A NEW POLARIZATION MEASURE AND SOME APPLICATIONS

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ABSTRACT

The paper proposes a new polarization measures that conceptually belong to the class of Gini-type functions. It is shown that the measure satisfies four key axioms. The measure is also easy to compute. Its properties are explored and the polarization index results are contrasted with those obtained from the Gini index using some simulated and real-world data. Inequality and polarization are shown to be related but distinct phenomena.

JEL classification: D31, D63

Key words: polarization, inequality.
1. Introduction

Recently, the issue of income polarization has received lots of attention. Polarization is, in principle, distinct from—if related to—inequality. While income inequality describes a situation when most, or even all, income is appropriated by a few persons, polarization describes the situation of a divided society. A perfectly unequal society, with one individual appropriating all income, is not a polarized society, simply because n-1 persons’ income is the same. It is a fairly homogeneous society. It is to this intuition that the need to define measures of polarization owes its existence.

Somewhat relatedly, the changing shape of the earnings distribution in the United States in the 1980’s and the 1990’s, characterized by increasing shares of people receiving both high and low wages (see e.g. Burtless (1997), and Beach, Chaykovski and Slotsve (1997) in an issue dedicated to inequality in the Nafta countries) has fuelled the discussion that a growing polarization—and not necessarily inequality—is occurring among the wage-earners. This has led to the popular view of the “disappearing the middle class” much discussed over the last decade and a half (see, for example, Bradbury 1985; Horrigan and Haugen, 1985; Levy 1987; Jenkins 1995). The existing inequality measures are not, it is held, adequate to describe this situation. More recently, similar observations have been made regarding income distribution in transition countries. It was argued that income distribution in Russia is becoming bi-modal, with lots of people bunched around a very low income level, a disappearing middle class, and then a sizable class of the rich “new Russians.”

This interest in polarization has led some authors to try to define measures to capture the phenomenon. Wolfson (1994) has proposed a relatively simple measure based on the distance of the Lorenz curve from the 45 degrees line, and the slope of the tangent to the Lorenz curve, both at the median income point. Esteban and Ray (1994) have used the axiomatic approach to derive a measure polarization. Unlike the Wolfson’s approach where the polarization measure is related to the Lorenz-Gini apparatus, Esteban and Ray’s measure is based on the number and size of “peaks” displayed by income distribution. Both measures have some weaknesses. Wolfson’s measure is essentially derived in an ad hoc manner. Its properties are left unexplored (e.g. how it reacts to transfers between recipients whose incomes differ). Esteban and Ray’s measure is
complicated and, to be computed, requires that a researcher make a rather arbitrary decision into how many groups (“peaks”) a given income distribution should be broken down. The result, as seen in Esteban, Gradin and Ray (1999) is that polarization measures for any given distribution are many, and when comparing several distributions, they are made to fit the Procrustean bed of a given number of groups.

The objective of this paper is to propose a new measure of polarization. It is based on the class of general Gini-type functions and is thus close to the Gini coefficient. I follow an axiomatic approach in the derivation of the measure. In Section 2, I define the desirable properties of a polarization measure. In Section 3, I define the new measure. In Section 4, I show whether it satisfies the axioms, how it behaves when progressive or regressive transfers among individuals take place, and how it compares with the Gini index. In section 5, I illustrate some of its properties by using several hypothetical distributions, and then a series of real world income distributions from Eastern European countries.
2. Deriving a polarization measure: axiomatic approach

We begin, axiomatically, with the desirable properties that a polarization measure should possess.

**Axiom 1. Range.** Polarization measure (P) ranges from 0 (no polarization) to 1, or 100 if expressed in percentage.

**Axiom 2. Maximum polarization.** Maximum polarization is reached when a society is divided into two halves such that each member of a half has the same income, with all members of the upper half having income equal to twice the mean, and all members of the bottom half having income 0.

As the number of “peaks” increases, polarization is reduced. But also if the density function has only one peak (so that everybody’s income is the same) polarization is zero.

