

Chapter 8

WEALTH AND PRODUCTION

One of the recurring themes in the sustainability literature has been the legitimacy of using an economic framework to account for natural resources. Those critical of such an approach contend that wealth accounting assumes that produced assets, such as human and physical capital, can substitute for natural-resource assets on a dollar-for-dollar basis. This, they argue, does not capture the limited degree to which such substitution is possible. A loss of some natural capital, such as an entire ecosystem, surely cannot be made up with an increase in physical capital if the very basis of social existence and well-being are destroyed in the areas affected by that system. This makes them skeptical of the kind of wealth accounts we are constructing here.

While we cannot hope to disentangle the full set of issues embedded in this line of reasoning, we can at least start by focusing on the degree of substitutability between the different assets. Underlying any wealth accounts is an implicit *production function*, which is a blueprint of the combinations of different assets with which we can achieve a given level of output. These blueprints are usually written as a mathematical function, which describes the precise relationship between the availability of different amounts of *inputs*, such as physical and human capital services, and the maximum output they could produce. The substitutability between inputs is then measured as an *elasticity of substitution*. In general terms, this captures the ease with which a decline in one input can be compensated by an increase in another, while holding output constant. More precisely, it measures how much the ratio of two inputs (for example, physical capital and land) changes when their relative price changes (for example, the price of land goes up relative to the price of capital).¹ The greater the elasticity, the easier it is to make up for the loss

of one resource by using another. Generally, an elasticity of less than one indicates limited substitution possibilities.

A commonly used production function, which implies elasticities of one between the inputs, is the Cobb-Douglas form, written as:

$$Y_t = A_t K^\alpha L^\beta \quad (8.1)$$

Income or output (Y) is expressed as a function of the levels of capital input (K), labor input (L), an exogenous technological factor (A) and the parameters α and β , which give the returns to capital and labor respectively. If the national production options could be captured by such a function, with natural capital services included, it would have considerable implications for sustainability. First, it would imply a degree of substitutability between natural and produced capital that would give some comfort to those who argue we can lose some natural capital without seriously compromising our well-being. Related to that it would validate the Hartwick rule, which states that when exploiting natural resources, consumption can be sustained at its highest possible level if net saving just equals the rent from exploiting those resources (Hartwick 1977; Hamilton 1995). The Hartwick rule is a useful sustainability policy since it is open to monitoring. We can check whether or not it has been adhered to.

Economists have devoted a considerable amount of effort to estimating these elasticities for inputs such as capital, labor, and energy but *not* natural resources. Although, starting in the 1970s, there were theoretical studies that modeled neoclassical economic growth with nonproduced capital, such as natural resources, as factors in production (Stiglitz 1974a, b; Mitra 1978),² the empirical estimation of the underlying production functions was never carried out, largely because of a lack of data.

This chapter is a preliminary attempt in that direction. As mentioned in the earlier chapters, a database of new wealth estimates has been developed, including both produced and nonproduced capital—renewable and nonrenewable resources and human resources—which allows us to estimate a production function that includes the services from these different resources as inputs. This chapter examines, therefore, the economic relationship between total wealth and income generation and takes advantage of the new wealth estimates to estimate a production function based on a larger set of assets. Section 2 presents the estimation of the production function. Section 3 concludes.

Estimation of Nested CES Production Function

The estimation carried out here uses national-level data on gross national income (GNI) or economic output and sees the extent to which variations in GNI across countries, at any point in time, can be explained in terms of the national availability of produced capital, human resources, and natural resources (energy and land resources). A Cobb-Douglas production function of the form shown above is not appropriate for this estimation because it restricts the elasticity between factors to be one. In fact, one of our objectives is to estimate the elasticity of substitution between factors or groups of factors. A form that holds the elasticity constant but allows it to take values different from one is the *constant elasticity of substitution* (CES) production function. In particular, this chapter uses a *nested CES* production function. For example, a two-level nested CES with three inputs takes the form:³

$$X = F[X_{AB}(A, B), C] \quad (8.2)$$

where X is the gross output; A , B , and C are inputs; and X_{AB} represents the joint contribution of A and B to production. The first level of the estimation involves A and B ; while the second level models the production of output by X_{AB} and C . A special feature of the nested CES function is that the elasticity of substitution between the first-level inputs, A and B , can be different from the elasticity of substitution between the second-level inputs, X_{AB} and C . In other words, by placing natural resources and other inputs in different levels of the function, we effectively allow for different levels of substitutability. So, for example, natural assets may be critical (low substitutability) while other inputs are allowed to be more substitutable among themselves.

