Financing Risk and Bubbles of Innovation*

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First Draft Sep 2009
Current Draft July 2010

Abstract

Investors in risky startups who stage their investments face *financing risk* - that is, the risk that later stage investors will not fund the startup, even if the fundamentals of the firm are still sound. We show that financing risk is part of a rational equilibrium where investors can flip from investing to not investing in certain sectors of the economy. We further demonstrate that financing risk has the greatest impact on firms with the most real option value. Hence, the mix of projects funded and type of investors who are active varies with the level of financing risk in the economy. We also highlight that some extremely novel technologies may in fact need ‘hot’ financial markets to get through the initial period of diffusion. Our work underscores that financial markets may play a much larger and under-studied role in creating and magnifying bubbles of innovation in the real economy.

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Introduction

Startup firms are a central part of the technological revolutions that lead to Schumpeter’s (1942) waves of creative destruction. For example, the introduction of motor cars, semiconductors and computers, the internet, biotechnology, and clean technology have all been associated with ‘startup’ firms. Since these waves are believed to be a fundamental driver of productivity growth in the economy (Aghion and Howitt (1992), King and Levine (1993)), entrepreneurs’ ability to get financing for these startups is central to the process of creative destruction and a key aspect of the innovation cycle in the real economy.

An inherent aspect of these young, innovative firms is that they have a high possibility of failure.\(^1\) A natural consequence of this uncertainty is that investors tend to stage their investments - i.e., provide limited capital to the firm in each round, and learn more about the firm’s potential before providing more financing (see Gompers (1995), Bergemann and Hege (2005), Bergemann et al. (2008)).

We show in this paper that the act of staging investments introduces a risk that finance theory has not previously considered: Investors with limited capital must forecast the probability that their investment may not find follow-on funding from other investors at its next stage, even if the fundamentals of the project at the next stage are sound.

We introduce what we call financing risk and explore its implications for the innovation cycle. We emphasize that financing risk is part of a rational equilibrium and all investors in our model will use an $\text{NPV} \geq 0$ investing rule. Even with this standard rule we show that spikes in investing activity are more common in the innovative sectors of the economy. These spikes in investment by financiers can both cause or magnify waves of innovation in the real economy by impacting new firm formation and the resources for R&D and patenting.

The intuition behind financing risk is relatively straightforward: when a project requires multiple periods of investment that are cumulatively more than any individual investor has (or is willing to allocate), current investors need to rely on future investors to continue funding the project in order to realize the value of their investment. We document that there is a class

\(^1\)For example, over 50% of venture backed startups are either liquidated or fail to receive follow-on funding, despite the extensive due diligence, help and support provided by the venture capitalists (Gompers and Lerner (2004)).
of projects where the extent to which a given investment is positive or negative NPV depends not only on the fundamentals of the project (that is, a high enough probability of realizing a good outcome), but also on the current investors’ beliefs that future investors will choose to participate in follow on funding for the project. For these projects, financing risk is the risk that future investors will not fund the project at its next stage even if the fundamentals of the project have not changed.

To any one investor, financing risk is exogenous; however, in equilibrium it becomes endogenous. Each investor becomes less willing to make an investment because they are worried that others won’t support the investment in the future. Thus, like in a bank run, if current investors believe that future investors will withdraw financing from such a project, they should also withdraw their investment, even though all investors would be better off in the equilibrium in which everyone invests. This is not an irrational decision and furthermore, does not depend on information asymmetries. There are simply two equilibria – one in which everyone invests in a sector and one in which no one does.

An important facet of this model is that each equilibrium is inherently unstable as it depends on the beliefs of others. Even when investors are in the ‘good’ financing equilibrium, investors realize that there is a potential to jump to the other equilibrium. In fact, financing risk is precisely the risk that the ‘good equilibrium’ switches after a given investor has funded a project but before returns can be realized. Investors thus estimate a transition probability that the state switches from the ‘good’ to the ‘bad’ financing equilibrium or vice versa.

Our paper is related to the literature exploring the link between financial markets and the real economy. The literature on venture capital has documented the extreme variation in venture capital investment (Gompers and Lerner (2004)) and fund-raising (Gompers and Lerner (1998)), that are correlated with high market values, hot IPO markets or past returns (Kaplan and Schoar (2005)). Furthermore, technological revolutions seem to be associated with ‘hot’ financial markets (Perez (2002)). Prior work has suggested that these correlations could be overreaction by investors (Gompers and Lerner (1998)), rational reactions to fundamentals (Gompers et al. (2008), Pastor and Veronesi (2009)), herd behavior for reputational concerns (Scharfstein and Stein (1990)) or even reverse causality (Hobijn and Jovanovic (2001)). Shleifer (1986) even
considers the idea that manager expectations about future economic variables may matter.\(^2\)

Our model provides a mechanism for the variation in venture capital markets, endogenizes ‘hot’ and ‘cold’ financial markets and links them to spikes of innovation in the real economy. While some of the implications are similar to past work accounting for financing risk in the innovation cycle provides several novel insights and implications that do not arise from previous models of innovation waves.

One important contribution of our work is that we don’t simply demonstrate the possibility that there are multiple funding equilibria, but we go further and endogenize the investors response to the possibility of financing risk. In a world where the equilibrium may jump, investors forecast the likelihood of the switch and provide more or less financing as insurance against the low funding equilibrium. For some firms investors can provide enough ‘insurance’ to eliminate financing risk altogether. However, this insurance is much more expensive for innovative firms where the value generated by providing limited funding and waiting to learn more is the greatest. Thus, investors face a tradeoff between insuring against financing risk and maximizing the real option value.

This leads directly to the first implication of our model - financing risk has the greatest impact on the most innovative projects in the economy, or ones that have the most real option value for investors. In fact, when financing risk is high, our model suggests that the mix of projects that get funded should shift towards less innovative ones and that conditional on being funded in a time of high financing risk, these less innovative projects should receive larger ex-ante investments (relative to their burn rate) as compared to times when financing risk is low.

We should note that it is still true in our model that on average, ‘better’ projects are funded during a ‘bad’ funding equilibrium. This occurs because only the very best projects can attract financing even in bad times and hence are positive NPV even in the low funding equilibrium. However, it also shows why fundamentally sound projects, particularly those with high real option value, can go unfunded in some periods but be funded in others.

