

Some Tools for Fiscal Sustainability Analysis

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In the previous chapter we introduced the concept of the government budget constraint. We derived two versions of the government budget constraint: the flow version and the lifetime version. In this chapter we will use both versions of the constraint as tools for fiscal sustainability analysis.

In the first section we consider a simple tool based on the lifetime budget constraint: a long-run version of this constraint that *assumes* constant values of (i) the flow variables in the constraint relative to some measure of real output, say GDP, and (ii) the real interest rate and the growth rate of real output. This tool determines the stock of initial debt, relative to output, that a government can sustain, given particular values of the flow variables. It turns out that—given the assumptions on which this tool is based—the debt to GDP ratio will remain constant at its initial level, if that initial level is consistent with the lifetime budget constraint being satisfied. In this sense, the first tool can be interpreted in the following way: given the initial level of debt, it determines the size of the primary balance the government would need to run in order to keep its debt stock constant as a fraction of GDP.

In the second section we consider a simple variant of the first tool. This variant is designed to capture the fact that some governments set stronger fiscal targets for themselves than mere solvency. In some cases, a government might have a legal obligation to attempt to achieve a particular debt target in steady state. Another example, of particular relevance in recent years, is the set of fiscal rules imposed by the European Union (EU) on countries seeking EU accession. These rules require that a particular debt to GDP ratio must be achieved for a country to be accepted into the union. The second tool for fiscal sustainability analysis that we consider modifies the first tool, by calculating the size of the primary balance needed to achieve a particular debt to GDP target by a particular date.

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In the third section, we will discuss how the lifetime budget constraint can be used to form the basis of formal statistical tests of fiscal sustainability.

In the fourth section, we introduce the concept of debt dynamics. Simply stated, the term debt dynamics is used for analysis that attempts to relate changes in measured debt to GDP ratios to the flows that appear in the government budget constraint. As we will see, understanding changes in the debt to GDP ratio requires that we introduce a number of new features to the budget constraint to deal with such issues as the currency denomination of debt, the effects of recessions, and the effects of inflation on the real value of nominal debt.

The fifth section shows how some of the complications described in the section on debt dynamics could be used to modify the simple tools described in the first two sections. In particular, we will illustrate forward looking fiscal sustainability analysis that considers currency risk, time varying interest rates, and the effects of recessions.

Finally, the sixth section explores more thoroughly a variety of issues related to the measurement of debt and interest payments that arise in the earlier sections and Chapter 1.

1. The Long-Run Fiscal Sustainability Condition

In the previous chapter we assumed that time is discrete, that all debt has a maturity of one period, that debt is real, in the sense that its face value is indexed to the price level, and pays a constant real rate of interest, r . In this case the government's flow budget constraint can be written as

$$b_t = (1 + r)b_{t-1} - x_t - \sigma_t, \quad (1.1)$$

where b_t is the end-of-period t stock of real debt, x_t is the real primary surplus and σ_t is the real value of seigniorage revenue.

We showed that forward iteration on this equation combined with the condition

$$\lim_{j \rightarrow \infty} (1 + r)^{-(j+1)} b_{t+j} = 0 \quad (1.2)$$

implies

$$b_{t-1} = \sum_{i=0}^{\infty} (1 + r)^{-(i+1)} (x_{t+i} + \sigma_{t+i}). \quad (1.3)$$

We called this equation the government's lifetime budget constraint. This equation states that the government finances its debt at the end of period $t - 1$, by, from date t forward, raising seigniorage revenue and running primary surpluses with an equal present value.

The most basic tool for fiscal sustainability analysis uses a steady state version of the lifetime budget constraint, (1.3). To begin, consistent with the presentation in most World Bank and IMF documents, it is useful to rewrite (1.3) in terms of stocks and flows expressed as fractions of GDP. Letting y_t represent real GDP, define $\bar{b}_t = b_t/y_t$, $\bar{x}_t = x_t/y_t$ and $\bar{\sigma}_t = \sigma_t/y_t$. Given this notation we can rewrite (1.3) as

$$\bar{b}_{t-1}y_{t-1} = \sum_{i=0}^{\infty} (1+r)^{-(i+1)} (\bar{x}_{t+i} + \bar{\sigma}_{t+i}) y_{t+i} \quad (1.4)$$

or

$$\bar{b}_{t-1} = \sum_{i=0}^{\infty} (1+r)^{-(i+1)} (\bar{x}_{t+i} + \bar{\sigma}_{t+i}) \frac{y_{t+i}}{y_{t-1}}. \quad (1.5)$$

Imagine a steady state in which (i) real GDP grows at a constant rate g , so that $y_t/y_{t-1} = 1+g$, (ii) the primary surplus as a fraction of GDP is a constant \bar{x} , and (iii) seigniorage as a fraction of GDP is a constant $\bar{\sigma}$. In this case, (1.5) reduces to

$$\bar{b}_{t-1} = \sum_{i=0}^{\infty} \left(\frac{1+g}{1+r} \right)^{i+1} (\bar{x} + \bar{\sigma}). \quad (1.6)$$

Assuming that $r > g$, (1.6) reduces to

$$\bar{b}_{t-1} = \bar{b} \equiv (\bar{x} + \bar{\sigma})/\bar{r}, \quad (1.7)$$

where $\bar{r} = (r-g)/(1+g)$.

We will consider two ways in which one could imagine using (1.7) to assess fiscal sustainability. First, one could imagine making assumptions about “reasonable” values of \bar{x} , $\bar{\sigma}$, r and g . These assumptions might be based on historical trends in the country’s fiscal accounts, as well as typical historical values of seigniorage revenue, the real interest rate and the real growth rate. Together these assumptions could be mapped into an estimate of \bar{b} using (1.7). If the government’s actual stock of debt exceeded this estimate, then the government’s finances could be argued to be unsustainable.

Alternatively, (1.7) could be rearranged as

$$\bar{x} = \bar{r}\bar{b}_{t-1} - \bar{\sigma}. \quad (1.8)$$

Given estimates of \bar{r} and $\bar{\sigma}$ and data on the size of the government’s actual debt stock, \bar{b}_{t-1} , (1.8) could be used to determine the necessary size of the primary balance to ensure fiscal sustainability. That is, rather than setting \bar{x} equal to some historical average, one could

determine the value that \bar{x} would need to take in the future in order to maintain sustainable finances.

A final interpretation of (1.8) is obtained by noting that if \bar{x} were set consistent with it, then the debt to GDP ratio would remain constant in the steady state described earlier. In other words, when \bar{x} , $\bar{\sigma}$, r and g are constant, and \bar{x} is given by (1.8), not only will the government's finances be sustainable, but it will also be true that \bar{b}_t will be constant and equal to \bar{b} . This follows trivially from (1.1) and the definition of \bar{b}_t .

An Example To put the tools discussed in this section into practice, we will consider an example, Bulgaria, which, at the end of 2001, had a debt stock estimated to be about 72 percent of GDP. So we could think of period t as 2002, and set $\bar{b}_{t-1} = 0.72$. We will use equation (1.8) to determine the primary balance Bulgaria would have needed to run from that date forward in order to achieve fiscal sustainability. To do this, we need benchmark values for the parameters that determine the right-hand side of (1.8): the real interest rate, r , and the real growth rate, g , that determine \bar{r} , and the flow of seigniorage relative to GDP, $\bar{\sigma}$. The analysis will be conducted from the perspective of an economist working with available data in early 2002.

Interest Rates To determine a benchmark value of the real interest rate, we will first consider the nominal interest rate R , and the inflation rate, π , and will compute the real interest rate from the equation $r = (R - \pi)/(1 + \pi)$. In 2001, Bulgaria was fortunate in that the cost of servicing its debt was relatively modest: the public sector interest bill was well below 10 percent of the public sector debt stock between 1998 and 2001. In 2001, total interest payments were projected to be about 4.1 percent of GDP, which represented about 5 percent of the debt stock at the end of 2000. This might suggest that R should be set equal to 0.05, but, given the structure of Bulgaria's debt at the end of 2001, we must be more careful.

Most of Bulgaria's public debt at the end of 2001 was external debt owed to official creditors, or was in the form of Brady bonds, with only a small fraction of the debt being denominated in lev. Much of this external debt paid interest at rates that should be regarded as somewhat concessional. In the longer-run, as Bulgaria made the transition towards integration with the EU, and being a more fully developed market economy, two important factors were likely to influence interest rates. First, less and less of Bulgaria's debt would

be concessional, especially once the Brady bonds were retired. Since this older debt was likely to be rolled into newer debt at higher interest rates, this would tend to raise Bulgaria's interest costs. On the other hand, Bulgaria's credit rating in private markets was likely to improve, so yields on its privately held debt were likely to come down.

This discussion implies that benchmarking an interest rate for Bulgaria that would apply to a long-run steady state could be quite difficult. In late 2001 secondary market yields on Bulgarian Bradys were at or near 12 percent. On the other hand, because of its substantial amount concessional debt, Bulgaria's average interest cost was much lower. Here we will assume that $R = 0.085$, an assumption that lies above Bulgaria's effective interest cost in 2001, but well below its marginal borrowing cost in private markets.

Inflation The inflation rate in Bulgaria was 7 percent in 1999, 11 percent in 2000 and about 4.5 percent in 2001. Bulgaria had (and has) a fixed exchange rate vis-a-vis the euro, so some further convergence towards European inflation rate might have been expected.¹ In early 2002, inflation projections for Bulgaria for the coming two years ran at about 3.5 percent. Although some further modest decline might have been expected beyond that horizon, a working assumption is that the inflation rate, π , would be 0.035 in perpetuity. Together with the assumption about the nominal interest rate, this implies a real interest rate, $r \approx 0.048$.

Growth In the 1998-2000 period, Bulgaria averaged real GDP growth of 3.9 percent, despite the poor external environment it faced during this period resulting from the Russian, Kosovo, and Turkish crises. Thus, it would have been reasonable, in 2002, to assume an average real GDP growth rate of at least 4 percent. So we will consider $g = 0.04$.² Our assumptions about the real interest rate and growth imply that $\bar{r} \approx 0.008$.

¹At the time euro-zone inflation rates had been in the low single digit range: in 2000-01 inflation in the euro-zone was about 3 percent, and was expected to average about 2 percent in 2002-03.

²At the time, the government targeted a higher growth rate on the order of 5 to 6 percent.

Seigniorage Seigniorage is the change in reserve money.³ In Bulgaria, in 1998, 1999 and 2000, the change in reserve money represented about 1.0, 1.5 and 1.2 percent of GDP respectively. In 2001, reserve money grew much faster (from 3.0 to 4.0 billion lev), meaning that seigniorage was well over 3 percent of GDP. An economist looking forward from 2001, would have considered it unlikely that seigniorage would be collected at the pace seen in 2001, especially in a long-run steady state. The argument is simple: suppose base money remained constant as a fraction of GDP, i.e. $M_t = \bar{m}P_t y_t$, where P_t is the price level and y_t is real GDP. If inflation and growth were constant and equal to π and g respectively, then, seigniorage revenue would be a constant fraction of GDP:

$$\bar{\sigma} = \frac{\pi + g + \pi g}{(1 + \pi)(1 + g)} \bar{m} \quad (1.9)$$

In early 2002, base money was about 14 percent of GDP. If we were to assume that $\bar{m} = 0.14$ and, as above, that $\pi = 0.035$, and $g = 0.04$, then seigniorage would be about 1 percent of GDP, i.e. $\bar{\sigma} \approx 0.01$.

Summary Assessment Combining the assumptions that $R = 0.085$, $\pi = 0.035$, $g = 0.04$, and $\bar{m} = 0.14$, along with the fact that $\bar{b}_{t-1} = 0.72$, imply, from (1.8) and (1.9), that $\bar{x} \approx -0.0042$. This would mean that Bulgaria could achieve fiscal sustainability, along with a constant debt to GDP ratio of 72 percent, by running a primary balance of -0.42 percent of GDP.

