monetary and fiscal policy coordination. Later we will show that in the fiscal theory of the price level (1.9) is reinterpreted as an equation that prices government debt.⁴

³In deriving (1.9) we imposed the condition (1.8) without stating its origin. Absent a theoretical model, of course, there is no reason for imposing this condition, as it is not obvious, without a theory, why the stock of debt should evolve in a way that satisfies (1.8). The end of this chapter discusses the appropriate interpretation of (1.8).

⁴See Sims (1994), Woodford (1995) and Cochrane (2001a, b), which discuss the fiscal theory in a closed economy context. Dupor (2000), Daniel (2001), and Corsetti and Mackowiak (2002) analyze the implications of the fiscal theory for open economies.

In Chapter 2 we use the lifetime budget constraint to derive our a simple tool for assessing fiscal sustainability: the long-run sustainability condition. We will also show how it can be used as the basis of formal statistical tests of (1.8), as in Hamilton and Flavin (1986).

2. Fiscal and Monetary Policy and the Effects of Government Deficits

In this section we will discuss the effects of government deficits on inflation. We will also discuss the issue of fiscal and monetary policy coordination. At first, it may not seem obvious that these questions and concerns are closely related to fiscal sustainability. In fact, they are intimately related to fiscal sustainability. As was argued in the introduction, it is possible that a government will violate (1.9) by defaulting on its debt obligations. However, in many cases, the role of fiscal sustainability analysis is not to point out concerns about default. Rather, the role of the analysis is to discuss the macroeconomic consequences of alternative policies which happen to all be consistent with (1.9). The lifetime budget constraint can be satisfied by generating large primary surpluses, but it can also be satisfied by generating a lot of seigniorage revenue. Obviously these policies will have different consequences for macroeconomic outcomes, and the timing of when surpluses occur may also be important.

This section take a first step in the direction of understanding the impact of different policy choices. Much of the discussion is closely related to the arguments outlined in Sargent and Wallace (1981), and Sargent (1983, 1985). We begin by returning to the real version of the government budget constraint, (1.3):

$$b_t = (1 + r)b_{t-1} - x_t - (M_t - M_{t-1})/P_t.$$

Notice that we can rearrange this equation as

$$P_t(b_t - b_{t-1}) + M_t - M_{t-1} = P_t(rb_{t-1} - x_t). \quad (2.1)$$

The right-hand side of (2.1) is the government's financing requirement: the nominal value of real interest payments plus the primary deficit. The left-hand side of (2.1) is the government's financing, which is a mix of net issuance of debt, and net issuance of base money.

If one thinks of a fiscal authority (a legislature together with the ministry of finance) as the determiner of x_t then, given that rb_{t-1} is pre-determined, (2.1) appears to define the role of the monetary authority as essentially a debt manager: it picks the financing mix between debt and monetary obligations.

Price Level Determination Conditional on a value for the price level P_t , we might think of the fiscal authority choosing x_t , and then the monetary authority choosing M_t and b_t consistent with (2.1). Not surprisingly, however, the price level, itself, will be influenced by the monetary authority's choice. The link between the monetary authority's decision making and the price level is usually made by writing down a model of money demand.

There are several models of money demand that we might use. For example, we might use the quantity theory of money, whereby the demand for real money balances is simply, $M_t/P_t = y_t/v$, where v represents a constant value for the velocity of money and y_t represents real GDP. Alternatively we might use a variant of the Cagan money demand representation in which the demand for real balances depends negatively on the nominal interest rate: $M_t/P_t = Ay_t \exp(-\eta R_t)$, where R_t represents the nominal interest rate. Alternatively, we could use any simple model of money demand consistent with the basic assumptions in a standard intermediate macroeconomics textbook, or perhaps a complicated general equilibrium model. For the remainder of this chapter we will use either the quantity theory or a variant of Cagan money demand, but mainly for analytical convenience. The qualitative findings we get with these assumptions would be robust to other specifications.

To obtain our first results on the effects of policy on the price level we take the Cagan money demand specification described above assuming that (i) the transactions motive is constant, i.e. $y_t = y$ for all t , and (ii) the nominal interest rate is just $R_t = r + E_t \ln(P_{t+1}/P_t)$. In this case we have

$$\ln(M_t/P_t) = a - \eta E_t \ln(P_{t+1}/P_t) \quad (2.2)$$

where $a = \ln(Ay) - \eta r$.