The second implication of this model is that the mix of investors should change in periods of high financing risk, relative to periods of low financing risk. Early round investors of very

\(^2\)In Shleifer (1986) implementation of an innovation brings imitation that quickly destroys the value of the innovation. So managers forecast a ‘good’ time to bring out an innovation - during a boom. Managers share expectations and therefore bring out inventions at the same time and the innovations create the boom.
innovative projects are subject to a greater amount of financing risk as the chance that the state switches to a ‘bad’ equilibrium after they have invested but before a liquidity event is realized, is higher. Their investing activity should be particularly impacted by hot and cold financial markets. Our model also predicts that the mix of investors should shift towards smaller investors (with less capital to deploy) when financing risk is low, as the smaller and more frequent investments in periods of low financing risk are particularly well suited to smaller investors.

A third implication of accounting for financing risk is that any given investor should not rush to invest into all projects in a sector that is out of favor. Conventional wisdom (and most past work) suggests that when money leaves a sector it is a good time to invest, and when a lot of money enters it is just the time to leave. This intuition arises because the flood of money lowers the discipline of external finance and allows lower quality projects to get capital (Gompers and Lerner (2000); Nanda (2008)). However, accounting for financing risk makes it clear that investors cannot rush to invest into all projects in a sector that is out of favor. In particular, innovative projects have a low probability of receiving future funding and become NPV negative once financing risk is taken into account.

The fourth implication of our model is that some extremely novel but NPV positive technologies or projects may in fact need ‘hot’ financial markets to get through the initial period of diffusion, because otherwise the financing risk for them is too extreme. This provides a more positive interpretation to waves of financial activity and may also explain the historical link between the initial diffusion of many very novel technologies (e.g. canals, railways, telephones, motor cars, internet, clean technology) being associated with heated financial market activity (Perez (2002)). Related to this, our model also provides a non-behavioral explanation for why asset prices can fall precipitously after rising steadily for long periods, even when the fundamentals of a firm have not changed (Pastor and Veronesi (2009)). If a sector stays in the ‘good’ equilibrium longer than expected or if the expected probability of remaining in the ‘good’ equilibrium increases, then asset prices will rise and returns will be high, even if the fundamentals remain similar. When the ‘bad’ equilibrium eventually occurs, returns will be far lower than that predicted simply by looking at fundamentals since the low funding equilibrium implies a fall in NPV and hence asset prices, but no change in fundamentals.

Our model provides an explanation for bubbles of investing activity that does not need to
depend on mispriced assets. In ‘good times’, when financing risk is low, project NPVs and thus asset prices should be high, as investors impute low financing risk into higher prices. In bad times the opposite is true. In our model, this boom and bust activity does not imply an irrational asset pricing bubble. Rather, the boom bust cycle is the inevitable outcome of multiple potential equilibria. We suggest that what may look like (or be partially driven by) over or under reaction or even a reaction to changing fundamentals may instead be a jump from a high investing to a low investing equilibrium.

Our final implication relates to direct measures of innovation such as patenting that occur in great waves of activity (see Griliches (1990)). There are many explanations for why innovative output might cluster in certain periods of time even though we expect ideas to occur at random.\(^3\)

While these traditional explanations clearly have merit, combining our model of financing risk with the direct evidence on the link between financial market activity and innovation (Kortum and Lerner (2000), Mollica and Zingales (2007), Samila and Sorenson (2010)) suggest that financial markets may play a much larger and under-studied role in the creation and magnification of innovation waves in the real economy.

The remainder of the paper is organized as follows. Section I. outlines a simple model of investing and illuminates the existence of the two potential investing equilibria. Section II. expands the model to a general equilibrium and shows how accounting for the transition probabilities from one equilibrium to the other affects the funding strategy of investors. Section III. allows complete, state contingent contracts and commitment among investors in an attempt to overcome financing risk, and shows why in a world of incomplete contracts, it is the innovative projects in the economy that are most impacted by financing risk. Section IV. summarizes the key implications and extensions of our model and Section V. concludes.

I. A Model of Investment

The central goal of our model is to delineate the impact of financing risk on investment decisions. Financing risk is the risk that future investors will not fund a firm at its next stage even if the

\(^3\)The classical explanations focus on sudden breakthroughs that lead to a cascade of follow-on inventions (e.g. Schumpeter (1939); Kuznets (1940); Kleinknecht (1987); Stein (1997)) or on changes in sales and profitability (or potential profitability) that stimulate investment in R&D and then drive concentrated periods of innovation (e.g. Schmookler (1966)). See Stoneman (1979) for a discussion of the supply versus demand considerations.
fundamentals of the project have not changed, leading a viable firm with good fundamentals to go bankrupt. We emphasize that financing risk is part of a rational equilibrium and show why innovative projects are particularly susceptible to financing risk.

A. Setup

We model a single early stage project inside a broader economy. For simplicity, we equate this project with a firm. By early stage we aim to capture the idea that the firm does not have the cash flows to be self sufficient and hence requires outside investment to survive. A second aspect of early stage firms is that it is not yet clear that the project will ‘work’. That is, investment in an early stage firm may produce positive results, negative results or more research may be needed. Furthermore, even when the initial results are positive, more investment may be needed to get over the next hurdle. For example, a new biotech firm may do initial studies to determine how well a compound works in mice. Then, depending on the results, money may be spent to start primate trials, the project may be shut down, or more studies on mice may be needed.