A summary assessment such as this provides a useful benchmark, but is obviously very dependent on the assumptions of the analysis. In reality, one would want to be cautious and examine the sensitivity of \bar{x} to the assumptions being made. What if growth were slower? What if the nominal interest rate on debt were greater? These questions would need to be addressed. Furthermore, as we will see in the next section, simple analysis like this may well miss important policy goals of the government. In Bulgaria's case, the government wished to achieve a much more ambitious goal than simple constancy of the debt to GDP ratio: it set the EU's 60 percent debt limit as its target to be achieved within 4 years. Achieving this

³Some readers will be familiar with the fact that Bulgaria's fixed exchange rate is supported by a currency board arrangement. Under a currency board reserve money is backed with foreign assets. This does not have any direct implications for the government budget constraint—equation (1.1) still holds—but it does have implications for the government's debt composition. In particular, more reserve money implies greater holdings of liquid foreign assets by the consolidated public sector. In this sense, seigniorage is a revenue source which is tied to a particular use—the acquisition of foreign assets. This issue aside, seigniorage is still a revenue source for the public sector.

goal would require a much tighter primary balance.

2. Achieving a Target Debt Level

Suppose that a country has a debt to GDP ratio at the end of period t equal to \bar{b}_t and wishes to achieve some specified debt target within J periods, equal to \bar{b}^* . We can divide both sides of (1.1) by real GDP to describe the evolution of the debt to GDP ratio:

$$\bar{b}_t = (1 + r)\bar{b}_{t-1}y_{t-1}/y_t - \bar{x}_t - \bar{\sigma}_t.$$

Under the assumption of constant real growth at the rate g , we have

$$\bar{b}_t = (1 + \bar{r})\bar{b}_{t-1} - \bar{x}_t - \bar{\sigma}_t, \quad (2.1)$$

where, as above, $\bar{r} = (r - g)/(1 + g)$.

If we iterate on (2.1) from time t to time $t + J$, we obtain the equation

$$\begin{aligned} \bar{b}_t &= (1 + \bar{r})^{-1}\bar{b}_{t+1} + (1 + \bar{r})^{-1}(\bar{x}_{t+1} + \bar{\sigma}_{t+1}) \\ &= (1 + \bar{r})^{-2}\bar{b}_{t+2} + (1 + \bar{r})^{-1}(\bar{x}_{t+1} + \bar{\sigma}_{t+1}) + (1 + \bar{r})^{-2}(\bar{x}_{t+2} + \bar{\sigma}_{t+2}) \\ &= \dots = (1 + \bar{r})^{-J}\bar{b}_{t+J} + \sum_{i=1}^J (1 + \bar{r})^{-i}(\bar{x}_{t+i} + \bar{\sigma}_{t+i}). \end{aligned} \quad (2.2)$$

Assume that the government runs a constant primary balance, \bar{x} , and obtains constant seigniorage revenue, $\bar{\sigma}$, between period $t + 1$ and period $t + J$. If the government is to achieve the debt target \bar{b}^* by period $t + J$, then it is clear from (2.2) that its primary balance must be at least as large as

$$\bar{x} = \bar{r} \frac{(1 + \bar{r})^J \bar{b}_t - \bar{b}^*}{(1 + \bar{r})^J - 1} - \bar{\sigma}. \quad (2.3)$$

Notice that if $\bar{r} = 0$ we must replace (2.3) with the equation $\bar{x} = (\bar{b}_t - \bar{b}^*)/J - \bar{\sigma}$.

A Modified Example Suppose we reconsider the example of Bulgaria from the previous section. There we solved for the value of \bar{x} consistent with fiscal sustainability in a steady state. This was equivalent to solving for the value of \bar{x} consistent with maintaining a constant debt to GDP ratio at 72 percent of GDP.

In 2002, the Bulgarian government set a goal of achieving the EU's debt ceiling (60 percent of GDP) prior to 2007, the date at which it hoped to achieve EU accession. In fact, the date it set as its target for achieving this target was 2005. Thus, we might think of using (2.3) to solve for \bar{x} , while maintaining some of the other assumptions of the previous section.

If we treat time t as 2001, and time $t + J$ as 2005, we have $J = 4$. We also have $\bar{b}_t = 0.72$ and $\bar{b}^* = 0.6$. Consistent with our example in the previous section, we can set $\bar{\sigma} = 0.01$, and $\bar{r} \approx 0.008$. Solving for \bar{x} using (2.3) we obtain $\bar{x} \approx 0.0255$. Thus, to achieve its goal, the government would need to run a primary surplus of about 2.55 percent of GDP. This result stands in marked contrast to the one in our previous example because here the government has a goal of reducing its debt stock by 12 percentage points of GDP in 4 years, while there the government only wanted to hold its debt stock constant as a percentage of GDP.

At this stage it makes some sense to consider how sensitive the estimated value of \bar{x} is to the assumptions underlying it. One might expect that the most important of these assumptions are the ones that matter for \bar{r} : the assumptions about the real interest rate, r , and the economy's real growth rate, g . In the baseline example we assumed these were 4.8 and 4 percent respectively. Table 1 provides some idea of how sensitive the results are to various assumptions about real interest rates and real growth. It computes \bar{x} for a variety of combinations of r and g .

Table 1 shows that the estimated primary surplus is very sensitive to the assumptions about growth and interest rates. Roughly speaking, for each additional percentage point of growth the government has room to run a smaller primary surplus by about 0.8 percent of GDP. For each additional percentage point on the interest rate, the required primary surplus rises by about 0.65 percent of GDP. This indicates that if a country in Bulgaria's position were to suffer a sustained rise in interest rates, or a sustained decline in its growth, it would need to run a substantially larger primary surplus to meet its debt target.

There are a number of other dimensions along which we could be critical of the model we have used to assess sustainability to this point. First, despite Bulgaria's changing debt profile we have assumed that the real interest rate is constant—we should probably have modeled it as increasing over time. Second, we ignored the fact that Bulgaria does not have real debt—it either has debt denominated in lev (the local currency), euro, or, for the most part, US dollars. Yet our analysis did not allow for currency risk or the effects of unexpected inflation on debt. Third, we did not examine issues concerning the maturity and revaluation of debt. Some of these issues will be dealt with later in this chapter. However, next we turn to formal statistical tests of fiscal sustainability.

3. Formal Statistical Tests

Nearly all formal statistical tests of fiscal sustainability stem from Hamilton and Flavin's (1986) analysis, which was applied to the US government's debt. To understand their approach, we return to the government's flow budget constraint described in Chapter 1. There we saw the simple accounting identity

$$B_t - B_{t-1} = I_t - X_t - (M_t - M_{t-1}). \quad (3.1)$$

where B_t is the end of period stock of debt measured in local currency, I_t is interest payments, X_t is the primary balance and $M_t - M_{t-1}$ is seigniorage revenue. In Chapter 1 we were not careful to define what we meant by "the stock of debt." Later, when we defined $b_t = B_t/P_t$ and derived the real flow budget constraint, we made a number of assumptions about government debt—that time is discrete, that bonds have a constant maturity of one year and bear a constant real interest rate—that Hamilton and Flavin avoid.

If we were to put (3.1) in real terms by defining $i_t = I_t/P_t$, $x_t = X_t/P_t$ and $\sigma_t = (M_t - M_{t-1})/P_t$ then we would get:

$$b_t - \frac{1}{1 + \pi_t} b_{t-1} = i_t - x_t - \sigma_t, \quad (3.2)$$

where $\pi_t = P_t/P_{t-1} - 1$. Suppose, rather than assuming there is a constant real interest rate, that we define r as the *average ex-post* real interest rate on government debt over a one year horizon. Then we can rewrite (3.2) as

$$b_t = (1 + r)b_{t-1} - x_t - \sigma_t + i_t - \left(\frac{r + \pi_t + r\pi_t}{1 + \pi_t} \right) b_{t-1}. \quad (3.3)$$

Notice that in (3.3), because we are not assuming that the ex-post real interest rate is constant, there is an error term $i_t - (r + \pi_t + r\pi_t)b_{t-1}/(1 + \pi_t)$ that does not appear in (1.1). Hamilton and Flavin (1986) further point out that if one is careful to define b_t as the real *market value* of government debt held by the public at the end of period t , then one may need an additional term on the right-hand side that measures the capital loss (if positive) or gain (if negative) from any changes in the market value of existing government debt.⁴ One also needs a term to deal with any asymmetry of timing, within the year, between the

⁴We will see, in Section 6, that if the market value of the government's debt is used in defining B_t , then (3.1) only holds exactly if I_t is defined using an accounting procedure that takes into account capital gains or losses on debt. Since this procedure is unlikely to be used in practice, an error term must be added to (3.3).

issuance of debt, and the flows that are debt creating or reducing. In conclusion, Hamilton and Flavin argue that, regardless of the structure of the debt and the behavior of interest rates,

$$b_t = (1 + r)b_{t-1} - x_t - \sigma_t + v_t, \quad (3.4)$$

where v_t is an error term. Given the discussion above, it is arguable that this error term should be mean zero.

Forward iteration on (3.4) implies that

$$b_t = (1 + r)^{-j}b_{t+j} + \sum_{i=1}^j (1 + r)^{-i}(x_{t+i} + \sigma_{t+i} - v_{t+i}). \quad (3.5)$$

Hamilton and Flavin point out that the hypothesis that the government is subject to a present-value borrowing constraint

$$b_t = E_t \sum_{i=1}^{\infty} (1 + r)^{-i}(x_{t+i} + \sigma_{t+i} - v_{t+i}) \quad (3.6)$$

is equivalent to the hypothesis that

$$\lim_{j \rightarrow \infty} (1 + r)^{-j} E_t b_{t+j} = 0. \quad (3.7)$$

The condition in (3.6) is similar to the lifetime budget constraint we derived in Chapter 1, yet it is expressed in expectational form. Of course, (3.6) is also the pricing equation for government debt according to the fiscal theory of the price level.

Hamilton and Flavin propose a test of the null hypothesis that (3.6) holds. We could interpret such a test as a test of fiscal sustainability. Formally, Hamilton and Flavin test the equivalent null hypothesis, that (3.7) holds against an alternative:

$$\lim_{j \rightarrow \infty} (1 + r)^{-j} E_t b_{t+j} = a(1 + r)^t$$

with $a \neq 0$. So their test boils down to whether $a = 0$ or $a \neq 0$. Notice that if the alternative hypothesis is correct we can write

$$b_t = E_t \sum_{i=1}^{\infty} (1 + r)^{-i}(x_{t+i} + \sigma_{t+i} - v_{t+i}) + a(1 + r)^t. \quad (3.8)$$

Defining $\eta_t = -E_t \sum_{i=1}^{\infty} (1 + r)^{-i} v_{t+i}$ and $z_t = x_t + \sigma_t$, (3.8) can be written as

$$b_t = E_t \sum_{i=1}^{\infty} (1 + r)^{-i} z_{t+i} + a(1 + r)^t + \eta_t. \quad (3.9)$$

In related work, Flood and Garber (1980) proposed a test for bubbles in prices. They considered the Cagan (1956) model of money demand, in which the fundamentals-based solution for the price level is the conditional expectation of a forward looking discounted sum of future money supplies.⁵ Their fundamentals-based solution for prices is analogous to the expression for debt in our equation (3.6). They also considered a nonfundamental solution for prices in which the fundamental solution is augmented with a bubble term like the $a(1+r)^t$ term that appears on the right-hand side of (3.9). Thus, Hamilton and Flavin's tests for fiscal sustainability are similar to tests for bubbles in prices.

