

# Public Initiatives to Support Entrepreneurs: Credit Guarantees versus Co-Funding

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# **Public Initiatives to Support Entrepreneurs: Credit Guarantees versus Co-Funding**

## **Abstract**

We analyze state-sponsored credit guarantees in a setting where entrepreneurs are capital-constrained and subject to moral hazard. In our model, guarantees can raise welfare because they reduce the cost of capital faced by entrepreneurs, and so potentially enhance entrepreneurial effort incentives. Overly generous guarantees have perverse incentive effects, however: they induce lenders to reduce lending standards and to lower their collateral requirements. Co-funding schemes do not suffer from the put option feature inherent in guarantees, but they may encourage entry by unproductive entrepreneurs who plan simply to consume subsidies, without investing. This limits the guarantee fund's ability to support productive entrepreneurs, and thereby destroys value. Based on these trade-offs, we show how the optimal design of state-sponsored loan subsidy schemes depends on the fund's technological expertise and the degree of creditor rights protection. We explore the dynamic efficiency effects of public subsidies, and show that they may impair the private sector's initiative to uncover cost savings in order to diminish future financial constraints.

*Keywords:* Partial Credit Guarantees, Co-funding and Loan Subsidies, Private Sector Initiative, Lending Standards

## 1. Introduction

A substantial literature supports the hypothesis that many firms face credit rationing problems.<sup>1</sup> If this is the case, can a government agency relax credit constraints, and so raise welfare? Government agencies have certainly attempted to do so. Public credit guarantee programs are commonplace: Gudger (1998) documents credit guarantee programs in most Western European countries and in several East European countries, all over Asia, in parts of Africa, and in Latin America. In this paper we consider the optimal design of credit guarantee schemes, and we ask under what circumstances they might be expected to raise welfare.

Some evidence suggests both that credit guarantees can ameliorate credit rationing problems, and also that they can enhance welfare. Zia (2008) examines the consequences for Pakistani exporters of cotton yarn of the 2001 withdrawal of cotton yarn exporters from the state-sponsored Export Finance Scheme. He finds that the consequence of this withdrawal was a significant reduction in exports by privately owned firms, while those of large publicly quoted exporters were unaffected. Zia argues that this indicates first that private firms were credit rationed, and second that the credit guarantee scheme reduced their rationing. Benavente, Galetovic, and Sanhueza (2006) study Chile's Fogape loan guarantee program, and present evidence that guarantees have increased access to credit for high-quality firms. Uesugi, Sakai, and Yamashiro (2006) discuss Japan's Special Credit Guarantee Program for Financial Stability, under which small and medium sized Japanese enterprises received government-sponsored credit guarantees between 1998 and 2001. Users of the program received more credit than non-users and experienced an increase in profitability. And Banerjee and Duflo (2004) find evidence that government-sponsored direct lending programs in India tend to complement rather than substitute for other forms of credit.

Notwithstanding this evidence, several questions arise concerning credit guarantees. First, guarantees may help to resolve a credit rationing problem, but is there a danger that they will worsen price signals, and so blunt borrower incentives? Lelarge, Sraer, and Thesmar (2008) present evidence from the French government loan guarantee program (Sofaris) that supports these con-

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<sup>1</sup>Important papers are Fazzari, Hubbard, and Petersen (1988) and Hoshi, Kashyap, and Scharfstein (1990), presenting evidence that firms invest more when their businesses generate more returns. Hubbard (1998) surveys the related literature.

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cerns. While they argue that the loan guarantees are effective in helping young French firms to find finance and to grow, they also find that guaranteed firms are more likely to adopt risky strategies, and that these firms file more often for bankruptcy.

A second question is, given that the state wishes to provide the entrepreneurial sector with subsidized funding, is a credit guarantee scheme the most effective way to do so? An alternative would be to offer a direct subsidy to entrepreneurs. This type of subsidy, which is sometimes referred to as a *co-funding scheme*, is also commonplace. For example, although there is a \$100 billion worldwide venture capital industry, the OECD (1997) reports that OECD governments invest about \$3 billion per year in small, innovative firms. Lerner (1999) documents massive levels of investment by government-sponsored credit agencies in the United States: in 1995, public venture capital initiatives invested \$2.4 billion in the United States, as against \$3.9 billion invested by private venture funds. Moreover, public venture funding has had some major successes in the United States: Apple Computer, Chinron, Compaq, FedEx, and Intel all received support at an early stage from Federal programs. In a more general survey article, Hall (2002) summarizes evidence on the funding gap for R&D, and argues that government seed capital and subsidy programs are, at the least, worthy of much more thorough investigation.

How should resources be divided between guarantee schemes and co-funding schemes? Li (1998) argues that this question is generally answered politically, suggesting that the benefits of guarantees are spread more evenly across the population than those of co-funding schemes, and hence that they may be easier to legitimize. Li's work also points to dynamic efficiency concerns: if, as she states, credit programs are mostly devices for redistributing cash between sectors that inevitably discourage private saving, then it is reasonable to ask whether their introduction might result in a redistribution of resources towards unproductive rent-seeking. In line with this concern, Johnson and Mitton (2003) presents evidence from the Asian financial crisis that indicates that, at least in Malaysia, those with close relationships to the government are more likely to receive government subsidies.

Public support of the entrepreneurial sector clearly has far-reaching welfare and policy implications. Yet surprisingly little formal work addresses the topic. Williamson (1994) uses two

models of adverse selection to do so: in his first, a costly state verification set-up à la Townsend (1978) and Gale and Hellwig (1985), direct government lending simply displaces private lending and hence does not raise welfare. In his second model, in which payoffs are verifiable but borrower types can be determined only using a costly ex ante screening technology, government guarantees can reduce intermediary screening costs, although they never generate a Pareto improvement. Benavente, Galetovic, and Sanhueza (2006) note that, when the guarantee fund disburses too many funds, it will tend to undermine bank screening incentives. Hence its budget constraint should be sufficiently tight. At the same time, Benavente et. al. argue that individual borrowers should obtain close to 100% coverage, so that the bank is able to fund good borrowers with few liquid assets.

We approach credit guarantees from a different angle. We present a model of entrepreneurial financing in which there is moral hazard between the entrepreneur and the financier, but not adverse selection. The entrepreneur is cash-constrained, and needs to raise funds to perform a project whose success depends upon the entrepreneur's costly, and unobservable, effort. Because the entrepreneur is unable to commit to a particular effort level, he will not in general select a socially optimal effort when he relies upon outside funding. As a result, he may be denied access to credit, and the value of the project may be lost to society.