We consider a firm that must get over two hurdles (the ‘initial’ hurdle, called hurdle A, and the ‘final’ hurdle, called hurdle B) in order to reach its potential expected payoff, V, which one can think of as an IPO. These hurdles could represent several rounds of technological uncertainty, or customer adoption risk, or scaling issues, etc. In reality, a project may need to get over many more than just two hurdles, but two will demonstrate the issues, and we will later discuss the impact of more hurdles. For simplicity, we refer to firms that have yet to cross the first hurdle as early stage and firms that are trying to get over the second hurdle as later stage or late stage firms. By spending $x^A$ the firm can attempt to get over the first hurdle. With a probability $\gamma^A_f$ the results are negative and the project fails, where the $f$ subscript represents failure and the $A$ indicates this is the initial hurdle. Failure means that some information is learned about the firm that makes any new investment negative NPV regardless of the financing environment. It might be the case that its technology does not work, its new processes is not cost effective or estimates of the target market are smaller than initially hoped, etc. With probability $\gamma^A_s$ the results are positive and there is initial success, where the $s$ represents success. And with probability $1 - \gamma^A_f - \gamma^A_s$ there is neither success nor failure. When there is neither success nor failure then spending $\$x^A$ again gives the firm another attempt to get over the hurdle.
If the firm successfully clears the initial hurdle, then by spending $x^B$ the firm can attempt to get over the second (and final) hurdle. With a probability $\gamma_f^B$ the results are negative and the project fails, where the $B$ indicates this is the second hurdle. With probability $\gamma_s^B$ the results are positive and the firm has an IPO with payoff $V$. And with probability $1 - \gamma_f^B - \gamma_s^B$ there is neither success nor failure, in which case spending $x^B$ again gives the firm another attempt to get over the hurdle. We will refer to the NPV of a firm that has yet to cross the first hurdle as $\Pi_t^A$ and to the NPV of a firm that has yet to cross the second hurdle as $\Pi_t^B$, where the $t$ subscript indicates the period.

It is interesting to note that in any period in which there is neither success nor failure the firm may continue to attempt to get over the hurdle. While in theory this means that a firm could continue for a nearly infinite number of periods, we would never expect to actually see this in the data. For example a firm with a 33% chance of neither succeeding or failing each period would only have an 11% chance of neither making it over the hurdle or failing after two periods and only a 0.4% chance after 5 periods. Thus, we might occasionally see a firm struggle on, never quite making it and never quite failing for 5 or even 10 years, but this would be extremely rare and anything much longer would essentially never occur. However, the notion that it may always be possible to try for one more period captures the idea that while at the start of a project we can be confident that the project will yield a positive or negative result within about 7 years we can never be sure when the project will end. Thus, conditional on a firm making it 7 years without failing or succeeding investors cannot be sure how much more investment will be needed to get an answer one way or the other.

We model the decision of investors willing to invest in early and later stage firms, which we call venture capitalists (VCs). In each period, VCs choose whether or not to invest the $x^i \in \{A,B\}$ to support the firm through the next period. Firms that do not receive capital go bankrupt. For

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4This could also represent the NPV of the project cash flows without an IPO.
5Eventually the $t$ subscript will be dropped due to the stationarity inherent in the model, i.e., the NPV of a project that has not yet crossed hurdle $A$ is the same no matter how many times it has failed to cross the hurdle in the past - sunk costs are sunk. More realistically, work that resulted in neither clear success nor failure could still reveal some small amount of information (rather than no information as we have assumed) which could cause the value of the firm to drift up or down but would not fundamentally alter the value of the project. In which case the value of the project would depend on the number of periods of investment. This would dramatically increase the difficulty of the model but the key insights would remain.
6This is consistent with the view that VCs are thought to have a number of skills relating to the finding and nurturing new companies (Hsu (2004); Kaplan et al. (2009); Hellmann and Puri (2002); Sorensen (2007)).
simplicity, we assume that firms that go bankrupt are worth nothing. Initially we will consider VCs who can fund the firm for only one period, and later we relax this assumption. However, capital is never scarce in the model as we assume that there are always enough VCs to support the firm for one or more periods. And although each individual VC is capital constrained, we assume that there are enough VCs so that all positive NPV projects get done. Therefore, the entrepreneur captures any expected rent from the firm. These assumptions maximize the chance that the VC will invest as we want to make sure our results do not arise from any exogenous capital constraints.

VCs require an expected rate of return of, $r$. VCs are rational and use a positive NPV rule for investing and they expect other VCs to also rationally use a positive NPV rule. Since VCs compete away all rents leaving the entrepreneur with any positive NPV, a VC investing in period 1 gets a fraction $x^A/(\Pi^A_1 + x^A)$ of the firm. This fraction is then diluted down in the next period as the next investor gets a fraction of either $x^A/(\Pi^A_2 + x^A)$ or $x^B/(\Pi^B_2 + x^B)$ depending on whether the firm has progressed. Of course, the present discounted value of the VCs fraction in each future period times the expected payoff in each future period exactly equals $\sum_{i \in \{A,B\}} x_i$. This ensures that the firm will get an investment as long as the firm is not NPV negative. Therefore, as we proceed, in order to determine if the VC will invest we will simply need to determine if the firm is not NPV negative.

To ensure that none of our results are driven by illiquidity we assume that the firm can be sold for its NPV at any time. In addition, however, we allow for strategic acquirers who

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7VCs have limited pools of capital and are often further restricted by the contract with their limited partners to invest no more than a given percent in any one deal.
8Pastor and Veronesi (2009) provide an interesting explanation of asset price increases and decreases in innovative sectors based on changing discount rates. Our focus is on real activity in innovative sectors and the changing nature of that activity.
9For example consider a simple firm with only one hurdle that requires an investment of $1 but pays $4 with a 50% probability or zero. If it pays zero then another $1 investment will pay $4 with a 50% probability and with a 50% probability the firm ends. The NPV of the firm, $Y = 0.5 \times 4 + 0.5 \times (0.5 \times 4 - 1) - 1 = 2 + 0.5 - 1 = $1.50, is captured by the entrepreneur. Therefore, the VC who invests the first $1 gets $1/(Y + 1) = 1/2.5 = 2/5$. The NPV of the second investment $\$1 is $Z = 0.5 \times 4 - 1 = $1, so the VC who invests the second $1 gets $1/(Z + 1) = 1/2$ of the firm. If the second investment occurs, then the first VC who originally owned 2/5th of the company gets diluted down to 1/5th. Thus the first VC gets an expected payoff of $0.5 \times 4 (2/5) + 0.5 \times 0.5 \times 4 (1/5) = 0.8 \times 4 + 0.2 = $1 which is exactly what he invested. Therefore, the investment including expected dilution is NPV zero for the VCs. This ensures a VC will invest as long as the firm has NPV $\geq 0$, i.e., the fraction $x/(NPV + x)$ is less than or equal to 1.
10Allowing investors to sell the firm to other investors for the current NPV of the firm at any time changes nothing as the new investors face the same issues as the old. The probability discussed below relates to the probability that a potential acquirer arrives who values the firm more than the NPV in the hands of the current
value the firm more than it is worth to the current investors. We assume that these strategic acquirers are present in any period with a probability, \( \alpha < 1 \).\(^{11}\) The probability of arrival is less than one because it only includes the arrival of potential acquirers with values greater than the current NPV, and because a probability less than one captures the idea that it is costly and time consuming for potential acquires to find and determine their value for a target (particularly small private targets), so potential acquires only arrive in a given period with a less than 100% probability.\(^{12,13}\)