Notice that (2.2) represents a linear first order difference equation in $\ln P_t$. If we treat M_t as an exogenous stochastic process controlled by the central bank, (2.2) implies the following solution for $\ln P_t$:

$$\ln P_t = -a + \frac{1}{1 + \eta} \sum_{j=0}^{\infty} \left(\frac{\eta}{1 + \eta} \right)^j E_t \ln M_{t+j} \quad (2.3)$$

Importantly, P_t depends on the current money supply as well as the expected path of the money supply. If the interest semi-elasticity of money demand, η , is very small, say approximately 0, then the price level depends mainly on the current money supply: $\ln P_t \approx -a + \ln M_t$. On the other hand, if η is very large the discount factor in (2.3) will be close to 1, and the price level will depend a lot on what agents expect the money supply to be well

into the future.

Are Government Deficits Inflationary? In this section we use the solution for the price level, (2.3), to ask whether government deficits are inflationary. We will see that this depends on how monetary and fiscal policy, together, are conducted. We will consider different policy regimes which have very different implications for inflation.

Regime 1 Suppose the government follows a regime in which it issues debt to finance deficits, and money is never printed:

$$M_t = M \text{ for all } t. \quad (2.4)$$

Under this regime, the lifetime budget constraint, (1.9), implies

$$b_{-1} = \sum_{t=0}^{\infty} (1+r)^{-(t+1)} x_t \quad (2.5)$$

so that the present value of the primary balance is the initial debt stock.

In essence, if we abstract from the stock of initial debt, the monetary policy $M_t = M$ for all t implies that running a primary deficit at time 0, $x_0 > 0$, forces there to be future primary surpluses in present value terms: i.e. $\sum_{t=1}^{\infty} (1+r)^{-(t+1)} x_t > 0$. In this policy regime monetary policy is rigid, and future fiscal policy must tighten if current fiscal policy becomes looser.

Furthermore, and as a consequence of this, notice that inflation is zero in this regime: $P_t = e^{-a} M$ for all t . So running a deficit at time 0 causes no inflation at time 0. This is precisely because agents in the economy understand the nature of monetary policy. They know that the deficit at time 0 will not be monetized at time 0, nor at any time in the future.

Regime 2 In the previous example there is no connection between the current primary deficit and the inflation rate. Now imagine a policy regime in which the government never issues debt:

$$b_t = 0 \text{ for all } t. \quad (2.6)$$

Of course, in this setting the flow budget constraint, (1.3), implies

$$M_t - M_{t-1} = -P_t x_t. \quad (2.7)$$

Not surprisingly, this policy regime is much more likely to be one where there is a connection between the current deficit and current inflation. For example, in the extreme case where the interest rate elasticity is zero ($\eta = 0$), (2.3) and (2.7) imply $P_t = e^{-a}M_{t-1}/(1 + e^{-a}x_t)$. This means that the smaller the primary surplus is, the higher today's price level is, given the value of M_{t-1} .

The important point is that under policy regime 2, the government prints money to meet its current financing need. This translates a short-run need for financing into inflation, something that does not occur under regime 1. Under regime 1, the government, instead, meets its financing needs through borrowing, and at some later date implements a fiscal reform that allows it to avoid using monetary financing.

Of course, in reality, there are all sorts of other policy regimes that fit somewhere between our two polar cases. The important lesson from our simple analysis is that the time series correlation between deficits and inflation depends critically on which policy regime we are in. In regime 1, deficits and inflation are uncorrelated: inflation is always zero no matter what the primary deficit is. In regime 2, primary deficits and the inflation rate are strongly positively correlated.

Policy Lessons A lack of correlation between deficits and inflation might be naively interpreted as indicating that somehow inflation is driven by something other than the government's fiscal policy, and that it has a life of its own. It is important that policy makers should not be misled into believing this. Even if it does not coordinate with the fiscal policy maker, the monetary authority can smooth the effects of fiscal policy on the price level by avoiding a monetary policy similar to (2.7). However it cannot prevent inflation occurring if the government is fiscally irresponsible.