The first test that Hamilton and Flavin perform is based on Diba and Grossman (1984) and Hamilton and Whiteman's (1985) critique of Flood and Garber's tests. Hamilton and Whiteman propose a test for bubbles based on tests for nonstationarity. In our context, these tests can be summarized as follows. Suppose that $E_t \sum_{i=1}^{\infty} (1+r)^{-j} z_{t+i}$ and η_t are stationary process. Then b_t will be stationary if and only if $a \neq 0$. So Hamilton and Flavin propose a simple test of fiscal sustainability requiring the following steps:

1. Assume that η_t is stationary (this seems like a reasonable assumption given its definition) and test for a unit root in z_t using a Dickey and Fuller (1979)-type test statistic. If you can reject the presence of a unit root in z_t , then z_t is apparently stationary, one can assume that $E_t \sum_{i=1}^{\infty} (1+r)^{-j} z_{t+i}$ is also stationary, and one should proceed to the second step.

2. Test for a unit root in b_t . If you cannot reject the presence of a unit root, then (3.6) is rejected in favor of (3.9) with $a \neq 0$. If you can reject the presence of a unit root, then (3.6) apparently holds.

Of course, this simple test suffers from all the well-known drawbacks of unit root tests. In particular, that they are generally weak against alternatives close to a unit root.

The second test proposed by Hamilton and Flavin is based on a regression. Here they assume that time t expectations of v_{t+i} and z_{t+i} are rationally formed by projecting them on current and lagged values of z_t and lagged values of b_t . In this case we can rewrite (3.9) as

$$b_t = \beta_0 + \sum_{i=1}^{k_b} \gamma_i b_{t-i} + \sum_{i=0}^{k_z} \kappa_i z_{t-i} + a(1+r)^t + \varepsilon_t$$

where ε_t is a white noise error term. The constants k_b and k_z and the parameters γ_i , $i = 1, \dots, k_b$ and κ_i , $i = 1, \dots, k_z$ are determined as functions of r and the coefficients in the rules

⁵Their model is the same as the one we considered in Chapter 1, with the additional feature of uncertainty regarding future money supplies.

for rationally forecasting v_t and z_t . A test for fiscal sustainability, in this case, is simply a test of whether $a = 0$ or not.

Obviously, the formal statistical tests described here are much more complex than the simple methods proposed in Sections 1 and 2. The question is whether they are more useful. Their main advantage of statistical tests is that they do not involve the arbitrary assumptions we made in previous sections: there is no need to assume the economy is in a long-run steady state, or that growth and the real interest rate are constant.

On the other hand, statistical methods have a number of drawbacks:

1. Since these methods are based on testing for unit roots or explosive trends in the data, they suffer from the inherent weakness of tests for these phenomena against alternative hypotheses that are nearly, but not, explosive.

2. The tests are based on historical data and are, therefore, fundamentally backward looking. They identify, in a sense, violations of fiscal sustainability in the past, but say little, in a forward looking way, about the state of the government's finances. They also require substantial amounts of historical data, something that is frequently lacking for developing and industrializing countries.

3. Ultimately, these tests are very narrowly focused on the question of whether (??) holds. Suppose we look at the data for a country and find no evidence against (??) in the historical data. Does this mean fiscal sustainability is a non-issue? Surely the answer to this question is "no." In the end, the question of fiscal sustainability comes down to whether *future* surpluses will be sufficient to service the most recently observed debt level. As we argued in Chapter 1, fiscal sustainability analysis is also meant to deal with broader issues than whether the government will satisfy its lifetime budget constraint or not. So, in the end, we must step beyond the statistical tests presented in this section if we are to provide useful policy analysis.

To conduct policy analysis in developing countries, the simple approaches in Sections 1 and 2 are perhaps more useful than statistical tests. Nonetheless they are, perhaps, too simple. In Section 5 we will explore extensions of these methods that abstract from a number of the simplifying assumptions we have made to this point.

4. Debt Dynamics

The term *debt dynamics* refers to the study of the evolution of the measured debt to GDP ratio. Often it is useful to describe how debt levels have evolved in the past, as this provides a useful perspective on the future. It can be helpful in describing why a country that has found itself with a debt problem got into that situation in the first place. It also helps identify what a country might have to do in order to get itself out of a debt problem. Debt dynamics are also useful in illustrating sources of risk to a government's finances.

4.1. Theoretical Background

A Baseline Case In Chapter 1 we began from the simple budget identity, expressed in local currency units:

$$\text{net debt issued} = I_t - X_t - (M_t - M_{t-1}). \quad (4.1)$$

Here I_t represents interest payments, X_t is the primary surplus and $M_t - M_{t-1}$ represents seigniorage revenue. We will begin by making the following assumptions: (i) debt is issued for one period only, (ii) all debt is issued in local currency units at nominal interest rates, which may vary from period to period. The second assumption is different than in our analysis of Chapter 1, and in Sections 1 and 2 of this chapter, where we assumed debt was real (or indexed to the price level) and issued at constant real interest rates.

As Section 6 shows, if we define B_t to be the market value of the government's stock of debt at the end of period t , then the market value and book value of the government's debt are the same, and net debt issued equals $B_t - B_{t-1}$. In this case, we can write

$$B_t - B_{t-1} = I_t - X_t - (M_t - M_{t-1}). \quad (4.2)$$

Typically we do not want to describe the evolution of the nominal stock of government debt. Instead we want to describe the evolution of its ratio to GDP. So, as in Chapter 1, we will let P_t and y_t represent the GDP deflator and real GDP, and define $\bar{b}_t = B_t/(P_t y_t)$, $\bar{i}_t = I_t/(P_t y_t)$, $\bar{x}_t = X_t/(P_t y_t)$ and $\bar{\sigma}_t = (M_t - M_{t-1})/(P_t y_t)$. If we divide both sides of (4.2) by $P_t y_t$ we obtain

$$\bar{b}_t - \frac{P_{t-1} y_{t-1}}{P_t y_t} \bar{b}_{t-1} = \bar{i}_t - \bar{x}_t - \bar{\sigma}_t. \quad (4.3)$$

As in Chapter 1, we define the inflation rate, $\pi_t = P_t/P_{t-1} - 1$, and the growth rate of real GDP, $g_t = y_t/y_{t-1} - 1$. Suppose, for the moment, that these are both zero. Then

$$\bar{b}_t - \bar{b}_{t-1} = \bar{i}_t - \bar{x}_t - \bar{\sigma}_t. \quad (4.4)$$

In this case, the change in the debt-to-GDP ratio consists of three components, (i) interest payments, (ii) the primary balance and (iii) seigniorage, all expressed as ratios to GDP.

Nonzero Inflation Suppose that $\pi_t \neq 0$, but $g_t = 0$. Then, with some manipulation, (4.3) can be rewritten as

$$\bar{b}_t - \bar{b}_{t-1} = \bar{i}_t - \bar{x}_t - \bar{\sigma}_t - \frac{\pi_t}{1 + \pi_t} \bar{b}_{t-1}. \quad (4.5)$$

In this case, the change in the debt-to-GDP ratio includes a fourth term, which we will refer to as the *inflation effect*. What is the inflation effect? Recall that we have assumed all debt is issued in local currency, and is issued at a fixed interest rate. This means that interest payments at time t are given by $I_t = R_{t-1}B_{t-1}$. Notice that this also means $\bar{i}_t = R_{t-1}\bar{b}_{t-1}/(1 + \pi_t)$, given our assumption that $g_t = 0$. The sum of the interest payments term and the inflation effect term is $(R_{t-1} - \pi_t)\bar{b}_{t-1}/(1 + \pi_t)$. In a sense, this sum represents ex-post *real* interest payments on the debt issued at time $t - 1$. The inflation effect captures the fact that inflation erodes the real cost to the government of servicing its local currency debt.

Nonzero Growth Now suppose we assume $\pi_t = 0$, but $g_t \neq 0$. Then, with some manipulation, (4.3) can be rewritten as

$$\bar{b}_t - \bar{b}_{t-1} = \bar{i}_t - \bar{x}_t - \bar{\sigma}_t - \frac{g_t}{1 + g_t} \bar{b}_{t-1}. \quad (4.6)$$

In this case, the change in the debt-to-GDP ratio includes a term that we will refer to as the *growth effect*. The growth effect captures the fact that in and of itself, if GDP gets bigger this makes the ratio of any previously accumulated debt to GDP smaller.

The General Case Unfortunately, when $\pi_t \neq 0$ and $g_t \neq 0$, it is impossible to perfectly decompose the extra term into inflation and growth effects. This is because the two effects interact with one another. For notational convenience we define the growth rate of nominal GDP, $z_t = (1 + \pi_t)(1 + g_t) - 1$. Using this definition, we can manipulate (4.3) to obtain

$$\bar{b}_t - \bar{b}_{t-1} = \bar{i}_t - \bar{x}_t - \bar{\sigma}_t - \frac{z_t}{1 + z_t} \bar{b}_{t-1}. \quad (4.7)$$

We will rewrite (4.7) as

$$\bar{b}_t - \bar{b}_{t-1} = \bar{i}_t - \bar{x}_t - \bar{\sigma}_t - \frac{\pi_t}{1 + \pi_t} \bar{b}_{t-1} - \frac{g_t}{1 + z_t} \bar{b}_{t-1}. \quad (4.8)$$

Now the change in the debt-to GDP ratio is the sum of five components: (i) interest payments, (ii) the primary balance, (iii) seigniorage, (iv) the inflation effect and (v) the growth effect. Notice, however, that we have redefined the growth effect as $-g_t \bar{b}_{t-1}/(1+z_t)$, which depends on the rate of inflation as well as the real growth rate.

Indexed Debt Suppose that some part of the government's debt is indexed to the price level. In particular, suppose that at time t the government issues indexed debt, B_t^I , and this implies that at time $t+1$ the government repays $(1+r_t)(1+\pi_{t+1})B_t^I$ to the holders of the debt. If we use the notation B_t^N to represent nonindexed debt, then we have

$$B_t^N + B_t^I = (1+R_{t-1})B_{t-1}^N + (1+r_{t-1})(1+\pi_t)B_{t-1}^I - X_t - (M_t - M_{t-1}). \quad (4.9)$$

Hence, with some algebra we can write

$$\bar{b}_t - \bar{b}_{t-1} = \bar{i}_t - \bar{x}_t - \bar{\sigma}_t - \frac{z_t}{1+z_t} \bar{b}_{t-1} + \left[\frac{R_{t-1}}{1+z_t} \bar{b}_{t-1}^N + \frac{r_{t-1} + \pi_t(1+r_{t-1})}{1+z_t} \bar{b}_{t-1}^I - \bar{i}_t \right], \quad (4.10)$$

where $\bar{b}_t = \bar{b}_t^N + \bar{b}_t^I$. As in the previous examples, the inflation and growth effects, which add up to $-z_t \bar{b}_{t-1}/(1+z_t)$, still apply to the entire stock of debt. However, the full breakdown of the change of debt into its components depends on the government's definition of interest payments, \bar{i}_t . This definition may or may not correspond to

$$\frac{R_{t-1}}{1+z_t} \bar{b}_{t-1}^N + \frac{r_{t-1} + \pi_t(1+r_{t-1})}{1+z_t} \bar{b}_{t-1}^I.$$

If it does, then the analysis is unaffected by the indexation of debt. If it does not then the last term on the right-hand side of (4.10) will be nonzero.

First Accounting Rule If $I_t = R_{t-1}B_{t-1}^N + r_{t-1}B_{t-1}^I$ then $\bar{i}_t = (R_{t-1}\bar{b}_{t-1}^N + r_{t-1}\bar{b}_{t-1}^I)/(1+z_t)$. Hence we can write

$$\bar{b}_t - \bar{b}_{t-1} = \bar{i}_t - \bar{x}_t - \bar{\sigma}_t - \frac{z_t}{1+z_t} \bar{b}_{t-1} + \frac{\pi_t(1+r_{t-1})}{1+z_t} \bar{b}_{t-1}^I. \quad (4.11)$$

Now, in addition to interest payments, the primary balance, seigniorage, the inflation effect and the growth effect, we need a term $\pi_t(1+r_{t-1})\bar{b}_{t-1}^I/(1+z_t)$ to capture the fact that investors in indexed bonds are compensated for inflation, both in the interest and principal, and this is not captured in \bar{i}_t .