The entrepreneur in our model is able to address the credit rationing problem by pledging as collateral personal assets not related to the project. Nevertheless, collateral on its own may be insufficient to generate access to credit market. If this is the case, we show that a credit guarantee can enhance welfare by ensuring that both the entrepreneur's incentive compatibility constraint and the lender's participation constraint can be simultaneously satisfied, so that the project can go ahead.

We also demonstrate that, for sufficiently small guarantees, the borrower's incentives are actually *increasing* in the size of the guarantee, and hence so is welfare. To understand why this is the case, note that increases in the credit guarantee have two effects. First, they make the financier willing to lend at better terms, and so reduce the entrepreneur's cost of capital; second, because some of the losses from the project are borne by the State, increases in the credit guarantee reduce the jointly efficient effort level for the entrepreneur and the financier. So long as the guarantee is

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small, the former effect outweighs the latter, and increased guarantees translate into greater entrepreneurial effort. For higher levels of guarantee, the second effect is dominant: further increases in the guarantee are used by the entrepreneur to substitute for collateral, and thus attenuate his effort.

A simple policy point emerges from this discussion. In many countries, guarantee agencies are subject to political and other pressures, such as lobbying or outright bribery by entrepreneurs and banks, to deploy all of their resources. To the extent that this is the case, it is important that guarantee agencies are not able to provide support beyond the point at which it starts to damage entrepreneurial incentives. Hence guarantee agencies should be subject to hard budget constraints. A similar point is made for slightly different reasons by Benavente, Galetovic, and Sanhueza (2006).

Even when a credit guarantee agency has a sufficiently tight hard budget constraint, it is still necessary to explain why it could ever outperform a co-funding scheme. To see why this might be the case, we extend our model to allow for two types of entrepreneur: productive entrepreneurs, who have potentially valuable projects that require effort to succeed, and “copy-cat” entrepreneurs, who have no productive project. Financial intermediaries are able in our set-up to distinguish between these types of entrepreneur, but the government credit agency, due to its limited technological expertise, may not. Hence, in our extended model government co-funding will cause some resources to be directed to unproductive entrepreneurs. This represents a straightforward redistribution of wealth, which in our model is damaging because it takes resources away from productive entrepreneurs who could make better use of them. In this case, we demonstrate that co-funding is valuable only after the credit guarantee agency has provided the maximum useful level of guarantees.

Our extended model yields one simple policy prescription. Credit subsidies and credit guarantees are the product of a single optimization problem, and hence should be designed together. This implies that the guarantee and subsidy agencies should be coordinated, and that they should be subject to a common budget constraint. One simple way to accomplish this would be to delegate guarantee and direct subsidy provision to a single institution.

We consider the dynamic efficiency consequences of our model in a further extension. We

allow entrepreneurs to exert effort to find productive projects, and also to uncover cost savings for their projects. As in our discussion of co-funding, unproductive entrepreneurs attract public funds that could be better employed elsewhere and hence, as in the co-funding discussion, an optimally designed public support scheme for entrepreneurs concentrates funds upon credit guarantees first, and then upon co-funding. Hence, entrepreneurs with productive projects extract more from the public purse. We conclude that well-designed public support programmes should foster entrepreneurial innovation. However, public support schemes have a deleterious impact upon (X-) efficiency. This is because credit-constrained entrepreneurs have a strong incentive to find ways to cut costs, because it is only by doing so that they can hope to raise the funds that they need to carry out their projects. Public schemes reduce the extent of credit rationing, and hence diminish the entrepreneur's incentive to reduce costs.

The remainder of the paper is organized as follows. Section 2 presents our basic model. We show how public credit schemes can ameliorate credit rationing, and how they can enhance entrepreneurial incentives. In section 3 we analyze the entry incentives provided by co-funding, and discuss the appropriate mix of co-funding and credit guarantees in a public support package. Section 4 presents our analysis of the dynamic efficiency effects of credit guarantees, and section 5 discusses some possible extensions of our results, and addresses some robustness issues. Section 6 concludes.

## 2. Basic Model

We start by considering a simple model of entrepreneurial financing. In our basic set-up we consider the design of credit guarantees; we extend our analysis to incorporate co-funding in section 3.

We consider a risk-neutral entrepreneur, who is endowed with a single project. The project requires an investment of  $I$ , and will either succeed, in which case it will return  $\Pi > 0$ , or it will fail, in which case its return will be zero. The probability that the project succeeds depends upon the entrepreneur's effort, which is unobservable and hence uncontractible: if the entrepreneur exerts effort  $e \in [0, 1]$  then the probability of project success is equal to  $e$ . The entrepreneur experiences a

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private cost  $\psi(e)$  from effort  $e$ , where  $\psi'(\cdot) > 0$ , and  $\psi''(\cdot) > 0$ .<sup>2</sup> We assume that, in the absence of financing constraints, the project has a positive NPV:

$$\max_e \{e\Pi - \psi(e)\} > I. \quad (1)$$

We write  $Q$  for the entrepreneur's endowment of liquid funds for investment in the project, and we assume that  $Q < I$ , so that the entrepreneur is unable to finance the project himself.<sup>3</sup> We assume that there are many potential risk neutral investors, who compete à la Bertrand to invest in the project. Because the entrepreneur is unable to commit to an effort level, financing is only possible if he extracts at least a minimal level of information rent, so that some socially viable projects will not be financed.

In addition to any liquid funds  $Q$ , the entrepreneur is also endowed with some illiquid assets  $A$ , which he can choose to pledge as collateral against his project. The assets  $A$  are not created by the project, but are already owned by the entrepreneur; an example would be the entrepreneur's home. We write  $\beta A$  for the level of collateral that the entrepreneur pledges where  $\beta \in [0, 1]$ . Collateral that is pledged will be liquidated in the event of project failure. In our initial model, the only active role for the bank is in enforcing collateral; later in the paper, we discuss an extension in which banks can actively monitor their loans. We assume that there is insufficient collateral to render the bank's claim riskless:

$$I - Q > A. \quad (2)$$

A contract in this set-up consists of a payment  $R$ , which the entrepreneur makes to the bank in the event of project success, and a level  $\beta$  of collateralization, which entitles the bank to seize entrepreneurial assets to the value of  $\beta A$  in the wake of project failure. Given a contract  $(R, \beta)$ , the entrepreneur will derive the following expected utility from an effort level  $e$ :

$$U(R, \beta, e) \equiv e(\Pi + A - R) + (1 - e)(1 - \beta)A - \psi(e). \quad (3)$$

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<sup>2</sup>We also adopt the following technical assumptions, which ensure that the maximization problem is well-behaved, and that it yields interior solutions:  $\psi(0) = \psi'(0) = 0$ ,  $\psi'(1)$  is sufficiently large, and  $\psi'''(e) \geq 0$ . A simple cost function  $\psi(e)$  satisfying all of these assumptions would be the quadratic cost function  $\alpha e^2$ , with  $\alpha$  large enough.