To see this, notice that the central bank is always free to adopt a constant money growth rule, regardless of the government's choice for the path of x_t . Notice that if $\Delta \ln M_t = \mu$, it is easy to show that $P_t = e^{\mu\eta - a}M_t$ and, therefore, that $\Delta \ln P_t = \mu$. So the monetary authority can smooth inflation, avoiding the fluctuations in inflation that would be inherent in policy regime 2.

Despite its ability to *smooth* inflation, however, the central bank cannot *suppress* it if the fiscal authority is irresponsible. With constant money growth, the flow of real seigniorage

revenue is constant, allowing us to rewrite (1.9) as

$$\sigma = rb_{t-1} - r \sum_{i=0}^{\infty} (1+r)^{-(i+1)} x_{t+i}$$

where

$$\sigma = \frac{M_t - M_{t-1}}{P_t} = e^{a-\mu\eta} (1 - e^{-\mu}).$$

Notice that the less fiscally responsible the government is, the smaller $\sum_{i=0}^{\infty} (1+r)^{-(i+1)} x_{t+i}$ is. This implies that the less fiscally responsible the government is, the larger σ must be, and hence, the bigger μ and, thus, the more rapid, inflation must be. A simple graph of the relationship between σ and μ is found in Figure 1.

Thus, when we look across countries, we should expect to see higher inflation in countries with less fiscally responsible governments, as long as we measure inflation and primary balances over reasonably long horizons.

Fiscal and Monetary Policy Coordination In this section we extend the analysis of the previous section by considering the coordination of monetary and fiscal policy. In the previous section we saw that for a given country fiscal deficits and inflation need not be correlated with one another if the monetary authority chooses a constant money growth rule. Conditional on such a policy, we saw that the fiscal authority's choice for the path of the primary surplus determines the money growth rate and the inflation rate.

It is clear that one choice for the government and the central bank is to coordinate policy. They could agree on a desired inflation target, π , and the central bank could set the money growth rate consistent with π . The government, in turn, could ensure that its choice of the path for the primary surplus would be consistent with (1.9), given π .

Alternatively, it is interesting to considering the case of uncoordinated policy. In this case, like Sargent and Wallace (1981) we could imagine a fiscal authority that chooses $\{x_t\}_{t=0}^{\infty}$ without regard to any coordinated policy goals. The monetary authority, on the other hand, attempts to do what most central banks do: fight inflation. Initially the central bank fights inflation by setting a low value of the money growth rate. Eventually, however, in the world Sargent and Wallace imagined, the central bank has to face the reality of the fiscal authority's dominance. It must eventually accommodate the government's financing needs by printing more money. We are interested in the consequences of such a policy.

To determine the implications of uncoordinated policy it is helpful to adopt the quantity

theory specification of money demand, so that $M_t/P_t = y_t/v$.⁵ We will assume that the transactions motive for money demand is constant, i.e. $y_t = y$, so that real balances are also constant: $M_t/P_t = m = y/v$. In this setting if money growth is constant at some rate μ , it is clear that the inflation rate will be $\pi = \mu$, and that the real value of seigniorage will be constant: $\sigma_t = \sigma$, where

$$\sigma = \frac{M_t - M_{t-1}}{P_t} = \frac{(1 + \mu)M_{t-1} - M_{t-1}}{(1 + \mu)P_{t-1}} = \frac{\mu}{1 + \mu}m. \quad (2.8)$$

We will model the central bank's initial desire to be tough on inflation by assuming that from period 0 to some period T , it sets the money growth rate to some arbitrary "low" value, μ . However, at date T , the central bank accepts the inevitable, that it will have to print more money to ensure the government's solvency. Therefore, from date T forward, the central bank sets the money growth rate to a constant, μ' , consistent with satisfying the government's lifetime budget constraint.

With these assumptions we can easily solve for, and characterize, the path of inflation for $t \geq 0$. We will assume, for simplicity, that the fiscal authority sets $x_t = x$ for all t , and that $x < rb_{-1}$. The second assumption implies that some seigniorage revenue will be required in order for the lifetime budget constraint to be satisfied.