There are several studies that have estimated the nested CES production function between three or four production inputs, such as capital, labor, energy, and nonenergy materials at the firm level (Prywes 1986; Manne and Richels 1992; Chang 1994; Kemfert 1998; Kemfert and Welsch 2000). A common interest among these studies is examining the capital-energy substitution in manufacturing industries. For example, Manne and Richels (1992) estimated the substitution possibilities between the capital and labor *nest* and energy to be about 0.4; while Kemfert (1998) estimated the same to be about 0.5. On the other hand, Prywes (1986) found the elasticity of substitution between the capital and energy *nest* and labor to be less than 0.5.

In this chapter we use related variables to estimate aggregate national-level production functions. The variables used are:⁴

- *Produced capital (K)* is an aggregate of equipments, buildings, and urban land.
- *Human capital (H)* has two alternative measures—human capital, which relates educational attainment with labor productivity (*HE*); or intangible capital residual (*HR*), which is obtained as the difference between a country's total wealth and the sum of produced and natural assets. Part of the intangible capital residual captures human capital in the form of raw labor and stock of skills. For further discussion of this variable and its rationale see chapters 2 and 7.
- *Production and net imports of nonrenewable energy resources (E)* includes oil, natural gas, hard coal, and lignite.⁵
- *Land resources (L)* refers to the aggregated value of cropland, pastureland, and protected areas. Land is valued in terms of the present value of the income it generates rather than its market value.

The GNI and all inputs mentioned above are measured in per capita values at 2000 prices and are taken at the national level for 208 countries. GNI data are obtained from the *World Development Indicators* (World Bank 2005). *HE* is derived based on the work by Barro and Lee (2000); *E* is a flow measure and is obtained using the same data that underpin the wealth estimates; while the remaining variables, *K*, *HR*, and *L* are the components of wealth as described in chapter 2.

The relationships of the production inputs to income are expressed in nested CES production functions described in the chapter annex. Three different nested CES approaches are examined:

- One-level function, with two inputs
- Two-level function, with three inputs
- Three-level function with four inputs

The combinations of the variables in the different CES approaches were varied to further investigate any possible differences among substitution elasticities for pairs of inputs.

The production function approach taken so far neglects an important set of factors that influence differences in national income. These

relate to the efficiency with which productive assets are utilized and combined, and include both institutional as well as economic factors. In this study, we consider the following institutional indicators, which capture the efficiency with which production can take place, as well as economic indicators, which also capture the efficiency of economic organization:

- *Institutional development indicators*—indices on voice and accountability (VA), political instability and violence (PIV), government effectiveness (GE), regulatory burden (RB), rule of law (RL); and control of corruption (CC). An increase in a given index measures an improvement in the relevant indicator. Hence, they are expected to have a positive impact on income and possibly growth (Kaufmann and others 2005).⁶
- *Economic indicators*—trade openness (TOPEN) is calculated as the ratio of exports and imports to GDP (World Bank 2005); and the country's domestic credit to the private sector as proportion of GDP (PCREDIT), which represents private sector investments (Beck and others 1999).⁷

Two methods of incorporating the impact of these institutional and economic indicators were investigated. The first method involved the derivation of residuals from the regression of a nested CES production function. The residuals are the part of income not explained by the wealth components—physical capital, human capital, land resources, and energy resources, and are regressed on the identified institutional and economic indicators. By using this method, however, a statistically significant correlation between the residuals and any indicator would imply that relevant variables have been omitted in the estimation of the nested CES production function. Thus, the estimated coefficients of the nested CES production function derived earlier will be biased and inefficient (Greene 2000). Hence another method is considered to be more appropriate. The influences of the institutional and economic indicators on income will be incorporated into the efficiency parameter of the production function, A (see annex 2). Depending on the available data for the variables of the nested CES production function, the number of countries drops in the range of 67 to 93 countries. In the complete case method, for a given nested CES approach, the reduction is caused by considering only those countries that have nonmissing observations for their corresponding dependent and explanatory variables.⁸

Regression Results

The nested CES production functions are estimated using a nonlinear estimation method.⁹ The sample size in each CES approach differs because countries with missing observations in any of the variables had to be dropped. Table A8.1.1 in annex 1 shows the estimated substitution elasticities corresponding to the case where human capital is part of the measured intangible capital residual (*HR*). All statistically significant substitution elasticity estimates have a positive sign, which is encouraging.¹⁰ The lowest is that between *K* and *E* at 0.37 in the three-level production function. It is also interesting to note that most of the significant elasticities of substitution are close to one.