A corollary of this Axiom is that if income distance between the two peaks is the same, P must decrease as the distribution of individuals between the two peaks diverges from half and half. In other words, society is still divided into two groups (the poor and the rich) with members of each group having the same income, but the difference between the two groups is less. Polarization will be thus greater when the distribution looks as in Figure 1a than as in Figure 1b.
Axiom 3. Distance. The distance between the peaks matters too. If society is divided into two equal halves but the difference in their incomes is small, polarization will be less than if society is divided into two halves with large income differences.

For example, polarization measure in a situation A will be greater than in situation B (Figure 2). Note that A could be a situation where one-half of the population has zero incomes, and other half incomes equal to twice the mean. If incomes of all the poor increase equally, and incomes of all rich go down equally, P must decrease.
Figure 2. Two equal peaks with difference in income distance

Axiom 4. Unit-independence. If all incomes increase in the same proportion, $P$ should not change. In other words, $P$ is a relative measure: it is affected by changes in relative not absolute distances between the individuals. This is a typical requirement imposed on most inequality measures.

A corollary of this Axiom is that a given absolute increase of all incomes should reduce the polarization measure.
3. A new polarization measure defined

A general class of Gini-type functions was defined by Milanovic (1994) as

\[ G^* = w' (u - m) \quad (1) \]

where \( G^* \) indicates a general class of Gini-type functions, \( u \) = the criterion vector, \( m \) = the outcome vector, and \( w' \) = row vector of weights. The standard Gini coefficient is obtained when we write \( u = 1 \) (unit column vector), \( m = y / \bar{y} \) where \( y \) = ordered vector of individual incomes going from the lowest to the highest, and \( \bar{y} \) = mean income, so that \( m \) is the ordered vector of mean-standardized incomes. Finally weights are \( w_i = \frac{2}{n(n+1)} (n-i+1) \), where \( n \) = total number of recipients, so that they decrease as income rank goes up. \(^1\) Writing it out, the Gini index \((G)\) becomes:

\[ G = \sum_{i=1}^{n} w_i (1 - \frac{1}{y}) \]

or

\[ G = \frac{2}{n(n+1)} [n \ n-1 \ n-2 \ldots \ 1] \left( \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} - \frac{1}{y} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_n \end{bmatrix} \right) \quad (2) \]

The unit column vector in (2) represents the yardstick against which we measure the outcome. In the case of the Gini index, the yardstick is equal income for all \((u=1)\). The weighted departure from that standard gives the Gini index.

Formulation (1) is, of course, quite flexible. If we are interested in a tax progressivity index, we can simply set the criterion vector \( u = (1/\bar{y}) \ y \), and outcome vector \( m = (1/\bar{t}) \ t \) where

\(^1\) Note that the sum of weights adds up to 1. The weight of the poorest person will be \( 2/(n+1) \); the weight of the richest person, \( 2/(n^2+n) \).
\( \mathbf{t} \) is vector of taxes paid, and \( \bar{t} \) = the average tax. The implication is that progressivity is zero if one’s relative tax (his tax divided by the mean tax) is the same as one’s relative income (his income divided by mean income). This and a few other applications are given in Milanovic (1994).

Consider now how this approach can be applied to derive a polarization measure. Polarization measure must be median-based because it addresses the issue of how individuals are divided into different income groups. As indicated by Axiom 1, polarization measure should attain the maximum value of 1 if society is divided into two halves with all members of the poor group having income 0 and all members of the rich group having income \( 2 \bar{y} \). Then if we set the criterion vector \( \mathbf{u} \) to be 0 for all individuals whose income is less than the median (\( \mu \)), and 2 for all those whose income is greater than the median, and the outcome vector simply to be the ordered mean-standardized income vector, \( \mathbf{m} = \frac{y}{\bar{y}} \), the polarization measure can be written as

\[
P = 1 - \left[ \sum_{i=1}^{n} w_i (m_i - 0) + \sum_{i=\mu+1}^{n} w_i (2 - m_i) \right]
\]

or in terms of the generalized Gini function as

\[
P = 1 - \mathbf{w}' (|\mathbf{m} - \mathbf{u}|).
\]

Note that the formula (4) highlights the fact that we are summing the absolute distance of the outcome vector from the criterion vector. The criterion vector itself takes the value of zero up to the median income recipient (whose rank is denoted by \( p \)) and value 2 afterwards (for all recipients with the rank greater than \( p \)).