<sup>3</sup>In section 3 we will endogenise  $Q$ , by allowing the government credit support agency to select it as a co-funding provision.

He will therefore select his effort level to satisfy the following first order condition:

$$\Pi - (R - \beta A) = \psi'(e). \quad (4)$$

The contract  $(R, \beta)$  will beat the lender's outside option precisely when the effort level  $e$  that satisfies equation (4) satisfies the following constraint:

$$eR + (1 - e)\beta A \geq I - Q. \quad (5)$$

In the absence of any form of public subsidy, the entrepreneur will maximize  $U(R, \beta, e)$  subject to equations (4), (5), and the requirement that  $\beta \in [0, 1]$ .

We now adopt the following parameter assumption:

**Assumption 1.** *The value of collateral  $A$  is sufficiently low to ensure that for every  $e \in [0, 1]$ , condition (6) is satisfied:*

$$I - Q - A > e(\Pi - \psi'(e)). \quad (6)$$

We demonstrate in proposition 1 that assumption 1 ensures that, without any form of public support, entrepreneurs will be unable to raise finance for their projects. Hence we introduce a public institution into our model that can provide *credit guarantees*. Under a credit guarantee of *strength*  $\phi \in [0, 1]$ , an investor in a failed entrepreneurial project receives a payment of  $\phi(I - Q)$ . The credit guarantee agency awards a guarantee to an entrepreneur, who is then free to provide it to whichever bank provides him with funding.

Proposition 1 demonstrates that credit guarantees are necessary to resolve a credit rationing problem.

**Proposition 1.** *Suppose that assumption 1 is satisfied. Then there exists a minimum level of credit guarantee  $\underline{\phi}$  such that for levels of guarantee  $\phi$  below  $\underline{\phi}$ , it is impossible to satisfy both the investor's participation constraint and the entrepreneur's incentive compatibility constraint simultaneously. Hence the entrepreneur is credit rationed when  $\phi < \underline{\phi}$ . For  $\phi \geq \underline{\phi}$ , the entrepreneur can raise external finance. Moreover, the entrepreneur's effort is initially increasing in  $\phi$ .*

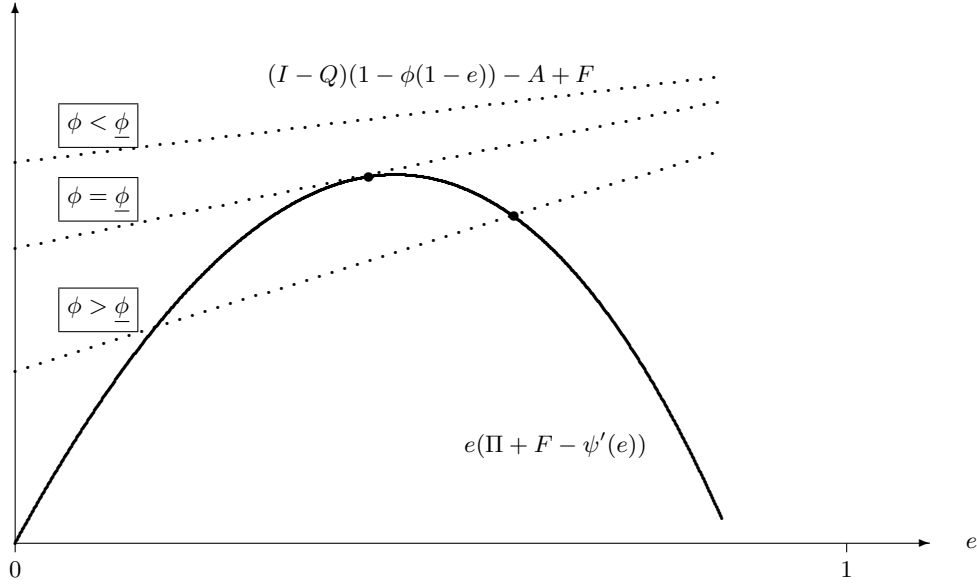


Figure 1: **Credit Guarantees, Incentive Compatibility, and Access to Credit.** The figure illustrates the incentive compatibility constraint (8). The bold line depicts the term  $e(\Pi + F - \psi'(e))$ , while the dotted lines depict the term  $(I - Q)(1 - \phi(1 - e)) - A + F$  for varying levels  $\phi$  of the credit guarantee. The entrepreneur's incentive compatibility constraint is satisfied when the two lines intersect. For  $\phi < \underline{\phi}$ , the two lines do not intersect, and hence it is impossible to satisfy the entrepreneur's IC constraint and the bank's IR constraint simultaneously. As a result, the entrepreneur is denied access to credit. For  $\hat{\phi} > \phi \geq \underline{\phi}$ , the equilibrium effort level is given by the right-hand intersection between the two lines. Hence, for  $\phi$  in this range, effort is increasing in  $\phi$ .

The proof of Proposition 1 appears in the appendix. The intuition for the result is however straightforward. With a credit guarantee, the lender's participation constraint (5) becomes:

$$eR + (1 - e)(\beta A + \phi(I - Q)) \geq I - Q. \quad (7)$$

We show below that for sufficiently low  $\phi$ , the entrepreneur optimally pledges all of his assets as collateral, i.e.,  $\beta = 1$ . Because the entrepreneur extracts all of the rent from the project, the investor participation constraint (7) must bind. Setting  $\beta = 1$  and using equation (7) to substitute into the incentive compatibility constraint yields the following expression:

$$\psi'(e) = \Pi - \frac{(I - Q)(1 - \phi(1 - e)) - A}{e}. \quad (8)$$

In other words, the investor's participation constraint and the entrepreneur's incentive constraint can be satisfied simultaneously precisely when there exists  $e \in [0, 1]$  such that

$$e(\Pi - \psi'(e)) = (I - Q)(1 - \phi(1 - e)) - A; \quad (9)$$

when no such  $e$  exists, the entrepreneur cannot raise financing. Equation (9) is illustrated in figure 1; the bold line depicts the left hand side of the equation, and the dotted lines depict the dotted lines for various levels of credit guarantee,  $\phi$ . For  $\phi < \underline{\phi}$  the two lines do not meet, and hence financing is impossible. For values of  $\phi$  above  $\underline{\phi}$  but still sufficiently low for the collateral level  $\beta$  optimally to be set equal to 1, an equilibrium with lending occurs at the point where the two lines meet. Increases in  $\phi$  shift the dotted line down, without affecting the solid line. Hence, provided  $\beta = 1$ , increases in  $\phi$  shift the intersection point to the right, and with it the equilibrium effort level.