Notice that the government's lifetime budget constraint as of period $T + 1$ is

$$b_T = \sum_{t=T+1}^{\infty} (1+r)^{-(t-T)}(x_t + \sigma_t). \quad (2.9)$$

Since $\sigma_t = \mu'm/(1+\mu')$ for $t > T$, and $x_t = x$ for all t , this means that (2.9) can be rewritten as

$$b_T = \frac{x + \mu'm/(1 + \mu')}{r}. \quad (2.10)$$

Notice that if we solve for μ' we obtain

$$\mu' = \frac{rb_T - x}{m - (rb_T - x)}. \quad (2.11)$$

Clearly the higher the level of debt at date T , the higher μ' must be, since

$$\frac{d\mu'}{db_T} = \frac{rm}{[m - (rb_T - x)]^2} > 0.$$

⁵The results in this section, regarding the qualitative properties of the path of inflation, hold true for more general money demand specifications.

The budget constraint rolled forward from period 0 to period T is

$$b_{-1} = (1+r)^{-(T+1)}b_T + \sum_{t=0}^T (1+r)^{-(t+1)}(x_t + \sigma_t)$$

which we can rewrite as

$$b_T = (1+r)^{T+1}b_{-1} - \frac{(1+r)^{T+1} - 1}{r} \left(x + m \frac{\mu}{1+\mu} \right)$$

Notice that

$$\frac{\partial b_T}{\partial T} = \ln(1+r)(1+r)^{T+1} \left[b_{-1} - \frac{1}{r} \left(x + m \frac{\mu}{1+\mu} \right) \right], \quad (2.12)$$

which is positive if the central bank sets μ low enough. Also

$$\frac{\partial b_T}{\partial \mu} = -\frac{(1+r)^{T+1} - 1}{r} m(1+\mu)^{-2} < 0 \quad (2.13)$$

So the government accumulates more debt with lower μ .

Together these results imply that the tougher the monetary authority is initially (the lower it sets μ) and the longer it is tough (the higher is T), the greater the stock of debt (b_T) will be when it finally recognizes the necessity of satisfying the government's need for financing. But the greater the stock of debt b_T , the higher the inflation rate will eventually be.

The basic policy message that emerges from this discussion is that, the tougher the monetary authority tries to be on inflation in the near term, the higher inflation will be in the long term. Having stably low inflation requires the coordination of fiscal and monetary policies.

Getting Inflation Under Control The discussion so far has described inflation as a problem that stems from loose fiscal policy. In particular, when the government sets its primary balance too low, so that

$$\sum_{t=0}^{\infty} (1+r)^{-(t+1)} x_t \ll b_{-1}$$

the central bank is forced, at some point, to print money. The logical consequence of printing money is inflation. This suggests that in order for the government to reduce inflation it must impose some degree of fiscal discipline. In a world with uncertainty, the government must

not only impose fiscal discipline, it must convince other agents that it will remain disciplined in the future.⁶

Policy makers frequently argue that inflation stems from other root causes, and that it is very difficult for them to eliminate inflation once it becomes part of people's everyday lives. One often hears of the role played by private expectations. While it is possible to construct examples in which outcomes can depend on self-fulfilling changes in agents' expectations, the role played by expectations is often over-emphasized. As Sargent (1983) argues, the fundamentals, namely fiscal policy, are often the important determinant of private expectations. In particular, if the government announces a credible policy regime shift that involves a shift to a permanently better primary balance, inflation can be brought down, and it can be brought down quickly.

Sargent's argument is based on the shared experiences of Austria, Hungary, Poland and Germany after World War I. All four countries ran large deficits after the war, and experienced hyperinflations. All four countries used fiscal rather than strong monetary measures to end their hyperinflations. They renegotiated war debts and reparations payments, which represented a substantial part of their fiscal burdens, and they announced other fiscal measures to contain their budget deficits. However, even after the hyperinflations were over, the four governments continued, for several months, to print money at a healthy pace. By following this policy mix all four governments quickly stabilized their price levels and exchange rates.

The lesson from these case studies is straightforward. In the face of credible fiscal reforms, private expectations of future money growth will adjust and the price level and inflation will stabilize. While the example of the central European economies in the interwar period may not be perfectly analogous to today's developing and emerging market economies, the story is still valuable. If private expectations of inflation are entrenched in these markets, why are they entrenched? Our theory suggests that it must stem from fiscal difficulties. Our theory also suggests that without fundamental steps being taken to correct fiscal imbalances, no monetary policy that attempts to control inflation can be successful indefinitely.