Second Accounting Rule If $I_t = R_{t-1}B_{t-1}^N + (1 + \pi_t)r_{t-1}B_{t-1}^I$ then $\bar{v}_t = [R_{t-1}\bar{b}_{t-1}^N + (1 + \pi_t)r_{t-1}\bar{b}_{t-1}^I]/(1 + z_t)$. This implies that

$$\bar{b}_t - \bar{b}_{t-1} = \bar{v}_t - \bar{x}_t - \bar{\sigma}_t - \frac{z_t}{1 + z_t}\bar{b}_{t-1} + \frac{\pi_t}{1 + z_t}\bar{b}_{t-1}^I \quad (4.12)$$

Now the extra term in the debt dynamics is $\pi_t\bar{b}_{t-1}^I/(1 + z_t)$. It captures the fact that the definition of \bar{v}_t includes the interest payment component, but not the amortization component, of the inflation compensation paid to investors in indexed bonds.

Third Accounting Rule If $I_t = R_{t-1}B_{t-1}^N + [\pi_t + (1 + \pi_t)r_{t-1}]B_{t-1}^I$ then $\bar{v}_t = \{R_{t-1}\bar{b}_{t-1}^N + [\pi_t + (1 + \pi_t)r_{t-1}]\bar{b}_{t-1}^I\}/(1 + z_t)$. This implies that

$$\bar{b}_t - \bar{b}_{t-1} = \bar{v}_t - \bar{x}_t - \bar{\sigma}_t - \frac{z_t}{1 + z_t}\bar{b}_{t-1}. \quad (4.13)$$

Now there is no need for an extra term in the debt dynamics, because the definition of \bar{v}_t already captures all inflation compensation paid to investors in indexed bonds.

In summary, indexed debt affects our analysis depending on how the government treats inflation compensation in its accounts. If all such compensation is included in interest payments, then debt dynamics calculations are unaffected. Otherwise, we need to explicitly account for inflation compensation in the analysis.

Floating Rate Debt Floating rate debt is debt issued at a flexible interest rate. Typically floating rate debt is medium or long-term debt on which the interest payments are somehow linked to shorter-term market interest rates. As we saw for indexed debt, how our accounting is affected depends largely on how the government measures interest payments. Since it is likely that the government's measure of interest payments fully reflects adjustments, due to floating rate contracts, to the amount of interest actually paid to investors, there is no need to change the way we do our debt dynamics accounting for floating rate debt.

Foreign Currency-Denominated Debt If governments issue debt in domestic and foreign currency this affects our calculations. Movements in the exchange rate between local and foreign currency will affect debt valuation much like inflation and growth do.

In this section we ignore indexed and floating rate debt and assume that all debt is issued in nominal currency units, either domestic or foreign, at fixed interest rates. Domestic currency debt will be denoted B_t^D , while foreign currency denominated debt will be denoted

B_t^F . The government's total stock of debt at the end of period t is $B_t = B_t^D + B_t^F S_t$, where S_t is the end-of-period t exchange rate measured in local currency units per foreign currency unit. The reason we use the end-of-period exchange rate in defining B_t is that this is the standard method in debt accounting: debt is typically measured at the end of a year, and the conversion of foreign to domestic currency is typically done at the same point in time.

We have assumed that all debt is issued for one period. It is convenient to assume that external debt is issued steadily throughout a year, so that the funds raised at time t by issuing external debt, B_t^F , are equivalent to $B_t^F S_{\bar{t}}$ in local currency units, where $S_{\bar{t}}$ is the average exchange rate at time t . The distinction between the average exchange rate and the end of period exchange rate is important in that the funds raised by issuing the debt may be more or less than the value of the debt at the end of the period, depending on how the exchange rate changes within the period. Similarly, we will assume that the stock of previously issued foreign currency debt, B_{t-1}^F , is amortized at a steady pace during period t . So the local currency value of amortization payments is $B_{t-1}^F S_{\bar{t}}$.

We will assume that the definition of interest payments includes all interest paid on domestic debt, as well as all interest payments made on external debt, valued in local currency at the time the payments were made. For this reason, we can write

$$B_t^D - B_{t-1}^D + (B_t^F - B_{t-1}^F)S_{\bar{t}} = I_t - X_t - (M_t - M_{t-1}). \quad (4.14)$$

Dividing both sides by $P_t y_t$, and defining $\bar{b}_t^D = B_t^D / (P_t y_t)$ and $\bar{b}_t^F = B_t^F S_t / (P_t y_t)$ we obtain

$$\bar{b}_t^D - \frac{P_{t-1} y_{t-1} \bar{b}_{t-1}^D}{P_t y_t} + (\bar{b}_t^F - \frac{S_t}{S_{t-1}} \frac{P_{t-1} y_{t-1} \bar{b}_{t-1}^F}{P_t y_t}) \frac{S_{\bar{t}}}{S_t} = \bar{i}_t - \bar{x}_t - \bar{\sigma}_t. \quad (4.15)$$

Thus, the change in the debt-to-GDP ratio is

$$\bar{b}_t - \bar{b}_{t-1} = \bar{i}_t - \bar{x}_t - \bar{\sigma}_t - \frac{z_t}{1 + z_t} \bar{b}_{t-1}^D + \left(1 - \frac{S_{\bar{t}}}{S_t}\right) \bar{b}_t^F + \left(\frac{S_{\bar{t}}}{S_{t-1}} \frac{1}{1 + z_t} - 1\right) \bar{b}_{t-1}^F. \quad (4.16)$$

While (4.16) is useful, the last two terms on the right-hand side of the equation, at first, appear to have little intuitive content. However, some further algebra reveals that we can rewrite (4.16) as:

$$\begin{aligned} \bar{b}_t - \bar{b}_{t-1} &= \bar{i}_t - \bar{x}_t - \bar{\sigma}_t - \frac{\pi_t}{1 + \pi_t} \bar{b}_{t-1} - \frac{g_t}{1 + z_t} \bar{b}_{t-1} \\ &\quad + \frac{S_t - S_{\bar{t}}}{S_t} \bar{b}_t^F + \frac{S_{\bar{t}} - S_{t-1}}{S_{t-1}} \frac{1}{1 + z_t} \bar{b}_{t-1}^F. \end{aligned} \quad (4.17)$$

or as

$$\begin{aligned}\bar{b}_t - \bar{b}_{t-1} &= \bar{v}_t - \bar{x}_t - \bar{\sigma}_t - \frac{\pi_t}{1 + \pi_t} \bar{b}_{t-1}^D - \frac{g_t}{1 + z_t} \bar{b}_{t-1} \\ &\quad + \frac{S_t - S_{\bar{t}}}{S_t} \bar{b}_t^F + \left(\frac{S_{\bar{t}} - S_{t-1}}{S_{t-1}} \frac{1}{1 + z_t} - \frac{\pi_t}{1 + \pi_t} \right) \bar{b}_{t-1}^F.\end{aligned}\quad (4.18)$$

Notice that (4.17) and (4.18) agree as to the first, second, third and fifth components of the change in debt: interest payments, the primary balance, seigniorage and the growth effect. However, they differ as to their definitions of the fourth and sixth components of the change in the debt-to-GDP ratio. In (4.17) these components are the inflation effect, $-\pi_t \bar{b}_{t-1}/(1 + \pi_t)$, which applies to the entire stock of debt, and a nominal revaluation effect, which is nonzero if and only if there are changes in the nominal exchange rate during the period. If local currency depreciates during the period then the revaluation effect is positive: the depreciation of the currency tends to add to the government's effective indebtedness.

By default, in what follows, we will use the decomposition described by (4.18) where the fourth component is the inflation effect, $-\pi_t \bar{b}_{t-1}^D/(1 + \pi_t)$, which applies only to the stock of domestic debt, and a *real* revaluation effect, which, roughly speaking, is nonzero if the changes in the nominal exchange rate differ from the inflation rate during the period.

Some Intuition from our Example in Continuous Time The reader familiar with analysis in continuous time will be aware that (4.17) limits to

$$\partial \bar{b}_t / \partial t = \bar{v}_t - \bar{x}_t - \bar{\sigma}_t - \pi_t \bar{b}_t - g_t \bar{b}_t + \delta_t \bar{b}_t^F \quad (4.19)$$

if we shrink the time interval between measurements of the stock of debt. Here the flows \bar{v}_t , \bar{x}_t and $\bar{\sigma}_t$ represent instantaneous versions of interest payments, the primary balance and seigniorage revenue, while $\pi_t = (\partial P_t / \partial t) / P_t$, $g_t = (\partial y_t / \partial t) / y_t$ and $\delta_t = (\partial S_t / \partial t) / S_t$ represent the instantaneous inflation, growth and depreciation rates. The continuous time equivalent of (4.18) is simply a rearrangement of (4.19):

$$\partial \bar{b}_t / \partial t = \bar{v}_t - \bar{x}_t - \bar{\sigma}_t - \pi_t \bar{b}_t^D - g_t \bar{b}_t + (\delta_t - \pi_t) \bar{b}_t^F \quad (4.20)$$

If we define the *real exchange rate* as $E_t = S_t P_t^* / P_t$, where P_t^* is the foreign price level, then the real rate of depreciation is $\varepsilon_t = \delta_t + \pi_t^* - \pi_t$. This allows us to rewrite (4.20) as

$$\partial \bar{b}_t / \partial t = \bar{v}_t - \bar{x}_t - \bar{\sigma}_t - \pi_t \bar{b}_t^D - g_t \bar{b}_t + (\varepsilon_t - \pi_t^*) \bar{b}_t^F. \quad (4.21)$$

Notice that (4.21) can be interpreted as follows. Domestic debt, which is denominated in units of local currency, depreciates in real value at the rate π . Foreign currency debt, which is denominated in units of foreign currency, would depreciate in real value at the rate π_t^* , the foreign rate of inflation, if there were no changes in the real exchange rate. To the extent that the real exchange rate depreciates (i.e. ε_t is positive), however, the burden of external debt increases.

4.2. Debt Dynamics in Practice

There are a number of obstacles to performing the calculations we have described above. First, one has to have reasonably accurate fiscal data. As mentioned in Chapter 1, we are typically interested in working with some definition of the consolidated nonfinancial public sector, which, importantly, should include the central bank. Often such data are not readily available. Sometimes good data exist for the central government, or perhaps the general government, but an accurate consolidation of the fiscal accounts, especially the debt statistics, across the entire public sector is less prevalent. Sometimes it is difficult to bring the central bank into the consolidated figures for debt. Also, it is important to be aware of the fact that government accounting frequently places extraordinary items below the line; i.e. there are items that are not included in standard budget categories and, instead, are placed outside these categories in order to fully explain the government's cash flow.

Not only must measures of stocks and flows correspond, in terms of the definition of the public sector, but we also need debt data that are broken down by currency of denomination. Frequently, domestic debt data are hard to come by. Sometimes the currency denomination of foreign debt is not widely published. In addition, as we will see in Section 6, accurately accounting for all changes in the stock of debt requires that we know a lot more than simply the currency denomination of debt. The structure of the various payments associated with the government's debt is also important.

Finally, it should be pointed out that by categorizing the change in the debt-to-GDP ratio into 6 components, we hide interesting phenomena that may be occurring within these components. For example, if there are dramatic changes in the primary balance, it is helpful to analyze revenue and expenditure separately.