Figure 3 shows the criterion vector (in bold) and the relationship between the progressivity index, \( P \), and the Lorenz curve. \( P \) reaches its maximum (=1), when the Lorenz curve coincides with the criterion vector—the bold line. The Lorenz curve is then horizontal up to the value of \( \frac{1}{2} \) on the x-axis, and slopes upward with the slope 2 (since everybody’s income is twice the mean) until it reaches the point (1,1) in the upper right corner. We can note that the Gini coefficient will, in that case, take the value of \( \frac{1}{2} \) (or 50 if expressed in percentages). This can be shown by
noticing that the area of the triangle AB1 is equal to ¼; hence the area OAB must also be equal to ¼ and the Gini coefficient is ½.  

\[ \text{Figure 3. The polarization index and the Lorenz curve} \]

\[ \text{Cumulative percentage of income} \]

\[ \text{Cumulative percentage of income} \]

\[ A=1/2 \]

\[ B \]

\[ \text{slope 2} \]

\[ 0 \]

\[ 1 \]

\[ \text{Cumulative percentage of recipients} \]

\[ \text{Cumulative percentage of recipients} \]

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\[ ^2 \text{The reverse does not hold: the Gini coefficient can be 50 without P being at its maximum.} \]
4. Properties of the new polarization measure

The polarization measure \( P \) satisfies Axiom 2. We know that when the population is divided into two equal halves such that incomes of the two respective groups are 0 and \( \bar{y} \), polarization index should, according to our Axiom 2, reach its maximum. If we replace \( m_i=0 \) if \( y_i<\mu \), and \( m_i=2 \) if \( y_i>\mu \), in equation (3), the whole term between brackets vanishes and \( P=1 \).

If all incomes are the same, (3) reduces to

\[
P = 1 - \left[ \sum_{i=1}^{p} w_i + \sum_{i=p+1}^{n} w_i \right] = 1 - \frac{2}{n(n+1)} \sum_{i=1}^{n} (n-i+1) = 1 - \frac{2}{n(n+1)} n + 1 = 0
\]

We thus see that \( P \)'s range is \([0,1]\), as requested by Axiom 1.

That Axiom 3 holds is also easy to prove. Let still society be divided into two equal halves with members of each half having identical incomes but with incomes respectively \( m_1 \) (\( m_1>0 \)) and \( m_2 \) (\( m_2<2 \)) where subscripts 1 and 2 refer to the poor and the rich group respectively. Substituting this in (3) we get

\[
P = 1 - \left[ \sum_{i=1}^{p} w_i (m_i - 0) + \sum_{i=p+1}^{n} w_i (2 - m_2) \right] = 1 - \left[ m_1 \sum_{i=1}^{p} w_i + (2-m_2) \sum_{i=p+1}^{n} w_i \right] < 1
\]

The expression between brackets in (5) must be positive because \( m_2<2 \) (if the poor’s income is greater than 0, income of the rich must be less than \( 2 \bar{y} \)).

It is easy to prove that Axiom 4 is satisfied. Note that \( m \)'s in the formula (3) are defined in relative —with respect to the mean—income terms. Hence, a proportionate increase of all incomes must increase the mean income in the same proportion, leaving the mean-adjusted incomes (\( m \)'s) unchanged. The measure therefore does not change.
The corollary of Axiom 4 is that a given across-the-board absolute increase in income must reduce the polarization measure. An equal infinitesimal increase in income yields

\[ dP = -\frac{1}{y} \left( \sum_{i=1}^{n} w_i dy_i - \sum_{i=p+1}^{n} w_i dy_i \right) = -\frac{dy_i}{y} \left( \sum_{i=1}^{p} w_i - \sum_{p+1}^{n} w_i \right) = \]