Proposition 1 states that, for low enough  $\phi$ , entrepreneurial effort in a lending equilibrium is increasing in  $\phi$ . Proposition 2 provides a complete characterization of equilibria with credit guarantees.

**Proposition 2.** *Suppose that an entrepreneur has internal funds  $Q$  and a credit guarantee of  $\phi(I - Q)$ . Define  $\hat{\phi}(Q)$  as follows:*

$$\hat{\phi}(Q) \equiv 1 - \frac{A}{I - Q}. \quad (10)$$

and notice that  $\underline{\phi} < \hat{\phi}(Q)$ . Then:

1. For  $\phi < \underline{\phi}$ , the entrepreneur is credit-rationed (Proposition 1);
2. For  $\underline{\phi} \leq \phi < \hat{\phi}(Q)$ , the optimal contract is as follows:

$$\begin{aligned} \beta &= 1; \\ R &= \frac{1}{e} \{(I - Q)(1 - \phi(1 - e)) - (1 - e)A\}, \end{aligned} \quad (11)$$

where the effort level  $e$  solves equation (9). For these levels of guarantee, entrepreneurial effort is increasing in  $\phi$ ;

3. For  $\phi \geq \hat{\phi}(Q)$ , the optimal contract is as follows:

$$\beta = \frac{(I - Q)(1 - \phi)}{A}; \quad (12)$$

$$R = I - Q, \quad (13)$$

where the effort  $e$  solves equation

$$\psi'(e) = \Pi - \phi(I - Q). \quad (14)$$

For these levels of guarantee, entrepreneurial effort is decreasing in  $\phi$ .

Proposition 2 is proved in the appendix. Its intuition is as follows. As in Proposition (1), no credit is extended for  $\phi < \underline{\phi}$ . For higher  $\phi$  the investor is prepared to invest. The effect of an increase in the credit guarantee on the entrepreneur's incentive to exert effort then hinges upon the relative importance of two opposing effects. First, an increase in the credit guarantee makes the financier willing to lend at better terms, and so reduces the entrepreneur's cost of capital. This makes it easier to induce the entrepreneur to exert high effort. Second, because some of the losses from the project are borne by the State, an increase in the credit guarantee reduces the jointly *efficient* effort level for the entrepreneur and the financier. So long as the guarantee is small, the former effect outweighs the latter, and increased guarantees translate into greater entrepreneurial effort. For higher levels of guarantee, the second effect is dominant: at these levels of guarantee, the jointly efficient effort level is so small such that the effort level that the entrepreneur would exert if the contract were fully collateralized would actually *exceed* the jointly efficient effort level. To ensure that the entrepreneur does not overexert effort, the optimal contract stipulates that the loan is partially, rather than fully, collateralized. In equilibrium, the entrepreneur exerts the jointly efficient level, which happens to be decreasing in the credit guarantee due to its put option feature.

The proposition is illustrated in figure 2. As indicated in the figure, no lending occurs for  $\phi < \underline{\phi}$ , fully collateralized lending with effort increasing in  $\phi$  occurs for  $\underline{\phi} \leq \phi < \hat{\phi}(Q)$ , and partially collateralized lending with collateral levels and effort both dropping in  $\phi$  occurs for  $\hat{\phi}(Q) \leq \phi \leq 1$ .

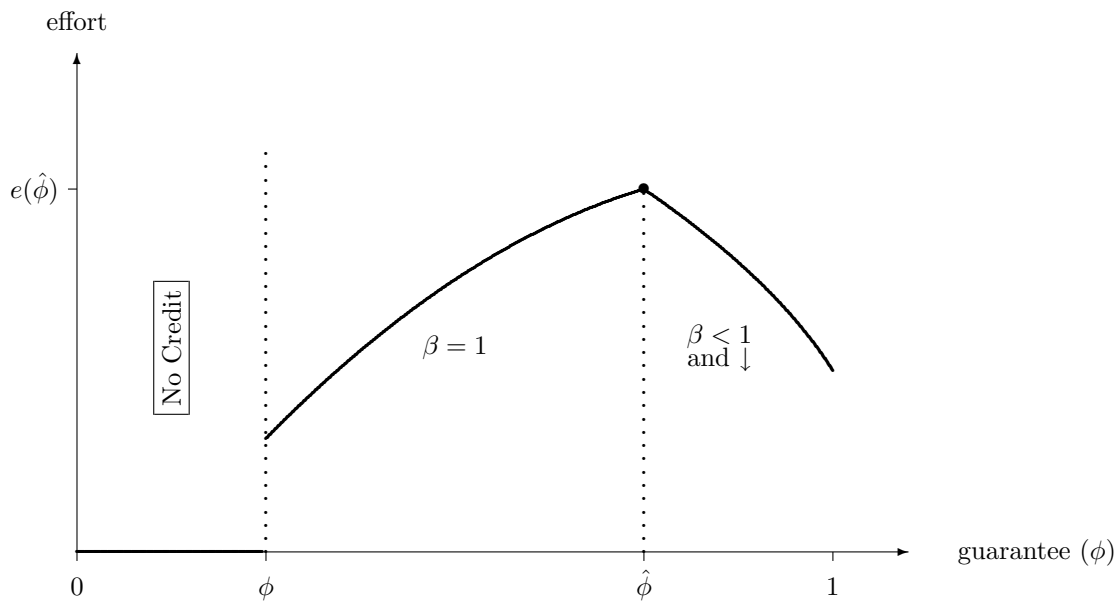


Figure 2: Credit guarantees and effort incentives.

### 3. Credit Guarantees and Co-Funding

Well-designed credit guarantees are welfare-enhancing in the simple model of the previous section, where every entrepreneurial project has a positive NPV. In this situation, however, it is reasonable to ask what is special about a credit guarantee: a system of co-funding under which the guarantee agency made investments that increased the entrepreneur's level  $Q$  of internal funds would also raise welfare, and would not impair incentives. In practice, of course, co-funding could result in excessive entrepreneurial entry if it attracted copy-cat entrants without a productive project: this would restrict the supply of funds to productive entrepreneurs. In this section, we examine the optimal mix of credit guarantee provision and co-funding in a world where entry by unproductive entrepreneurs is possible.