4.3. Some Examples of Debt Dynamics

In this section we will illustrate the use of debt dynamics using examples from Turkey and Argentina. In the case of Turkey, we will use debt dynamics to try to understand how Turkey's debt went from 45 percent of GNP in 1994 to 93 percent of GNP by 2001. We will also attempt to assess whether Argentina's crisis, at the end of 2001, was fiscally driven. We will also ask questions about policy coordination and credibility.

Turkey 1995–2002 Between the end of 1995 and the end of 2001, Turkey's debt stock more than doubled in terms of the debt to GNP ratio. As Table 2 shows, at the beginning of the period, debt was about 41 percent of GNP, with over two-thirds of the stock being external debt. By the end of the period, debt amounted to 95 percent of GNP. In 2002, the debt stock shrank somewhat to 80 percent of GNP, but it remained at almost twice its level in 1995, and external debt represented just 40 percent of the stock.

As Table 2 indicates, a sizeable fraction of Turkey's domestic debt stock is now denominated in foreign currency, but that this is a relatively new phenomenon. Until 2001, the vast majority of Turkey's domestic debt was denominated in local currency and was issued at fixed interest rates. In 2001 this changed, and now a substantial fraction of Turkey's domestic debt is either indexed to foreign currency, denominated in foreign currency, or issued at a floating interest rate.

Looking at Table 2, it is clear that Turkey's debt stock was relatively stable from the end of 1995 through the end of 1998. Essentially, it increased in two steps, from 43.7 to 61.0 percent of GNP in 1999, and from 57.4 to 95.0 percent of GNP in 2001. Why was Turkey's debt stable in the earlier period? And why did indebtedness rise so quickly in 1999 and 2001?

Table 3 decomposes Turkey's changes in debt into the components described above. In the period 1995-98, on average, Turkey's public sector ran a tiny primary surplus of about 0.1 percent of GNP. Nonetheless, the overall fiscal balance $\bar{v}_t - \bar{x}_t$, was very negative in this period, averaging a deficit of 11.1 percent of GNP. Given these large deficits, it may seem puzzling that Turkey's debt stock stayed roughly constant over the period. Even once we account for seigniorage, which averaged 2.7 percent of GNP in this period, we would predict very rapid debt accumulation, at an average pace of 8.4 percentage points of GNP per year, if we only considered the standard terms in the debt dynamics.

Of course, a major part of the story is that the 3 extra terms we described above played an important role in Turkey. A substantial fraction of Turkey's interest burden was eased by the fact that the real face value of TL-denominated debt was being eroded through inflation (the inflation effect averaged -7.5 percent of GNP) and the real value of FX-denominated debt was being eroded through real appreciation of the TL (the revaluation effect averaged -2.6 percent of GNP). Turkey was also fortunate to have positive growth throughout the period at an average rate of 6.8 percent per annum, implying an average growth effect of about -1.5 percent of GNP. So, together, these three effects contributed 11.6 percent of GNP per year to the stability of Turkey's debt stock.

But the 1995-98 period is also interesting because it looks like a period in which the government's generally loose fiscal policy was accommodated with loose monetary policy. In Chapter 1 we saw that the central bank can fight inflation in the face of loose fiscal policy, but at the cost of an increasing stock of debt, and higher inflation later. In the 1995-98 period, one could argue that the central bank was not fighting inflation. Money was printed at a rapid pace (as the numbers on seigniorage attest) and inflation was rampant, averaging about 80 percent per annum. Had the central bank fought inflation more vigorously, the stock of debt would probably not have been so stable in this period.

In 1999, the various factors that had helped to keep Turkey's debt in check in the 1995-98 period were reversed. The primary balance deteriorated to -2.0 percent of GNP from an average of 0.1 percent of GNP in the 1995-98 period. This was largely due to two factors: the economy slipping into recession and the fiscal easing that occurred after the Marmara earthquake in August. The real exchange rate depreciated, implying a revaluation effect of 1.2 percent of GNP, as opposed to an average effect of -2.6 percent of GNP in the 1995-98 period. These two factors raised the financing burden by 5.5 percent of GNP compared to the 1995-98 period.

Nominal interest rates remained high in 1999, despite a decline in inflation. One interpretation is that this was the result of a monetary policy stance geared at consolidating the disinflation that began in 1998. Another is that real interest rate movements were the result of a loss of confidence in the domestic debt market. Regardless of the explanation, the net impact on the *real* domestic interest burden was substantial—on the order of 9.7 percent of GNP.

The policy mix and other economic events thus created a financing nightmare: the overall

balance that needed to be financed deteriorated sharply, but none of the implicit sources of financing (inflation, revaluation and growth) helped offset this increase to the extent they previously had. With seigniorage revenue already near its natural limit an enormous increase in net debt was inevitable. The lesson from 1999 appears to be a very strong one: a policy geared towards reduced inflation must be coordinated with credible fiscal reform.

There was a slight improvement in the debt situation in 2000 that coincided with a rebound in the real exchange rate, and the resumption of growth. However, after the financial tremor of November 2000, the economy fell into a full-scale financial and economic crisis in February 2001. Much as in 1999, this crisis played havoc with the government's finances.

In 2001 the debt stock rose by 35.9 percent of GNP. One reason for this increase was that the government issued new bonds in order to recapitalize failing banks. These bonds are accounted for below the line; i.e. the cost of bailing out the banks is not included as an expenditure item in computing the primary balance. A rough estimate of the quantity of bonds issued for this purpose is around 18.2 percent of GNP. So the direct impact of the financial crisis only explains about half the increase in the ratio of debt to GNP. In other respects 2001 was similar to 1999, but with some important differences.

Unlike 1999, the primary fiscal balance did not deteriorate relative to the stance of the mid 1990s. Indeed, the government successfully implemented a sharp fiscal contraction despite the recession, with the primary fiscal balance improving to 5.5 percent of GNP (compared to 0.1% of GNP, in 1995-98).

In 2001 there was a sharp depreciation of the lira brought on by the financial and fiscal crisis. Inflation did not bring the price level in line with the exchange rate by the end of the year, so there was a sizeable real depreciation and a revaluation effect on the order of 13.2 percent of GNP (compared to -2.6% of GNP in 1995-98). The revaluation effect was so large that it completely offset the inflation effect on domestic debt.

As in 1999 the government faced an enormous financing challenge. With seigniorage kept to just 1.4 percent of GNP, and with sharply negative growth, the government was forced to issue a great deal of new debt. Unlike 1999, when the domestic market absorbed most of the government's financing requirement, only about half the new debt was issued domestically. Furthermore, almost all net financing obtained from the domestic market was raised by issuing debt denominated in or indexed to foreign currency.

In 2002, the debt picture improved, mainly due to the economic rebound. Growth re-

turned, and was unusually strong given the depth of the preceding recession. The exchange rate appreciated substantially in real terms, after the sharp real depreciation of 2001. Interest rates moderated somewhat. Together these factors implied a sharp reduction in Turkey's public debt from 95 to 79.8 percent of GNP.

While we can draw some lessons from Turkey's experience that relate to the theory we discussed in Chapter 1, it is clear that there are other important factors that come into play. Debt levels can fluctuate wildly depending on the value of the real exchange rate if a country has significant amounts of external or foreign currency denominated debt. The primary budget, as in 1999, responds to economic conditions. Finally, financial crises, which lie outside our theoretical discussion so far, can have a dramatic impact on the government's finances. Crises will be discussed in more detail in Chapters 8–10.

Argentina, 1996–2002 Table 4 presents debt data for Argentina, from 1993 through 2002, while Table 5 illustrates Argentina's debt dynamics from 1994 through 2002. Both tables present the accounts of the consolidated nonfinancial public sector. As Table 4 shows, Argentina gradually accumulated debt, beginning from a stock of just under 30 percent of GDP in 1993. By 1998 the stock of debt had grown to 37.6 percent of GDP. But the process accelerated in 1999, as the economy entered recession, and again in 2001, as the recession deepened, and by the end of that year the stock of debt had grown to 53.8 percent of GDP.

Many observers of the Argentine economy during the 1990s argued that Argentina did not have a fiscal problem because it was generally running a primary surplus. This fact is confirmed by Table 5. But, as we have seen in earlier sections, running a primary surplus is not a sufficient criterion for achieving fiscal sustainability. Indeed, we have seen the long-run condition that states that a government must run a primary balance $\bar{x} = \bar{r}\bar{b} - \bar{\sigma}$ to achieve fiscal sustainability.

>From the end of 1993 to the end of 2001, Argentina's debt stock, as a percentage of GDP, grew in every year except 1997. A country can accumulate debt, and continue to have sustainable finances, but to do so, it must eventually implement a fiscal adjustment, loosen monetary policy or be lucky enough to begin growing faster.⁶ As we can see from Table 5, Argentina made no fiscal adjustment significant enough to raise its primary surplus over this period. Under the currency board loosening monetary policy was not an option—the size of

⁶If the economy began to grow faster the \bar{r} term in the fiscal sustainability condition would become smaller.

the monetary base was dictated by the current account. To make matters worse after 1998 the economy began to shrink, rather than growing faster. Thus, rather than reversing the trend of a growing debt stock and interest burden, Argentina's government found itself in a situation where the trend worsened.

It is difficult to argue that by the end of 2001 Argentina had achieved some magic number for debt that meant it was bound to suffer a crisis. However, it is not surprising that in the context of a growing fiscal problem and a stagnant economy, investors lost confidence in the government's ability to sustain its finances.

Two aspects of Table 5 that deserve mention are the large residuals in the 1994-98 period, and the huge residual in 2002. For the former period, the large positive residuals imply that there were important factors contributing to the accumulation of debt that do not appear in the debt dynamics detailed in Table 5. This suggests that the primary balance was probably overstated and omitted off balance sheet items. For 2002, the large residual is explained by the government's default on its external debt. Notice, from Table 4, that the end of 2002 stock of external debt was actually smaller than it was at end of 2001. Given the large real depreciation of the peso in 2002, there would have been an enormous increase in external debt as a percentage of GDP, had the government not explicitly defaulted. This is reflected in the estimated revaluation effect equal to 78.7 percent of GDP.

In summary, Argentina looks like a country that in the early 1990s had public finances that were close to sustainable. The government might have been able to correct the situation via a small fiscal adjustment. However, it did not do so, and its debt stock began to grow. This combined with the deep recession entered in 1999, led to an acceleration of the fiscal problem.

5. Using Debt Dynamics in Forward-Looking Analysis

In Sections 1 and 2 we presented simple tools that could be used to assess fiscal sustainability in a forward looking way. In Section 4 we considered debt dynamics, a tool for analyzing historical debt data. In this section we show how some of the details of Section 4 can be adapted for use in the type of forward looking analysis used in Sections 1 and 2.

Let us reconsider the basic accounting identity

$$\text{net debt issued} = I_t - X_t - (M_t - M_{t-1}).$$

In Chapter 1 and in Sections 1 and 2 we adopted the assumptions that all debt has a maturity of one period and debt is real and pays a constant real rate of interest, r . In this case we were able to rewrite the budget constraint in real terms as $b_t = (1 + r)b_{t-1} - x_t - \sigma_t$. Rather than proceeding in this fashion, we can, instead adopt the budget constraint, measured in units of local currency, that was derived in the section of debt dynamics:

$$B_t^D - B_{t-1}^D + (B_t^F - B_{t-1}^F)S_t = I_t - X_t - (M_t - M_{t-1}).$$

We will maintain the assumption that all debt has a maturity of one period, but we will allow the interest rates on domestic and external debt, R_t^D and R_t^F to be time varying.