\[ = -\frac{dy_i}{y} \left( \frac{2}{n(n+1)} \cdot \frac{n + n - p + 1}{2} p - \frac{2}{n(n+1)} \cdot \frac{n - p + 1}{2} p \right) = \]

\[ = -\frac{dy_i}{y} \left( \frac{3n + 2 - n - 2}{4(n+1)} \right) = -\frac{dy_i}{y} \left( \frac{n}{2(n+1)} \right) = -\frac{dy_i}{2y} \]  \( (5) \)

where we use \( w_i = \frac{2}{n(n+1)} (n-i+1), p=\frac{n}{2} \), and for large \( n \), write approximation \( \frac{n}{n+1} = 1 \). Equation (5) shows that, if there is an across-the-board increase in income, the polarization measure decreases by one-half of the mean-adjusted increase. In other words, if we give to every and each recipient the same increase of income equal to (say) 20 percent of overall mean income, \( \frac{dy_i}{y} = 0.2 \) then the polarization measure will drop 0.1 or 10 points.

**Income transfers and polarization**

Consider now the change in our measure if there is a progressive (Daltonian) or regressive income transfer. We need to consider two types of transfers. First, between two individuals whose incomes are respectively higher and lower than the median; second, within each group (those below, and those above the median) between two individuals with unequal incomes.

Let first there be an infinitesimal transfer of income from a person of income rank \( k \) (with income above the median) to a person of lower income rank \( d \) (with income below the median). This is a progressive transfer spanning the two groups, those below and those above, the median. (Transfer is throughout supposed to keep ranks of recipients unchanged, and only to reduce income distance between them.)
Total differentiation of 3 (where we write out $m_i = y_i / \bar{y}$) with respect to income yields:

$$dP = 0 - \left( w_d \frac{dy}{y} + w_k (0 + \frac{dy}{y}) \right) = - \left( \frac{dy}{y} (w_d + w_k) \right) = - \frac{dy}{y} (w_d + w_k) < 0 \quad (6)$$

Progressive transfer between two individuals at the two sides of the median lowers $P$ by the sum of their weights ($w_k + w_d$) multiplied by the relative amount of transfer. Relation (6) shows that the change in $P$ will not only depend on the amount of the transfer $dy / \bar{y}$, but on between whom the transfer takes place. Poorer people have higher weights ($w_i > w_j$ if $i < j$). Hence if $d$ is a very poor person, a given transfer will reduce $P$ by more than if he is just under the median.

Consider now a progressive transfer among people whose income is less than the median. Total differentiation of (3) now yields

$$dP = 0 - \left( w_d \frac{dy}{y} - w_k \frac{dy}{y} \right) = - \left( \frac{dy}{y} (w_d - w_k) \right) = - \frac{dy}{y} (w_d - w_k) < 0 \quad (7)$$

Equation (7) must be negative because $w_d > w_k$. However, compared to a progressive transfer between two individuals who are at two different sides of the median, here the same transfer will reduce $P$ by less because $w_d - w_k$ is less than the sum of $w_d$ and $w_k$.

Finally, let now a progressive transfer take place between individuals whose incomes are both higher than the median.

$$dP = 0 - \left( 0 - w_d \frac{dy}{y} + w_k \frac{dy}{y} \right) = - \left( \frac{dy}{y} (w_k - w_d) \right) = - \frac{dy}{y} (w_k - w_d) > 0 \quad (8)$$

Now, a progressive transfer will increase $P$. How to explain this seemingly paradoxical result? Consider its obverse, namely that a regressive transfer among those with incomes above
the median will reduce polarization. Think of a situation where society is divided into two equal halves with all individuals belonging to each half having the same income. Let now regressive transfers take place among the rich so that income of some declines almost all the way to the income of the poor, while income of a few others increases. But while inequality would go up, polarization—as implied by our Corollary of Axiom 2—would not: there would be simply more people who are poor and have similar incomes.