We now assume that there is a unit mass of productive entrepreneurs, each of whom has no internal funds, illiquid collateralizable assets of  $A$ , and a project of the type analyzed in the previous section. There is also a mass  $\xi \leq 1$  of copy-cat entrepreneurs, who do not have a productive project, and whose only investment opportunity is a riskless storage technology. Although the lender in our setup can distinguish between productive and copy-cat entrepreneurs, we assume that the credit guarantee agency, due to its limited technological expertise, is only imperfectly able

to do so. Specifically, we assume that the credit guarantee agency is able to identify a copy-cat entrepreneur with probability  $\theta$  when evaluating a support request, while with probability  $1 - \theta$  it is not able to so. The mass of copy-cat entrepreneurs being able to attract co-funding is then given by  $\rho = \xi(1 - \theta)$ . Hence, the lower the guarantee agency's technological expertise, the larger will be the transfer of wealth to unproductive copy-cat entrepreneurs. Notice that while copy-cat entrepreneurs may attract co-funding, they do not benefit from credit guarantees. This is because they are not endowed with real investment projects, and hence they cannot possibly go bankrupt.

Two strategies are available to the credit guarantee agency. First, it can co-fund, in which case it provides the entrepreneur with co-funding  $Q$ ; second, it can provide a credit guarantee of  $\phi(I - Q)$ , as in section 2. We assume that the credit guarantee agency faces the following budget constraint:

$$\underbrace{\rho Q}_{\text{Funds allocated to copy-cat investors}} + \underbrace{Q + (1 - e)(I - Q)\phi}_{\text{Funds allocated to productive entrepreneurs}} \leq B < (1 + \rho)(I - A) \equiv \bar{B}. \quad (15)$$

To see why we assume  $B < \bar{B}$ , notice that if the fund had financial resources  $B \geq \bar{B}$ , then the fund would be able to implement the first best by offering co-funding  $Q = I - A$  to every entrepreneur. Assuming that  $B < \bar{B}$  allows us to rule out this rather uninteresting case.

Given a support package  $(Q, \phi)$  from the credit guarantee agency, a productive entrepreneur will solve the problem analyzed in section 2. The credit guarantee agency therefore selects  $(Q, \phi)$  to solve

$$\begin{aligned} \max_{\{Q, \phi\}} \quad & e(Q, \phi)\Pi - \psi(e(Q, \phi)) \\ \text{s.t.} \quad & \end{aligned}$$

$$\rho Q + Q + (1 - e(Q, \phi))(I - Q)\phi \leq B \quad (16)$$

$$\phi \leq \hat{\phi}(Q) = 1 - \frac{A}{I - Q} \quad (17)$$

where  $e(Q, \phi)$  is characterized by expression (9).

The solution to the agency's problem is described in Proposition 3.

**Proposition 3.** *Suppose that there is a fixed mass 1 of productive entrepreneurs, that there is a mass  $\rho$  of unproductive copy-cat entrepreneurs, that neither entrepreneurs nor copy-cats have any*

wealth, and that the credit guarantee agency is able to provide co-funding or credit guarantees, subject to the budget constraint (15), and define

$$\underline{B} \equiv \left(1 - e(0, \hat{\phi}(0))\right) (I - A).$$

Then:

1. If  $B < \underline{B}$  then the credit guarantee agency should provide no co-funding, and should provide the maximum credit guarantee consistent with its budget constraint, i.e.,  $Q = 0$  and  $\phi = B / ((1 - e)I)$ , where  $e$  is the largest solution of

$$\psi'(e) = \Pi - \frac{I - A + B}{e}$$

2. If  $B \geq \underline{B}$  then the credit guarantee should provide co-funding

$$Q = \frac{B - (1 - e)(I - A)}{\rho + e}$$

and a credit guarantee of  $\hat{\phi}(Q) = 1 - A / (I - Q)$ , where  $e$  is the largest solution of

$$\psi'(e) = \Pi - (I - Q - A)$$

In the limit, as  $B$  approaches  $\bar{B}$ , we have  $Q = I - A$  and  $\phi = 0$ .

The proof of Proposition (3) appears in the appendix. To understand the result, note first that credit guarantees are used to subsidize productive projects only, while co-funding induces entry by unproductive copy-cat entrepreneurs, who therefore take a proportion of any co-funding. Hence, if the government agency faces a very tight budget constraint, then it should provide no co-funding and spend its entire resources on credit guarantees. As the government agency's budget increases, it will continue to devote its entire resources to guarantees up to point where the level of guarantee hits the incentive-compatibility threshold  $\hat{\phi}(0)$ . As the agency's budget increases beyond  $\underline{B}$ , its budget constraint would be slack if it continued to provide a guarantee of  $\hat{\phi}(0)$  and no co-funding. The agency responds by devoting some resources to co-funding, but at the same time reducing the guarantee to  $\hat{\phi}(Q) < \hat{\phi}(0)$ . The amount of co-funding is then increasing in the agency's budget, while the level of guarantee is decreasing.

It is worth noting that a decrease in the value of the collateral  $A$  leads to a decrease in the level of guarantee when the credit guarantee agency operates under a sufficiently generous budget constraint. The reason for that is that with high private collateral value a lower level of guarantee is needed to achieve the jointly efficient effort level for the entrepreneur and the bank. At the same time, given the fund's budget a higher  $A$  also results in more co-funding. Creditors rights protection is often one of the most important determinants of collateral value. Hence, we suggest that in countries where creditors rights are well protected we should observe less generous guarantees and more co-funding. The fund's technological expertise, measured by  $\rho$ , also affects the design of support package. In particular, we find that funds with more expertise should provide more co-funding and, correspondingly, less guarantees.

Note that the analysis of this section is predicated upon the assumption that a single agency is responsible both for direct subsidies of investment, in the form of co-funding, and also for the provision of credit guarantees. Only when this is the case can the optimal joint policy be adopted: if the two forms of entrepreneurial support are not subject to a single budget constraint then there is no *a priori* reason to assume that correct policies will be adopted. Hence our analysis indicates that governments should optimally combine the provision of credit guarantees and of co-funding within a single agency.

#### **4. Dynamic Efficiency**

We have shown that properly designed government support schemes can ensure that lenders' participation constraints are satisfied, while providing entrepreneurs with better effort incentives. But our analysis thus far has taken place in a one-period model and, as we noted in the introduction, several authors have expressed concern that a system of public support to private entrepreneurs may have damaging long-term effects. In particular, if public funds are channeled towards entrepreneurial projects, entrepreneurs may be less concerned to manage costs. On the other hand, if innovation improves access to public subsidies, public support may encourage entrepreneurs to be creative. In this section we present a simple extension of our basic model in which we can analyze this trade-off.

