The Global Diffusion of Ideas

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PRELIMINARY AND INCOMPLETE

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Abstract

We provide a tractable theory of innovation and diffusion of technologies to explore the role of international trade and foreign direct investment (FDI). We model innovation and diffusion as a process involving the combination of new ideas with insights from other industries or countries. We provide conditions for the equilibrium distribution of productivity in each country to be Frechet, and derive a system of differential equations describing the evolution of the scale parameters of these distributions, i.e., the stock of ideas. In particular, the growth of the stock of ideas in a country is a weighted sum of the stock of ideas in all countries, where the weights are given by trade and FDI shares. We use this framework to quantify the dynamics gains from trade and FDI.
Economic miracles are characterized by protracted growth in productivity, per-capita income, and increases in trade and FDI flows. The experiences of Japan and South Korea in the postwar period and the recent performance of China are prominent examples. These experiences suggest an important role played by openness in the process of development. Yet quantitative trade models relying on standard static mechanisms imply relatively small gains, and therefore, cannot account for the experiences of growth miracles. These findings call for alternative channels through which openness can affect development. In this paper we present and analyze a model of an alternative mechanism: the impact of openness on the creation and diffusion of best practices across countries.

We model innovation and diffusion as a process involving the combination of new ideas with insights from other industries and countries. Insights occur randomly due to the local interactions that domestic producers have. In our theory openness affects the creation and diffusion of ideas by determining the distribution from which domestic producers draw their insights. Our theory is flexible enough to incorporate different channels through which ideas are diffused across countries. We focus on two main channels: (i) insights are drawn from the set of sellers to a country, (ii) insights are drawn from the set of technologies used domestically, both when are domestically or foreign own. In our model, openness to trade and Foreign Direct Investment (FDI) affects the quality of the insights drawn by domestic producers by selecting different sellers to a country and/or affecting the technologies used to produce domestically.

We use the model to explore several questions. First, we study how barriers to trade and FDI alter the learning process. At the micro level, the insights one draws depend on local interactions. At the aggregate level, the growth of a country’s stock of knowledge depends on its trade and FDI shares and the stocks of knowledge of its trading partners, and that of foreign firms operating domestically. Starting from autarky, opening up to trade and FDI

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1 See Feyrer (2009a,b) for recent estimates of the impact of trade on income, and a review of the empirical literature. See also the discussion in Lucas (2009b).

results in a higher growth rate of the stock of knowledge, as the producers are exposed to more productive ideas. Nevertheless, the case of costless trade does not necessarily result in the highest growth rate of the stock of knowledge. For example, a country can change the growth rate of its stock of knowledge by altering the composition of its trading partners. If learning from sellers is important, a country could increase the growth rate of its stock of knowledge by tilting imports toward its higher-wage trade partners; imports from higher-wage countries tend to be produced at higher productivity, as the high wage must be overcome with a low unit labor requirement. In this case, the growth rate of the stock of knowledge is maximized when import costs perfectly offset the wage differences among its trading partners. However, this generically conflicts with maximizing the static gains from trade.

We next use our model to quantify the dynamic gains from openness, studying in particular how opening to trade and FDI shape the diffusion of ideas. In a world that is generally open, if a single closed country opens to trade, it will experience an instantaneous jump in real income, a mechanism that has been well-studied in the trade literature. Following that jump, this country’s stock of knowledge will gradually improve as the liberalization leads to an improvement in the composition of insights drawn by its domestic producers. Here, the speed of convergence depends on the nature of learning process. If insights are drawn from goods that are sold to the country, then convergence will be faster, as opening to trade allows producers to draw insight from the relatively productive foreign producers. In contrast, if insights are drawn from technologies that are used locally, the country’s stock of knowledge grows more slowly. In that case, a trade liberalization leads to better selection of the domestic producers, but those domestic producers have low productivity relative to foreign firms. Opening to FDI also tends to lead to faster convergence than opening to trade. Opening to FDI provides more immediate access to the ideas of foreign producers. Over time, the exposure of more productive foreign multinational producing domestically leads to faster growth of the stock of knowledge. In the case of opening to FDI, the stock of knowledge grows faster when learning is from technologies used locally.
Finally, we study whether trade and FDI are substitutes or complements in learning. For both the static and dynamic gains from opening, this depends crucially on the correlation of multinationals’ productivities across potential production locations. When this correlation is high, trade and FDI are substitutes in increasing the speed of learning and raising real incomes.

**Literature Review**  
Our work builds on a large literature modeling innovation and diffusion of technologies as a stochastic process, starting from the earlier work of Jovanovic and Rob (1989), Jovanovic and MacDonald (1994), Kortum (1997), and recent contributions by Alvarez et al. (2008) and Luttmer (2012). We are particularly related to recent applications of these frameworks to study the connection between trade and the diffusion of ideas (Lucas, 2009a; Alvarez et al., 2013; Perla et al., 2013; Sampson, 2014).

Our theory captures the models in Kortum (1997), Alvarez et al. (2008, 2013), Perla et al. (2013) and Sampson (2014) as special cases. When the contribution of insights to the development of new technologies is zero, \( \beta = 0 \) in our notation, our framework simplifies to a version of Kortum (1997) with exogenous search intensity. As in that paper, ours is a model with semi-endogenous growth. When insights from domestic sellers are the only input to the development of new technologies, \( \beta = 1 \), our framework simplifies to the model in Alvarez et al. (2008, 2013) with stochastic arrival of ideas. Beyond analyzing the intermediate cases, \( \beta \in (0,1) \), we consider alternative sources of insights, e.g., learning from sellers to a country, learning from producers in a country, and study the role of both trade and FDI in determining the distribution of insights. Perla et al. (2013) and Sampson (2014) study the case with \( \beta = 1 \), where learning is from domestic producers and countries only interact through trade.

Eaton and Kortum (1999) also build a model of the diffusion of ideas across countries in which the distribution of productivities in each country is Frechet, and where the evolution

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3Lucas and Moll (2014) and Perla and Tonetti (2014) extends these models by studying the case with endogenous search effort, a dimension that we abstract from.
of the scale parameter of the Frechet distribution in each country is governed by a system of differential equations. In their work insights are drawn from the distribution of potential producers in each country, according to exogenous diffusion rates which are estimated to be country-pair specific, although countries are assumed to be in autarky otherwise. Therefore, changes in trade and FDI costs do not affect the diffusion of ideas.

The model shares some features with Oberfield (2013) which models the formation of supply chains and the economy’s input-output architecture. In that model, entrepreneurs discover methods of producing their goods using other entrepreneurs’ goods as inputs.\textsuperscript{4}

\section{Technology Diffusion given a General Source Distribution}

We begin with a description of technology diffusion in a single country given a general source distribution. The source distribution describes the set of insights that domestic producers might access. In the specific examples that we explore later in the paper, the source distribution will be a function of the profile of distributions of productivity across all countries in the world, but in this section we take it to be a general function satisfying weak tail properties.

Given the assumption on the source distribution, we show that the equilibrium distribution of productivity in a given economy is Frechet, and derive a differential equation describing the evolution of the scale parameter of this distribution.

We consider an economy with a continuum of goods \( s \in [0,1] \). Each good can be manufactured by using the same labor-only, linear technology

\[ y(s) = q(s)l(s), \]  

(1)

\textsuperscript{4}Here, the evolution of the distribution of marginal costs depends on a differential equation summarizing the history of insights that were drawn. In Oberfield (2013), the distribution of marginal costs is the solution to a fixed point problem, as each producer’s marginal cost depends on her potential suppliers’ marginal costs.
where $l(s)$ is the labor input and $q(s)$ is the productivity associated with good $s$. The state of technology in the economy is therefore described by the function $F_t(q)$, the CDF of the distribution of productivity.

We now turn to a description of the dynamics of the distribution of productivity $F_t(q)$. We model diffusion as a process involving the random interaction among product managers of different goods or countries. We assume that technology managers draw insights from others at rate $\tilde{\alpha}$. Insights are drawn from the distribution $G_t(q')$, which we denote the source distribution. When a manager with productivity $q$ draws insight from someone with productivity $q'$, she learns an idea to produce her own good with productivity $z^{1-\beta}q'^\beta$, where $z$ is drawn from a distribution with CDF $H(z)$. The resulting idea is adopted provided $z^{1-\beta}q'^\beta > q$. We refer to $H(z)$ as the exogenous distribution of ideas.5

This process captures the fact that interactions with more productive individuals tend to lead to more useful insights, but it also allows for randomness in the adaptation of more productive technologies to the production of other goods, or similar goods in other countries. This last dimension is captured by the random variable $z$. An alternative interpretation of the model is that $z$ represents an “original” random idea had by an innovator, which is combined with random insights obtained from other technologies.6

Given the distribution of productivity at time $t$, $F_t(q)$, the source distribution, $G_t(q')$, and the exogenous distribution of ideas, $H(z)$, the distribution of productivity at time $t + \Delta$

$$F_{t+\Delta}(q) = F_t(q) \left[ (1 - \tilde{\alpha}\Delta) + \tilde{\alpha}\Delta \int_0^\infty G_t \left( \left( \frac{q}{z^{1-\beta}} \right)^\frac{1}{\beta} \right) dH(z) \right]$$

The first term is the distribution of productivity at time $t$, which gives the fraction of goods with productivity less than $q$. The second terms is the probability that there were no insights.

5From the perspective of this section, both $G_t(q)$ and $H(z)$ are exogenous. The distinction between these distributions will become clear once we consider specific examples of source distributions, in which the source distribution will be an endogenous function of the profile of the distribution of productivity of all countries in the world.

6Along this second interpretation, if $\beta = 0$ our framework simplifies to a version of the model in Kortum (1997) with exogenous search intensity. With $\beta = 1$ and $G_t = F_t$, the framework simplifies to the model of diffusion in Alvarez et al. (2008) with stochastic arrival of ideas.
between time $t$ and $t + \Delta$ which resulted in an improvement in the technology to a productivity better than $q$. This can happen because there were no insights in an interval of time $\Delta$, an event with probability $1 - \tilde{\alpha}\Delta$, or because none of the insights resulted in a technology with productivity greater than $q$, an event with probability $\int_0^\infty G_t \left( \left( \frac{q}{z^\theta} \right)^{\frac{1}{\beta}} \right) dH(z)$.

Taking the limit as $\Delta \to 0$ we obtain

$$\frac{d}{dt} \ln F_t(q) = -\tilde{\alpha} \int_0^\infty \left[ 1 - G_t \left( \frac{q^{1/\beta}}{z^{(1-\beta)/\beta}} \right) \right] dH(z).$$

To gain analytical tractability, we assume that the arrival of ideas with idiosyncratic component greater than $z$ is given by

$$\tilde{\alpha} \left[ 1 - H(z) \right] = \alpha z^{-\theta}.$$

Formally, this is a limit of a sequence of economies under the following assumptions: We assume that $H(z) = 1 - \left( \frac{z}{z_0} \right)^{-\Theta}$ for $z > z_0$, and define $\theta \equiv \frac{\Theta}{1-\beta}$. We also define $\alpha \equiv \tilde{\alpha} z_0^\Theta$ to be a normalizing constant. Then

$$\frac{d}{dt} \ln F_t(q) = -\alpha \int_{z_0}^\infty \left[ 1 - G_t \left( \frac{q^{1/\beta}}{z^{(1-\beta)/\beta}} \right) \right] \Theta z^{-\Theta-1} dz$$

$$= -\alpha q^{-\theta} \int_0^{\infty} \left[ 1 - G_t(x) \right] \beta \theta x^{\beta \theta - 1} dx.$$

Consider a sequence of economies with $z_0 \to 0$, holding fixed $\alpha$. In that sequence, $\tilde{\alpha} \to \infty$; the economy with lower $z_0$ has more ideas, but each draw is worse. In the limit we obtain

$$\frac{d}{dt} \ln F_t(q) = -\alpha q^{-\theta} \int_0^\infty \left[ 1 - G_t(x) \right] \beta \theta x^{\beta \theta - 1} dx.$$

If $\lim_{x \to \infty} \left[ 1 - G_t(x) \right] x^{\beta \theta} = 0$, as will be the case later in our applications, this reduces to

$$\frac{d}{dt} \ln F_t(q) = -\alpha q^{-\theta} \int_0^\infty x^{\beta \theta} dG_t(x).$$
Then the solution is
\[ F_t(q) = e^{-\lambda_t q^{-\theta}}, \]
where
\[ \dot{\lambda}_t = \alpha \int_0^\infty x^{\beta \theta} dG_t(x). \]  

(2)

Thus, the distribution of productivities in this economy is Frechet and the dynamics of the scale parameter is governed by the differential equation (2). To simplify the analysis, we have resorted to rather special assumptions about the distribution \( H(z) \) and the arrival rate \( \tilde{\alpha} \). In Appendix A we show that the same result is obtained asymptotically under the weaker condition that the distribution \( H(z) \) has a Pareto tail and the arrival of ideas grows at a constant rate.