A second round of regressions was carried out using the other measure of human capital that is related to schooling and labor productivity, *HE*. Table A8.1.2 in annex 1 shows the statistically significant elasticities of substitution, which also have a positive sign. An elasticity of substitution approximately equal to one is likewise found for most of the nested functions.

The results provide some interesting findings. First, there is no sign that the elasticity of substitution between the natural resource (land) and other inputs is particularly low. Wherever land emerges as a significant input, it has an elasticity of substitution approximately equal to or greater than one. Second, the *HE* variable performs better in the estimation equations than the *HR* variable. Third, the best-determined forms, with all parameters significant, are those using *HE*, involving four factors and containing the combinations:

- *K*, *HE*, and *L* are nested together and then combine with *E*, or
- *K*, *HE*, and *E* are nested together and then combine with *L*.

It is hard to distinguish between these two versions, and so they are both used in the further analysis reported below.

From the nested CES production function estimations, the elasticity estimates of the institutional and economic indicators can be derived. Table A8.1.3 and table A8.1.4 in annex 1 show the results for the four-factor production functions $[(K,HE,L)/E]$ and $[(K,E,HE)/L]$ of table A8.1.2, respectively. In both tables, the variables on trade openness

and private sector investment are found to be statistically significant. The elasticity estimates of these two variables are not very different from each other. The results imply that for every percent increase in trade openness, gross national income per capita (GNIPC) increases by approximately 0.5 percent. None of the institutional indicators, on the other hand, has a statistically significant elasticity estimate.¹¹

Simulation

The predicted value of the dependent variable can be calculated by using the estimated coefficient estimates of the production function and the mean values of the explanatory variables. Through this method, we try to predict what will happen to the economic output per capita (GNIPC) if there is significant natural resource depletion. The natural resource considered in this exercise is land resources (L); and the four-factor nested CES production functions used are $[(K, HE, L)/E]$ and $[(K, E, HE)/L]$ of table A8.1.2. Table A8.1.5 in annex 1 presents the predicted average GNIPC, as well as the change in GNIPC given a reduction in the amount of land resources, other things being equal. Based on the production function $[(K, HE, L)/E]$, economic output is reduced by 50 percent when the amount of L declines by about 92 percent, while holding other variables constant. For the production function $[(K, E, HE)/L]$, on the other hand, it takes a reduction in the amount of L by about the same percentage, other things being equal, to halve the economic output relative to the baseline.

Conclusions

In this chapter, we looked at the potential for substituting between different inputs in the generation of GNI. Among these are land resources, one of the most important natural resources. The estimation of a well-known production function form, which allows the elasticities of substitution to be different from one, was carried out. The resulting elasticities involving land resources (between L and other inputs such as physical capital, human capital, and energy resources) were

generally around one or greater, which implies a fairly high degree of substitutability. Moreover, it validates the use of a Hartwick rule of saving the rents from the exploitation of natural resources if we are to follow a maximum constant sustainable consumption path.

This result, not surprisingly, has many caveats. Land resources as measured here include cropland, pastureland, and protected areas. Each has been valued in terms of present value of the flow of income that it generates. Such flows, however, underrepresent the importance of, for example, protected areas, which provide significant nonmonetary services, including ecosystem maintenance services that are not included. Further work is needed to include these values, and if this were done, and if the GNI measure were adjusted to allow for these flows of income, the resulting estimates of elasticities of substitution might well change. We intend to continue to work along these lines and to improve the estimates made here.

Another shortcoming of the method applied here is the limited number of factors included in the original estimation. Generating national income depends not on the stock of assets, but on the amounts of the stocks that are used in production and the way in which they are used. For physical and human capital and land, we assume the rate of use is proportional to the stock. That assumption should be improved on, to allow for different utilization rates.