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We suppose that entrepreneurs make unobservable effort decisions along two dimensions. First, they select an *innovation effort*  $a \in [0, 1]$ :  $a$  is the probability that the entrepreneur uncovers a productive project. Second, entrepreneurs select a *cost-cutting effort*  $b \in [0, 1]$ :  $b$  is the probability that the investment outlay for a productive project is low, rather than high. For the sake of simplicity, we assume that, conditional upon finding a productive project, an entrepreneur who exerts cost-cutting effort  $b$  will have an investment outlay of zero with probability  $b$ , and will otherwise have an investment outlay of  $I > 0$ . Entrepreneurs select the effort levels  $(a, b)$  that they devote to innovation cost-cutting at the start of the game, and the outcome of their efforts realizes before the funding stage. The cost to the entrepreneur of an effort pair  $(a, b)$  is  $c(a, b) = c(a) + c(b)$ , where  $c(\cdot, \cdot)$  is increasing in both arguments, and is strictly convex.

Three types of entrepreneurs emerge in this setting. Type 1 entrepreneurs have a productive project that requires external funding; type 2 entrepreneurs have a productive project but do not require external funding; and type 3 entrepreneurs do not have a productive project. An entrepreneur that makes effort choice  $(a, b)$  will be of type 1 with probability  $a(1 - b)$ , of type 2 with probability  $ab$ , and of type 3 with probability  $1 - a$ .

In line with the analysis of section 3, we assume that lenders are able to distinguish between the various types of entrepreneurs, but that the credit guarantee agency is not. We also assume that the entrepreneur and the lender will not collude together, so that type 2 and 3 entrepreneurs will not enter into bogus contracts with banks intended to induce bankruptcy. However, both type 2 and 3 entrepreneurs are able to access co-funding provided by the government agency.

With these informational assumptions, the type 1 entrepreneurs in this set-up correspond to the productive entrepreneurs of section 3, and the mass of copy-cat entrepreneurs of section 3 corresponds in this section to the type 2 and the type 3 entrepreneurs combined: the former do not require public funds to undertake their project, and the latter have no project, so public funds cannot increase the productivity of either. When the guarantee agency has a hard budget constraint, the presence of types 2 and 3 entrepreneurs therefore reduces the quantity of funds that can be distributed to type 1 entrepreneurs, and hence reduces the effectiveness of credit support schemes.

Now suppose that entrepreneurs anticipate that a support package  $(Q, \phi)$  will become available

at the funding stage. A type 2 entrepreneur does not require outside finance and therefore reaps the total product of his investment. As a result, he exerts the first best effort level  $e^{FB}$  and hence earns an expected continuation payoff, net of personal assets  $A$ , of

$$V_2 \equiv e^{FB}\Pi - \psi(e^{FB}) + Q. \quad (18)$$

A type 1 entrepreneur will earn the following continuation payoff

$$V_1 \equiv e^*\Pi + (1 - e^*)\phi(I - Q) - \psi(e^*) - I + Q, \quad (19)$$

where  $e^*$  is the equilibrium managerial effort level from section 3.  $V_1$  is the sum of the project's net present value net of effort costs and the expected subsidy.

Entrepreneurs therefore select effort levels  $(a, b)$  in order to maximize the following objective function:

$$V(a, b) \equiv a(bV_2 + (1 - b)V_1) + (1 - a)Q - c(a, b).$$

Proposition 4 shows how effort  $(a, b)$  varies with public support.

**Proposition 4.** *Provided the cost function  $c(\cdot, \cdot)$  is sufficiently convex, public support strengthens entrepreneurial innovation incentives and diminishes cost-cutting incentives.*

This result is proved in the appendix. The intuition is straightforward. Public support initiatives reduce the cost of capital to type 2 entrepreneurs, and hence increase the expected value of a productive project. This effect strengthens entrepreneurs' incentive to innovate. At the same time, however, public support reduces the proportion of the project's costs borne by the entrepreneur, and so impairs his incentives to uncover cost savings so as to diminish the financial constraints that he faces.

## 5. Further Extensions and Robustness

In this section we discuss some possible extensions to our model, and address some robustness issues.

### 5.1 *Bank Monitoring*

The only active role played by the banks in our basic set-up is in realizing collateral assets. In reality, it is well-understood that banks play a valuable role as active monitors of entrepreneurs: banks that form close relationships with the entrepreneurs to whom they lend are in a position to influence their activities. If they have the requisite incentives, then, banks are able to help entrepreneurs to resolve the commitment problem that they face with regards to their effort problem. Although we do not formally model their monitoring activities, it is interesting to ask what the effect of public credit guarantees and co-funding would be upon bank monitoring incentives.

To address this question, consider an extension to our model in which first, the bank is able at a cost to force the entrepreneur to select a specific effort level, and second, the entrepreneur is risk-averse. The bank's active monitoring ability is valuable to the entrepreneur if, as in the seminal work of James (1987) and Datta, Iskandar-Datta, and Patel (1999), the entrepreneur wishes to use bank debt to reduce its cost of cost of funds from other sources. The entrepreneur can therefore commit to a particular effort level in either of two ways: the first, upon which we have concentrated throughout the paper, is by designing appropriate incentive payments into a financial contract; the second is to use bank financing. The first method is costly because the entrepreneur is risk-averse; the second, because the bank experiences a cost of monitoring. Intuitively, the cost of bank monitoring should be greater, the higher the discrepancy between the effort level that the entrepreneur would select absent monitoring, and the effort level to which the entrepreneur wishes to commit.

Suppose that the bank is unable to commit to a particular level of monitoring. Then the entrepreneur can achieve bank commitment only by designing an appropriate lending contract. A credit guarantee scheme reduces the sensitivity of the bank's returns to the entrepreneur's effort level, and hence diminishes the bank's incentives to monitor. As a result, if the entrepreneur wishes to commit to a particular effort level, it must design a greater degree of cash-flow sensitivity into its own cash flows. In other words, credit guarantees will be accompanied by higher-powered incentive contracts for entrepreneurs. Since co-funding does not affect bank incentives, it will not have this effect.

### *5.2 Risky Copy-Cat Projects*

The copy-cat entrepreneurs in section 3 are not risky. Hence they can benefit from co-funding opportunities, but not from credit guarantees. As a result, credit guarantees are a more effective way of using the limited resources of the guarantee fund, because they direct those resources towards productive projects.

Suppose instead that copy-cat entrepreneurs were endowed with risky projects. In this case, they would be in a position to profit from the put option provided by the credit guarantee fund. If they were riskier than the productive entrepreneurs then the relative cost in terms of resources diverted to unproductive uses of credit guarantees would exceed that of co-funding. In this case, our simple model appears to suggest that the credit guarantee agency should concentrate upon co-funding.