In the rest of the paper we analyze alternative models for the source distribution \( G_t \). A simple example that illustrates basic features of more general cases is \( G_t(q) = F_t(q) \). This corresponds to the case in which diffusion opportunities are randomly drawn from the set of domestic best practices across all sectors. In a closed economy this set equals the set of domestic producers and sellers. In this case equation (2) becomes
\[ \dot{\lambda}_t = \alpha \Gamma(1 - \beta) \lambda_t^\beta. \]

Growth in the long-run is obtained in this framework by assuming that the arrival rate of insight is growing over time, \( \alpha(t) = \alpha_0 e^{\gamma t} \). In this case, the scale of the Frechet distribution \( \lambda_t \) grows asymptotically at the rate \( \gamma/(1 - \beta) \), and per-capital GDP grows at the rate \( \gamma/[(1 - \beta)\theta] \). For this case, the evolution of the de-trended stock of ideas \( \hat{\lambda}_t = \lambda_t e^{\gamma/(1 - \beta)t} \)
\[ \dot{\hat{\lambda}}_t = \alpha_0 \Gamma(1 - \beta) \hat{\lambda}_t^\beta - \frac{\gamma}{1 - \beta} \hat{\lambda}_t, \]
and on a balanced growth path the de-trended stock of ideas

\[ \hat{\lambda} = \left[ \frac{\alpha_0 (1 - \beta)}{\gamma} \Gamma(1 - \beta) \right]^{\frac{1}{1-\beta}}. \]

## 2 International Trade

We first consider a world where \( n \) economies interact through trade, and ideas diffuse through the contact of domestic producers with domestic and foreign sellers to a country, and other domestic producers. Given the results from the previous section, the static trade theory is given by the standard Ricardian model in Eaton and Kortum (2002) and Alvarez and Lucas (2007), which we briefly introduce before deriving the equations which characterize the evolution of the profile of the distribution of productivities of countries in the world economy.

In each country, consumers have identical preferences over a continuum of goods. We use \( c(s) \) to denote the consumption of an agent of each of the \( s \in [0, 1] \) goods. The utility function is given by

\[ C = \left[ \int_0^1 c(s)^{1-1/\epsilon} ds \right]^{\epsilon/(\epsilon-1)} \]

so goods enter symmetrically and exchangeably. In each country, individual goods can be manufactured by many producers, each using the same labor-only, linear technology (1). Given the setup, it is natural to assume that all the identical producers of goods \( s \) behave competitively. We maintain this assumption throughout the analysis of the paper as it simplifies the notation substantially.\(^7\)

In the \( n \)-country world, each good is characterized by the vector \( q(s) = (q_1(s), ..., q_n(s)) \) denoting the productivity with which a good can be produced in each country. Using the symmetry of the utility function and the assumed competitive behavior we group goods by

\(^7\)See Alvarez et al. (2013) for an extension of the model with Bertrand competition.
their productivity $q$ and write the utility function as

$$C = \left[ \int_0^1 c(q)^{1-1/\epsilon} dF(q) \right]^{\epsilon/(\epsilon-1)}$$

where $c(q)$ is the consumption of any good $s$ that has profile of productivities $q$ and $F(q) = \prod_{i=1}^n F_i(q_i)$ is the CDF of the distribution of the profile of productivities.

In turn, as discussed in the previous section, provided countries share the same exogenous distribution of ideas $H(z)$, the distribution of productivity in each country is described by a Frechet distribution with curvature $\theta$ and a country-specific scale $\lambda_i$, $F_i(q) = e^{-\lambda_i q^\theta}$. Transportation costs are given by the standard “iceberg” assumption, where $\kappa_{ij}$ denotes the units that are needed to be shipped from country $j$ to deliver a unit of the good in country $i$, with $\kappa_{ii} = 1$ and $\kappa_{ij} \geq 1$. We now briefly present the basic equations describing a static trade equilibrium given the vector of scale parameters $\lambda = (\lambda_1, ..., \lambda_n)$.

Given these assumptions, and denoting by $w_i$ the wage in country $i$, the price of good $s$ in country $i$

$$p_i(s) = \min \left\{ \frac{w_1 \kappa_{i1}}{q_1(s)}, ..., \frac{w_n \kappa_{in}}{q_n(s)} \right\}$$

and the price index country $i$

$$P_i = \left[ \int_0^1 p_i(s)^{1-\epsilon} ds \right]^{\frac{1}{1-\epsilon}}$$

$$= \left[ \sum_{j=1}^n \int_0^\infty \left( \frac{w_j \kappa_{ij}}{q} \right)^{1-\epsilon} \prod_{k \neq j} F_k \left( \frac{w_k \kappa_{ik}}{w_j \kappa_{ij}} q \right) dF_j(q) \right]^{\frac{1}{1-\epsilon}}$$

$$= \Gamma \left( 1 - \frac{\epsilon - 1}{\theta} \right)^{\frac{1}{1-\epsilon}} \left\{ \sum_j \lambda_j (w_j \kappa_{ij})^{-\theta} \right\}^{-\frac{1}{\theta}}. \quad (3)$$

where $\Gamma(1 - \frac{\epsilon - 1}{\theta}) = \int_0^\infty t^{-\frac{\epsilon - 1}{\theta}} e^{-t} dt$ is the gamma function. To interpret the price index, note that $\prod_{k \neq j} F_k \left( \frac{w_k \kappa_{ik}}{w_j \kappa_{ij}} \right) dF_j(q)$ is the probability that country $j$ can produce a good with
productivity $q$ and is the lowest cost provider of that good to $i$. In the case that a producer from country $j$ with productivity $q$ is the lowest cost provider, its price in country $i$ equals $w_j \kappa_{ij}/q$.

Denoting by $\pi_{ij}$ the share of country $i$’s expenditure that is spent in goods from country $j$

$$\pi_{ij} = \frac{1}{P_i^{1-\varepsilon}} \int_0^\infty \left( \frac{w_j \kappa_{ij}}{q} \right)^{1-\varepsilon} \prod_{k \neq j} F_k \left( \frac{w_k \kappa_{ik}}{w_j \kappa_{ij}} q \right) dF_j(q)$$

$$= \frac{\lambda_j (w_j \kappa_{ij})^{-\theta}}{\sum_{k=1}^n \lambda_k (w_k \kappa_{ik})^{-\theta}},$$

A static equilibrium is given by a profile of wages $\mathbf{w} = (w_1, ..., w_n)$ such that labor market clears in all countries

$$w_i L_i = \sum_j w_j L_j \pi_{ji}, \quad i = 1, ..., n.$$ 

As a simple benchmark, it is useful to consider the case with costless trade, $\kappa_{ij} = 1$, all $j$, and countries of equal size $L_i = L_j$, all $j \neq i$. In this case, relative wages

$$\frac{w_i^{FT}}{w_i'^{FT}} = \left( \frac{\lambda_i}{\lambda_i'} \right)^{\frac{1}{1+\theta}}$$

and the relative expenditure shares

$$\frac{\pi_{ij}^{FT}}{\pi_{ij'}^{FT}} = \left( \frac{\lambda_j}{\lambda_j'} \right)^{\frac{1}{1+\theta}}.$$ (4)

Given the static equilibria, we next solve for the evolution of the profile of scale parameters $\lambda = (\lambda_1, ..., \lambda_n)$ by specializing (2) for alternative assumptions about source distributions. We consider source distributions that encompass two cases: (i) domestic producers learn from sellers to the country, (ii) domestic producers learn from other producers in the country.
2.1 Learning from Sellers

Following the framework introduced in Section 1, we model the evolution of technologies as the outcome of a process where technology managers combine “own ideas” with random insights from technologies in other sectors or countries. We first consider the case in which insights are drawn from sellers to the country. In particular, we assume that insights are randomly drawn from the distribution of sellers’ productivity in proportion to the expenditure on each good.\(^8\) In this case, the source distribution is given by the expenditure weighted distribution of productivity of sellers

\[
G(q) = G^S_i(q) = \int_0^q \sum_{j=1}^n \left( \frac{w_j K_{ij}}{P_i} \right)^{1-\varepsilon} x^{\varepsilon-1} \prod_{k \neq j} F_k \left( \frac{w_k K_{ik} x}{w_j K_{ij}} \right) dF_j(x). 
\]

Using that \( F_i(q) = e^{-\lambda_i q^{-\theta}} \)

\[
G^S_i(q) = \sum_{j=1}^n \frac{\Gamma \left( 1 - \frac{\varepsilon-1}{\theta}, \frac{\lambda_i}{\pi_{ij}} q^{-\theta} \right)}{\Gamma \left( 1 - \frac{\varepsilon-1}{\theta} \right)} \pi_{ij}. \tag{5}
\]

where \( \Gamma \left( 1 - \frac{\varepsilon-1}{\theta}, \frac{\lambda_i}{\pi_{ij}} q^{-\theta} \right) = \int_{\pi_{ij} q^{-\theta}}^{\infty} t^{-\frac{\varepsilon-1}{\theta}} e^{-t} dt \) is the incomplete gamma function.

The CDF \( G^S_i(q) \) summarizes the quality of ideas to which producers in \( i \) are exposed. It is relatively straightforward to see that the quality of ideas improves, i.e., the CDF decreases, as a country starts trading with other countries, i.e., \( \partial G^S_i(q) / \partial \pi_{ij} |_{\pi_{ij}=0} < 0 \). Nevertheless, the quality of ideas is not necessarily maximized in the case of free trade. To maximize the quality of insights countries must bias their trade towards those countries with higher technologies. In particular, the expenditure shares that provide country \( i \) with the best

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\(^8\)For the case of learning from sellers, the assumption that insights are randomly drawn in proportion to the expenditure is not very central. Alternative assumptions, e.g., insights are uniformly drawn from the set of sellers, give the same law of motion for the scale parameters up to a constant. See Appendix B.2.1.
distribution of insights is

\[
\frac{\pi_{ij}}{\pi_{ij'}} = \frac{\lambda_j}{\lambda_{j'}}.
\]  

(6)

While in equilibrium, country \(i\)'s expenditure share will be

\[
\frac{\pi_{ij}}{\pi_{ij'}} = \frac{\lambda_j (w_j \kappa_{ij})^{-\theta}}{\lambda_{j'} (w_{j'} \kappa_{ij'})^{-\theta}}.
\]  

(7)

Notice that (6) and (7) coincide only if differences in trade costs perfectly offset differences in trading partners’ wages.

Specializing equation (2) to the source distribution in (5), the evolution of the scale of the Frechet distribution, i.e., the stock of ideas, is described by

\[
\dot{\lambda}_i = \alpha \int_0^\infty x^{\beta \theta} dG_i^S(q)
\]

\[
= \alpha \frac{\Gamma (1 - \beta - \frac{\varepsilon - 1}{\theta})}{\Gamma (1 - \frac{\varepsilon - 1}{\theta})} \sum_{j=1}^n \pi_{ij} \left( \frac{\lambda_j}{\pi_{ij}} \right)^\beta
\]  

(8)

That is, the evolution of the stock of ideas is a simple weighted sum of the stock of ideas in all countries, where the weights are given by expenditure shares. Again, it is easy to see that the growth of the stock of ideas of a country is maximized when expenditure shares are proportional to the stock of ideas of trading partners. For example, for a country that is the technological leader, this can be implemented by increasing the cost of trading with less technologically developed countries. In turn, countries that are not at the technological frontier need to subsidize trade with technological advanced countries to maximize the growth rate of the stock of ideas.

Equation (8) shows that trade shapes how a country learns in two ways. Trade gives a country access to the ideas of sellers from other countries. In addition, trade leads to tougher competition, so that there is more selection among the producers from which insights are drawn. In fact, the less a country is able to sell, the stronger selection is among its producers.
The amount \( i \) learns from \( j \) is given by \((\lambda_j / \pi_{ij})^\beta\). Holding fixed \( j \)’s stock of knowledge, a smaller \( \pi_{ij} \) reflects more selection into selling, which means that the insights drawn from sellers from \( j \) are likely to be higher quality insights.