In the section on debt dynamics we assumed that amortization, the issuance of new debt, and interest payments took place at a steady pace within the year, so that the period average exchange rate, S_t , was the appropriate one to use in valuing foreign currency transactions. For simplicity, in this section, we will assume that the period average exchange rate is the same as the end of period exchange rate so that $S_t = S_t$. Then we can rewrite the budget constraint as

$$B_t^D - B_{t-1}^D + S_t(B_t^F - B_{t-1}^F) = R_{t-1}^D B_{t-1}^D + S_t R_{t-1}^F B_{t-1}^F - X_t - (M_t - M_{t-1}).$$

Dividing both sides by nominal GDP, $P_t y_t$, we get

$$\bar{b}_t^D + \bar{b}_t^F = \left(\frac{1 + R_{t-1}^D}{1 + z_t} \right) \bar{b}_{t-1}^D + \frac{S_t}{S_{t-1}} \frac{1 + R_{t-1}^F}{1 + z_t} \bar{b}_{t-1}^F - \bar{x}_t - \bar{\sigma}_t. \quad (5.1)$$

Using assumptions about nominal interest rates, inflation, real growth, the primary balance, the monetary base, and the exchange rate, (5.1) can be used to simulate the path of debt for an economy. If, in the simulation, the debt to GDP ratio continues to grow faster than the effective nominal interest rate net of the inflation and growth rates—i.e. an appropriate weighted average of $(1 + R_{t-1}^D)/(1 + z_t)$ and $(1 + R_{t-1}^F)/(1 + z_t)$ —then debt will not be sustainable. This is the analog to verifying whether the long-run sustainability condition of Section 1 holds. Alternatively one could simulate the path of debt using (5.1) to see whether a debt target, such as the one we illustrated in Section 2, could be achieved. But (5.1) is especially useful, because one can simulate many paths of debt using random paths for specific variables, in order to assess risk factors. We will illustrate the possibilities using two extensions of our Bulgaria example from Section 2.

Time-Varying Interest Rates In Section 2 we illustrated a case study of Bulgaria, where we calculated the primary balance that Bulgaria would need in order to achieve a debt target of 60 percent of GDP by 2005. To get our estimate of the necessary primary balance we assumed that the nominal interest rate would be $R = 0.085$ in perpetuity while the inflation rate would be $\pi = 0.035$. This implied a real interest rate, $r = 0.048$. We assumed that the real growth rate was $g = 0.04$. Finally, we assumed that the monetary base would stay constant as a percentage of GDP at $\bar{m} = 0.14$, so that $\bar{\sigma} = z\bar{m}/(1+z) \approx 0.01$ where $z = \pi + g + \pi g$. We found that in order for Bulgaria to reduce its debt stock from 72 percent of GDP at the end of 2001 to 60 percent of GDP at the end of 2005, would require a primary balance $\bar{x} = 0.0255$, or 2.55 percent of GDP.

In Table 1 we examined the sensitivity of this result to our assumptions about the real interest rate and growth. Recall, however, that our concern with the interest rate assumption was based on the fact that Bulgaria's debt profile is changing. In 2001, Bulgaria was fortunate in that it was servicing its debt, which was largely concessional, at an effective nominal interest rate of about 5 percent. On the other hand, when borrowing in private markets it was borrowing at rates closer to 12 percent. Since Bulgaria is expected to retire a great deal of its concessional debt by 2007, it might be desirable to assume that Bulgaria's effective borrowing cost will rise over time.

So rather than choosing a constant interest rate between 5 and 12 percent, it might be interesting to assume that the effective nominal interest rate would start out at 5 percent in 2002, but gradually rise to, say, 10 percent, by 2006. After that it might be assumed to be constant at 10 percent.

Making this small change in the assumptions of Section 2, and ignoring the issues raised by external debt, we will rewrite (5.1) as

$$\bar{b}_t = \left(\frac{1 + R_{t-1}}{1 + z} \right) \bar{b}_{t-1} - \bar{x} - \frac{z}{1 + z} \bar{m}. \quad (5.2)$$

We have assumed $\pi = 0.035$, $g = 0.04$, and $\bar{m} = 0.14$ so $z = 0.0764$ and $\bar{\sigma} \approx 0.01$. It turns out that if the primary surplus is set to $\bar{x} = 0.0255$, as before, then Bulgaria would more than meet the goal of the 60 percent debt rule by 2005. In fact it would achieve a 54.9 percent debt-to-GDP ratio, as illustrated in Table 6. In fact, Bulgaria could run a primary balance of just 1.27 percent of GDP and achieve the debt target by that date. Of course, this result is due to our new assumption that the nominal interest rate, while growing, would lie below 8.5 percent through 2004. In Section 2 we had assumed a constant nominal interest

rate of 8.5 percent. Not surprisingly this implied a much bigger primary surplus.

Exchange Rate Risk At the end of 2000, 65 percent of Bulgaria’s external public sector debt was denominated in U.S. dollars. On the other hand, Bulgaria’s currency board maintains a peg between local currency (the lev) and the euro. Hence, the government faces considerable exchange rate risk due to movements in the euro-dollar exchange rate.

To gauge the risk that movements in the euro-dollar exchange rate pose to the sustainability of Bulgaria’s debt, we could consider a simple simulation exercise that uses (5.1) to allow for the varied currency composition of the debt.

Suppose that we maintain the assumption that inflation, growth, the primary balance and the monetary base (as a percentages of GDP) are constant, so that $\pi_t = \pi$, $g_t = g$, $z_t = z = \pi + g + \pi g$, $\bar{x}_t = \bar{x}$ and $\bar{\sigma}_t = \bar{\sigma} = \bar{m}z/(1+z)$. Let us further suppose that there are no expected exchange rate movements, and no risk premium to borrowing in either currency so that $R_t^D = R_t^F = R_t$ for all t . We treat lev or euro borrowing as borrowing in domestic currency, while treating US dollars as foreign currency. We can rewrite (5.1) as

$$\bar{b}_t^D + \bar{b}_t^F = \left(\frac{1 + R_{t-1}}{1 + z}\right) \bar{b}_{t-1}^D + \frac{S_t}{S_{t-1}} \frac{1 + R_{t-1}}{1 + z} \bar{b}_{t-1}^F - \bar{x} - \bar{\sigma}. \quad (5.3)$$

Notice that if θ_t represents the fraction of the debt that is denominated in US dollars at the end of time t , then we can rewrite (5.3) as

$$\bar{b}_t = (1 + \delta_t \theta_{t-1}) \left(\frac{1 + R_{t-1}}{1 + z}\right) \bar{b}_{t-1} - \bar{x} - \frac{z}{1 + z} \bar{m}, \quad (5.4)$$

where δ_t is the rate of depreciation of the euro against the dollar.

Table 7 illustrates the result of using (5.4) to simulate 1000 debt paths that assume an initial 65 percent dollar composition of debt, while maintaining the assumptions used in making the projections in Table 6: $\pi = 0.035$, $g = 0.04$, $\bar{m} = 0.14$, $\bar{x} = 0.0255$ and the two tables use identical time-varying interest rate assumptions. The simulated paths for debt depend on simulated values of the dollar-euro exchange rate drawn from a random number generator. These simulations assume that δ_t is i.i.d., and normal, with mean 0 and standard deviation equal to 0.128.⁷ The simulations also assume that in each period the government acts to ensure that 65 percent of the debt is dollar-denominated.

⁷From 1973 to 2000 the average depreciation of the deutsche mark/euro against the dollar was close to zero, while the standard deviation of the rate of depreciation was about 12.8 percent, with a moderate degree of serial correlation (0.28).

The simulations in Table 7 illustrate that exchange rate risk could have been a serious issue for Bulgaria. In 5% of the simulations debt rises to almost 76 percent of GDP by the year 2007. In just over 8 percent of the simulations debt remains above its 2001 level in 2007. In addition, in about 33 percent of the cases the EU accession target is not reached by 2005, although the target is reached 76 percent of the time by 2007.

Simulations based on (5.4), of course, do not take into account the possibility that the primary surplus, growth, or seigniorage could be affected by dollar-euro exchange rate movements. This may or may not be a reasonable assumption depending on the structure of government spending and government revenue. However, it seems reasonable to assume that Bulgaria is more tightly integrated into the European economy and that growth and government finance are more likely to be affected by real exchange rate movements between the lev and the euro rather than by changes in the dollar-euro nominal exchange rate.

In the end, these simulations could have been used to argue that the government should consider hedging some of the exchange rate risk it faced. One could conduct similar simulations to take into account the effects of recessions. This would imply making assumptions about the stochastic process governing g_t , and, perhaps, making assumptions about the interaction between growth and the primary balance.

6. Measuring Debt

This section highlight issues in the measurement of debt. In particular, this section shows how the structure of the government's debt creates problems in defining the stock of debt relevant in our analysis. To keep things simple this section focuses on examples where the government only issues nonindexed debt denominated in units of local currency. It would be straightforward to extend the analysis to more complex debt structures.

Baseline Example: Single Period Debt Suppose that the government issues all its debt in the form of single period bonds denominated in local currency units, which, for convenience, I will call *pesos*. The government issues two types of bonds: (i) zero coupon bonds and (ii) coupon bonds.

The government uses the following accounting rules. If, at time t , the government issues a zero coupon bond with a face value of 1 peso, with an effective yield to maturity given by R_t , then this bond contributes (i) $1/(1 + R_t)$ to the book value of government debt at time

t , (ii) $1/(1 + R_t)$ to time $t + 1$ amortization payments and (iii) $R_t/(1 + R_t)$ to time $t + 1$ interest payments. If, on the other hand, the government issues a coupon bond with a face value of 1 peso, and coupon rate c_t , with an effective yield to maturity given by R_t , then this bond contributes (i) $(1 + c_t)/(1 + R_t)$ to the book value of government debt at time t , (ii) $(1 + c_t)/(1 + R_t)$ to time $t + 1$ amortization payments and (iii) $(1 + c_t)R_t/(1 + R_t)$ to time $t + 1$ interest payments. These accounting rules imply that coupon bonds and zero-coupon bonds receive symmetric treatment in the government's books. They also imply that the book value of any debt is the same as its market value at the time of issuance.

We have the accounting identity (4.1), which we repeat here as

$$\text{net debt issued} = I_t - X_t - (M_t - M_{t-1}). \quad (6.1)$$

Net debt issued is proceeds raised from the sale of debt at time t , which we denote with B_t , minus any amortization payments at time t , which we denote A_t . So we can write

$$B_t - A_t = I_t - X_t - (M_t - M_{t-1}). \quad (6.2)$$

Notice, given our description of the baseline example, that $A_t = B_{t-1}$, so that

$$B_t - B_{t-1} = I_t - X_t - (M_t - M_{t-1}). \quad (6.3)$$

Also, notice that if we assume that all debt issued by the government at time t is issued at the same effective yield, R_t , then $I_t = R_{t-1}B_{t-1}$. Hence

$$B_t - B_{t-1} = R_{t-1}B_{t-1} - X_t - (M_t - M_{t-1}). \quad (6.4)$$

In the baseline example, (6.4) holds regardless of whether B_t represents gross proceeds from issuing debt, the book value of the government's debt, or the market value of the government's debt. The simplicity of the example with only single period debt is precisely why we adopted the single period debt assumption throughout Chapters 1 and 2.

Two-Period Debt Once we introduce two-period debt our accounting becomes more complicated. We saw, above, that as long as the accounting was done carefully, there was no distinction between a one-period zero coupon bond and a one-period coupon bond. Both are promises to pay a lump-sum one period ahead, and that lump-sum can be divided between interest and amortization in a way that blurs the distinction between the two types of bonds.

This is no longer the case with two-period bonds. A zero-coupon bond issued for two periods postpones the entire cash flow for two periods, whereas a coupon bond does not. Hence, these types of bonds are fundamentally different.