Finally, the chapter also examines how the institutional and economic indicators affect the generation of GNI. Estimation results show that income generation is significantly influenced by changes in trade openness and private sector investment. The institutional indicators, however, have no statistically significant impact on income generation.

Annex 1 Tables

Table A8.1.1 Elasticities of Substitution ($\hat{\sigma}_i$), Using Human Resources (HR)

Inputs	Elasticity of substitution		<i>R</i> -squared	Adj. <i>R</i> -squared	Sample size
	$\hat{\sigma}_i$	Standard error			
A. Two factors (one-level CES production function)					
(1) <i>K/HR</i>	1.00*	3.88E-10	0.9216	0.9131	93
(2) <i>K/E</i>	-0.48	2.02	0.9958	0.9951	78
B. Three factors (two-level CES production function)					
(1) (<i>K,HR</i>)/ <i>L</i>			0.9375	0.9290	93
> <i>K/HR</i>	6.79	13.92			
> (<i>K,HR</i>)/ <i>L</i> ^a	1.00*	4.33E-10			
(2) (<i>K,HR</i>)/ <i>E</i>			0.9089	0.8916	70
> <i>K/HR</i>	-0.78	1.31			
> (<i>K,HR</i>)/ <i>E</i> ^a	1.00*	5.37E-10			
(3) (<i>K,E</i>)/ <i>HR</i>			0.87667	0.8533	70
> <i>K/E</i>	0.65	0.69			
> (<i>K,E</i>)/ <i>HR</i> ^a	1.00*	3.96E-09			
C. Four factors (three-level CES production function)					
(1) (<i>K,HR,L</i>)/ <i>E</i>			0.3435	0.1911	70
> <i>K/HR</i>	-0.90	0.70			
> (<i>K,HR</i>)/ <i>L</i> ^a	0.97*	0.01			
> (<i>K,HR,L</i>)/ <i>E</i> ^b	1.00*	5.46E-12			
(2) (<i>K,HR,E</i>)/ <i>L</i>			0.9958	0.9951	78
> <i>K/HR</i>	-0.13	0.17			
> (<i>K,HR</i>)/ <i>E</i> ^a	0.93*	0.18			
> (<i>K,HR,E</i>)/ <i>L</i> ^b	1.00*	6.52E-09			
(3) (<i>K,E,HR</i>)/ <i>L</i>			0.9350	0.9200	70
> <i>K/E</i>	0.37*	0.20			
> (<i>K,E</i>)/ <i>HR</i> ^a	-0.64	0.55			
> (<i>K,E,HR</i>)/ <i>L</i> ^b	1.00*	1.27E-09			

Source: Authors.

Notes:

Legend: *K*=physical capital; *HR*=human capital (captures raw labor and stock of skills); *L*=land resources; *E*=energy resources.

Inputs in parentheses imply that they are nested.

a. Two inputs in a nested function.

b. Three inputs in a nested function.

(*) denotes statistical significance at 5 percent level.

The elasticities of substitution and their corresponding standard errors are rounded off to the nearest hundredth.

Table A8.1.2 Elasticities of Substitution ($\hat{\sigma}_i$), Using Human Capital Related to Schooling (HE)

Inputs	Elasticity of substitution		R-squared	Adj. R-squared	Sample size
	$\hat{\sigma}_i$	Standard error			
A. Two factors (one-level CES production function)					
(1) K/HE	1.00*	2.50E-08	0.9061	0.8942	81
B. Three factors (two-level CES production function)					
(1) $(K,HE)/L$			0.9203	0.9076	81
> K/HE	1.01*	0.01			
> $(K,HE)/L^a$	1.00*	2.23E-10			
(2) $(K,HE)/E$			0.8952	0.8742	67
> K/HE	1.65*	0.12			
> $(K,HE)/E^a$	1.00*	6.76E-11			
(3) $(K,E)/HE$			0.7674	0.7209	67
> K/E	0.17	0.19			
> $(K,E)/HE^a$	1.00*	8.22E-08			
C. Four factors (three-level CES production function)					
(1) $(K,HE,L)/E$			0.9037	0.8081	67
> K/HE	1.78*	0.11			
> $(K,HE)/L^a$	1.14*	0.02			
> $(K,HE,L)/E^b$	1.00*	2.52E-12			
(2) $(K,HE,E)/L$			0.9059	0.8828	67
> K/HE	-8.55	12.61			
> $(K,HE)/E^a$	0.48*	0.17			
> $(K,HE,E)/L^b$	1.00*	4.60E-11			
(3) $(K,E,HE)/L$			0.9062	0.8831	67
> K/E	1.57*	0.37			
> $(K,E)/HE^a$	0.92*	0.02			
> $(K,E,HE)/L^b$	1.00*	6.41E-11			

Source: Authors.