As discussed before, to obtain growth in the long-run we assume that the arrival rate of insights grow over time, \( \alpha(t) = \alpha_0 e^{\gamma t} \), in which case it is convenient to analyze the evolution of the de-trended stock of ideas \( \hat{\lambda}_{it} = \lambda_{it} e^{-\frac{\gamma}{1-\beta} t} \)

\[
\dot{\hat{\lambda}}_{it} = \alpha_0 \frac{\Gamma(1 - \beta - \frac{\varepsilon-1}{\theta})}{\Gamma(1 - \frac{\varepsilon-1}{\theta})} \sum_{j=1}^{n} \pi_{ij}^{1-\beta} \hat{\lambda}_{jt}^\beta - \frac{\gamma}{1 - \beta} \hat{\lambda}_{it}, \quad (9)
\]

On a balanced growth path where the arrival rate of insights grows at rate \( \gamma \), the de-trended stock of knowledge solves the system of non-linear equations

\[
\hat{\lambda}_i = \frac{(1 - \beta) \alpha_0 \Gamma(1 - \beta - \frac{\varepsilon-1}{\theta})}{\gamma \Gamma(1 - \frac{\varepsilon-1}{\theta})} \sum_{j=1}^{n} \pi_{ij}^{1-\beta} \hat{\lambda}_{jt}^\beta. \quad (10)
\]

### 2.1.1 Static and Dynamic Gains from Trade

Specializing equation (10) for the case of costless trade with symmetric countries, the de-trended stock of ideas on a balanced growth path

\[
\hat{\lambda}^{FT} = \left[ \frac{(1 - \beta) \alpha_0 \Gamma(1 - \beta - \frac{\varepsilon-1}{\theta})}{\gamma \Gamma(1 - \frac{\varepsilon-1}{\theta})} \sum_{j=1}^{n} \pi_{ij}^{1-\beta} \hat{\lambda}_{jt}^\beta \right]^{\frac{1}{1-\beta}} n^{\frac{\beta}{1-\beta}}. \quad (11)
\]

Similarly, the de-trended stock of ideas in a symmetric world as a function of the trade cost \( \kappa \)

\[
\hat{\lambda}(\kappa) = \left[ \frac{(1 - \beta) \alpha_0 \Gamma(1 - \beta - \frac{\varepsilon-1}{\theta})}{\gamma \Gamma(1 - \frac{\varepsilon-1}{\theta})} \sum_{j=1}^{n} \pi_{ij}^{1-\beta} \hat{\lambda}_{jt}^\beta \right]^{\frac{1}{1-\beta}} \left[ 1 + (n - 1) \kappa^{-\theta(1-\beta)} \right]^{\frac{1}{1-\beta}}. \quad (11)
\]
The de-trended real per-capita income is obtained by substituting (11) into (3)

\[ \hat{y}(\kappa) = \Gamma \left( 1 - \frac{\varepsilon - 1}{\theta} \right)^{-\frac{1}{1-\varepsilon}} \hat{\lambda}(\kappa)^\frac{1}{\theta} \left( 1 + (n - 1)\kappa^{-\theta} \right)^\frac{1}{\theta} \]  

(12)

Using equation (12), we derive a simple expression for the gains from trade. In our model, gains from trade has a static and dynamic component. The static component, holding each country’s stick of knowledge fixed, is the familiar gains from trade in standard Ricardian models, e.g., Eaton and Kortum (2002). The dynamic gains from trade are the ones that operate through the effect of trade on the flow of ideas. For the case of a world with symmetric populations, the gains from trade as measured by the per-capita income on a balanced growth path with costless relative to autarky, can be decomposed as follows

\[ \frac{y_{FT}}{y_{AUT}} = n^{\frac{1}{\theta}} \left( \frac{\lambda_{FT}}{\lambda_{AUT}} \right)^{\frac{1}{\theta}} \]

\[ = n^{\frac{1}{\theta}} \underbrace{n^{1-\beta \theta}}_{\text{static}} \underbrace{n^{\beta \theta}}_{\text{dynamic}} \].

(13)

The gains from trade depend on three parameters \( n, \theta, \) and \( \beta \). As in standard trade models, the gains from trade depend on the size of the country \( 1/n \) and the curvature of the distribution of productivity \( \theta \). The smaller each individual economy in this symmetric world, the more they gain by having access to the best producers abroad. In turn, the higher the curvature \( \theta \), the thinner the right tails of productivities. That is, there are fewer highly productive producers abroad which individuals can buy from under free-trade. The novel parameter determining the gains from trade is \( \beta \). The parameter \( \beta \) controls the importance of insights from sellers in determining the evolution of individual’s productivity, i.e., the extent of technological spillovers associated with trade. With higher \( \beta \), insights from sellers are more important, and therefore, more is gained by being exposed to more productive sellers in a world with free trade. In the limit as \( \beta \) goes to 1, holding fixed \( \theta \), the gains from trade relative to autarky are arbitrarily large. This limiting case is the one analyzed by Alvarez.
2.1.2 Asymmetric Example: Technology and Income Differences

To analyze the ability of the model to account for cross-country income differences, we explore numerically an asymmetric version of the model with \( n - 1 \) open countries, \( i = 1, \ldots, n - 1 \), and a single deviant economy, \( i = n \). We assume that the \( n - 1 \) open countries can freely trade among themselves, \( \kappa_{ij} = \kappa_1 = 1, \ i, j < n \), but that trade to and from the deviant economy incurs transportation cost \( \kappa_{nj} = \kappa_{jn} = \kappa_n \geq 1, \ j < n \).

![Figure 1: The Stock of Ideas and Openness of the Deviant Economy. Alternative Diffusion Coefficients, \( \beta \).](image)

Figure 1 shows how the stocks of ideas in the \( n - 1 \) open countries (left panel) and the single deviant economy (right panel) change with the degree of openness of the deviant economy. The x-axis measures openness as the inverse of the cost to trade to and from the deviant economy, \( 1/\kappa_n \). On the y-axis we report the stock of ideas relative to the case.

---

\(^{9}\)We consider a world with \( n = 50 \) economies with symmetric populations, so that each country is of the size of Canada or South Korea. We set \( \theta = 5 \), the curvature of the Frechet distribution, which itself equals the tail of the distribution of exogenous ideas. This value is in the range consistent with estimates of trade elasticities. See Simonovska and Waugh (2014), and the references therein. Given a value of \( \beta \), the growth rate of the arrival rate of ideas is calibrated so that on the balanced growth path each country grows at \( 2\% \), \( \gamma/(1 - \beta) = 0.02 \). The parameter \( \alpha_0 \) is normalized so that in the case of costless trade, \( \kappa_n = 1 \), the de-trended stock of ideas equals 1.
with costless trade, $\kappa_n = 1$. The different lines correspond to alternative values of $\beta$, which controls the importance of insights from sellers. As we consider balanced growth paths in which the deviant economy becomes more isolated, the stock of ideas of this economy contracts relatively to that of the balanced growth path of $n$ economies engaging in costless trade, and also relative to the stock of ideas in the $n - 1$ open economies trading with 1 deviant country. As can be seen by comparing the left and right panels, while the isolation of one economy has global effects, these are less important than the direct impact on the isolated country.

![Figure 2: Per-Capita Income and Openness of the Deviant Economy.](image)

As discussed earlier, through their effect on the stock of ideas trade costs have effects on per-capita income that goes beyond the static gains from trade. Figure 2 illustrates these effects by showing the per-capita income in the $n - 1$ open countries (left panel) and the single deviant economy (right panel) as a function of the degree of openness in the single deviant economy. On the y-axis we report the per-capita income relative to that of a balanced growth path with costless trade, $\kappa_n = \kappa_1 = 1$. The solid line shows the effect of openness in the case with no spillovers, $\beta = 0$, which also equals the effect in the standard static trade theory of Eaton and Kortum (2002). The other two curves correspond to cases with positive
technological spillovers from sellers, and therefore, larger gains from trade.

2.2 Learning from Production

Another natural source of ideas is given by the interaction of technology managers with other domestic producers, or workers employed by these producers. Along these lines, in this section we consider the case in which the insights are drawn from the distribution of productivity among domestic producers, in proportion to the labor used in the production of each good.\(^\text{10}\) Under this assumption, the source distribution is given by the labor weighted distribution of productivity of domestic producers

\[
G_i(q) = G_i^P(q) = \sum_{j=1}^{n} \int_{0}^{q} \frac{L_j w_j}{L_i w_i} \left( \frac{w_i K_{ji}}{P_j} \right)^{1-\varepsilon} x^{\varepsilon-1} \prod_{k \neq j} F_k \left( \frac{w_k K_{jk}}{w_i K_{ji}} x \right) dF_i(x)
\]

To interpret this, if a producer in \(i\) is the lowest cost supplier of good \(s\) to country \(j\), she will use \(\frac{L_j w_j}{w_i} \left( \frac{w_i K_{ji}}{P_j} \right)^{1-\varepsilon} x^{\varepsilon-1}\) units of labor to produce and export the good to \(j\). Using that \(F_i(q) = e^{-\lambda_i q^{-\theta}}\)

\[
G_i^P(q) = \sum_{j=1}^{n} \frac{\Gamma \left( 1 - \frac{\varepsilon-1}{\theta}, \frac{\lambda_i}{\pi_{ji}} q^{-\theta} \right)}{\Gamma \left( 1 - \frac{\varepsilon-1}{\theta} \right)} \frac{L_j w_j}{L_i w_i} \pi_{ji} \tag{14}
\]

\[
= \sum_{j=1}^{n} \frac{\Gamma \left( 1 - \frac{\varepsilon-1}{\theta}, \frac{\lambda_i}{\pi_{ji}} q^{-\theta} \right)}{\Gamma \left( 1 - \frac{\varepsilon-1}{\theta} \right)} r_{ji} \tag{15}
\]

where \(r_{ji} = \frac{\pi_{ji} w_j L_j}{\sum_k \pi_{ki} w_k L_k}\) is the share of \(i\) revenue coming from sales to country \(j\). Thus, the source distribution of a country \(i\) is a function of the fraction of domestic goods purchased by other countries, \(\pi_{ji}\), the income of these countries relative to that in country \(i\), \(L_j w_j / (L_i w_i)\),

\(^\text{10}\) For this case, the assumption that insights are randomly drawn in proportion to the labor used or the value added produced, instead of been randomly drawn from the set of producers, is more important. See Appendix B.2.1 for a characterization of the dynamics of the stock of ideas under alternative assumptions.
and the domestic stock of knowledge, $\lambda_i$. Specializing equation (2) to the source distribution in (14), the evolution of the scale of the Frechet distribution, i.e., the stock of ideas, is described by

$$
\dot{\lambda}_it = \alpha \int_0^{\infty} x^{\beta\theta} dG_i^P(q) = \frac{\Gamma (1 - \beta - \frac{\varepsilon-1}{\theta})}{\Gamma (1 - \frac{\varepsilon-1}{\theta})} \sum_{j=1}^{n} r_{ji} \left( \frac{\lambda_j}{\pi_{ji}} \right)^\beta.
$$

In autarky, insights are drawn from all domestic producers, including very unproductive ones. As a countryopens up to trade the set of domestic producers improves as the unproductive technologies are selected out, and the growth rate of the stock of ideas increases. Still, the growth rate of the stock of ideas is not maximized in the case of free trade. In this example, this is not even true for the case of symmetry countries, i.e., $\lambda_i = \lambda_j$, $L_i = L_j$, all $i \neq j$.$^{11}$ In this version of the model, starting from a free trade allocation, there is a first order improvement in the quality of insights by shifting labor towards the most efficient producers to produce exports, which outweighs the deterioration of the quality of the set of domestic producers.$^{12}$

As before, if the arrival rate of insights grows at the rate $\gamma$, the evolution of the de-trended scale $\hat{\lambda}_it = \lambda_it/e^{\gamma/(1-\beta)t}$ is given by

$$
\dot{\hat{\lambda}}_it = \alpha_0 \frac{\Gamma (1 - \beta - \frac{\varepsilon-1}{\theta})}{\Gamma (1 - \frac{\varepsilon-1}{\theta})} \sum_{j=1}^{n} r_{ji} \left( \frac{\hat{\lambda}_jt}{\pi_{ji}} \right)^\beta - \frac{\gamma}{(1 - \beta)} \hat{\lambda}_it, \quad (16)
$$

$^{11}$Recall that for the case in which insights are drawn from the sellers to a country, equations (4) and (6) coincide when $\lambda_j = \lambda_j'$.

$^{12}$If insights were to be randomly drawn from the set of active domestic producers and trade barriers satisfy the triangle inequality, the evolution of the stock of knowledge is $\dot{\lambda}_it = \Gamma (1 - \beta)(\lambda_i/\pi_{ii})^\beta$. In this case, the growth rate of the stock of ideas would be maximized in the case of costless trade, $\pi_{ii} = 1/n$, provided we abstract from import subsidies. The only effect of higher trade costs in this version of the model is to worsen the selection of domestic producers.
and on a balanced growth path it solves the following system of non-linear equations

\[ \hat{\lambda}_i = \frac{(1 - \beta)\alpha_0}{\gamma} \Gamma \left( 1 - \beta - \frac{\varepsilon - 1}{\sigma} \right) \frac{\Gamma (1 - \frac{\varepsilon - 1}{\sigma})}{\sum_{j=1}^{n} r_{ji} \left( \frac{\hat{\lambda}_{jt}}{\pi_{ji}} \right)^{\beta}}. \]

It is relatively easy to see that the implications for symmetric examples are the same as in the case of learning from sellers. In particular, the de-trended stock of knowledge and per-capita income on a balanced growth path as a function of the (symmetric) trade cost \( \kappa \) is given by (11) and (12), respectively. Similarly, the static and dynamic gains from trade can be decomposed as in (13).

### 2.2.1 Asymmetric Example: Technology and Income Differences

In asymmetric examples, the model with learning from domestic producers has very different implications, relative to those of the model with learning from sellers, when considering asymmetric examples. As in Section 2.1.2, we consider an asymmetric version of the model with \( n - 1 \) open countries, \( i = 1, \ldots, n - 1 \), and a single deviant economy, \( i = n \). The \( n - 1 \) open countries can freely trade among themselves, i.e., \( \kappa_{ij} = 1 \), \( i, j < n \), but trade to and from the deviant economy incurs transportation cost, i.e., \( \kappa_{nj} = \kappa_{jn} = \kappa_n \geq 1 \), \( j < n \).\(^{13}\)

Figure 3 shows how the stock of ideas varies across balanced growth paths associated with different degrees of openness of the single deviant country. The effect on the stock of ideas in the single deviant economy is shown in the right panel. Starting from the case of costless trade, the stock of ideas in the deviant economy initially improves as the trade cost to and from this country increases, and it becomes isolated. Initially, as the trade costs increase, labor in the deviant economy becomes more concentrated among efficient producers, and therefore, its stock of ideas in the balance growth path improves. Eventually, the negative selection effects of inefficient domestic producers entering dominates, and the stock of ideas in the deviant economy deteriorates. These effects are more pronounced the

\(^{13}\)We follow the calibration described in footnote 9.
higher the degree of spillovers as measured by the parameter $\beta$.