Conditional on finding each other, the potential acquirer and target negotiate the price for the transaction. The negotiation, if consummated, results in the target receiving an amount \( \Omega_t^A \) or \( \Omega_t^B \) depending on whether the firm has crossed the first hurdle or not. To determine this amount we must decide on a model for negotiations. While many different choices for the model of negotiations will work for our purposes, the simplest is the Nash bargaining solution. In the Nash bargaining solution the transaction price will depend on the potential acquirer’s value and the opportunity cost if each side walks away from the deal.

The potential strategic acquirer may value the target more than the target’s stand alone value because of positive synergies such as cost savings or better sales channels, or from a greater probability of success, or a lower discount rate or potentially because they simply overvalue the firm. For simplicity we assume the potential acquirer’s payoff conditional on success is \( \hat{V} > V \) so the project is worth more to the potential acquirer in any period.

If the target walks away from the negotiation, the target is worth the NPV from continuing to look for investors, either \( \Pi_t^A \) or \( \Pi_t^B \) depending on the stage of the firm. If the potential acquirer does not purchase the target then either the target at some point fails, leaving the value of the potential acquirer unchanged, or the target succeeds. If the target succeeds but is not purchased by the potential acquirer then it competes with the potential acquirer causing the potential acquirer’s value to fall. In this case we assume that the potential acquirer suffers a loss that is proportional to the value of the firm, i.e., \( \lambda \) times the value of the project.

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\(^{11}\)The potential for an acquirer to arrive and pay more than the investors NPV seems realistic given the large fraction of VC backed companies that are eventually sold to strategic acquirers.

\(^{12}\)The idea that acquires and target’s must search for one another is fully developed in Rhodes-Kropf and Robinson (2008).

\(^{13}\)For simplicity \( \alpha \) is not subscripted by whether or not the firm is over the first hurdle. However, It would be logical for \( \alpha \) to increase as the project progresses. All results in the model would hold with this extension.
captures the idea that the profits a firm would earn are likely to come at least partially from incumbent competitors.\textsuperscript{14}

The extensive form of the game is shown in figure 1.

Figure 1: Extensive Form Representation of the Model

\[ \text{WTP} = \text{willingness-to-pay} \]

There is one last aspect of the model. Since investors have only enough money to support the firm for one (or limited) periods investors deciding whether or not to invest must determine whether or not they believe other investors will continue to support the firm in the future. Since all investors are rational and all investors know that other investors are rational it would seem that the need to forecast the actions of others would not matter. But we will see that this is not the case and financing risk will have an impact even though fundamentals do not change.

B. Forecasts

In this section, we will show how the NPV of the project and thus each VC’s decision to invest depends on his rational forecast about the actions of future VCs. We hypothesize (and later confirm) that there are two equilibria in each period - one in which VCs choose to fund a viable

\textsuperscript{14}To the extent this is not true then \( \lambda = 0 \).
project and one when they do not. We will show that each equilibrium is inherently unstable as it depends on the beliefs of others. We assume that an exogenous signal causes investors to believe that the other investors are forecasting future funding or not. Since this common belief becomes self-fulfilling, the equilibrium will depend on this exogenous signal. We will call the signal $I \in [0, 1]$ where $I = 1$ is the ‘Invest’ signal and $I = 0$ is the ‘No-Invest’ signal. Examples of such signals might relate to a key invention in a sector, future industry growth expectations, a government proposal to improve technology in an area, or alternatively a signal that some other sector is hot and thus money will head there. We think of these signals as relating to an industry or area of investing such as bio-tech, green-tech, or high-tech but they could also occur at a more or less granular level. For example, we would argue that part of the dramatic decline in venture investing that began in late 2008 is due to an equilibrium that is economy wide in which investors cannot invest because they do not believe others will be there to support the firms.

In our model, the signal, and thus the state of the world has an exogenous transition probability $(1 - \theta)$ that an industry or sector shifts from the Invest to the No-Invest state and a probability $\phi$ that an industry transitions back to the Invest state. However, initially we will suppress the Markov chain ($\theta = 1$ and $\phi = 0$) to demonstrate the two equilibria in the simpler setting. In either case, for this to be a rational equilibrium all forecasts must be correct in expectation.

$I$ is the signal and thus it also represents the rational forecast of the VCs. When $I = 1$ the forecast is that the next round VC will invest and when $I = 0$ the next round VC is forecasted not to invest. Since all VCs are rational they will invest if the expected NPV of the project is

\[15\] One might think that the global games refinement proposed in Carlsson and van Damme (1993), and used in interesting papers such as Morris and Shin (1998), Goldstein and Pauzner (2005) and Goldstein and Pauzner (2004) would be useful here. The refinement results in a unique equilibria given the fundamentals rather than a unique equilibria given a signal. This refinement will not work in a model of investment across time because future investors know the actions of past investors and so there is no sense in which they are concerned about what action they may take or what signal they got. In the global games refinement investors today are concerned about the actions of other investors today because coordinated action can prevent or create the currency crises or bank run. With investment across time investors are concerned about the actions of future investors, so the assumptions required for the global games refinement do not hold. Our simpler set up also allows us to endogenize the response of investors to the potential of multiple equilibria.

\[16\] Economic logic dictates that $\theta > \phi$ since either state is more likely to occur in a subsequent period if investors are currently in that state.

\[17\] We will see that when this forecast is accurate the forecast will also determine whether or not VCs will invest today so $I = 1$ or 0 will also represent the current ‘state’ of the world in equilibrium.
positive. Let $\Pi_t^A|_{I=1}$ represent the NPV of the project when the forecast is ‘invest’ and let $\Pi_t^B|_{I=0}$ represent the NPV of the project when the forecast is for ‘no-investment’. And remember that for now each VC only has enough money to support the project for one period.