⁶As we have seen with the Cagan money demand examples, the current price level depends on current and expected future money supplies. Hence, even if the government eliminates its use of monetary financing today, it must convince other agents that it will avoid monetary financing in the future.

3. The Fiscal Theory of the Price Level

A branch of macroeconomic theory that has recently become more popular, the fiscal theory of the price level, differs in its interpretation of the government's lifetime budget constraint. In fact, this theory would not even admit that the equation (1.9) represents a constraint on the government.

Interpreting the Lifetime Budget Constraint Let us return to (1.9) again, and consider how we obtained it. We started from the flow budget constraint, (1.3), which represents a simple accounting identity that holds under certain assumptions about the real interest rate and the structure of debt. From (1.3) we derived an intertemporal equation, (1.7). If we imposed the condition, (1.8), which we repeat here as

$$\lim_{j \rightarrow \infty} (1+r)^{-(j+1)} b_{t+j} = 0, \quad (3.1)$$

then the lifetime budget constraint, (1.9), emerged.

Equation (3.1) is often referred to as a no-Ponzi scheme condition on the government. So imposing the lifetime budget constraint on the government's behavior is often thought of as being equivalent to not allowing the government to run a Ponzi scheme. However, as McCallum (1984) points out, if we consider theoretical settings in which optimizing households are the potential holders of government debt, any violation of the government lifetime budget constraint, (1.9), implies either that the households are not optimizing, or that they are violating a no-Ponzi scheme condition imposed on them. As Cochrane (2001b) points out, the transversality, or no-Ponzi scheme, condition that we are familiar with from dynamic consumer problems is applied to households so that dynamic trading opportunities do not broaden the household budget set relative to having a single contingent claims market at time 0. So, it would appear that an additional constraint on the government is not needed to justify (1.9). And it would appear that this condition is the consequence of imposing rationality, and a no-Ponzi scheme condition on households.

To see this worked through suppose that we let $x_t + \sigma_t = -k$, so that debt increases in each period by an amount

$$b_t - b_{t-1} = rb_{t-1} + k. \quad (3.2)$$

If $k > -rb_0$, then debt grows over time with

$$\lim_{t \rightarrow \infty} (1+r)^{-t} b_t = b_0 + \frac{k}{r} > 0. \quad (3.3)$$

In simple dynamic macroeconomic models this would immediately imply a violation of the type of transversality (or no-Ponzi scheme) condition that we usually impose on households.

Importantly, however, allowing the government's debt to grow fast enough that (1.8) is violated also implies a violation of optimizing behavior on the part of households. Notice that the household's flow budget constraint will usually look like

$$\text{income} - \text{purchases} = (1 + r)^{-1}b_t - b_{t-1} \tag{3.4}$$

Notice that, in our example, this means

$$\text{income} - \text{purchases} = k$$

Clearly this means the household is doing something suboptimal, since it is voluntarily giving up a constant stream of income that could be used to permanently increase its consumption. In essence, this is the nature of argument in McCallum that we need not think of the government budget constraint as an additional constraint on the government.

Cochrane (2001b) presents a much more general argument than McCallum's. His argument is more complex, but makes clear why we can think about the government budget constraint as an equation for valuing government debt, rather than as a budget constraint. In the next section we will consider a similar, but simple argument, put forward by Christiano and Fitzgerald (2000).

Fiscal Theory in a Nutshell Christiano and Fitzgerald (2000) use a simple one-period model to describe the fiscal theory of the price level. Their model assumes that the world lasts for just one period. Households enter the period with real claims on the government, b . Households demand end-of-period claims on the government, b' .

Clearly households will not pick $b' > 0$ because the world ends at the end of period 1. The argument is simple. Any household, no matter what the exact specification of its budget constraint is, that demands $b' > 0$ is wasting resources—in essence this is the same point being made by McCallum (1984) in the context of a dynamic model.

If households were completely unconstrained in their choice of b' , they would pick $b' = -\infty$. So usually we impose $b' \geq 0$. That is, we do not allow households to end life with unpaid debt. This constraint, in a one-period model, is the natural analog to the no-Ponzi scheme condition in a dynamic infinite horizon model. Since optimizing households will not

choose $b' > 0$ and we constrain them to have $b' \geq 0$, it is clear that households will pick $b' = 0$ no matter what government policy is in the period.