The effect on per-capita income is shown in Figure 4. As the stock of ideas in the single deviant economy initially increases as this economy becomes isolated starting from costless trade, the dynamic gains from trade are negative in this range. But eventually, the dynamic gains from trade are positive, and can be very large as the deviant economy approaches
autarky.

2.3 Trade Liberalization

We now study how a country’s stock of knowledge and per-capita GDP change when it opens to trade. In particular, we consider a world economy that starts with \( n - 1 \) open and 1 deviant economies that are in a balance growth path. Initially, the \( n - 1 \) open economies can trade at no cost among themselves, \( \kappa_1 = 1 \), but trade to and from the deviant economy face large trade costs, \( \kappa_n = 100 \). The initial balanced growth path corresponds to a point close to the origin in figures 1-4. We then trace the evolution of the stock of ideas and per-capita income as trade costs to and from the (former) deviant economy are eliminated, \( \kappa_n = 100 \rightarrow \kappa'_n = 1 \). The paths of the (de-trended) stock of knowledge solve the differential equations in (9) and (16), depending on whether insights are drawn from sellers or producers.\(^{14}\)

![Graph](image)

Figure 5: Dynamics Following a Trade Liberalization for Alternative Sources of Insights.

Figure 5 shows the evolution of the stock of ideas (left panel) and per-capita income (right panel) in the initially deviant economy, following the elimination of trade costs. The solid line corresponds to the transitional dynamics in the model where learning is from sellers.

\(^{14}\)We set \( \beta = 0.5 \). The rest of the parameters follow the calibration in footnote 9.
to a country, while the dashed line gives the dynamics for the model in which insights are drawn from domestic producers.

Following the elimination of trade costs, the stock of ideas in the deviant economy slowly converges to that of the \( n - 1 \) open countries. This process is substantially faster for the case of learning from sellers. When insights are drawn from sellers, a trade liberalization gives immediate access to insights from goods sold by high productivity foreign producers. In contrast, when insights are drawn from domestic producers, the insights are initially low quality, although they become more selected, and only gradually improve as the country’s stock of knowledge increases.

On impact real income jumps as it would in a static model. Furthermore, over time, it continues to increase as the stock of knowledge improves. The speed of this process depends on the sources of insights, as discussed above. Notice that for this example, the long-run effect on per-capita income is substantially larger than the effect on impact, and that the transition is very protracted. These examples suggest that our model provides a promising theory of growth miracles fueled by openness, and underscores the importance of investigating the particular mechanism for the diffusion of ideas.

3 Trade and Multinational Production (VERY PRELIMINARY)

The set of producers in a country may be influenced by multinationals’ decisions of where to locate production. Some countries have policies to encourage FDI, and this might be related to positive spillovers to local producers. In this section we extend the basic model to allow for multinational production, and insights drawn from foreign technologies used domestically.

We follow Ramondo and Rodriguez-Clare (2013) in modeling a multinational as a producer that can manufacture a single variety with productivity that varies with the loca-
tion of production. For variety $s$, country $i$ is characterized by a vector of productivities $q_i(s) = \{q_{ji}(s)\}_j$, where $q_{ji}(s)$ is the best productivity available to multinationals based in $i$ when producing variety $s$ in country $j$.

Producing abroad is further subject to iceberg costs. Suppose a producer of variety $s$ based in $i$ can produce in $j$ with productivity $q$. That producer’s marginal cost will be $\frac{w_j \delta_{ji}}{q_{ji}(s)}$.

We assume $\delta_{ji} \geq 1$ and $\delta_{ii} = 1$.

We first study learning from a general source distribution. Among all varieties, let $F_i(\{q_1, ..., q_n\})$ be the joint distribution of productivities across locations among multinationals based in $i$. Suppose an individual production manager draws an insight from a source distribution, $G_i(q)$. Upon drawing insight from an idea with productivity $\tilde{q}$, the manager is provided with a vector of ideas with productivities $\{z_{ij}^{1-\beta} \tilde{q}^\beta\}_j$ across locations. The idiosyncratic portion of the productivities, $\{z_j\}$, are drawn from a multivariate distribution with joint CDF $H(\{z_1, ..., z_n\})$.

Extending the analysis in Section 1, the distribution of productivity of technologies based on country $i$

\[
\frac{d}{dt} \ln F_i(\{q_1, ..., q_n\}) = -\tilde{\alpha} \int_0^\infty \left[ 1 - H \left( \left\{ \left( \frac{q_1}{\tilde{q}^\beta} \right)^{\frac{1}{1-\beta}}, ..., \left( \frac{q_n}{\tilde{q}^\beta} \right)^{\frac{1}{1-\beta}} \right\} \right) \right] dG_i(\tilde{q})
\]

We assume that the idiosyncratic components of new ideas are drawn from the following distribution

\[
H(\{z_1, ..., z_n\}) = \max \left\{ 1 - \left( \sum_i \left( \frac{z_i}{z_0} \right)^{-\frac{\alpha}{1-\rho}} \right)^{1-\rho}, 0 \right\}
\]

with $\rho \in [0, 1]$. This can be thought of as a multivariate Pareto because each marginal distribution is Pareto. As before, define $\alpha \equiv \tilde{\alpha} z_0^\theta$ and $\theta \equiv \frac{\theta}{1-\beta}$. Using the same logic as the baseline analysis, we can show that in the limiting economy as $z_0 \to 0$, the law of motion for
the distribution of ideas in $i$ is

$$\frac{d}{dt} \ln F_{it} \left( \{q_1, \ldots, q_n\} \right) = -\alpha \left( \sum_j q_j^{-\frac{\theta}{1-\rho}} \right)^{1-\rho} \int_0^\infty \tilde{q}^{\beta \theta} dG_{it}(\tilde{q})$$

Suppose initial conditions are such that $\ln F_{i0} \left( \{q_1, \ldots, q_n\} \right) = -\lambda_{i0} \left( \sum_j q_j^{-\frac{\theta}{1-\rho}} \right)^{1-\rho}$. Then the solution to this differential equation is

$$F_{it} \left( \{q_1, \ldots, q_n\} \right) = e^{-\lambda_{it} \left( \sum_j q_j^{-\frac{\theta}{1-\rho}} \right)^{1-\rho}}$$

with

$$\dot{\lambda}_{it} = \alpha \int_0^\infty \tilde{q}^{\beta \theta} dG_{it}(\tilde{q})$$

These can be called a multivariate Frechet because each marginal distributions is Frechet.

We now describe the equilibrium, leaving the derivation of the equations to Appendix C. Define $\pi_{ijk}$ to be the share of $i$’s expenditure on imports from $j$ that were produced by multinationals based in $k$. Similarly, let $r_{ijk} = \frac{\pi_{ijk} w_i L_i}{w_j L_j}$ be the share of $j$’s revenue from sales to $i$ of goods produced by multinationals based in $k$. Thus $1 = \sum_j \pi_{ij} = \sum_j \sum_k \pi_{ijk}$ and $1 = \sum_i r_{ij} = \sum_i \sum_k r_{ijk}$. Given wages, the trade shares and the price level are

$$\pi_{ijk} = \frac{\lambda_k \left( \sum_m \left[ w_m \kappa_{im} \delta_{mk} \right]^{-\frac{\theta}{1-\rho}} \right)^{-\rho} \left[ w_j \kappa_{ij} \delta_{jk} \right]^{-\frac{\theta}{1-\rho}}}{\sum_l \lambda_l \left( \sum_m \left[ w_m \kappa_{im} \delta_{ml} \right]^{-\frac{\theta}{1-\rho}} \right)^{1-\rho}}$$

and

$$P_i = \Gamma \left( 1 - \frac{\varepsilon - 1}{\theta} \right)^{\frac{1}{1-\varepsilon}} \left( \sum_k \lambda_k \left( \sum_j \left[ w_j \kappa_{ij} \delta_{jk} \right]^{-\frac{\theta}{1-\rho}} \right)^{1-\rho} \right)^{-\frac{1}{\theta}}.$$
The set of wages that clear each country’s labor market solve the set of equations

\[ w_j L_j = \sum_i w_i L_i \pi_{ij}. \]

If individuals learn from sellers, the evolution of country \( i \)'s stock of knowledge can be summarized by the following differential equation:

\[ \dot{\lambda}_i = \frac{\Gamma (1 - \beta - \frac{\varepsilon - 1}{\theta})}{\Gamma (1 - \frac{\varepsilon - 1}{\theta})} \alpha \sum_j \sum_k \pi_{ijk} \left( \frac{\lambda_k}{\pi_{ijk}^{1-\rho} [\sum_l \pi_{ilk}]^\rho} \right)^\beta. \]

If individuals learn from domestic producers, country \( i \)'s stock of knowledge evolves as

\[ \dot{\lambda}_i = \frac{\Gamma (1 - \beta - \frac{\varepsilon - 1}{\theta})}{\Gamma (1 - \frac{\varepsilon - 1}{\theta})} \alpha \sum_j \sum_k \pi_{jik} \left( \frac{\lambda_k}{\pi_{jik}^{1-\rho} [\sum_l \pi_{jlk}]^\rho} \right)^\beta. \]

We parameterize learning so that it is a convex combination of these two processes, letting the importance of learning from sellers be \( \psi \). Thus the evolution of country \( i \)'s stock of knowledge can be summarized by the following differential equation:

\[ \dot{\lambda}_i = \frac{\Gamma (1 - \beta - \frac{\varepsilon - 1}{\theta})}{\Gamma (1 - \frac{\varepsilon - 1}{\theta})} \alpha \left\{ \psi \sum_j \sum_k \pi_{ijk} \left( \frac{\lambda_k}{\pi_{ijk}^{1-\rho} [\sum_l \pi_{ilk}]^\rho} \right)^\beta \right. \\
\left. + (1 - \psi) \sum_j \sum_k \pi_{jik} \left( \frac{\lambda_k}{\pi_{jik}^{1-\rho} [\sum_l \pi_{jlk}]^\rho} \right)^\beta \right\}. \]

There are a number of ways that production managers in \( i \) can get insights from production managers in \( k \). When learning from sellers is important, a manager in \( i \) can draw insight from multinationals based in \( k \) from goods produced in any location. When learning from producers is important, a manager in \( i \) can learn from multinationals based in \( k \) that produce goods in \( i \) to export to \( j \).

When a multinational’s productivities are uncorrelated across production locations \( (\rho = 0) \), the logic of selection of ideas is exactly the same as the case without FDI: holding
fixed $k$’s stock of knowledge, for each combination of multinational’s home and production location, the smaller $i$’s expenditure, the more likely it is that insights are drawn from higher productivity goods. When a multinational’s ideas are correlated across locations ($\rho > 0$), the logic is similar.

Whether trade and FDI are complements or substitutes in learning and production depends on the correlation of multinationals’ productivities across locations. This can be seen most easily in the case of symmetric economies. Let $y(\kappa, \delta)$ be real income when trade costs between any pair of countries is $\kappa$ and FDI costs between any pair of countries is $\delta$. In this setting, real income relative to costless trade can be summarized concisely for two polar cases:

$$\lim_{\rho \to 0} \frac{y(\kappa, \delta)}{y(1, 1)} = \left[ \left( \frac{1 + (n - 1) \kappa^{-(1-\beta)}}{n} \right) \left( \frac{1 + (n - 1) \delta^{-\theta(1-\beta)}}{n} \right) \right]^\frac{1}{\beta(1-\beta)}$$

and

$$\lim_{\rho \to 1} \frac{y(\kappa, \delta)}{y(1, 1)} = \max \left\{ \left( \frac{1 + (n - 1) \kappa^{-\theta(1-\beta)}}{n} \right), \left( \frac{1 + (n - 1) \delta^{-\theta(1-\beta)}}{n} \right) \right\}^\frac{1}{\beta(1-\beta)}$$

When multinationals’ productivities are uncorrelated across locations ($\rho \to 0$), lower worldwide trade costs and lower worldwide FDI costs are complementary. In contrast, when each multinational has the same productivity in every location, trade and FDI are perfect substitutes. Consider a multinational that has the highest productivity for its variety. If trade costs are lower than FDI costs, it will produce in its home country and export. If FDI costs are lower than trade costs, it will produce in each destination country.

For the case of a symmetric world, the gains from openness (trade and FDI) as measured by the per-capita income on the balanced growth path with costless trade and FDI relative to autarky can be characterized for an arbitrary correlation of multinationals’ productivities.
across locations:

\[
\frac{y^O}{y_{AUT}} = n^{\frac{1}{\theta}} \left( \frac{\lambda^{FT}}{\lambda^{AUT}} \right)^{\frac{1}{\theta}} = n^{\frac{2-\rho}{\theta}} \frac{(2-\rho)^{\beta}}{\theta}^{\frac{(1-\beta)^{\theta}}{\theta}} \]

Notice that both, the static and dynamic, gains from openness are larger than those in (13), provided \( \rho < 1 \).