Conditional on a rational forecast of the VCs’ actions in the future, the NPV of the project is either

\[
\Pi_t^A|_{I} = \frac{1 - \gamma_f^A - \gamma_s^A}{1 + r} \left[ I(1 - \alpha)\Pi_{t+1}^A|_{I} + \alpha\Omega_t^A|_{I} \right] + \frac{\gamma_s^A}{1 + r} \left[ I(1 - \alpha)\Pi_{t+1}^B|_{I} + \alpha\Omega_t^B|_{I} \right] - x^A
\]

or

\[
\Pi_t^B|_{I} = \frac{1 - \gamma_f^B - \gamma_s^B}{1 + r} \left[ I(1 - \alpha)\Pi_{t+1}^B|_{I} + \alpha\Omega_t^B|_{I} \right] + \frac{\gamma_s^B}{1 + r} \left[ I(1 - \alpha)\Pi_{t+1}^B|_{I} + \alpha\Omega_t^B|_{I} \right] - x^B
\]

depending on whether the project is early or later stage. Each of these equations forecasts the actions of VC in the next period. Since a rational forecast must be correct in equilibrium, the next period NPV, $\Pi_{t+1}^A|_{I=1}$ and $\Pi_{t+1}^B|_{I=1}$ must be greater than or equal to zero when the forecast is that the next period VC will invest, and $\Pi_{t+1}^A|_{I=0}$ and $\Pi_{t+1}^B|_{I=0}$ must be less than or equal to zero when the forecast is that the next period VC will not invest. Since investors have limited liability, when $I = 0$, $\Pi_{t+1}^A|_{I=0}$ and $\Pi_{t+1}^B|_{I=0}$ drop out of the equation.\(^{18}\)

Exploiting the stationarity in the model we can drop the time subscripts and solve for equilibrium NPV.

\[
\Pi_t^A|_{I} = \frac{(1 - \gamma_f^A - \gamma_s^A)\alpha\Omega^A|_{I} + \gamma_s^A[I(1 - \alpha)\Pi_{t+1}^B|_{I} + \alpha\Omega_{t}^B|_{I}]}{(1 + r) - (1 - \gamma_f^A - \gamma_s^A)I(1 - \alpha)} - (1 + r)x^A
\]

\(^{18}\)It is also possible that VCs rationally forecast that next period VCs will fund the project only if it gets over the initial hurdle. In which case

\[
\Pi_{t+1}^A|_{I=1|f_B} = \frac{1 - \gamma_f^A - \gamma_s^A}{1 + r} \alpha\Omega_t^A|_{I=0} + \frac{\gamma_s^A}{1 + r} \left[ (1 - \alpha)\Pi_{t+1}^B|_{I=1} + \alpha\Omega_t^B|_{I=1} \right] - x^A
\]

and $\Pi_{t+1}^B|_{I=1}$ is the same as in equation (2) above.

Or even that the VC will fund the project next period only if it does not get over the initial hurdle. In which case

\[
\Pi_{t+1}^A|_{I=0|f_B} = \frac{(1 - \gamma_f^A - \gamma_s^A)\alpha\Omega^A|_{I=1} + \gamma_s^A\alpha\Omega^B|_{I=0} - (1 + r)x^A}{(1 + r) - (1 - \gamma_f^A - \gamma_s^A)(1 - \alpha)}
\]

We will consider these cases below.
or
\[
\Pi^B\big|_I = \frac{(1 - \gamma^B_f - \gamma^B_s)\alpha\Omega^B|_I + \gamma^B_s V - (1 + r)x^B}{(1 + r) - (1 - \gamma^B_f - \gamma^B_s)I(1 - \alpha)}
\]

(5)

The above equations demonstrate the effect on the current NPV of the forecast of the VC. Comparing equation (1) when \( I = 1 \) to the same equation when \( I = 0 \) we see that the NPV when the project is expected not to get funding is impacted in two ways. First, the project no longer accrues value from all future investments so \( \Pi^A_{t+1}|_I \) and \( \Pi^B_{t+1}|_I \) fall out of the NPV equation. And second, the negotiations with a potential acquirer are affected (\( \Omega^A|_{I=1} \) becomes \( \Omega^A|_{I=0} \)) because the outside opportunities of both the potential acquirer and the target change.

Thus to understand the impact of the different forecasts we need to understand how negotiations are impacted.

C. Negotiations

Acquisition negotiations under the Nash bargaining solution depend on the potential acquirer’s value and the outside opportunities of each party.

As assumed above the acquirer’s payoff conditional on success is \( \hat{V} \), but, of course the acquirer still has to get the firm over any unmet hurdles. Therefore, at the point the potential acquirer is negotiating with the target the NPV of the potential acquirer’s expected gain either is

\[
\hat{\Pi}^A = \frac{\gamma^A_s \gamma^B_s \hat{V} - (1 + r)\gamma^A_s x^B}{(r + \gamma^A_f + \gamma^A_s)(r + \gamma^B_f + \gamma^B_s)} - \frac{(1 + r)x^A}{r + \gamma^A_f + \gamma^A_s}
\]

(6)

or

\[
\hat{\Pi}^B = \frac{\gamma^B_s \hat{V} - (1 + r)x^B}{r + \gamma^B_f + \gamma^B_s}
\]

(7)

if they buy the target, where \( \hat{\Pi}^j\in[A,B] \) represents the NPV to an acquirer with payoff \( \hat{V} \) (the \( \hat{\text{hat}} \) will signify the acquirer throughout the paper).\(^{19}\)

If, however, the firm succeeds but was not purchased by the potential acquirer then the potential acquirer’s value is reduced by \( \lambda \hat{V} \). Since this loss only occurs if the project succeeds the expected loss depends on the stage of the company and on the company receiving enough financ-

\(^{19}\)Note that it is implicitly assumed that the potential acquirer does not face financing risk because he has an asset that generates enough per period to support the project. Furthermore, it is also assumed that the potential acquirer will not sell the project before fruition. Neither assumption is required but they simplify the exposition.
ing to either make it over both hurdles or to be sold to someone else in the future. Therefore, the potential acquirer expects to lose

$$\hat{C}^A = -\frac{\gamma_s^A \hat{C}^B}{r + \gamma_f^A + \gamma_s^A}$$

(8)

or

$$\hat{C}^B = -\frac{\gamma_s^B \lambda \hat{V}}{r + \gamma_f^B + \gamma_s^B}$$

(9)

if they do not buy the target and the target receives enough funding to get to fruition, where $$\hat{C}^j \in \{A, B\}$$ represents the present value of the expected cost of not buying the target. However, if the firm will not be funded next period then the potential acquirer expects to lose nothing if he does not acquire the target.