This example illustrates the philosophical difference between polarization and inequality measures. In the latter, we reach maximum inequality when the entire income is held by one person; in polarization measures, we reach the maximum when the society is split “along the middle.” In political parlance, inequality is a measure of despotism: it has long been observed, since at least Montesquieu, that despotism makes all members of the society equal powerless—but for one. Such a society is not polarized. Differently, polarization occurs when a society is split into two camps of approximately equal size. Using politico-economic analogies, the Weimar Germany was a polarized society, while today’s Brazil is unequal.

Table 1 summarizes the change in the polarization measure due to a given infinitesimal progressive transfer. Of course, by changing the sign, we obtain the effects of a regressive transfers.
Table 1. Change in the polarization measure caused by a progressive transfer equal to \( \frac{dy}{m} \)

<table>
<thead>
<tr>
<th>Transfers</th>
<th>Polarization measure</th>
<th>Change in P</th>
</tr>
</thead>
<tbody>
<tr>
<td>Among those with incomes below the median</td>
<td>Decreases</td>
<td>(- (w_d - w_k) = \frac{-2}{n(n+1)} (k - d) &lt; 0)</td>
</tr>
<tr>
<td>Between a person below, and a person above the median</td>
<td>Decreases</td>
<td>(- (w_k + w_d) = \frac{-2}{n(n+1)} (2n + 2 - k - d) &lt; 0)</td>
</tr>
<tr>
<td>Among those with incomes above the median</td>
<td>Increases</td>
<td>(- (w_k - w_d) = \frac{-2}{n(n+1)} (d - k) &gt; 0)</td>
</tr>
</tbody>
</table>

Notes: \( w_i = \frac{-2}{n(n+1)} (n - i + 1) \).
\( k \geq d \).

**Relationship with Gini index**

We have seen above that both \( P \) and Gini index belong to the class of general Gini-functions. There is an explicit, even if complex, relationship between the two. We write Gini from equation (2) as

\[
G = \sum_{i=1}^{n} w_i (1 - \frac{1}{y}) = \sum_{i=1}^{n} w_i + \sum_{p+1}^{n} w_i - \sum_{i=p+1}^{n} miwi - \sum_{i=p+1}^{n} miwi
\]

Writing out \( P \) from (3), and substituting from equation (9), yields

\[
P = 1 - \sum_{i=1}^{p} w_i m_i - \sum_{i=p+1}^{n} 2w_i + \sum_{i=p+1}^{n} w_i m_i = 1 - G + \left( \sum_{i=1}^{p} w_i - \sum_{i=p+1}^{n} w_i \right) - 2 \sum_{i=1}^{p} w_i m_i
\]

The two terms between brackets on the RHS of (10) can be written

\[
\left( \sum_{i=1}^{\mu} w_i - \sum_{i=\mu+1}^{n} w_i \right) = \frac{2}{n(n+1)} \frac{n+n-p+1}{2} p - \frac{2}{n(n+1)} \frac{n-p+1}{2} p = \frac{n^2}{2n(n+1)} = \frac{1}{2}
\]
if \( n \) is relatively large so that \( \frac{n^2}{n(n+1)} = 1 \).

Also, note that

\[
\sum_{i=1}^{p} w_i m_i = \frac{2}{n(n+1)} \sum_{i=1}^{p} (n - i + 1)m_i = \frac{2}{n(n+1)} \left( n^2 Q(p) - \sum_{i=1}^{p} im_i + nQ(p) \right) = \frac{2}{n(n+1)} \left( n(n+1)Q(p) - \sum_{i=1}^{p} im_i \right) = 2Q(p) - \frac{2}{n(n+1)} \sum_{i=1}^{p} im_i \quad (12)
\]

where we use the fact that the sum of \( m_i \)'s, that is \( \sum_{i=1}^{p} m_i \) is equal to the share of total income received by people with income below the median, denoted \( Q(p) \), multiplied by \( n \).