Nonetheless, we can still summarize all debt by how much it promises to pay in what period. When debt can be issued for at most two periods, then at the end of period t , the government will have commitments to pay the nominal amounts $D_{t-1,2}$, $D_{t,1}$ and $D_{t,2}$ in periods $t+1$ and $t+2$ respectively. The notation $D_{t,j}$ represents debt contracted at date t , payable at date $t+j$. The market value of the government's debt is

$$B_t^m = \frac{D_{t-1,2} + D_{t,1}}{1 + R_{t,1}} + \frac{D_{t,2}}{(1 + R_{t,2})^2}, \quad (6.5)$$

where $R_{t,j}$ is the yield to maturity of a j -period zero coupon bond issued at date t . The book value of the government's debt is

$$B_t^b = \frac{D_{t-1,2}}{(1 + R_{t-1,2})^2} + \frac{D_{t,1}}{1 + R_{t,1}} + \frac{D_{t,2}}{(1 + R_{t,2})^2} \quad (6.6)$$

The government's gross proceeds from issuing debt at time t are

$$G_t = \frac{D_{t,1}}{1 + R_{t,1}} + \frac{D_{t,2}}{(1 + R_{t,2})^2}, \quad (6.7)$$

while amortization plus interest at time t is

$$A_t + I_t = D_{t-1,1} + D_{t-2,2}. \quad (6.8)$$

We will assume, in all cases, that the net issuance of debt is defined as $G_t - A_t$ (the gross issuance of debt net of amortization) so that

$$G_t - A_t = I_t - X_t - (M_t - M_{t-1}). \quad (6.9)$$

At this stage we can demonstrate the circumstances under which the net issuance of debt corresponds to the change in the stock of debt, measured either as B_t^m or B_t^b . Notice that

$$B_t^m - B_{t-1}^m = G_t - A_t \iff B_t^m = G_t + B_{t-1}^m - A_t.$$

If these statements are true then, given (6.5), (6.7) and (6.8), we have

$$\frac{D_{t-1,2} + D_{t,1}}{1 + R_{t,1}} + \frac{D_{t,2}}{(1 + R_{t,2})^2} = \frac{D_{t,1}}{1 + R_{t,1}} + \frac{D_{t,2}}{(1 + R_{t,2})^2} + B_{t-1}^m + I_t - (D_{t-1,1} + D_{t-2,2})$$

or, equivalently,

$$D_{t-1,1} + D_{t-2,2} + \frac{D_{t-1,2}}{1 + R_{t,1}} = B_{t-1}^m + I_t. \quad (6.10)$$

From (6.5) we can see that

$$D_{t-1,1} + D_{t-2,2} + \frac{D_{t-1,2}}{1 + R_{t,1}} = (1 + R_{t-1,1})B_{t-1}^m + \left[\frac{1}{1 + R_{t,1}} - \frac{1 + R_{t-1,1}}{(1 + R_{t-1,2})^2} \right] D_{t-1,2}.$$

Hence (6.10) can be rewritten as

$$I_t = R_{t-1,1}B_{t-1}^m + \left[\frac{1}{1 + R_{t,1}} - \frac{1 + R_{t-1,1}}{(1 + R_{t-1,2})^2} \right] D_{t-1,2}. \quad (6.11)$$

So, $B_t^m - B_{t-1}^m = I_t - X_t - (M_t - M_{t-1})$ if and only if the government's measure of interest payments corresponds to (6.11). Notice that this measure of interest payments is the one period interest rate at time $t - 1$, $R_{t-1,1}$, times the market value of the debt at the end of period $t - 1$, B_{t-1}^m , plus a capital loss term that results from any change in the holding period yield that applies to the interval between period t and period $t + 1$. Notice that $(1 + R_{t-1,2})^2 / (1 + R_{t-1,1})$ represents the portion of the gross return on a 2-period zero coupon bond, purchased at time $t - 1$, that accrues between periods t and $t + 1$. The gross return on a 1-period zero coupon bond held between t and $t + 1$ is $1 + R_{t,1}$. Notice that if the latter is less than the former, the second term on the right-hand side of (6.11) is positive. This is because the government takes a capital loss resulting from the decline in the cost of borrowing money between periods t and $t + 1$. The total loss is proportional to the amount of money that was tied down for two periods at time $t - 1$.

On the other hand, when we consider the book value of debt

$$B_t^b - B_{t-1}^b = G_t - A_t \iff B_t^b = G_t + B_{t-1}^b - A_t.$$

If these statements are true then

$$\frac{D_{t-1,2}}{(1 + R_{t-1,2})^2} + \frac{D_{t,1}}{1 + R_{t,1}} + \frac{D_{t,2}}{(1 + R_{t,2})^2} = \frac{D_{t,1}}{1 + R_{t,1}} + \frac{D_{t,2}}{(1 + R_{t,2})^2} + B_{t-1}^b + I_t - (D_{t-1,1} + D_{t-2,2})$$

or

$$D_{t-1,1} + D_{t-2,2} + \frac{D_{t-1,2}}{(1 + R_{t-1,2})^2} = B_{t-1}^b + I_t.$$

Using (6.6) we can substitute out B_{t-1}^d and solve for I_t :

$$I_t = R_{t-1,1} \frac{D_{t-1,1}}{1 + R_{t-1,1}} + [(1 + R_{t-2,2})^2 - 1] \frac{D_{t-2,2}}{(1 + R_{t-2,2})^2}. \quad (6.12)$$

So,

$$B_t^b - B_{t-1}^b = I_t - X_t - (M_t - M_{t-1})$$

if and only if the government's measure of interest payments corresponds to (6.12). Notice that this measure of interest payments takes the book value of the payments being made at time t —i.e. $D_{t-1,1}/(1 + R_{t-1,1})$ and $D_{t-2,2}/(1 + R_{t-2,2})^2$ —and computes the accumulated book interest on these amounts given the periods over which the funds were borrowed.

Multi-Period Debt When debt is issued for at most K periods, then at the end of period t , the government will have commitments to pay the nominal amounts $(D_{t-K+1,K}, D_{t-K+2,K-1}, \dots, D_{t-1,2}, D_{t,1})$, $(D_{t-K+2,K}, \dots, D_{t-1,3}, D_{t,2})$, \dots , $(D_{t-1,K}, D_{t,K-1})$, $D_{t,K}$ in periods $t+1, t+2, \dots, t+K-1, t+K$, respectively. The market value of the government's debt is

$$B_t^m = \sum_{j=1}^K (1 + R_{t,j})^{-j} \sum_{i=j}^K D_{t+j-i,i}. \quad (6.13)$$

The book value of the government's debt is

$$B_t^b = \sum_{j=1}^K \sum_{i=j}^K (1 + R_{t+j-i,i})^{-i} D_{t+j-i,i}. \quad (6.14)$$

The government's gross proceeds from issuing debt at time t are

$$G_t = \sum_{j=1}^K (1 + R_{t,j})^{-j} D_{t,j}, \quad (6.15)$$

while amortization plus interest at time t is

$$A_t + I_t = \sum_{j=1}^K D_{t-j,j}. \quad (6.16)$$

As before, the net issuance of debt is $G_t - A_t$, and is given by (6.9).

As in the previous section $B_t^m - B_{t-1}^m = G_t - A_t$ if $B_t^m = G_t + B_{t-1}^m - A_t$, or

$$\sum_{j=1}^K (1 + R_{t,j})^{-j} \sum_{i=j}^K D_{t+j-i,i} = \sum_{j=1}^K (1 + R_{t,j})^{-j} D_{t,j} + B_{t-1}^m + I_t - \sum_{j=1}^K D_{t-j,j}. \quad (6.17)$$

Notice that

$$B_{t-1}^m = \sum_{j=1}^K (1 + R_{t-1,j})^{-j} \sum_{i=j}^K D_{t-1+j-i,i}$$

so that

$$(1 + R_{t-1,1})B_{t-1}^m = \sum_{j=1}^K \frac{1 + R_{t-1,1}}{(1 + R_{t-1,j})^j} \sum_{i=j}^K D_{t-1+j-i,i}.$$

Hence we can rewrite (6.17) as

$$I_t = R_{t-1,1}B_{t-1}^m + \sum_{j=1}^K \left\{ (1 + R_{t,j})^{-j} \left[\sum_{i=j}^K D_{t+j-i,i} - D_{t,j} \right] - \frac{1 + R_{t-1,1}}{(1 + R_{t-1,j})^j} \sum_{i=j}^K D_{t-1+j-i,i} + D_{t-j,j} \right\}. \quad (6.18)$$

Considerable algebra shows that (6.18) can be rewritten as

$$I_t = R_{t-1,1}B_{t-1}^m + \sum_{j=1}^{K-1} \left[(1 + R_{t,j})^{-j} - \frac{1 + R_{t-1,1}}{(1 + R_{t-1,j+1})^{j+1}} \right] \sum_{i=j}^{K-1} D_{t-i,i+j}. \quad (6.19)$$

So, $B_t^m - B_{t-1}^m = I_t - X_t - (M_t - M_{t-1})$ if and only if the government's measure of interest payments corresponds to (6.19). This measure of interest payments is the one period interest rate at time $t - 1$, $R_{t-1,1}$, times the market value of the debt at the end of period $t - 1$, B_{t-1}^m , *plus* a capital loss term that results from any change in the market holding period yields that are relevant in valuing pre-existing debt that matures at date $t + 1$ or later.

As before, $B_t^b - B_{t-1}^b = G_t - A_t$ if $B_t^b = G_t + B_{t-1}^b - A_t$, or

$$\sum_{j=1}^K \sum_{i=j}^K (1 + R_{t+j-i,i})^{-i} D_{t+j-i,i} = \sum_{j=1}^K (1 + R_{t,j})^{-j} D_{t,j} + B_{t-1}^b + I_t - \sum_{j=1}^K D_{t-j,j}.$$

Using (6.14) we can substitute out B_{t-1}^d and solve for I_t :

$$I_t = \sum_{j=1}^K \left[\sum_{i=j}^K (1 + R_{t+j-i,i})^{-i} D_{t+j-i,i} - (1 + R_{t,j})^{-j} D_{t,j} + D_{t-j,j} - \sum_{i=j}^K (1 + R_{t-1+j-i,i})^{-i} D_{t-1+j-i,i} \right].$$

Considerable algebra shows that this can be simplified considerably:

$$I_t = \sum_{j=1}^K \left[(1 + R_{t-j,j})^j - 1 \right] \frac{D_{t-j,j}}{(1 + R_{t-j,j})^j}. \quad (6.20)$$

So, $B_t^b - B_{t-1}^b = I_t - X_t - (M_t - M_{t-1})$ if and only if the government's measure of interest payments corresponds to (6.20). As in the case of two-period debt, this measure of interest payments takes the book value of the payments being made at time t —i.e. $D_{t-j,j}/(1 + R_{t-j,j})^j$ —and computes the accumulated book interest on these amounts given the periods over which the funds were borrowed.

Which Measure of Debt and Interest is Relevant? >From the point of view of computing debt dynamics, it is likely that B_t^b is the relevant measure. Why? Largely because it is likely that government accounting methods are consistent with the definition of

B_t^b in (6.14) and the definition of I_t given in (6.20). Thus as long as one uses these definitions consistently, the identity (6.3) will hold, at least up to any measurement error.

On the other hand, to calculate B_t^m using the definition in (6.13) requires that a sophisticated and active market for government debt exists, thus enabling the evaluation of the interest rate components of the formula. Frequently such markets do not exist, and governments rarely publish up to date revaluations of their own debt stocks, consistent with (6.13). Furthermore, it is unlikely that the government will use a definition of interest payments, like (6.19), which includes a capital loss term that marks the government's future debt payments to market.