If the negotiation is not consummated then the project’s value to the target shareholders is either $$\Pi^j \in \{A, B\} | I = 1$$ or zero (since $$\Pi^j \in \{A, B\} | I = 0 < 0$$). Thus, $$\Pi^j \in \{A, B\} | I = 1$$ represents the target’s outside option or reservation value because if $$I = 0$$ then the target’s outside option becomes zero.

Therefore, the set of possible acquisition agreements, $$\hat{\Omega}^j | I$$ for the acquirer and $$\Omega^j | I$$ for the target is $$\Omega = \{ (\hat{\Omega}^j | I, \Omega^j | I) : I \Pi^j | I \leq \Omega^j | I \leq \hat{\Pi}^j - I \hat{C}^j \text{ and } \hat{\Omega}^j | I = \hat{\Pi}^j - I \hat{C}^j - \Omega^j | I \}$$ where $$j \in \{A, B\}$$ and $$I \in [0, 1]$$. Note that since the potential acquire expects to lose value if they face the target as a competitor they are willing (but may not have to) pay more than $$\hat{\Pi}^j$$ to acquire the target to prevent the loss. Note also that the acquirer’s expected loss, $$\hat{C}^j$$ is multiplied by $$I$$. This is because the potential acquirer only expects losses if the firm succeeds when it is not bought which can happen only if the firm gets funded.

Using the Nash bargaining solution, the equilibrium split is just the solution to

$$\max_{(\hat{\Omega}^j | I, \Omega^j | I) \in \Omega} (\hat{\Omega}^j | I - I \hat{C}^j)(\Omega^j | I - I \Pi^j | I))$$

(10)

20 These are the expected costs when $$\theta = 1$$. Equations (22) and (23) define the costs more generally for $$\theta \leq 1$$. Remember that if the NPV were positive then the VC would invest and it would not be rational to forecast the no-investment outcome.

21 i.e. $$\hat{C}^j$$ is a negative number.

22 More generally one might expect that if the firm could not find future funding its bargaining position might be affected in ways other than just through the reservation values. In the generalized Nash bargaining solution, for example, one might think the bargaining power exponent parameters also shifted to favor the acquirer. This effect would magnify the results presented here.
where $I \in [0,1]$ and $j \in [A,B]$. The well known solution to the bargaining problem is presented in the following Lemma.

**Lemma 1**  In equilibrium the resulting merger share for the target is

$$\Omega^j|_I = \frac{1}{2}(\hat{\Pi}^j - I\hat{C}^j + I\Pi^j|_I) \quad (11)$$

where $\hat{\Pi}^j$ is defined by equations (6) and (7), $\hat{C}^j$ is defined by equations (8) and (9), and $\Pi^j|_I$ is defined by equations (4) and (5).

Plugging this solution into equations (4) and (5) we find that

$$\Pi^A|_I = \frac{(1 - \gamma_f^A - \gamma_s^A)\frac{9}{2}(\hat{\Pi}^A - I\hat{C}^A) + \gamma_s^A[I(1 - \alpha/2)\Pi^B|_I + \frac{9}{2}(\hat{\Pi}^B - I\hat{C}^B)] - (1 + r)x^A}{(1 + r) - (1 - \gamma_f^A - \gamma_s^A)I(1 - \alpha/2)} \quad (12)$$

and

$$\Pi^B|_I = \frac{(1 - \gamma_f^B - \gamma_s^B)\left[\frac{9}{2}(\hat{\Pi}^B - I\hat{C}^B)\right] + \gamma_s^BV - (1 + r)x^B}{(1 + r) - (1 - \gamma_f^B - \gamma_s^B)I(1 - \alpha/2)} \quad (13)$$

where

$$\hat{\Pi}^A - I\hat{C}^A = \frac{\gamma_s^A\gamma_s^B\hat{V}(1 + I\lambda) - (1 + r)\gamma_s^Ax^B}{(r + \gamma_f^A + \gamma_s^A)(r + \gamma_f^B + \gamma_s^B)} - \frac{(1 + r)x^A}{r + \gamma_f^A + \gamma_s^A} \quad (14)$$

$$\hat{\Pi}^B - I\hat{C}^B = \frac{\gamma_s^B\hat{V}(1 + I\lambda) - (1 + r)x^B}{r + \gamma_f^B + \gamma_s^B} \quad (15)$$

This leads directly to our understanding that there are potentially two equilibria 24

**Proposition 1**  There are some firms $\{V, \hat{V}, x^j, \gamma^j_s, \gamma^j_f, \lambda, \alpha, r\}$ at some stages (early and/or late) whose funding does not depend on the funding signal, $I$, (they either always get funding or never do). However, there are some firms at some stages for which there are two possible equilibria - one in which the VCs invest (and they forecast other VCs will invest) and another in which VCs do not invest (and they forecast other VCs will not invest).

**Proof.** See Appendix A.i. ■

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24When we say equilibria we mean pure strategy equilibria as mix strategy equilibria have no economic meaning here since we have assumed there are an infinite number of investors in order to insure capital is always available and investors only earn their required return.
It is only rational for a VC to forecast that a future VC will invest even if the firm has not improved if it is an NPV positive investment, $\Pi^j_{|I=1} \geq 0$. On the other hand, it is only rational to forecast other VCs will not invest if $\Pi^j_{|I=0} < 0$. However, for both equilibria to simultaneously hold for a firm at a given stage it must be the case that $\Pi^j_{|I=1} \geq 0$ and $\Pi^j_{|I=0} \leq 0$. Thus, for some parameters and stages a firm that is ‘good enough’ will get funded no matter what the signal is and a forecast of no funding is not rational. For other ‘weak’ firms the firm never receives funding as it is always NPV negative. But there are some firms and stages where both equilibria are possible because when future funding is not expected the firm NPV drops from positive to negative.