Then the substitution of (11) and (12) into (10) yields the final expression linking the polarization and the Gini index

\[
P = \frac{3}{2} - G - 4Q(p) + \frac{4}{n(n+1)} \sum_{i=1}^{p} im_i \quad (13)
\]

Now, if \( P=1 \), we know that the cumulative income up to the median is equal to 0 (see Figure 1). Hence the two last terms vanish, and \( G \) must be \( \frac{1}{2} \) as we already noted. Similarly, when \( P=0 \), we know that all recipients have the same income. Hence \( Q(p) = \frac{1}{2} \) and

\[
\sum_{i=1}^{p} i = \frac{n(n+2)}{8}
\]

so that the last term in (13), for a large \( n \), becomes \( \frac{1}{2} \). Then, for \( P=0 \), \( G \) must be 0.

Equation (13) illustrates complex relationship between our polarization index and the Gini. When \( G \) increases, \( P \) goes down in virtue of direct relationship between \( P \) and \( G \). But, in addition, if increase in Gini implies, as it often down, a decline in the income share of the recipients below the median, both \( Q(p) \) and the sum of \( m_i \)'s will decrease. Now, the decline in
Q(p) will increase P, while the decrease in the sum of m_i’s (the last term in 13) will reduce P. The overall effect on P will be impossible to predict, in general case, because of the offsetting forces.

There are three links between inequality and polarization:

- (1) direct effect of G on P (negative);
- (2) effect through total income share of those with income below the median Q(p): if it increases, P goes down;
- (3) effect through distribution among the “poor”. Even if total income share of those below the median remains unchanged, but there is a regressive transfer among them, P will increase (note that in (13) to each m_i is attached a weight equal to its rank: increase of higher incomes therefore increases the last term and thus P).\(^3\)

The implication is that if increase in the Gini entirely occurs through redistribution between those with incomes above the median, so that neither Q(p) nor m_i’s below the median are not affected, polarization must go down. We thus reestablish the result noted above: regressive transfers among the “rich” reduce polarization, by making some of those with incomes above the median “closer” to those below the median. They thus represent the movement toward what we termed a “despotic” solution to polarization where all income is appropriated by one or a few people. Polarization is thus low (since almost all are poor) even if inequality is high. If increase in Gini occurs through decrease in the share of those below the median, P will decrease both through the direct effect (1) and because the share of Q(p) goes down (effect 3).

\(^3\) This is equivalent to stating that a regressive transfer between the people to the left of the median will increase polarization (a finding reported in Table 1).
5. Some simulations and application

A hypothetical example

Table 2 shows how the measure of polarization changes with different distributions of incomes. Distribution A is bi-polar with the maximum distance between the poles. The poors’ income is zero; all the rich have the same income. P reaches its maximum value 100 (expressed as a percentage), Gini index is 50.

When the distribution remains bi-polar but the distance between the poles diminishes (distribution B), the measure decreases; in our example, P drops to 66.7 (Axiom 3).

Distribution C is obtained from the distribution B through a series of regressive transfers among the those whose income is above the median. Hence, polarization of C must be less than polarization of B (see Table 1), even if Gini increases.

With a greater number of peaks, as in distribution D, polarization drops further: P decreases to 44.4.
Table 2. Features of a new polarization measure

<table>
<thead>
<tr>
<th></th>
<th>Distribution</th>
<th>Total income</th>
<th>Polarization measure</th>
<th>Gini index</th>
<th>Explanation</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>(0, 0, 0, 0, 0, 0, 6, 6, 6, 6, 6, 6)</td>
<td>36</td>
<td>100</td>
<td>50</td>
<td>P reaches maximum (Axiom 2)</td>
</tr>
<tr>
<td>B</td>
<td>(1, 1, 1, 1, 1, 1, 5, 5, 5, 5, 5, 5)</td>
<td>36</td>
<td>66.7</td>
<td>33.3</td>
<td>P decreases because the distance between the two peaks goes down (Axiom 3)</td>
</tr>
<tr>
<td>C</td>
<td>(1, 1, 1, 1, 1, 1, 1, 7, 7, 7, 7)</td>
<td>36</td>
<td>56.4</td>
<td>44.4</td>
<td>Distribution C obtained through a series of regressive transfers above the median from distribution B. P goes down.</td>
</tr>
<tr>
<td>D</td>
<td>(1, 1, 1, 1, 3, 3, 3, 3, 5, 5, 5, 5)</td>
<td>36</td>
<td>44.4</td>
<td>29.6</td>
<td>P decreases because the number of peaks increases (Axiom 2)</td>
</tr>
<tr>
<td>E</td>
<td>(1, 1, 1, 1, 2, 2, 2, 2, 6, 6, 6, 6)</td>
<td>36</td>
<td>50.4</td>
<td>37.0</td>
<td>A combination of regressive transfers among the “poor” (increasing P), and among the rich (decreasing P). The former effect stronger because of greater weights among the poor and P increases compared to situation D.</td>
</tr>
<tr>
<td>F</td>
<td>(4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4, 4)</td>
<td>36</td>
<td>0</td>
<td>0</td>
<td>P equal zero as everybody’s income is the same.</td>
</tr>
</tbody>
</table>