Notes:

Legend: K =physical capital; HE =human capital related to educational attainment and labor productivity; L =land resources; E =energy resources.

Inputs in parentheses imply that they are nested.

a. Two inputs in a nested function.

b. Three inputs in a nested function.

(*) denotes statistical significance at 5 percent level; (**) at 10 percent level.

The elasticities of substitution and their corresponding standard errors are rounded off to the nearest hundredth.

Table A8.1.3 Elasticity Estimates of the Economic and Institutional Indicators, Using the [(K, HE, L)/E] Production Function

Variable	Elasticity	Standard error	t-statistic
<i>TOPEN</i>	0.47	0.10	4.53
<i>PCREDIT</i>	0.51	0.12	4.25
<i>VA</i>	0.01	0.04	0.28
<i>PIV</i>	-0.01	0.02	-0.28
<i>GE</i>	0.04	0.10	0.40
<i>RB</i>	0.03	0.07	0.39
<i>RL</i>	-0.07	0.10	-0.73
<i>CC</i>	0.01	0.09	0.17

Source: Authors.

Note: Legend: *TOPEN*=trade openness; *PCREDIT*=variable for private sector investment; *VA*=voice and accountability; *PIV*=political instability and violence; *GE*=government effectiveness; *RB*=regulatory burden; *RL*=rule of law; and *CC*=control of corruption.

Table A8.1.4 Elasticity Estimates of the Economic and Institutional Indicators, Using the [(K, E, HE)/L] Production Function

Variable	Elasticity	Standard error	t-statistic
<i>TOPEN</i>	0.50	0.09	5.27
<i>PCREDIT</i>	0.51	0.11	4.83
<i>VA</i>	0.02	0.03	0.45
<i>PIV</i>	-0.01	0.02	-0.44
<i>GE</i>	0.06	0.09	0.62
<i>RB</i>	0.03	0.07	0.37
<i>RL</i>	-0.08	0.09	-0.86
<i>CC</i>	-0.02	0.08	-0.24

Source: Authors.

Note: Legend: *TOPEN*=trade openness; *PCREDIT*=variable for private sector investment; *VA*=voice and accountability; *PIV*=political instability and violence; *GE*=government effectiveness; *RB*=regulatory burden; *RL*=rule of law; and *CC*=control of corruption.

Table A8.1.5 Level of Gross National Income per Capita, Given a Reduction in the Amount of Land

Prod. function	Baseline*	Reduction in the amount of land by			
		20%	50%	75%	92%
$(K, HE, L)/E$	\$8,638.10	\$8,068.84	\$7,019.27	\$5,774.25	\$4,297.16
<i>Difference from baseline**</i>		(-7%)	(-19%)	(-33%)	(-50%)
$(K, E, HE)/L$	\$9,096.20	\$8,540.27	\$7,477.97	\$6,147.62	\$4,455.06
<i>Difference from baseline**</i>		(-6%)	(-18%)	(-32%)	(-51%)

Source: Authors.

Notes:

*Predicted per capita GNI at the mean values of the explanatory variables.

**Rounded off to the nearest whole number.

Sample size of each production function = 67.

Annex 2 Three Different CES Approaches

1. A traditional CES production function with two inputs is written as:

(a) Physical capital (K) and human capital (H)

$$Y = A(aK^{-\beta} + bH^{-\beta})^{-1/\beta} \quad (\text{A.1})$$

(b) Physical capital (K) and energy resources (E)

$$Y = A(aK^{-\beta} + bE^{-\beta})^{-1/\beta} \quad (\text{A.2})$$

where Y is the per capita gross national income. A is an efficiency parameter. a and b are distribution parameters that lie between zero and one and β represents the substitution parameter. The elasticity of substitution (σ) is calculated as: $\sigma = 1/(1 + \beta)$. Values of β must be greater than -1 (a value less than -1 is economically nonsensical, although it has been observed in a number of studies [Prywes 1986]). If $\beta > -1$, the elasticity of substitution must, of course, be positive.