We do not think that this conclusion is appropriate. The argument of the previous paragraph ignores the screening abilities of banks. In our formal modeling, we allow for these by assuming that banks can identify low-quality borrowers, and that they will not advance them funds. Hence these borrowers can only profit from co-funding schemes, which do not require the active participation of the bank. In a model with risky copy-cat entrepreneurs, the public fund could use the banks' superior informational assets by compensating them based upon the aggregate result of their entire portfolio of loans. Hence, if this portfolio were large enough, a Diamond (1984)-style argument would preclude lending to risky borrowers, so that our results would still hold.

### *5.3 Institutional Issues*

We assume throughout our discussion that the government agency distributed funds equally amongst qualifying borrowers. However, as stated in our introduction, there is some evidence that suggests that some agencies may use their funds to finance government cronies. If this is the case then credit support for the entrepreneurial sector may have damaging dynamic effects not considered in section 4, in the form of rent-seeking, as productive assets are diverted towards grant-extraction (see Tullock (1967) and Krueger (1974) for the classic expositions of this problem). When these

effects are sufficiently pronounced, they may completely outweigh the benefits that we identify in this paper.

These observations do not obviate our conclusions. They do, however, suggest some pre-conditions for our conclusions to be valid. First, public support for private entrepreneurs should be provided by an agency that is sufficiently removed from the government of the day. Second, the credit agency should be forced to publish and to follow objective criteria for the allocation of loans, even if these criteria are second-best. More generally, public support for private business is unlikely to work in a country with weak institutions, and where the rule of law is weak.

## **6. Conclusion**

We study credit guarantees in a world where entrepreneurs are capital-constrained, suffer from a moral hazard-induced principal-agent problem, and are able to pledge costly collateral. In this world, state-sponsored credit guarantees can raise welfare, because they reduce the lender's need to extract success-state payouts from entrepreneurs, and so enhance entrepreneurial effort incentives. Guarantees may generate problems: for example, we argue in an extension to our model that they may undermine bank monitoring incentives so that, as banks accept less risk, entrepreneurs are forced to accept more. Nevertheless, they outperform co-funding schemes when the latter encourage excessive entry by unproductive entrepreneurs who plan simply to consume the co-funding, without investing.

Our model points to a number of ways in which state support of the entrepreneurial sector can raise welfare. Nevertheless, we identify several caveats to our reasoning. First, there is the danger, mentioned in the preceding paragraph, that credit guarantees may undermine the effectiveness of bank monitoring, and so force entrepreneurs to fall back upon second-best commitment devices. Second, our results rely upon an assumption that banks can screen out the worst borrowers ex ante, and that they have the incentives to do so. And finally, we note that state support for the entrepreneurial sector is likely to be positively damaging in countries whose legal and political institutions are under-developed. A more complete analysis of these points is left for further work.

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## Appendix

### *Proof of Propositions 1 and 2*

Recall that the investor's IR constraint with a credit guarantee  $\phi$  is given by equation (7). Since

the entrepreneur extracts all of the surplus from the project, this condition must bind, so that we can re-write the objective function (3) as follows:

$$U(\beta, e) \equiv e\Pi + A - (I - Q)(1 - \phi(1 - e)) - \psi(e), \quad (20)$$

and the incentive compatibility constraint (4) as follows:

$$\Pi - \psi'(e) - \frac{(I - Q)(1 - \phi(1 - e)) - \beta A}{e} = 0. \quad (21)$$

The entrepreneur's problem is therefore to maximize  $U(\beta, e)$  subject to the IC constraint (21) and the following limited liability constraint:

$$\beta \leq 1. \quad (22)$$

We write  $\lambda$  and  $\mu$  for the Lagrangian multipliers relating to conditions (21) and (22), respectively. Condition (22) gives rise to the following complementary slackness condition:

$$\mu(1 - \beta) = 0. \quad (23)$$

The first order conditions for the entrepreneur's problem are the following:

$$\Pi - \phi(I - Q) - \psi'(e) - \lambda\psi''(e) + \lambda\frac{(I - Q)(1 - \phi) - \beta A}{e^2} = 0; \quad (24)$$

$$\lambda A = \mu e. \quad (25)$$

Equations (21) and (24) together give us

$$\begin{aligned} \frac{1}{e} \{(I - Q)(1 - \phi(1 - e)) - A\} \\ + \frac{1}{e}(1 - \beta)A &= \Pi - \psi'(e) \\ &= \lambda \left[ \psi''(e) - \frac{1}{e^2} \{(I - Q)(1 - \phi) - \beta A\} \right] \\ &\quad + \phi(I - Q) \\ &= \lambda \left[ \psi''(e) - \frac{1}{e^2} \{(I - Q)(1 - \phi) - A\} \right] \\ &\quad + \phi(I - Q), \end{aligned} \quad (26)$$

where the last line follows from the preceding one by equations (23) and (25). Rearranging equation (26) gives us equation (27):

$$(I - Q)(1 - \phi) - A = \frac{\lambda e^2 \psi''(e) - e(1 - \beta)A}{\lambda + e}. \quad (27)$$

We consider two cases, according to the sign of the left hand side of (27).

*Case 1: left hand side of equation (27) positive.* This is the case precisely when  $\phi < \hat{\phi}$ , where  $\hat{\phi}$  is given by equation (10). In this case the right hand side of equation (27) must also be positive and, since  $\beta \leq 1$ , we must therefore have  $\lambda > 0$ . Hence, by equation (25),  $\mu > 0$  so that, by the complementary slackness condition (23),  $\beta = 1$ , as in part 1 of proposition (2). Then, using the binding participation constraint (7) with  $\beta = 1$ , we obtain equation (11). Substituting  $\beta = 1$  into the incentive compatibility constraint (21) gives us equation (8), which is illustrated in figure 1, and from which the existence of  $\underline{\phi}$  follows.

Differentiating equation (8) with respect to  $\phi$  gives us the following:

$$e'(\phi) = \frac{(1 - e)(I - Q)}{e\psi''(e) + \phi(I - Q) + \psi'(e) - \Pi}. \quad (28)$$

To determine the sign of  $e'(\phi)$ , substitute  $\beta = 1$  into equation (27):

$$\{(I - Q)(1 - \phi) - A\}(\lambda + e) = \lambda e^2 \psi''(e). \quad (29)$$

When  $\beta = 1$ , equation (21) can be re-written as follows:

$$(\Pi - \psi'(e))e - \{(I - Q)(1 - \phi) - A\} - e\phi(I - Q) = 0,$$

and combining this with equation (29) gives us the following:

$$\lambda e \{e\psi''(e) - \Pi + \psi'(e) + e\phi(I - Q)\} = e \{(I - Q)(1 - \phi) - A\},$$

which, because  $\phi < \hat{\phi}$ , is positive. Hence  $e'(\phi) > 0$ , as in the proposition.