### 3.1 Opening to Trade and FDI

Economic miracles are characterized by protracted growth in productivity and per-capita income, and are associated with increases in trade and FDI flows. Our theory features novel mechanisms through which trade and FDI liberalization could result in protracted growth in productivity and per-capita income. In this section we explore quantitatively the dynamic implications of our theory following trade and FDI liberalization. We also assess the relative importance of these flows for the diffusion of technologies.

We consider a world economy that starts with \( n-1 \) (relatively) open economies and one deviant economy that are on a balanced growth path. We calibrate the trade costs of the \( n-1 \) open economies so that their trade shares equal 0.50, \( \kappa_1 = 2.15 \). We set the FDI cost to be 40% higher than the trade costs, \( \delta_1 = 3 \), to be consistent with the estimates in Ramundo and Rodriguez-Clare (2013). Trade to and from, and operation in, the deviant economy face large trade and FDI costs, \( \kappa_n = \delta_n = 100 \). We trace the evolution of the stock of ideas and per-capita income as trade and FDI costs are eliminated, \( \kappa_n = \delta_n = 100 \rightarrow \kappa'_n = \delta'_n = 1 \). We also consider the cases in which only one of the costs are eliminated. In all these examples, we set \( \beta = 0.5, \rho = 0.5, \) and \( \psi = 0.1 \).\(^{15}\)

Figure 6 shows the dynamics of the stock of ideas (left panel) and per-capita income (right panel) following the reduction of trade and/or FDI cost with the deviant economy.

\(^{15}\)The rest of the parameters are calibrated as discussed in footnote 9.
The y-axis measures the variables relative to their values on a balanced growth path where all countries have the same, low trade and FDI costs, \( \kappa_1 = \kappa_n = 2.15 \) and \( \delta_1 = \delta_n = 3 \). The solid line shows the dynamics after trade and FDI costs with the deviant country are reduced, while the dashed and dotted lines correspond to the cases where only trade or FDI costs are reduced, respectively.

Following the opening to trade and FDI, the stock of ideas in the (formerly) deviant country undergoes a sustained period of growth, converging to that of the \( n - 1 \) open economies. This process is substantially slower when only trade costs are reduced, since in this example we consider the case where most of the insights are drawn from domestic producers, \( \psi = 0.1 \). If only trade costs are reduced, most of the improvements in the quality of insights come from the selection of better domestic producers, which are themselves relatively unproductive to start with. On the contrary, when FDI costs are reduced, the initial growth of the stock of knowledge is much faster, as more productive foreign multinational start producing in the (formerly) deviant country, resulting in a better distribution of insights. In the long run, since trade costs are reduced by more than FDI costs, the gains associated with a reduction in the former is larger, albeit these gains materialize at a slower pace.
In the right panel we present the dynamics of per-capita income following a the reduction in trade and/or FDI costs. On impact the gains are larger when FDI cost are reduced, as in this case the more advanced foreign technologies get to be used. This case attains most of the static gains from openness. Over time, the growth of per-capita income reflects the improvement in the technology shown in the left panel.

4 Conclusions

To be written.
A Arrival Rate of Ideas Grows, Pareto Tail

Here we don’t assume that the noise \( z \) follows a Pareto distribution, but rather we assume that the right tail is Pareto and that the meeting rate grows exponentially. We show that in the long run the distribution of productivities converges to a Frechet. We begin with the law of motion

\[
\frac{\partial}{\partial t} \ln F_t(q) = -\tilde{\alpha} \int_{0}^{\infty} \left[ 1 - H \left( \frac{q^{-\beta}}{x^{-\beta}} \right) \right] G_t'(x) \, dx
\]

The key assumptions are that the meeting rate grows exponentially:

\[
\tilde{\alpha} = \alpha e^{\gamma t}
\]

and that in the limit the idiosyncratic part of the draws has a power law right tail:

\[
\lim_{z \to \infty} \frac{1 - H(z)}{z^{-\Theta}} = \phi.
\]

Let \( \theta \equiv \frac{\Theta}{1 - \beta} \) and let \( \eta = \Theta \gamma \). Also, define \( g_t(q) = G_t(e^{\eta t} q) \) and \( f_t(q) = F_t(e^{\eta t} q) \). We want to express \( \frac{\partial}{\partial t} \ln F_t(q) \) in terms of \( f_t \). We can differentiate \( f_t(q) = F_t(e^{\eta t} q) \) with respect to \( t \) and \( q \) to get

\[
t : \quad \frac{\partial}{\partial t} f_t(q) = \frac{\partial}{\partial t} F_t(e^{\eta t} q) + \eta e^{\eta t} q F'_t(e^{\eta t} q)
\]

\[
q : \quad f'_t(q) = e^{\eta t} F'_t(e^{\eta t} q)
\]

\[
\Rightarrow \quad \frac{1}{F_t(e^{\eta t} q)} \frac{\partial}{\partial t} F_t(e^{\eta t} q) = \frac{\partial \ln f_t(q)}{\partial t} - \eta \frac{qf'_t(q)}{f_t(q)}
\]

We can also differentiate \( G_t(q) = g_t(e^{-\eta t} q) \) with respect to \( q \):

\[
dG_t(q) = e^{-\eta t} dg_t(e^{-\eta t} q)
\]
Now, back to the law of motion:

\[
\frac{\partial}{\partial t} \ln F_t(q) = -\dot{\alpha} \int_0^\infty \left[ 1 - H \left( q^{\frac{1}{\beta}} x^{-\frac{\beta}{\alpha}} \right) \right] dG_t(x)
\]

\[
= -\dot{\alpha} \int_0^\infty \left[ 1 - H \left( q^{\frac{1}{\beta}} x^{-\frac{\beta}{\alpha}} \right) \right] e^{-\eta t} dG_t(e^{-\eta t} x)
\]

\[
= -\dot{\alpha} \int_0^\infty \left[ 1 - H \left( q^{\frac{1}{\beta}} w^{-\frac{\beta}{\alpha}} e^{-\eta t} \right) \right] dG_t(w)
\]

Let \( \tilde{q} = q e^{-\eta t} \)

\[
\frac{\partial}{\partial t} \ln F_t(e^{\eta t} \tilde{q}) = -\dot{\alpha} \int_0^\infty \left[ 1 - H \left( \tilde{q}^{\frac{1}{\beta}} w^{-\frac{\beta}{\alpha}} e^{\eta t} \right) \right] dG_t(w)
\]

\[
\frac{\partial}{\partial t} \ln f_t(\tilde{q}) - \eta \frac{\tilde{q} f_t'(\tilde{q})}{f_t(\tilde{q})} = -\dot{\alpha} e^{-\Theta \eta t} \tilde{q}^{-\theta} \int_0^\infty \left[ 1 - H \left( \tilde{q}^{\frac{1}{\beta}} w^{-\frac{\beta}{\alpha}} e^{\eta t} \right) \right] \phi w^{\beta \theta} dG_t(w)
\]

\[
\frac{\partial}{\partial t} \ln f_t(\tilde{q}) - \eta \frac{\tilde{q} f_t'(\tilde{q})}{f_t(\tilde{q})} \to -\alpha \tilde{q}^{-\theta} \int_0^\infty \phi w^{\beta \theta} dG_t(w)
\]

On a balanced growth path, \( \frac{\partial ln f_t(q)}{\partial t} = 0 \), so that

\[
\frac{\partial}{\partial t} \ln f_t(\tilde{q}) \to \alpha \tilde{q}^{-\theta} \int_0^\infty \phi w^{\beta \theta} dG_t(w)
\]

\[
\eta \frac{\tilde{q} f_t'(\tilde{q})}{f_t(\tilde{q})} \to \alpha \phi \tilde{q}^{-\theta} \int_0^\infty w^{\beta \theta} dG_t(w)
\]

**B \hspace{1cm} Trade**

**B.1 General Equilibrium**

This section derives expressions for price indices, trade shares, and a set of equations that determine equilibrium wages. Total income and expenditure in \( i \) is \( w_i L_i \).

For a variety produced in \( j \) at productivity \( q \) that is exported to \( i \), the equilibrium price in \( i \) is \( p_i(s) = \frac{w_i \kappa_i}{q} \), the expenditure on consumption in \( i \) is \( \left( \frac{p_i(s)}{P_i} \right)^{1-\varepsilon} w_i L_i \); consumption is \( \frac{1}{p_i(s)} \left( \frac{p_i(s)}{P_i} \right)^{1-\varepsilon} w_i L_i \), and the labor used in \( j \) to produce variety \( s \) for \( i \) is \( \frac{1}{w_j} \left( \frac{p_i(s)}{P_i} \right)^{1-\varepsilon} w_i L_i \).
Define the measure $\mathcal{F}_{ij}$ so that

$$
\mathcal{F}_{ij}(q) \equiv \int_0^q \prod_{k \neq j} F_k \left( \frac{w_k \kappa_{ik}}{w_j \kappa_{ij}} x \right) dF_j(x)
$$

This is the fraction of varieties for which $j$ is the lowest cost provider to $i$ and for that can be produced in $j$ with productivity no greater than $q$. The price index and trade shares can be written in terms of this measure:

$$
\pi_{ij} = \int_0^\infty \frac{(w_j \kappa_{ij})^{1-\varepsilon} q^{\varepsilon-1}}{P_i^{1-\varepsilon}} d\mathcal{F}_{ij}(q)
$$

$$
P_i^{1-\varepsilon} \equiv \sum_j (w_j \kappa_{ij})^{1-\varepsilon} \int_0^\infty q^{\varepsilon-1} d\mathcal{F}_{ij}(q)
$$

We can use these to derive a labor clearing constraint for $j$. Total labor in $j$ used to produce goods for $i$ is

$$
\int_0^\infty \frac{1}{w_j} \frac{w_i L_i}{w_j \kappa_{ij}} \frac{(w_j \kappa_{ij})^{1-\varepsilon} q^{\varepsilon-1}}{P_i^{1-\varepsilon}} d\mathcal{F}_{ij}(q) = \frac{1}{w_j} \pi_{ij} w_i L_i
$$

Thus total labor is labor used to make goods for all destinations

$$
L_j = \sum_i \frac{1}{w_j} \pi_{ij} w_i L_i
$$

We begin with a lemma that will be useful in deriving several expressions.

**Lemma 1** With the functional form assumptions, for any $\tau \in [0,1)$,

$$
\int_0^\infty q^{\tau} d\mathcal{F}_{ij}(q) = \Gamma(1-\tau) \pi_{ij}^{1-\tau} \lambda_j^\tau
$$
In addition,

\[ \pi_{ij} = \frac{\lambda_j (w_{ji})^{-\theta}}{\sum_k \lambda_k (w_{ik})^{-\theta}} \]

\[ P_i = \Gamma \left( 1 - \frac{\varepsilon - 1}{\theta} \right)^{-\frac{1}{\varepsilon - 1}} \left\{ \sum_j \lambda_j (w_{ji})^{-\theta} \right\}^{-\frac{1}{\theta}} \]

**Proof.** Define \( D_{ij} = \frac{\lambda_j (w_{ji})^{-\theta}}{\sum_k \lambda_k (w_{ik})^{-\theta}} \).

\[
\int_{0}^{\infty} q^{\tau \theta} dF_{ij}(q) = \int_{0}^{\infty} q^{\tau \theta} \prod_{k \neq j} F_k \left( \frac{w_{ki}^{\varepsilon}}{w_{ij}^{\varepsilon}} q \right) dF_j(q)
\]

\[ = \int_{0}^{\infty} q^{\tau \theta} \prod_{k \neq j} \lambda_k \left( \frac{w_{ki}^{\varepsilon}}{w_{ij}^{\varepsilon}} q \right)^{-\theta} \lambda_j \theta q^{-\theta - 1} dq \]

\[ = \int_{0}^{\infty} q^{\tau \theta} e^{-D_{ij}^{\varepsilon - 1} \lambda_j q^{-\theta}} \lambda_j \theta q^{-\theta - 1} dq \]

\[ = D_{ij}^{1 - \tau} \lambda_j \int_{0}^{\infty} x^{-\tau} e^{-x} dx \]

\[ = D_{ij}^{1 - \tau} \lambda_j \Gamma(1 - \tau) \]