In a one-period model, the government's budget constraint is simply

$$b' = b - x - \sigma \tag{3.5}$$

because b now represents total claims on the government, i.e. it represents principal *plus* interest. This is just a notational difference, and is of no consequence for the arguments being made.

The fact that households will pick $b' = 0$ for any combination of government policies implies that the government budget constraint is

$$b = x + \sigma. \tag{3.6}$$

Real Debt Suppose we are in the world similar to the one we described earlier, where government debt is a claim to a real quantities of goods. This means the value of b is fixed in real terms. This means that if the government chooses a “loose” fiscal policy (i.e. in the sense that $x \ll b$) it is clear than the monetary authority must provide the necessary “loose” monetary policy ($\sigma \gg 0$) to finance the government.

Here we have the arguments made in the previous section of this chapter in an incredibly simple form. If the government sets fiscal policy without regard to the price level or rate of inflation, the central bank must accommodate by printing money. When we saw this before in a dynamic setting, there was an issue of timing. The central bank faced a choice between inflation now or later, but the central bank could not avoid printing money eventually in the face of loose fiscal policy.

Nominal Debt Now imagine, instead, that, as is the case for most of the debt issued by the United States government, the government's debt represents a claim to a certain number of units of local currency. Now the government budget constraint is

$$B' = B - P(x + \sigma). \tag{3.7}$$

Since, again, households will not want to hold government debt at the end of the period, we will have $B' = 0$. This implies $B = P(x + \sigma)$, or

$$B/P = x + \sigma. \tag{3.8}$$

Notice, now, that the government and the central bank are free to choose any $x + \sigma$ combination subject to $x + \sigma > 0$. Given a policy commitment to x and σ , a beginning of period market for government debt will *induce* a price level P that satisfies the “government budget constraint.” In this sense, the government is not constrained in its actions and (3.8) does not represent a government budget constraint. Instead, (3.8) provides a way of valuing the nominal government debt issued in the previous period.

Is the Fiscal Theory Relevant? The curious reader might wonder whether the fiscal theory of the price level has any relevance to real world policy making. The astute reader might also wonder how we can have two equations determining the price level: the lifetime budget constraint and the solution for the price level derived from a money demand specification. The answers to these two questions are related. The basic relevance of the fiscal theory is established by the fact that many governments issue nominal debt. But the fact that the fiscal theory does not overdetermine the price level depends crucially on the assumption that some monetary policy rules do not specify an exogenous path for the money supply. In particular, according to some monetary policy rules, the money supply is endogenous.

Consider, for example, the case where money demand conforms to the Cagan specification we saw above, so that $M_t = Ay_t \exp(-\eta R_t) P_t$, where y_t is real output, R_t is the nominal interest rate and P_t is the price level. Suppose that output is constant, $y_t = y$ and that the central bank pegs the nominal interest rate at some value R . Clearly this policy requires that whatever the price level, P_t , the central bank must ensure that the monetary base is equal to $M_t = Aye^{-\eta R} P_t$. But notice that since this rule specifies the money supply in terms of the price level, this policy rule does not allow us to solve for the price level using (2.3).

This is where the government’s lifetime budget constraint comes in. Notice that real balances are constant and equal to $m = Aye^{-\eta R}$. If the real interest rate is some constant, r , the central bank’s peg of the nominal interest rate also implies that the inflation rate must be constant, π and equal to $\pi = (R - r)/(1 + r)$. This implies that the real value of seigniorage is

$$\begin{aligned} \sigma &= \frac{M_t - M_{t-1}}{P_t} = \frac{M_t}{P_t} - \frac{1}{1 + \pi} \frac{M_{t-1}}{P_{t-1}} \\ &= \frac{\pi}{1 + \pi} m. \end{aligned} \tag{3.9}$$

If we return to the dynamic model where (1.9) must hold notice that we now have

$$b_{t-1} = \sum_{i=0}^{\infty} (1+r)^{-(i+1)} x_{t+i} + \frac{1}{r} \frac{\pi}{1+\pi} m.$$

If the government's debt at the beginning of time t is denominated in units of local currency, and equals B_{t-1} , this means that

$$\frac{B_{t-1}}{P_t} = \sum_{i=0}^{\infty} (1+r)^{-(i+1)} x_{t+i} + \frac{1}{r} \frac{\pi}{1+\pi} m, \quad (3.10)$$

as long as there is no uncertainty beyond date t .