*Case 2: left hand side of equation (27) non-positive.* This is the case precisely when

$$\phi \geq \hat{\phi}. \quad (30)$$

If in this case we had  $\lambda > 0$  then we would also have  $\mu > 0$  and hence, by the complementary slackness condition (23),  $\beta = 1$ , which would contradict the non-positivity of equation (27). Hence  $\lambda = \mu = 0$ , and equation (27) reduces to equation (31).

$$\beta = \frac{(I-Q)(1-\phi)}{A} \quad (31)$$

Inserting this expression into the participation constraint (7) gives us

$$R = I - Q \quad (32)$$

Setting  $\lambda = 0$  into equation (24) gives us equation (14), from which equation (33) follows immediately:

$$e'(\phi) = -\frac{I-Q}{\psi''(e)} < 0. \quad (33)$$

*Proof of Proposition 3*

For a given level  $Q$  of co-funding, we know from section 2 that the optimal choice of subsidy is  $\hat{\phi}(Q)$ , defined in equation (10). If this level violates the budget constraint (15) then by proposition 2, the optimal  $\phi$  is the one that makes the budget constraint bind. Hence, for a given  $Q$ , the guarantee agency will set  $\phi = \phi^*(Q)$ , where

$$\phi^*(Q) \equiv \min \left\{ \hat{\phi}(Q), \frac{B-Q(1+\rho)}{(1-e)(I-Q)} \right\}. \quad (34)$$

Equation (34) may also be written as follows:

$$\phi^*(Q) = \begin{cases} \frac{B-Q(1+\rho)}{(1-e)(I-Q)} & , \text{ if } B < Q(e+\rho) + (1-e)(I-A); \\ \hat{\phi}(Q) & , \text{ if } B \geq Q(e+\rho) + (1-e)(I-A). \end{cases} \quad (35)$$

The guarantee agency's problem therefore reduces to the following:

$$\max_{Q \geq 0} \mathcal{W}(Q, \phi^*(Q)). \quad (36)$$

For a given  $Q$  we consider the two cases that arise in equation (35).

First, suppose that  $B < Q(e + \rho) + (1 - e)(I - A)$ , so that  $\phi^*(Q) = \frac{B - Q(1 + \rho)}{(1 - e)(I - Q)} < \hat{\phi}$ . In this case the analysis performed under case 1 of section (2) applies. We therefore have

$$\begin{aligned} \frac{d\mathcal{W}}{dQ} &= e'(Q) (\Pi - \psi'(e)) \\ &= \frac{e'(Q)}{e} \{(I - Q)(1 - \phi^*(Q)(1 - e)) - A(1 - e)\} \\ &= \frac{e'(Q)}{e} \{I - B + Q\rho - A(1 - e)\} \end{aligned} \quad (37)$$

where the second line follows from the incentive compatibility constraint (8). We know from equation (15) that  $I - A - B > 0$ , so the curly bracketed term in equation (37) is positive. The functional form of  $\phi^*$  allows us to express  $Q$  in terms of  $\phi$ , so by the chain rule,  $e'(Q) = e'(\phi)\phi'(Q)$ . We know from equation (28) that  $e'(\phi) > 0$  and direct differentiation gives us

$$\phi'(Q) = \frac{-(1 + \rho)I + B}{(1 - e)(I - Q)^2};$$

using equation (15) again, the numerator of this expression is less than  $-A - \rho I < 0$ , so  $\phi'(Q) < 0$  and hence

$$\frac{d\mathcal{W}}{dQ} < 0. \quad (38)$$

It follows immediately from equation (38) that, when  $B < Q(e + \rho) + (1 - e)(I - A)$ , the agency should set  $Q = 0$  and set  $\phi = \phi^*(Q)$ , as in part 1 of proposition (3).

The second case arises when  $B \geq Q(e + \rho) + (1 - e)(I - A)$ , so that  $\phi^*(Q) = \hat{\phi}(Q)$ . Then

$$\begin{aligned} \frac{d\mathcal{W}}{dQ} &= e'(Q) (\Pi - \psi'(e)) \\ &= e'(Q) \hat{\phi}(I - Q) \\ &= e'(Q) (I - Q - A), \end{aligned}$$

where the second line follows from the incentive compatibility constraint (14). In this case, we know from equation (33) that  $e'(\phi) < 0$ , and straightforward differentiation gives us

$$\phi'(Q) = \hat{\phi}'(Q) = -\frac{A}{(I - Q)^2} < 0. \quad (39)$$

It follows that  $\frac{d\mathcal{W}}{dQ}$  has the same sign as  $(I - Q - A)$ . Hence, when the credit guarantee agency has sufficient funds to provide the optimal level  $\hat{\phi}(Q)$  of guarantees at a given level of co-funding

$Q < I - A$ , it will increase welfare further by increasing the level of co-funding. Equation (39) indicates that, as it does so, it will reduce the guarantee that it offers. Since  $\frac{dW}{dQ}$  changes sign as soon as either  $Q > I - A$  or as soon as the budget constraint binds, the guarantee agency will stop increasing  $Q$  as soon as either of these events occurs. Note that, when  $Q$  attains its socially maximal level of  $I - A$ ,  $\hat{\phi} = 0$ .

*Proof of Proposition 4*

The entrepreneur's maximization problem yields the following first order conditions:

$$\begin{aligned} b(e^{FB}\Pi - \psi(e^{FB})) + (1-b)X &= \frac{\partial c}{\partial a}; \\ a(e^{FB}\Pi - \psi(e^{FB})) &= \frac{\partial c}{\partial b}, \end{aligned}$$

where  $X = V_1 - Q$ . We wish to prove that  $\frac{\partial a}{\partial X} > 0 > \frac{\partial b}{\partial X}$ .

Assume that  $c(a, b) = c(a) + c(b)$ . We have

$$\begin{aligned} \frac{\partial a}{\partial X} &= \frac{(1-b)\frac{\partial^2 c}{\partial b^2} - a(V_2 - X)}{\frac{\partial^2 c}{\partial a^2} \frac{\partial^2 c}{\partial b^2} - (V_1 - X)^2}; \\ \frac{\partial b}{\partial X} &= \frac{(1-b)(V_2 - X) - a\frac{\partial^2 c}{\partial a^2}}{\frac{\partial^2 c}{\partial a^2} \frac{\partial^2 c}{\partial b^2} - (V_1 - X)^2}. \end{aligned}$$

The denominator in these two expressions is positive, so the result follows immediately.