Nonetheless, from the point of view of theory, the market value based measure of debt is the most relevant one. It is straightforward to demonstrate this in a finite horizon example. Suppose that the final period in which the government exists is period T . In this case, because the government cannot issue new debt in period T its time T budget constraint is

$$\sum_{j=1}^K D_{T-j,j} = X_t + M_t - M_{t-1}.$$

The government must retire its debt obligations by running a sufficiently large primary balance inclusive of seigniorage revenue. The market value of the government's debt at time $T - 1$ is

$$B_{T-1}^m = \frac{\sum_{j=1}^K D_{T-j,j}}{1 + R_{T-1,1}}.$$

Hence

$$B_{T-1}^m = \frac{X_t + M_t - M_{t-1}}{1 + R_{T-1,1}},$$

so that a simple version of the present value budget constraint holds for the market value based measure of debt.

On the other hand, the book value of debt is

$$B_{T-1}^b = \sum_{j=1}^K \frac{D_{T-j,j}}{(1 + R_{T-j,j})^j}.$$

Quite clearly, a simple version of the present value budget constraint does not hold for the book value of debt. There is a discount factor \bar{R} for which

$$B_{T-1}^b = \frac{X_t + M_t - M_{t-1}}{1 + \bar{R}}$$

but \bar{R} is a nonlinear average of the book yields on the government's debt:

$$1 + \bar{R} = \frac{\sum_{j=1}^K D_{T-j,j}}{\sum_{j=1}^K (1 + R_{T-j,j})^{-j} D_{T-j,j}}.$$

7. Conclusion

In this chapter we introduced several tools used in fiscal sustainability analysis. We started with the simplest tool: the long-run version of the government's lifetime budget constraint. This tool links the size of the primary balance the government must run to maintain fiscal sustainability to the size of its debt, the flow of seigniorage it raises, the real interest rate and the real growth rate. We then modified this tool to take into account the fact that government often set fiscal goals that go beyond mere sustainability. We showed how the first tool could be modified to calculate the primary balance needed to achieve a specific, quantitative, debt target.

In the third section we discussed related statistical tests for fiscal sustainability. We argued that these tests are more useful in examining historical data than they are for projecting the future.

In Section 4, we introduced the concept of debt dynamics. We showed how debt dynamics can be used to analyze historical changes in a country's public debt, and in Section 5, we showed how they can be adapted for assessing the future sustainability of a government's finances.

Finally, in Section 6 we grappled with a number of issues related to the measurement of debt. We argued that if one is working with historical data then it is most likely that the debt and interest payment series are based on book values. As long as this is true, the budget constraints we have used in our analysis should be valid for multi-period as well as single-period debt. On the other hand, we also argued that market value based measures of debt are the ones most relevant when working with theoretical models of the government's lifetime budget constraint.

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TABLE 1

SIZE OF THE PRIMARY SURPLUS NEEDED FOR BULGARIA TO
ACHIEVE A 60% DEBT TARGET BY 2005
(percent of GDP)

Real Growth Rate (percent)	Real Interest Rate (percent)					
	1	3	5	7	9	11
3	0.82	2.13	3.44	4.76	6.08	7.40
4	0.06	1.36	2.66	3.96	5.26	6.57
5	-0.68	0.60	1.88	3.17	4.46	5.75

Sources: Author's calculations described in the text.

TABLE 2

THE STOCK OF PUBLIC DEBT IN TURKEY, 1994–2002
(percent of GNP)

	94	95	96	97	98	99	00	01	02
Domestic debt	14.0	12.2	20.5	20.4	24.4	40.9	39.1	57.2	47.7
<i>TL-denominated</i>	36.4	36.8	32.4
<i>FX-denominated</i>	2.7	20.4	15.3
External debt	30.7	29.1	26.0	22.5	19.3	20.1	18.3	37.7	37.7
External + FX-den.	21.0	58.1	47.4
Total	44.7	41.3	46.5	42.9	43.7	61.0	57.4	95.0	79.8

Note: ‘..’ indicates data either not available or not calculated. TL is the standard abbreviation for Turkish lira.

Sources: Various IMF reports, and author's calculations.

TABLE 3
PUBLIC DEBT DYNAMICS IN TURKEY, 1995–2002
(percent of GNP)

	95	96	97	98	99	00	01	02
Change in debt	-3.4	5.2	-3.6	0.8	17.3	-3.6	35.9	-13.5
A) Sum of std. terms: $\bar{i} - \bar{x} - \bar{\sigma}$	1.6	8.8	10.2	12.9	20.9	17.8	16.2	10.9
Interest Payments, \bar{i}	7.3	10.0	11.0	16.2	22.1	21.9	23.5	16.3
Primary Balance, \bar{x}	2.7	-1.2	-2.1	0.9	-2.0	2.7	5.5	3.9
Seigniorage, $\bar{\sigma}$	3.0	2.4	2.9	2.4	3.2	1.8	1.4	1.5
B) Sum of extra terms:	-12.6	-8.8	-12.8	-12.1	-5.7	-20.1	4.1	-26.1
Growth Effect	-1.7	-1.5	-2.0	-0.9	1.8	-2.4	3.9	-4.7
Inflation Effect	-6.5	-5.3	-9.2	-8.8	-8.7	-13.8	-13.0	-11.2
Revaluation Effect	-4.4	-1.9	-1.6	-2.5	1.2	-3.8	13.2	-10.1
C) Net below-the-line terms:	0.0	0.0	-0.1	-0.5	-0.1	-1.6	16.3	1.8
Cost of financial sector bailout	0.0	0.0	0.0	0.0	0.0	0.0	18.2	1.9
Privatization revenue	0.0	0.0	0.1	0.5	0.1	1.6	1.9	0.1
Memo items:								
Standard budget deficit, $\bar{i} - \bar{x}$	4.6	11.2	13.1	15.3	24.1	19.6	17.6	12.4
Errors and discrepancy	7.6	5.2	-1.0	0.6	2.2	0.2	-0.7	-0.1

Notes: The growth effect is computed as $-g_t \bar{b}_{t-1} / (1 + z_t)$, where g_t is the growth rate of real GNP, \bar{b}_t is the end of period t debt-to-GNP ratio, and z_t is the growth rate of nominal GNP. The inflation effect is computed as $-\pi_t \bar{b}_{t-1}^D / (1 + \pi_t)$, where π_t is the inflation rate of the GNP deflator, and \bar{b}_t^D is the end of period t stock of TL-denominated domestic debt. The revaluation effect is computed as $\bar{\delta}_t \bar{b}_t^F + [\underline{\delta}_t / (1 + z_t) - \pi_t / (1 + \pi_t)] \bar{b}_{t-1}^F$ where \bar{b}_t^F is the end of period t stock of external debt plus foreign currency-denominated domestic debt, and $\bar{\delta}_t$ and $\underline{\delta}_t$ are defined as follows. Let S_{it} be the end of period t exchange rate in units of foreign currency i per TL, and let $S_{i\bar{t}}$ be the equivalent period average exchange rate. Then $\bar{\delta}_t = \sum_{i=1}^N \theta_i (S_{it} - S_{i\bar{t}}) / S_{it}$ and $\underline{\delta}_t = \sum_{i=1}^N \theta_i (S_{i\bar{t}} - S_{i\bar{t}-1}) / S_{i\bar{t}-1}$ where θ_i represents the average share of currency i in Turkey's external debt in periods $t-1$ and t . 'Errors and discrepancy' equals the difference between the change in debt, and the sum of (A), (B) and (C).

Sources: Various IMF reports, and author's calculations.

TABLE 4

THE STOCK OF PUBLIC DEBT IN ARGENTINA, 1993–2002
(percent of GNP)

	93	94	95	96	97	98	99	00	01	02
Domestic debt	2.3	3.3	2.3	3.6	4.0	5.1	5.7	4.6	4.4	1.8
External debt	27.5	27.9	31.5	32.0	30.5	32.4	37.3	40.5	49.4	37.3
Total	29.8	31.2	33.8	35.7	34.5	37.6	43.0	45.0	53.8	39.1

Sources: Argentina Ministry of Economy and author's calculations.

TABLE 5

PUBLIC DEBT DYNAMICS IN ARGENTINA, 1994–2002
(percent of GNP)

	94	95	96	97	98	99	00	01	02
Change in debt	1.4	2.6	1.9	-1.2	3.1	5.4	2.1	8.7	-14.7
A) Sum of std. terms: $\bar{i} - \bar{x} - \bar{\sigma}$	0.0	2.2	1.7	0.9	1.3	2.5	2.9	4.4	-4.0
Interest Payments, \bar{i}	1.2	1.6	1.7	2.0	2.2	2.9	3.4	3.8	2.2
Primary Balance, \bar{x}	0.8	0.6	-0.4	0.4	0.8	0.3	1.0	0.5	0.7
Seigniorage, $\bar{\sigma}$	0.4	-1.2	0.4	0.7	0.1	0.0	-0.5	-1.2	5.5
B) Sum of extra terms:	-2.4	-0.1	-1.8	-2.5	-0.7	2.0	-0.1	2.6	82.7
Growth Effect	-1.6	0.9	-1.8	-2.7	-1.3	1.3	0.3	2.1	5.0
Inflation Effect	-0.1	-0.1	0.0	0.0	0.1	0.1	-0.1	0.1	-1.0
Revaluation Effect	-0.8	-0.9	0.0	0.1	0.5	0.6	-0.4	0.4	78.7
C) Privatization revenue	0.3	0.5	0.1	0.1	0.0	0.9	0.1	0.0	0.0
Memo items:									
Standard budget deficit, $\bar{i} - \bar{x}$	0.4	1.0	2.1	1.6	1.4	2.6	2.4	3.3	1.5
Errors and discrepancy	3.6	0.1	1.8	0.4	2.5	-0.1	-0.8	1.7	-93.3

Notes: The growth and inflation effects are computed as for Turkey (see Table 3), except that GDP is the income concept. The revaluation effect is computed as $\delta_t \bar{b}_t^F + [\underline{\delta}_t / (1 + z_t) - \pi_t / (1 + \pi_t)] \bar{b}_{t-1}^F$ where \bar{b}_t^F is the end of period t stock of external debt plus foreign currency-denominated domestic debt, and $\bar{\delta}_t$ and $\underline{\delta}_t$ are defined as follows. Let S_t be the end of period t exchange rate in US dollars per peso, and let $S_{\bar{t}}$ be the equivalent period average exchange rate. Then $\bar{\delta}_t = (S_t - S_{\bar{t}}) / S_{\bar{t}}$ and $\underline{\delta}_t = (S_{\bar{t}} - S_{t-1}) / S_{t-1}$. 'Errors and discrepancy' equals the difference between the change in debt, and (A)+(B)-(C).

Sources: Argentina Ministry of Economy, Central Bank of Argentina and author's calculations.

TABLE 6
BULGARIA'S DEBT DYNAMICS WITH TIME-VARYING INTEREST RATES

	2001	2002	2003	2004	2005	2006	2007
Interest Rate	5	6	7	8	9	10	11
a. Assuming $\bar{x} = 0.0255$							
Public debt (% of GDP)	72.0	66.7	62.1	58.2	54.9	52.0	49.6
b. Assuming $\bar{x} = 0.0127$							
Public debt (% of GDP)	72.0	68.0	64.7	62.0	60.0	58.5	57.5

TABLE 7
BULGARIA'S DEBT DYNAMICS WITH EXCHANGE RATE RISK

	2001	2002	2003	2004	2005	2006	2007
Interest Rate	5	6	7	8	9	10	11
Public debt (% of GDP)							
Mean	72.0	67.2	62.9	59.3	56.1	53.6	51.5
Median	72.0	67.0	62.7	58.5	55.0	51.9	49.8
5th Percentile	72.0	58.3	50.9	45.1	40.3	36.3	32.7
95th Percentile	72.0	77.7	77.3	75.6	75.2	75.6	75.8
Percentage of observations in which debt is greater than 60% of GDP	100.0	89.8	61.9	42.9	32.7	28.7	23.9