The two equilibria have a similar flavor to the dual equilibria in the banking literature where depositors can ‘run’ on a bank as in Diamond and Dybvig (1983). Depositors leave money in the bank unless they believe others will withdraw. Once a depositor believes others will withdraw, the only rational response is to attempt to withdraw first. Depositors are better off in the ‘deposit’ equilibrium, but this equilibrium is inherently unstable, as anything that makes depositors think others will withdraw makes everyone withdraw and makes everyone worse off.\footnote{The bank run equilibrium is possible because investors cannot coordinate their actions (if they could they would not run). It is also easy to believe that coordinating investors across time would be quite difficult although we consider some commitment mechanisms below.}

Our argument is that when investors must rely on other investors to fund projects, a similar phenomena can occur. That is, if investors believe that other future investors will not invest in the firm, then they themselves will not invest, leading to a self fulfilling equilibria in which everyone is worse off.\footnote{One difference between our model and a bank or currency run model is in the time delay between investor actions. In a bank or currency run model each player is concerned about the current actions of other players and furthermore, simultaneous actions are strategic complements. In our model investors in the future know the actions of investors in the past but are concerned about investors further into the future.}

The central mechanism behind our theory is quite different than in a bank run. For the two equilibria to be possible the NPV of the project today must change from positive to negative NPV depending on whether or not the project is expected to get funding tomorrow. This is not as straight forward as it sounds. For example, if the ‘project’ is simply a series of NPV positive coin flips then failing to get funding tomorrow will reduce the total NPV of the project, however, \textit{it will still be positive NPV}. Thus, the investor today should still pay to see the coin flipped, and so should the investor tomorrow thus making the forecast of no future financing incorrect.
The only way for a ‘no-invest’ forecast to be correct even when fundamentals have not changed is if the forecast fundamentally alters today’s payoffs. The channel we have chosen to use to demonstrate this effect is the sale to a strategic buyer. The negotiation to sell the company today is fundamentally altered by the ‘no-invest’ forecast because the target’s bargaining power is reduced and because the potential acquirer is less worried about the firm as a competitive threat when future funding is not available.

We think the strategic sale channel is a fundamental and important force for venture investing. It is easy to believe that companies often wait to acquire firms, or at least become more aggressive, when they perceive the firm as a potential threat to their business. For example, the large oil companies and other energy firms have been slow to invest in alternative energies, while big pharma is a regular buyer of biotech start ups. We would argue that oil companies currently see little threat to their business, while big pharma knows that they will likely compete against products they don’t buy. We would also argue that this leads back to the VC community and leads to a great deal more capital flowing to biotech firms than alternative energy startups. In fact, the so called ‘valley of death’ that alternative energy firms must walk through to become successful may be partially a self fulfilling prophecy - since none of the firms can truly make it to a scale where they could compete with the big energy firms, the big energy firms feel no competitive threat and thus wont pay much for alternative energy companies. This in turn leads investors to not want to invest in alternative technologies particularly if it will take a lot of money to get it to scale (so more coordination of investors is required) thus the startup firms cant make it to scale and the big energy firms are not worried! But this equilibrium could flip at any time.

While we believe the sales channel is a central part of financing risk, this is not the only channel and we suggest that it is likely that forces work in concert to magnify financing risk. For example, another alternative channel is that employees today who forecast that financing wont be available in the next period leave or work less hard as they look for another job. This could fundamentally change the investment decision today making it rational to believe it won’t get financing. Furthermore, customers who do not think the company could get funding in the future may not want to buy a product today if it requires any future support. In fact, the balance sheet of most startups is a closely guarded secret and many firms hope for the statement
from their auditor that they are a ‘going concern’ i.e., they have enough money to last for a year.\textsuperscript{27}

There are certainly even more channels but to create financing risk they must work the same way. They must change the project from positive to negative NPV based on a forecast that the project will not get funding in the future. We will continue to use the sales channel as we go forward but we believe the other channels only magnify the results we present.

We have presented the main driver of financing risk but to fully understand its impact we must consider investors who have more than one period of funding and investor responses to financing risk. We must also include the possibility that the state can jump back and forth between the Invest and No-Invest equilibria. We will start by considering wealthier investors.

\textbf{D. Can a wealthier investor overcome the ‘No Invest’ equilibrium?}

If it is the reliance on other investors that leads to the problem, the question arises as to whether a VC with more money or a syndicate of VCs can overcome the No-Invest equilibrium. In this section we show that is not the case. To see this, consider a VC who has enough capital to fund the investment for two periods. One might imagine that this VC faces less financing risk in the first period they invest because they can be sure to invest in the next period. In this case one might think that even in the No-Invest equilibrium the expected value of the firm in the first period is

\[
\Pi_t^A = \frac{1 - \gamma^A f - \gamma^A s}{1 + r} \left[ (1 - \alpha)\Pi_{t+1}^A | I=0 + \alpha \Omega_t^A | I=1 \right] \\
+ \frac{\gamma^A s}{1 + r} \left[ (1 - \alpha)\Pi_{t+1}^B | I=0 + \alpha \Omega_t^B | I=1 \right] - x^A
\]

which is the NPV from equation (1) with I set to 1 only for period t in spite of the No-Invest equilibrium (because of the second \$x^j held by the VC). If this were the case, then the NPV in period t might be greater for a VC with enough funding for two period since the purchase offer would be larger if it occurs and the funding is sure to come if no offer arrives.

However, equation (16) demonstrates the fallacy of this argument. In equation (16) the extra

\textsuperscript{27}After the collapse in 2000 companies doing business with startups began asking about and only working with, entrepreneurial firms that were a going concern.
$x^j$ is assumed to be invested even though the forecast is still that no other investors will invest. However, when the VC with $2x^j$ gets to the second period she will only have one $x^j$. At that point, if she invests, she knows that no other investor will support the project. Therefore, she gets $\Pi_{t+1}^A|_{I=0}$ or $\Pi_{t+1}^B|_{I=0}$ by investing. However, the No-Invest equilibrium is only rational if $\Pi_{t+1}^A|_{I=0} < 0$ and $\Pi_{t+1}^B|_{I=0} < 0$. Therefore, the VC will not invest their second $x^j$.