Using this in the expression for \( \pi_{ij} \) gives

\[
\pi_{ij} = \frac{(w_{ji})^{1 - \varepsilon} D_{ij}^{\varepsilon - 1} \lambda_j \Gamma(1 - \frac{\varepsilon - 1}{\theta})}{\sum_l (w_{il})^{1 - \varepsilon} D_{il}^{\varepsilon - 1} \lambda_l \Gamma(1 - \frac{\varepsilon - 1}{\theta})} = \frac{\left[ \lambda_j (w_{ji})^{-\theta} \right]^{\varepsilon - 1} D_{ij}^{\varepsilon - 1}}{\sum_l \left[ \lambda_l (w_{il})^{-\theta} \right]^{\varepsilon - 1} D_{il}^{\varepsilon - 1}} = \frac{D_{ij}}{\sum_l D_{il}} = D_{ij}
\]
Finally, the price index is

\[ P_i^{1-\varepsilon} = \sum_l (w_l \kappa_{il})^{1-\varepsilon} \int_0^\infty q^{\varepsilon-1} d\mathcal{F}_{il}(q) \]

\[ = \sum_l (w_l \kappa_{il})^{1-\varepsilon} D_{il}^{1-\frac{\varepsilon-1}{\theta}} \lambda_l^{\frac{\varepsilon-1}{\theta}} \Gamma \left( 1 - \frac{\varepsilon - 1}{\theta} \right) \]

\[ = \Gamma \left( 1 - \frac{\varepsilon - 1}{\theta} \right) \left\{ \frac{\sum_l (w_l \kappa_{il})^{-\theta} \lambda_l}{\sum_k \lambda_k (w_k \kappa_{ik})^{-\theta}} \right\}^{\frac{\varepsilon-1}{\theta}} \]

\[ = \Gamma \left( 1 - \frac{\varepsilon - 1}{\theta} \right) \left\{ \sum_l (w_l \kappa_{il})^{-\theta} \lambda_l \right\}^{\frac{\varepsilon-1}{\theta}} \]

\[ B.2 \quad \text{Learning from Sellers} \]

Here we characterize the learning process when insights are drawn from sellers in proportion to the expenditure on each seller’s good. Consider a variety that can be produced in \( j \) at productivity \( q \). If \( i \) actually imports it, the fraction of \( i \)'s expenditure is \( \left( \frac{w_j \kappa_{ij}}{P_i} \right)^{1-\varepsilon} \). We thus have

\[ G_i^S(q) = \sum_j \left( \frac{w_j \kappa_{ij}}{P_i} \right)^{1-\varepsilon} q^{\varepsilon-1} d\mathcal{F}_{ij}(q) \]

We therefore have that

\[ \int_0^\infty q^{\beta \theta} dG_i^S(q) = \int_0^\infty q^{\beta \theta} \sum_j \left( \frac{w_j \kappa_{ij}}{P_i} \right)^{1-\varepsilon} q^{\varepsilon-1} d\mathcal{F}_{ij}(q) \]

\[ = \sum_j \left( \frac{w_j \kappa_{ij}}{P_i} \right)^{1-\varepsilon} \int_0^\infty q^{\beta \theta} q^{\varepsilon-1} d\mathcal{F}_{ij}(q) \]
Using the definition of $\pi_{ij}$ and Lemma (1), this becomes

$$
\int_{0}^{\infty} q^{\beta \theta} dG_i^S(q) = \sum_j \pi_{ij} \int_{0}^{\infty} q^{\beta \theta} q^{\varepsilon-1} dF_{ij}(q)
$$

$$
= \sum_j \pi_{ij} \int_{0}^{\infty} q^{\beta \theta} q^{\varepsilon-1} dF_{ij}(q)
$$

$$
= \sum_j \pi_{ij} \frac{\Gamma \left( 1 - \beta - \frac{\varepsilon - 1}{\theta} \right) \pi_{ij}^{1-\beta-\frac{\varepsilon-1}{\theta}} \lambda_j^{\frac{\varepsilon - 1}{\theta}}}{\Gamma \left( 1 - \frac{\varepsilon - 1}{\theta} \right) \pi_{ij}^{1-\frac{\varepsilon - 1}{\theta}} \lambda_j^{\frac{\varepsilon - 1}{\theta}}}
$$

$$
= \frac{\Gamma \left( 1 - \beta - \frac{\varepsilon - 1}{\theta} \right)}{\Gamma \left( 1 - \frac{\varepsilon - 1}{\theta} \right)} \sum_j \pi_{ij}^{1-\beta} \lambda_j^{\beta}
$$

### B.2.1 Alternative Weights of Sellers

Here we explore two alternative processes by which individuals can learn from sellers. In the first case, individuals are equally likely to learn from all active sellers, independently of how much of the sellers’ variety they consume. In the second case, insights are drawn from sellers in proportion to consumption of each sellers’ goods. In each case, the speed of learning is the same as our baseline (equation (17)) up to a constant.

#### Learning from All Active Sellers Equally

The measure of firms in $j$ with productivity $q$ that export to $i$ is $F_{ij}(q)$. The source distribution therefore satisfies $G_i^S(q) = \sum_j F_{ij}(q)$. Using Lemma (1), we have

$$
\int_{0}^{\infty} q^{\beta \theta} dG_i^S(q) = \sum_{j=1}^{n} \int_{0}^{\infty} q^{\beta \theta} dF_{ij}(q) = \Gamma(1 - \beta) \sum_j \pi_{ij}^{1-\beta} \lambda_j^{\beta}
$$

#### Learning from Sellers in Proportion to Consumption

Consider a variety that can be produced in $j$ at productivity $q$. If $i$ actually imports the variety from $j$, its consumption of that variety is $\left( \frac{(w_j \kappa_{ij})^{-\varepsilon}}{\bar{p}_i} \right)^{\varepsilon} w_i L_i$. Thus if insights are drawn in proportion to consumption, then

$$
G_i^S(q) = \sum_j \int_{0}^{\infty} \frac{(w_j \kappa_{ij})^{-\varepsilon}}{\bar{p}_i} x^{\varepsilon} dF_{ij}(x)
$$
We therefore have that

\[
\int_0^\infty q^{3\theta} \, dG^S_i(q) = \frac{\int_0^\infty q^{3\theta} \sum_j (w_j \kappa_{ij})^{-\varepsilon} q^\varepsilon \, dF_{ij}(q)}{\sum_j \int_0^\infty (w_j \kappa_{ij})^{-\varepsilon} q^\varepsilon \, dF_{ij}(q)} = \frac{\sum_j (w_j \kappa_{ij})^{-\varepsilon} \int_0^\infty q^{3\theta+\varepsilon} \, dF_{ij}(q)}{\sum_j (w_j \kappa_{ij})^{-\varepsilon} \int_0^\infty q^\varepsilon \, dF_{ij}(q)}
\]

Using Lemma (1), we have

\[
\int_0^\infty q^{3\theta} \, dG^S_i(q) = \frac{\sum_j (w_j \kappa_{ij})^{-\varepsilon} \Gamma \left( 1 - \beta - \frac{\varepsilon}{\theta} \right) \pi_{ij}^{\frac{1}{\theta}} \lambda_j^{\beta + \frac{\varepsilon}{\theta}}}{\sum_j (w_j \kappa_{ij})^{-\varepsilon} \Gamma \left( 1 - \frac{\varepsilon}{\theta} \right) \pi_{ij}^{\frac{1}{\theta}} \lambda_j^{\frac{\varepsilon}{\theta}}}
\]

Finally, using \(\pi_{ij} = \frac{\lambda_j (w_j \kappa_{ij})^{-\theta}}{\sum_{k=1}^n \lambda_k (w_k \kappa_{ik})^{-\theta}}\), we can write this as

\[
\int_0^\infty q^{3\theta} \, dG^S_i(q) = \frac{\Gamma \left( 1 - \beta - \frac{\varepsilon}{\theta} \right)}{\Gamma \left( 1 - \frac{\varepsilon}{\theta} \right)} \sum_j \pi_{ij}^{\frac{1}{\theta}} \lambda_j^{\beta}
\]

### B.3 Learning from Producers

Here we characterize the learning process when insights are drawn from domestic producers in proportion to labor used in production. Consider a variety that can be produced in \(i\) at productivity \(q\). If it exports to \(j\), the producer will use \(\frac{1}{w_i} \left( \frac{w_j \kappa_{ij}}{P_j} \right)^{1-\varepsilon} q^{\varepsilon-1} w_j L_j\) units of labor, or, using \(\left( \frac{w_j \kappa_{ij}}{P_j} \right)^{-\theta} = \Gamma \left( 1 - \frac{\varepsilon-1}{\theta} \right)^{-\frac{\theta}{\varepsilon-1}} \pi_{ji} \frac{w_j L_j (\pi_{ji}/\lambda_i)^{\frac{\varepsilon-1}{\theta}}}{\frac{w_i}{\Gamma(1-\frac{\varepsilon-1}{\theta})}} \right) q^{\varepsilon-1}\) units of labor. The source distribution for \(i\) is thus

\[
G^P_i(q) = \int_0^q = \frac{1}{L_i} \sum_j w_j L_j \left( \frac{\pi_{ji}/\lambda_i}{\lambda_j} \right)^{\frac{\varepsilon-1}{\theta}} \frac{w_i}{\Gamma \left( 1 - \frac{\varepsilon-1}{\theta} \right)} x^{\varepsilon-1} \, dF_{ji}(x)
\]
We can thus use Lemma (1) to write

\[ \int_0^\infty q^{\beta \theta} dG_i^P(q) = \frac{1}{L_i} \sum_j \frac{w_j L_j}{w_i} \frac{\pi_{ji}(q)}{\lambda_i} \frac{\epsilon^{\frac{1}{\theta}}}{\Gamma(1 - \frac{1}{\theta})} \int_0^\infty q^{\beta \theta} q^{\epsilon - 1} dF_{ji}(q) \]

\[ = \frac{1}{w_i L_i} \sum_j w_j L_j \frac{\pi_{ji}(q)}{\lambda_i} \frac{\epsilon^{\frac{1}{\theta}}}{\Gamma(1 - \frac{1}{\theta})} \Gamma \left( 1 - \beta - \frac{\epsilon - 1}{\theta} \right) \pi_{ji}^{1 - \beta} \frac{\epsilon^{\frac{1}{\theta}}}{\lambda_i^{\beta} + \epsilon^{\frac{1}{\theta}}} \]

\[ = \frac{\Gamma \left( 1 - \beta - \frac{\epsilon - 1}{\theta} \right)}{\Gamma(1 - \frac{1}{\theta})} \frac{1}{w_i L_i} \sum_j w_j L_j \pi_{ji}^{1 - \beta} \lambda_i^{\beta} \]

Using \( r_{ji} = \frac{w_j L_j \pi_{ji}}{w_i L_i} \), this is

\[ \int_0^\infty q^{\beta \theta} dG_i^P(q) = \frac{\Gamma \left( 1 - \beta - \frac{\epsilon - 1}{\theta} \right)}{\Gamma(1 - \frac{1}{\theta})} \sum_j r_{ji} \left( \frac{\lambda_i}{\pi_{ji}} \right)^{\beta} \]

### B.3.1 Alternative Weights of Producers

Here we briefly describe the alternative learning process in which insights are equally likely to be dawn from all active domestic producers. We consider only the case in which trade costs satisfy the triangle inequality \( \kappa_{jk} < \kappa_{ji} \kappa_{ik}, \forall i, j, k \) such that \( i \neq j \neq k \neq i \). We will show that, in this case, all producers that export also sell domestically. This greatly simplifies characterizing the learning process.

Towards a contradiction, suppose there is a variety \( s \) such that \( i \) exports to \( j \) and \( k \) exports to \( i \). This means that \( \frac{w_j \kappa_{ji}}{q_i(s)} \leq \frac{w_k \kappa_{ik}}{q_k(s)} \) and \( \frac{w_k \kappa_{ik}}{q_k(s)} \leq \frac{w_i \kappa_{ii}}{q_i(s)} \). Since \( \kappa_{ii} = 1 \), these imply that \( \kappa_{ji} \kappa_{ik} \leq \kappa_{jk} \), a violation of the triangle inequality and thus a contradiction.