A , the efficiency parameter, is assumed to be a function of the economic ($TOPEN$ and $PCREDIT$) and institutional indicators described in the text. Two functional forms of A have been tried:

$$(c) A = e^{\lambda_1 TOPEN + \lambda_2 PCREDIT + \lambda_3 VA + \lambda_4 PIV + \lambda_5 GE + \lambda_6 RB + \lambda_7 RL + \lambda_8 CC} \quad (\text{A.3})$$

$$(d) A = \lambda_1 TOPEN + \lambda_2 PCREDIT + \lambda_3 VA + \lambda_4 PIV + \lambda_5 GE + \lambda_6 RB + \lambda_7 RL + \lambda_8 CC \quad (\text{A.4})$$

and the second functional form of A was found to be more appropriate.

$TOPEN$ means trade openness; $PCREDIT$ is a variable for private sector investment; VA , voice and accountability; PIV , political instability and violence; GE , government effectiveness; RB , regulatory burden; RL , rule of law; and CC , control of corruption. The scores for each institutional indicator lie between -2.5 and 2.5 , with higher scores corresponding to better outcomes.

2. A two-level nested CES production function with three inputs is investigated for three cases:

(a) K and H in the nested function, X_{KH} is a substitute for land resources (L):

$$Y_1 = A_1 \left[a_1 \left(b_1 K^{-\alpha_1} + (1 - b_1) H^{-\alpha_1} \right)^{\beta_1/\alpha_1} + (1 - a_1) L^{-\beta_1} \right]^{-1/\beta_1} \quad (\text{A.5})$$

- (b) K and H in the nested function, X_{KH} is a substitute for energy resources (E):

$$Y_2 = A_2 \left[a_2 \left(b_2 K^{-\alpha_2} + (1-b_2) H^{-\alpha_2} \right)^{\beta_2/\alpha_2} + (1-a_2) E^{-\beta_2} \right]^{-1/\beta_2} \quad (\text{A.6})$$

- (c) K and E in the nested function, X_{KE} is a substitute for human capital (H):

$$Y_3 = A_3 \left[a_3 \left(b_3 K^{-\alpha_3} + (1-b_3) E^{-\alpha_3} \right)^{\beta_3/\alpha_3} + (1-a_3) H^{-\beta_3} \right]^{-1/\beta_3} \quad (\text{A.7})$$

where α_i and β_i are substitution parameters.

3. A three-level nested CES production function with four inputs is studied for these three cases:

- (a) K , H , and L in the nested function, and E as a substitute for X_{KHL} :

$$Y_4 = A_4 \left\{ a_4 \left[b_4 \left(c_4 K^{-\alpha_4} + (1-c_4) H^{-\alpha_4} \right)^{\rho_4/\alpha_4} + (1-b_4) L^{-\rho_4} \right]^{\beta_4/\rho_4} + (1-a_4) E^{-\beta_4} \right\}^{-1/\beta_4} \quad (\text{A.8})$$

- (b) P , H , and E in the nested function, and L as a substitute for X_{KHE} :

$$Y_5 = A_5 \left\{ a_5 \left[b_5 \left(c_5 K^{-\alpha_5} + (1-c_5) H^{-\alpha_5} \right)^{\rho_5/\alpha_5} + (1-b_5) E^{-\rho_5} \right]^{\beta_5/\rho_5} + (1-a_5) L^{-\beta_5} \right\}^{-1/\beta_5} \quad (\text{A.9})$$

- (c) K , E , and H in the nested function, and L as a substitute for X_{KEH} :

$$Y_6 = A_6 \left\{ a_6 \left[b_6 \left(c_6 K^{-\alpha_6} + (1-c_6) E^{-\alpha_6} \right)^{\rho_6/\alpha_6} + (1-b_6) H^{-\rho_6} \right]^{\beta_6/\rho_6} + (1-a_6) L^{-\beta_6} \right\}^{-1/\beta_6} \quad (\text{A.10})$$

where α_p , ρ_p , β_i are substitution parameters; and $0 < a_p, b_p, c_i < 1$.