Figure 3. The relationship between Gini and the polarization measure (hypothetical distributions)
Figure 3 graphs the Gini and polarization indexes from Table 2. Note that the two measures do not necessarily move in the same direction. For example, while polarization index declines as we move from distribution B to C, Gini coefficient rises substantially.

**Application to East European data**

Figure 4 shows the Gini coefficients and the new polarization measure on the data from the three transition economies (Bulgaria, Poland, Russia using income and expenditure data). We notice that, in almost all cases, Gini and the polarization measure move in the same direction (note the lines are upward sloping). The clearest case of almost identical movements of inequality and polarization is exhibited on the Russian expenditure data for the period 1994-98. However, in a few cases, the two measures do not move in unison. In Bulgaria, inequality in 1996 and 1997 is the same (Gini equal to 34), but polarization increased by about ½ points. In Poland, between 1995 and 1996, inequality increased by ½ points, but polarization stayed the same. Thus, while it seems that in most cases, the two measures move together, this is not the always the case. The two measures, as argued above, do describe similar but different phenomena.

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4 In the four examples shown in Figure 4, in 11 out of 13 cases, the two measures increase or decrease together.
Figure 4. Evolution of inequality (Gini) and new polarization measure in several transition economies

Bulgaria 1992-98

Poland 1995-98

Russia 1994-98 (expenditures)

Russia 1994-98 (income)

Sources: Bulgaria and Poland: Household budget surveys. Russia: Russian Longitudinal Monitoring Survey. In all cases, decile data (10 percent of individuals) used to calculate both measures. Data for Poland and Bulgaria are on per capita basis; data for Russia per equivalent adult.
6. Conclusions

The decreasing share of the wage-earners with wages around the mean, has fuelled the discussion of wage (and income) polarization. It was argued that inequality and polarization are distinct concepts. Inequality increases as fewer people appropriate a larger share of resources; but polarization increases as the society gets divided into two groups. This paper takes the same perspective. It proposes several axioms that a polarization measure should satisfy, and then derives a new polarization measure. The new polarization measure belong to the class of Gini-type functions as defined by Milanovic (1994). It is related, but of course, is different from the Gini index. The paper shows the sensitivity of polarization to regressive and progressive transfers of income between the recipients, to a uniform increase in all incomes etc. It shows, for example, that a progressive (Daltonian) transfer will affect progressivity differently depending on whether it takes place between individuals whose incomes are both below the median, or above the median. This, clearly, is different from what one normally requires from an inequality measure.

The new polarization measure is simple to calculate (an Excel spreadsheet is sufficient), and easy to understand. The behavior of the new measure is contrasted with that of the Gini coefficient on some simulated data, and on real income (and expenditure) data derived from Household budget surveys.

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5 For example Beach, Chaykowski and Slotsve (1997, p. 144) show that the share of male workers with wage below 50 percent of the mean increased from 15 percent in 1968 to 21 percent in 1990; the share of those with wage above 150 percent of the mean increased from 18 percent in 1968 to 27 percent in 1990. Clearly, the “middle class” (those with wages between 0.5 and 1.5 times the mean) shrunk from 67 percent of all male earners to slightly over a half.
REFERENCES