In this case, the measure of varieties with productivity \( q \) that are actually produced and sold domestically is \( dF_{ii}(q) \). Thus

\[ \int_0^\infty q^{\beta \theta} dG_i^P(q) = \frac{\int_0^\infty q^{\beta \theta} dF_{ii}(q)}{\int_0^\infty dF_{ii}(q)} \]
Using Lemma (1), this becomes

\[ \int_0^\infty q^{\beta \theta} dG_i^P(q) = \Gamma(1 - \beta) \left( \frac{\lambda_i}{\pi_{ii}} \right)^\beta \]

**B.4 Symmetric Countries**

If countries are symmetric, there are two possible values of \( \pi_{ij} \):

\[
\begin{align*}
\pi_{ii} &= \frac{1}{1 + (n - 1)\kappa^{-\theta}} \\
\pi_{ij} &= \frac{\kappa^{-\theta}}{1 + (n - 1)\kappa^{-\theta}}, \quad i \neq j
\end{align*}
\]

Normalizing the wage to unity, the price level is

\[ P = \Gamma \left( 1 - \frac{\varepsilon - 1}{\theta} \right) \lambda^{-\frac{1}{\theta}} \left( 1 + (n - 1)\kappa^{-\theta} \right)^{-\frac{1}{\theta}} \]

The de-trended scale parameter on a balance growth path is

\[ \hat{\lambda}(\kappa) = \left[ (1 - \beta)\frac{\alpha}{\gamma} \frac{\Gamma(1 - \beta - \frac{\varepsilon - 1}{\theta})}{\Gamma(1 - \frac{\varepsilon - 1}{\theta})} \Gamma \left( \frac{1 - \beta}{\gamma} \frac{\Gamma(1 - \beta - \frac{\varepsilon - 1}{\theta})}{\Gamma(1 - \frac{\varepsilon - 1}{\theta})} \right) \right]^\frac{1}{1 - \beta} \]

Using this expression, per-capita income, \( y_i = w_i / P_i \), is

\[ y(\kappa) = \Gamma \left( 1 - \frac{\varepsilon - 1}{\theta} \right) \lambda^{\frac{1}{\theta}} \left[ (1 - \beta)\frac{\alpha}{\gamma} \frac{\Gamma(1 - \beta - \frac{\varepsilon - 1}{\theta})}{\Gamma(1 - \frac{\varepsilon - 1}{\theta})} \right]^\frac{1}{1 - \beta} \left( 1 + (n - 1)\kappa^{\theta(1 - \beta)} \right)^{\frac{1}{1 - \beta}} \]

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The de-trended stock of knowledge and per-capita income relative to costless trade are

\[ \hat{\lambda}(\kappa) = \left[ 1 + (n - 1)\kappa^{-\theta(1-\beta)} \right] \frac{1}{n} n^{-\frac{\rho}{1-\beta}} \]

\[ \hat{y}(\kappa) = \left( \frac{1 + (n - 1)\kappa^{-\theta}}{n} \right)^{\frac{1}{\beta}} \left( \frac{\hat{\lambda}(\kappa)}{\hat{\lambda}(1)} \right)^{\frac{1}{\beta}} \]

In particular, per-capita income in autarky relative to the case with costless trade

\[ \frac{y(\infty)}{y(1)} = n^{-\frac{1}{\beta}} \frac{n^{-\frac{\rho}{1-\beta}}}{\text{static}} \frac{n^{-\frac{\rho}{1-\beta}}}{\text{dynamic}} \]

C    Multinationals

C.1 Learning and the Source Distribution

The law of motion for the distribution of ideas in \( i \) is

\[ \frac{d}{dt} \ln F_{it}(\{q_1, ..., q_n\}) = -\hat{\alpha} \int_0^{\infty} \left[ 1 - H_i \left( \left\{ \frac{q_1^{1-\beta}}{x^{1-\beta}}, ..., \frac{q_n^{1-\beta}}{x^{1-\beta}} \right\} \right) \right] dG_{it}(x) \]

With the functional form \( 1 - H \{z_1, ..., z_n\} = \min \left\{ \left( \sum_j \left( \frac{z_j}{z_0} \right)^{-\frac{\theta}{1-\rho}} \right)^{1-\rho} \right\} \) and defining \( \alpha \equiv \hat{\alpha}z_0^\Theta \) to be a normalizing constant, this becomes

\[ \frac{d}{dt} \ln F_{it}(\{q_1, ..., q_n\}) = -\alpha \int_0^{\infty} \min \left\{ \left( \sum_j \left( \frac{q_j}{x^{1-\beta}} \right)^{-\frac{\theta}{1-\rho}} \right)^{1-\rho}, z_0^{-\Theta} \right\} dG_{it}(x) \]

Consider a sequence of economies with \( z_0 \to 0 \) (holding fixed \( \alpha \)). In the limit, the law of motion is

\[ \frac{d}{dt} \ln F_{it}(\{q_1, ..., q_n\}) = -\alpha \int_0^{\infty} \left( \sum_j \left( \frac{q_j}{x^{1-\beta}} \right)^{-\frac{\theta}{1-\rho}} \right)^{1-\rho} dG_{it}(x) \]
Simplifying, this becomes

$$\frac{d}{dt} \ln F_{it}(\{q_1, \ldots, q_n\}) = -\alpha \left( \sum_j q_j^{\frac{\theta}{1-\rho}} \right)^{1-\rho} \int_0^\infty x^{\beta\theta} dG_{it}(x)$$

C.2 General Equilibrium

We define a measure $F_{ijk}$ so that $dF_{ijk}(q)$ is the measure of varieties whose home country is $k$ that can be produced in $j$ with pure productivity $q$ (or effective productivity $\delta^{-1}_{jk}q$) and are actually exported to $i$.

$$F_{ijk}(q) = \int_0^q \prod_{l \neq k} F_l \left( \frac{w_1 \kappa_{i1} \delta_{1l}}{w_j \kappa_{ij} \delta_{jk}}, \ldots, \frac{w_n \kappa_{in} \delta_{nl}}{w_j \kappa_{ij} \delta_{jk}} \right) x dx, \quad \frac{w_j \kappa_{ij} \delta_{jk}}{w_j \kappa_{ij} \delta_{jk}}$$

With this, we can begin to derive expressions for the price indices and trade shares. The price in $i$ of variety $s$ if it is imported from $j$ and produced in $k$ is $p_i(s) = \frac{w_j \kappa_{ij} \delta_{jk}}{q_{jk}(s)}$. The price level is thus

$$P_{i}^{1-\varepsilon} = \int_0^1 p_i(s)^{1-\varepsilon} ds = \sum_k \sum_j (w_j \kappa_{ij} \delta_{jk})^{1-\varepsilon} \int_0^\infty q^{-1} dF_{ijk}(q)$$

Let $\pi_{ijk}$ be the share of $i$’s expenditure on imports from $j$ that are produced by firms based in $k$, so that $\pi_{ij} = \sum_k \pi_{ijk}$. To compute this, first note that the fraction of $i$’s expenditure on good imported from $j$ from a multinational based in $j$ is $\left( \frac{w_j \kappa_{ij} \delta_{jk}}{q_{jk}(s)} \right)^{1-\varepsilon}$. $\pi_{ijk}$ equals

$$\pi_{ijk} = \int_0^\infty \left( \frac{w_j \kappa_{ij} \delta_{jk}}{q_{jk}(s)} \right)^{1-\varepsilon} q^{-1} dF_{ijk}(q)$$

The following lemma will be used repeatedly:
Lemma 2 With the functional form assumptions, for any \( \tau \in [0,1) \) and \( \rho \in [0,1) \),

\[
\int_0^\infty q^\tau dF_{ijk}(q) = \Gamma(1-\tau)\pi_{ijk} \left( \frac{\lambda_k}{\pi_{ijk}^{1-\rho} \left( \sum_l \pi_{ilk} \right)^\rho} \right)^\tau
\]

\[
\pi_{ijk} = \frac{\lambda_k \left( \sum_m \left[ w_m \kappa_{im} \delta_{mk} \right]^{-\frac{\rho}{1-\rho}} \left[ w_j \kappa_{ij} \delta_{jk} \right]^{-\frac{\theta}{1-\rho}} \right)}{\sum_l \lambda_l \left( \sum_m \left[ w_m \kappa_{im} \delta_{ml} \right]^{-\frac{\rho}{1-\rho}} \right)^{1-\rho}}
\]

\[
P_i = \Gamma \left( 1 - \frac{\varepsilon - 1}{\theta} \right)^{\frac{1}{1-\rho}} \left\{ \sum_k \lambda_k \left( \sum_j \left[ w_j \kappa_{ij} \delta_{jk} \right]^{-\frac{\theta}{1-\rho}} \right) \right\}^{1-\rho} \frac{1}{\theta}
\]

\[
\left[ \frac{w_j \kappa_{ij} \delta_{jk}}{P_i} \right]^{-\varepsilon} = \frac{1}{\Gamma \left( 1 - \frac{\varepsilon - 1}{\theta} \right)^{\frac{1}{1-\rho}} \left( \frac{\lambda_k}{\pi_{ijk}^{1-\rho} \left( \sum_l \pi_{ilk} \right)^\rho} \right)^{\frac{1-\varepsilon}{\theta}}}
\]

Proof. We begin by defining the variables

\[
\varphi_{ik} = \lambda_k \left( \sum_m \left[ w_m \kappa_{im} \delta_{mk} \right]^{-\frac{\rho}{1-\rho}} \right)^{1-\rho}
\]

\[
\varphi_i = \sum_k \varphi_{ik}
\]

\[
\chi_{ijk} = \frac{\left[ w_j \kappa_{ij} \delta_{jk} \right]^{-\frac{\theta}{1-\rho}}}{\sum_m \left[ w_m \kappa_{im} \delta_{mk} \right]^{-\frac{\theta}{1-\rho}}}
\]

Using \( F_l(\{q_1, \ldots, q_n\}) = e^{-\lambda_l \left( \sum_m q_m^{-\frac{\theta}{1-\rho}} \right)^{1-\rho}} \), we have

\[
F_l \left( \frac{w_1 \kappa_{i1} \delta_{1l}}{w_j \kappa_{ij} \delta_{jk}} q, \ldots, \frac{w_n \kappa_{in} \delta_{nl}}{w_j \kappa_{ij} \delta_{jk}} q \right) = e^{-\lambda_l \left( \sum_m \left( \frac{w_m \kappa_{im} \delta_{ml}}{w_j \kappa_{ij} \delta_{jk}} \right)^{-\frac{\theta}{1-\rho}} \right)^{1-\rho}} = e^{-\varphi_{il} \left( w_j \kappa_{ij} \delta_{jk} \right)^{\frac{\theta}{1-\rho}} q^{-\theta}}
\]
While the partial derivative is

$$F_k \left( \frac{w_1 \kappa_1 \delta_{1k}}{w_j \kappa_j \delta_{jk}} q, \ldots, \frac{w_j \kappa_j \delta_{jk}}{w_j \kappa_j \delta_{jk}} q, \frac{w_{j+1} \kappa_{j+1} \delta_{j+1k}}{w_j \kappa_j \delta_{jk}} q, \ldots, \frac{w_I \kappa_I \delta_{Ik}}{w_j \kappa_j \delta_{jk}} q \right) = e^{-\lambda_k \left( \sum_{n=1}^{l} \left( \frac{w_n \kappa_n \delta_{nk}}{w_j \kappa_j \delta_{jk}} q \right) \right)^{\frac{\theta}{1-\rho}}} \lambda_k \left( \sum_{n=1}^{l} \left( \frac{w_n \kappa_n \delta_{nk}}{w_j \kappa_j \delta_{jk}} q \right) \right)^{\frac{1-\rho}{1-\rho}} \theta q^{\frac{\theta}{1-\rho}-1} dq$$

Together, these imply that the measure can be written as

$$d \mathcal{F}_{ijk}(q) = e^{-\varphi_{ik} (w_j \kappa_{ij} \delta_{jk})^{\theta}} \varphi_{ik} \left( \frac{\varphi_{ik} \chi_{ijk}}{\varphi_i} \right) \left( \varphi_i (w_j \kappa_{ij} \delta_{jk})^{\theta} \right) \theta q^{\theta-1} dq$$

We can then integrate to get

$$\int_0^{\infty} q^{\tau \theta} d \mathcal{F}_{ijk}(q) = \int_0^{\infty} q^{\tau \theta} e^{-\varphi_{ik} (w_j \kappa_{ij} \delta_{jk})^{\theta}} \varphi_{ik} \chi_{ijk} \left( w_j \kappa_{ij} \delta_{jk} \right)^{\theta} \theta q^{\theta-1} dq$$

$$= \frac{\varphi_{ik} \chi_{ijk}}{\varphi_i} \left( \varphi_i (w_j \kappa_{ij} \delta_{jk})^{\theta} \right) \int_0^{\infty} x^{-\tau} e^{-x} dx = \frac{\varphi_{ik} \chi_{ijk}}{\varphi_i} \left( \varphi_i (w_j \kappa_{ij} \delta_{jk})^{\theta} \right) \Gamma (1 - \tau)$$ (17)

The price index is then

$$P_{i}^{1-\varepsilon} = \sum_k \sum_j (w_j \kappa_{ij} \delta_{jk})^{1-\varepsilon} \int_0^{\infty} q^{\varepsilon-1} d \mathcal{F}_{ijk}(q)$$

$$= \sum_k \sum_j (w_j \kappa_{ij} \delta_{jk})^{1-\varepsilon} \frac{\varphi_{ik} \chi_{ijk}}{\varphi_i} \left( \varphi_i (w_j \kappa_{ij} \delta_{jk})^{\theta} \right)^{\frac{\varepsilon-1}{\theta}} \Gamma \left( 1 - \frac{\varepsilon - 1}{\theta} \right)$$

$$= \frac{\varphi_{ik} \chi_{ijk}}{\varphi_i} \Gamma \left( 1 - \frac{\varepsilon - 1}{\theta} \right) \sum_k \sum_j \frac{\varphi_{ik} \chi_{ijk}}{\varphi_i}$$

$$= \frac{\varphi_{ik} \chi_{ijk}}{\varphi_i} \Gamma \left( 1 - \frac{\varepsilon - 1}{\theta} \right)$$ (18)
We now compute the trade-FDI share,

\[
\pi_{ijk} = \left( \frac{w_j \kappa_{ij} \delta_{jk}}{P_i} \right)^{1-\epsilon} \int_0^\infty q^{\epsilon-1} dF_{ijk}(q)
\]

\[
= \left( \frac{w_j \kappa_{ij} \delta_{jk}}{P_i} \right)^{1-\epsilon} \frac{\varphi_{ik} \chi_{ijk}}{\varphi_i} \left[ \varphi_i \left( w_j \kappa_{ij} \delta_{jk} \right)^\theta \right]^{\frac{\epsilon-1}{\theta}} \Gamma \left( 1 - \frac{\epsilon-1}{\theta} \right)
\]

\[
= \Gamma \left( 1 - \frac{\epsilon-1}{\theta} \right) \varphi_{ik} \chi_{ijk} \frac{\varphi_{ik}^{\frac{\epsilon-1}{\theta}}}{\varphi_i^{1-\epsilon}}
\]

\[
= \frac{\varphi_{ik} \chi_{ijk}}{\varphi_i}
\]