The substitution elasticities for these CES approaches can be described as follows:

$\sigma_{\alpha_i} = \frac{1}{1+\alpha_i}$	<p>Gives the elasticity of substitution between K and H when $i = 1,2,4,5$</p> <p>Gives the elasticity of substitution between K and E when $i = 1,6$</p>
$\sigma_{\rho_i} = \frac{1}{1+\rho_i}$	<p>Gives the elasticity of substitution between K/H and L when $i = 4$</p> <p>Gives the elasticity of substitution between K/H and E when $i = 5$</p> <p>Gives the elasticity of substitution between K/E and H when $i = 6$</p>
$\sigma_{\beta_i} = \frac{1}{1+\beta_i}$	<p>Gives the elasticity of substitution between K/H and L when $i = 1$</p> <p>Gives the elasticity of substitution between K/H and E when $i = 2$</p> <p>Gives the elasticity of substitution between K/E and H when $i = 3$</p> <p>Gives the elasticity of substitution between $K/H/L$ and E when $i = 4$</p> <p>Gives the elasticity of substitution between $K/H/E$ and L when $i = 5$</p> <p>Gives the elasticity of substitution between $K/E/H$ and L when $i = 6$</p>

The nested CES production functions are estimated using the nonlinear estimation method via the STATA program. The nonlinear estimation program uses an iterative procedure to find the parameter values in the relationship that cause the sum of squared residuals (SSR) to be minimized. It starts with approximate guesses of the parameter values (also called *starting values*), and computes the residuals and then the SSR. The starting values are a combination of arbitrary values and coefficient estimates of a nested CES production function. For example, the starting values of equation (A.1) are arbitrary. A set of numbers is tried until convergence is achieved. On the other hand, the starting values of

equation (A.5) are based on the coefficient estimates of equation (A.1). Next, it changes one of the parameter values slightly and computes again the residuals to see whether the SSR becomes smaller or larger. The iteration process goes on until there is convergence—it finds parameter values that, when changed slightly in any direction, cause the SSR to rise. Hence, these parameter values are the least squares estimate in the nonlinear context.

Endnotes

1. Where prices are not defined, we measure the change in the ratio of the inputs resulting from a change in the marginal rate at which one factor can be substituted for another (Chiang 1984).
2. A bibliographical compilation of studies can be found in Wagner (2004). One exception to the observation that there is little empirical work is Berndt and Field (1981), who did look at limited natural resource substitution between capital, labor, energy, and materials. The studies generally found low elasticities between capital and materials. They did not, however, look at land as an input in the way we do here. Nor did they work with national-level data.
3. This model makes the further assumption of *homothetic weak separability* for groups of inputs. *Homothetic weak separability* means that the marginal rate of substitution between inputs in a certain group is independent of output and of the level of inputs outside that group (Chiang 1984).
4. Per capita dollar values at nominal 2000 prices are used.
5. For energy it would be inappropriate to take the stock value of the asset, as what is relevant for production is the flow of energy available to the economy. This is given by production plus net imports. With the other assets (K , H , and L) it is also the flow that matters, but it is more reasonable to assume that the flow is proportional to the stock. We do note, however, in the conclusions that even this assumption needs to be changed in future work.
6. Data can be obtained from the website: <http://www.worldbank.org/wbi/governance/pubs/govmatters4.html>.
7. Hnatkovska and Loayza (2004) use openness and credit as a measure of financial depth, which they find to have a positive impact on growth. Data for this indicator can be obtained from the following website: <http://www.worldbank.org/research/projects/finstructure/database.htm>.

8. An *imputation method* was tried to fill the missing values for some of the countries to keep all 208 countries in the estimation. Most of the results, however, were not found to be reasonable. For example, the imputed value of physical capital for a low-income country turned out to be too high compared with the average value of physical capital of its income group. Hence, the imputation method was not used since it poses more problems in the estimates than using the *complete case method*.

9. See annex 2 for more details.

10. A negative elasticity of substitution is economically nonsensical—it implies a decline in the availability of one input can be *made up* by a decline in the availability of other factors. Nevertheless, some production function studies do find such negative values.

11. In the regression where the residuals are expressed as a function of the institutional variables, we did find significant values for a few institutional variables, especially the rule of law, which was encouraging as that variable also emerges as important in other evaluations of intercountry differences in this study. Unfortunately, the result did not hold when the more appropriate method was used.