Notice that since π and m are determined by the central bank's peg of the nominal interest rate, this means the price level is determined by the present value of the stream of future primary surpluses plus seigniorage, relative to the quantity of government debt in circulation. The money supply can then be obtained by multiplying P_t by m .

Does the Fiscal Theory Lead to a Different Policy Message? The short answer to this question is no. The fiscal theory still sends the message that government deficits are inflationary if the government does not explicitly default on debt obligations. The only distinction between the fiscal theory and the earlier literature that we discussed above is that inflation can happen in one of two ways.

First, consistent with the discussion where debt was real, inflation could result from the central bank using a money supply rule that preserves the real value of the government's debt at the time it was issued. Second, inflation could result from the government issuing nominal debt the real value of which it does not attempt to preserve. In the face of a fiscal shock that reduces the present value of the government's future primary surpluses, the price level might jump in order that (1.9) holds.

4. Conclusion

In this opening chapter we have derived the government's lifetime budget constraint under some simple assumptions about government debt: time is discrete, debt is issued for one period, is real, and bears a constant real interest rate. We showed that under these assumptions fiscal sustainability revolves around the fiscal and monetary authorities setting the paths of the primary surplus, x_t , and the supply of base money, M_t , consistent with

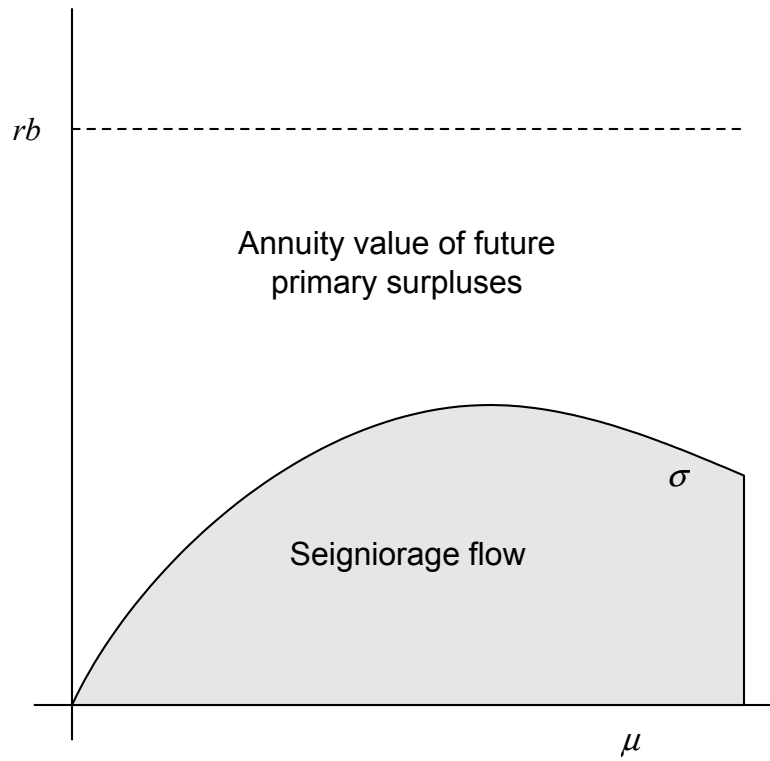
(1.9). We argued that there are many combinations of fiscal and monetary policy consistent with (1.9). However, we argued that the inevitable consequence of loose fiscal policy, $\sum_{t=0}^{\infty} (1+r)^{-(t+1)} x_t \ll b_{-1}$, is inflation. The monetary authority cannot fight inflation indefinitely without the cooperation of the fiscal authority. Thus, the goal of low inflation, combined with fiscal sustainability, can only be achieved if monetary and fiscal policy are coordinated. This message does not change when we consider alternative interpretations of the government's lifetime budget constraint, such as those provided by the fiscal theory of the price level.

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FIGURE 1

SEIGNIORAGE AS A FUNCTION OF THE MONEY GROWTH RATE



Note: Under the assumptions listed in the text, the constant seigniorage flow is σ , given a constant money growth rate μ .