(19)

The expressions for \( P_i \) and \( \pi_{ijk} \) come simply from plugging in the expressions for \( \varphi_i \), \( \varphi_{ik} \), and \( \chi_{ijk} \) into equations (18) and (19). Finally, can use equation (19) to write

\[
\pi_{ijk}^{1-\rho} \left[ \sum_l \pi_{ilk} \right]^\rho = \left( \frac{\varphi_{ik} \chi_{ijk}}{\varphi_i} \right)^{1-\rho} \left( \sum_l \frac{\varphi_{ik} \chi_{ilk}}{\varphi_i} \right)^\rho = \left( \frac{\varphi_{ik} \chi_{ijk}}{\varphi_i} \right)^{1-\rho} \left( \frac{\varphi_{ik}}{\varphi_i} \right)^\rho
\]

\[
= \frac{\varphi_{ik} \chi_{ijk}}{\varphi_i}
\]

\[
= \lambda_k \left( \sum_m \left[ w_m \kappa_{im} \delta_{mk} \right] \right)^{-\theta} \left( \sum_l \sum_m \left[ w_m \kappa_{im} \delta_{mk} \right] \right)^{1-\rho} \left( \frac{\varphi_i}{\varphi_{ik}} \right)^{1-\rho}
\]

(20)

Equation (17) can thus be written as

\[
\int_0^\infty q^{\tau-\theta} dF_{ijk}(q) = \frac{\varphi_{ik} \chi_{ijk}}{\varphi_i} \left[ \varphi_i \left( w_j \kappa_{ij} \delta_{jk} \right)^\theta \right]^{\tau} \Gamma \left( 1 - \tau \right)
\]

\[
= \Gamma(1 - \tau) \pi_{ijk} \left( \frac{\lambda_k}{\pi_{ijk}^{1-\rho} \left[ \sum_l \pi_{ilk} \right]^\rho} \right)^\tau
\]

Lastly, the expression for \( P_i \) can be combined with equation (20) to verify that

\[
\left( \frac{w_j \kappa_{ij} \delta_{jk}}{P_i} \right)^{1-\epsilon} \left( \frac{1}{\Gamma \left( 1 - \frac{\epsilon-1}{\theta} \right)} \right) \left( \frac{\lambda_k}{\pi_{ijk}^{1-\rho} \left[ \sum_l \pi_{ilk} \right]^\rho} \right)^{1-\epsilon}
\]
C.3 Learning from Sellers

Consider a variety that can be produced in \( j \) by a firm in \( k \) with pure productivity \( q \) (and thus effective productivity \( \delta_{jk}^{-1}q \)). If \( i \) actually imports the good from \( j \), the fraction of expenditure will be

\[
\left( \frac{w_j k_{ij} \delta_{jk}}{P_i} \right)^{1-\varepsilon} q^{\varepsilon-1} = \frac{1}{\Gamma(1-\frac{\varepsilon-1}{\theta})} \left( \frac{\lambda_k}{\pi_{ijk}^{1-\rho} \left[ \sum_l \pi_{ilk} \right]^{\rho}} \right)^{\frac{1-\varepsilon}{\theta}}
\]

The measure of such varieties that are actually imported is \( dF_{ijk}(q) \), so to find the flow of ideas, we sum over producer and home countries and integrate over productivities to get

\[
\int_0^\infty q^{\beta \theta} dG_i^S(q) = \int_0^\infty q^{\beta \theta} \sum_j \sum_k \frac{1}{\Gamma(1-\frac{\varepsilon-1}{\theta})} \left( \frac{\lambda_k}{\pi_{ijk}^{1-\rho} \left[ \sum_l \pi_{ilk} \right]^{\rho}} \right)^{\frac{1-\varepsilon}{\theta}} q^{\varepsilon-1} dF_{ijk}(q)
\]

\[
= \sum_j \sum_k \frac{1}{\Gamma(1-\frac{\varepsilon-1}{\theta})} \left( \frac{\lambda_k}{\pi_{ijk}^{1-\rho} \left[ \sum_l \pi_{ilk} \right]^{\rho}} \right)^{\frac{1-\varepsilon}{\theta}} \int_0^\infty q^{\beta \theta} q^{\varepsilon-1} dF_{ijk}(q)
\]

Using Lemma (2), this becomes

\[
\int_0^\infty q^{\beta \theta} dG_i^S(q) = \sum_j \sum_k \frac{\Gamma(1-\beta - \frac{\varepsilon-1}{\theta})}{\Gamma(1-\frac{\varepsilon-1}{\theta})} \left( \frac{\lambda_k}{\pi_{ijk}^{1-\rho} \left[ \sum_l \pi_{ilk} \right]^{\rho}} \right)^{\frac{1-\varepsilon}{\theta}} \pi_{ijk} \left( \frac{\lambda_k}{\pi_{ijk}^{1-\rho} \left[ \sum_l \pi_{ilk} \right]^{\rho}} \right)^{\beta + \frac{\varepsilon-1}{\theta}}
\]

\[
= \frac{\Gamma(1-\beta - \frac{\varepsilon-1}{\theta})}{\Gamma(1-\frac{\varepsilon-1}{\theta})} \sum_j \sum_k \pi_{ijk} \left( \frac{\lambda_k}{\pi_{ijk}^{1-\rho} \left[ \sum_l \pi_{ilk} \right]^{\rho}} \right)^{\beta}
\]

C.4 Learning from Producers

Consider a variety that would be produced in \( j \) by a firm based in \( k \) at pure productivity \( q \) (and hence effective productivity \( \delta_{jk}^{-1}q \)). If it is exported to \( i \) the fraction of \( j \)’s labor used is

\[
\frac{1}{w_j L_j} \frac{w_i L_i}{P_i} \left( \frac{w_j k_{ij} \delta_{jk}}{P_i} \right)^{1-\varepsilon} q^{\varepsilon-1} = \frac{1}{w_j L_j} \frac{1}{\Gamma(1-\frac{\varepsilon-1}{\theta})} \left( \frac{\lambda_k}{\pi_{ijk}^{1-\rho} \left[ \sum_l \pi_{ilk} \right]^{\rho}} \right)^{\frac{1+\varepsilon}{\theta}} q^{\varepsilon-1}
\]
To integrate over the source distribution, we sum over export destination and home countries and integrate over values of productivity:

\[
\int_0^\infty q^\beta dG_j^P(q) = \int_0^\infty q^\beta \sum_i \sum_k \frac{w_i L_i}{w_j L_j} \frac{1}{\Gamma (1 - \frac{\epsilon - 1}{\sigma})} \left( \frac{\lambda_k}{\pi_{ijk}^{1-\rho} \left[ \sum_l \pi_{ilk} \right]^\rho} \right)^{\frac{1-\rho}{\sigma}} q^{\frac{\epsilon}{\sigma}} dF_{ijk}(q)
\]

Using Lemma (2), this is

\[
\int_0^\infty q^\beta dG_j^P(q) = \sum_i \sum_k \frac{w_i L_i}{w_j L_j} \frac{1}{\Gamma (1 - \frac{\epsilon - 1}{\sigma})} \left( \frac{\lambda_k}{\pi_{ijk}^{1-\rho} \left[ \sum_l \pi_{ilk} \right]^\rho} \right)^{\frac{1-\rho}{\sigma}} \frac{\lambda_k}{\pi_{ijk}^{1-\rho} \left[ \sum_l \pi_{ilk} \right]^\rho}^{\beta + \frac{\epsilon - 1}{\sigma}}
\]

Defining \( r_{ijk} \equiv \frac{w_i L_i \pi_{ijk}}{w_j L_j} \) to be the fraction of \( j \)'s revenue accounted for sales to \( i \) by multinationals based in \( k \), we have

\[
\int_0^\infty q^\beta dG_j^P(q) = \frac{\Gamma (1 - \beta - \frac{\epsilon - 1}{\sigma})}{\Gamma (1 - \frac{\epsilon - 1}{\sigma})} \sum_i \sum_k r_{ijk} \left( \frac{\lambda_k}{\pi_{ijk}^{1-\rho} \left[ \sum_l \pi_{ilk} \right]^\rho} \right)^{\beta}
\]
C.5 Symmetric Countries

For symmetric countries with symmetric trade cost $\kappa$ and FDI cost $\delta$, the trade FDI shares can be written as

$$
\pi_{iii} = \frac{(1 + (n - 1)(\kappa \delta)^{-\frac{\theta}{1-\rho}})^{-\rho}}{(1 + (n - 1)(\kappa \delta)^{-\frac{\theta}{1-\rho}})^{1-\rho} + (n - 1)(\delta^{-\frac{\theta}{1-\rho}} + \kappa^{-\frac{\theta}{1-\rho}} + (n - 2)(\kappa \delta)^{-\frac{\theta}{1-\rho}})^{1-\rho}},
$$

$$
\pi_{ik} = \frac{(\delta^{-\frac{\theta}{1-\rho}} + \kappa^{-\frac{\theta}{1-\rho}} + (n - 2)(\kappa \delta)^{-\frac{\theta}{1-\rho}})^{-\rho}}{(1 + (n - 1)(\kappa \delta)^{-\frac{\theta}{1-\rho}})^{1-\rho} + (n - 1)(\delta^{-\frac{\theta}{1-\rho}} + \kappa^{-\frac{\theta}{1-\rho}} + (n - 2)(\kappa \delta)^{-\frac{\theta}{1-\rho}})^{1-\rho}},
$$

$$
\pi_{ijk} = \frac{(\delta^{-\frac{\theta}{1-\rho}} + \kappa^{-\frac{\theta}{1-\rho}} + (n - 2)(\kappa \delta)^{-\frac{\theta}{1-\rho}})^{-\rho}}{(1 + (n - 1)(\kappa \delta)^{-\frac{\theta}{1-\rho}})^{1-\rho} + (n - 1)(\delta^{-\frac{\theta}{1-\rho}} + \kappa^{-\frac{\theta}{1-\rho}} + (n - 2)(\kappa \delta)^{-\frac{\theta}{1-\rho}})^{1-\rho}},
$$

$$
\pi_{ij} = \frac{(1 + (n - 1)(\kappa \delta)^{-\frac{\theta}{1-\rho}})^{-\rho}}{(1 + (n - 1)(\kappa \delta)^{-\frac{\theta}{1-\rho}})^{1-\rho} + (n - 1)(\delta^{-\frac{\theta}{1-\rho}} + \kappa^{-\frac{\theta}{1-\rho}} + (n - 2)(\kappa \delta)^{-\frac{\theta}{1-\rho}})^{1-\rho}},
$$

The price index is

$$
P = \Gamma \left( 1 - \frac{\varepsilon - 1}{\theta} \right)^{\frac{1}{\theta}} \lambda^{-\frac{1}{\beta}} \left( 1 + (n - 1)(\kappa \delta)^{-\frac{\theta}{1-\rho}} \right)^{-\rho}
$$

$$
+ (n - 1) \left( \kappa^{-\frac{\theta}{1-\rho}} + \delta^{-\frac{\theta}{1-\rho}} + (n - 2)(\kappa \delta)^{-\frac{\theta}{1-\rho}} \right)^{1-\rho} \right)^{-\frac{1}{\beta}}
$$

D Technological Distance and Diffusion - Closed Economy Version

Let varieties be indexed by $s \in [0, 1]$. These can be partitioned into a finite number of sets, $\{S_n\}_{n=1}^N$. For a producer in of a variety in $S_n$, the arrival rate of a usable idea that is currently used by a variety in $S_m$ is $\tilde{\alpha}_{nm}$. Let $F_{nt}(q)$ be the fraction of type $n$ varieties with productivity no greater than $q$. The we have

$$
\frac{d}{dt} \ln F_{nt}(q) = - \sum_{m=1}^N \tilde{\alpha}_{nm} \int_0^\infty \left[ 1 - F_{mt} \left( \frac{q^{1/\beta}}{z^{(1-\beta)/\beta}} \right) \right] dH(z)
$$
This can be solved in general. But with symmetric initial conditions and the right parameter restrictions, this will map into the original model. Suppose that initial conditions such that

\[ F_{n0}(q) = e^{-\lambda_0 q^{-\theta}} \]

Suppose also that for each \( n \),

\[ \bar{\alpha} = \sum_{m=1}^{N} \bar{\alpha}_{nm} \]

It should be easy to show that for each \( t \), the distribution of productivities is the same:

\[ F_{nt}(q) = F_t(q) = e^{-\lambda_t q^{-\theta}} \]

\[ \frac{d}{dt} \ln F_t(q) = -\bar{\alpha} \int_{0}^{\infty} \left[ 1 - F_t \left( \frac{q^{1/\beta}}{z^{(1-\beta)/\beta}} \right) \right] dH(z) \]

References