Of course, using backward induction, the VC will realize that they will not invest the second $x^j$ and therefore, will reevaluate their decision to invest the first $x^j$. Since the second $x^j$ will only be invested in the Invest equilibrium, the decision to invest the first $x^j$ is the same for VCs with either $x^j$ or $2x^j$. This same backward induction tells us that (in the absence of commitment) only an investor with an amount of capital $X = \frac{x^j}{r}$ can break the No-Invest equilibrium.

We believe that showing this existence of multiple equilibria – that are not overcome purely by reasonable sized syndicates or wealthy investors – is a key contribution of our paper. We can also see why larger firms with cash generating assets greater than $X$ do not face financing risk. Our theory suggests that waves of investment activity are self-fulfilling and that hence innovation waves are the inevitable outcome for projects that rely on future investors to continue funding projects in order for them to be NPV positive for initial investors.

In the next section, we examine two important scenarios to see how they impact the financing strategy of VCs. First, what happens if everyone expects the equilibrium to flip from the No-Invest to the Invest equilibrium at some point in the future (or vice versa)? Second, what happens if investors can write complete contracts that commit themselves to continue investing in a firm even if they are in a No-Invest equilibrium in the future? We will turn first to the idea of transition probabilities from state-to-state and then to the idea of commitment. We will see how these additional ideas allow us to establish that financing risk is more important for innovative projects.

## II. Transitions from State to State

An important facet of this model is that each equilibrium is inherently unstable as it depends on the beliefs of others. Given this fact, VCs will also need to forecast the possibility of a jump to
the other equilibrium and a jump back when calculating the NPV of their investment. VCs that forecast a possibility of the No-Invest equilibrium will prepare for it. And if a project does not need to survive an infinite No-Invest period, then more money may help prevent the No-Invest equilibrium from affecting the firm.

We assume the signal follows a Markov chain. The transition matrix for the signal \( I \) is

\[
\begin{pmatrix}
I = 1 \\
I = 0 \\
\end{pmatrix} = S
\]

Given this transition matrix the NPV in period \( t \) can be written as

\[
\Pi^A|_I = \frac{1 - \gamma_f^A - \gamma_s^A}{1 + r} [Z(1 - \alpha)\Pi^A_{t+1}|_{I=1} + Z\alpha\Omega^A_{t}|_{I=1} + Y\alpha\Omega^A_{t}|_{I=0}] + \gamma_s^A[1 + r[Z(1 - \alpha)\Pi^B_{t+1}|_{I=1} + Z\alpha\Omega^B_{t}|_{I=1} + Y\alpha\Omega^B_{t}|_{I=0}] - x^A]
\]

or

\[
\Pi^B|_I = \frac{1 - \gamma_f^B - \gamma_s^B}{1 + r} [Z(1 - \alpha)\Pi^B_{t+1}|_{I=1} + Z\alpha\Omega^B_{t}|_{I=1} + Y\alpha\Omega^B_{t}|_{I=0}] + \gamma_s^B[1 + r[V - x^B]]
\]

where

\[
Z = I\theta + (1 - I)(1 - \phi)
\]

and

\[
Y = I(1 - \theta) + (1 - I)\phi.
\]

Note that both of these equation assume that if investors enter the ‘No-Invest’ state then no further investment will occur. This must be confirmed in equilibrium. If it is not true then the NPV equations reduce to equations (4) and (5), as though the state is always in the invest equilibrium and the signal is meaningless.

Exploiting the stationarity in the model we can solve for equilibrium NPV

\[
\Pi^A|_I = \frac{[(1 - \gamma_f^A - \gamma_s^A)\frac{1}{2}\hat{\Pi}^A + \gamma_s^A\frac{1}{2}\hat{\Pi}^B - (1 + r)x^A](1 + (Z - \theta)(1 - \gamma_f^A - \gamma_s^A)(1 - \alpha/2)}{(1 + r) - (1 - \gamma_f^A - \gamma_s^A)\theta(1 - \alpha/2) \\
+ \gamma_s^A[Z(1 - \alpha/2)\Pi^B|_{I=1} - \frac{1}{2}Z\hat{C}^B] - (1 - \gamma_f^A - \gamma_s^A)\frac{1}{2}Z\hat{C}^A}{(1 + r) - (1 - \gamma_f^A - \gamma_s^A)\theta(1 - \alpha/2)}
\]

(20)
and

\[
\Pi^B|_I = \left[ (1 - \gamma_f^B - \gamma_s^B) \frac{\hat{\Pi}^B + \gamma^B \hat{V} - (1 + r) x^B}{(1 + r) - (1 - \gamma_f^B - \gamma_s^B) \theta(1 - \alpha/2)} \right] (1 + (Z - \theta)(1 - \gamma_f^B - \gamma_s^B)(1 - \alpha/2)) \\
- \frac{(1 - \gamma_f^B - \gamma_s^B)Z \alpha \hat{C}^B}{(1 + r) - (1 - \gamma_f^B - \gamma_s^B) \theta(1 - \alpha/2)}
\]

where \( Z = I\theta + (1 - I)(1 - \phi) \) and \( \hat{C}^A \) and \( \hat{C}^B \) now equal

\[
\hat{C}^A = -\frac{\gamma_s^A \theta \hat{C}^B}{(1 + r) - \theta(1 - \gamma_f^A - \gamma_s^A)} \quad (22)
\]

\[
\hat{C}^B = -\frac{\gamma_s^B \lambda \hat{V}}{(1 + r) - \theta(1 - \gamma_f^B - \gamma_s^B)} \quad (23)
\]

when \( \theta \leq 1 \).

The potential for the No-Invest state to end improves the value of an investment in the No-Invest state and reduces financing risk. Thus the following proposition is similar to the first proposition, but must account for the probability that the No-Invest equilibrium might not last forever.

**Proposition 2** There are some firms \( \{V, \hat{V}, x^j, \gamma^j_f, \gamma^j_s, \lambda, \alpha, r\} \) at some stages (early and/or late) whose funding does not depend on the funding signal, \( I \), (they either always get funding or never do). However, as long as \( \phi \) and \( \theta \) are large enough, there are some firms at some stages for which there are two possible equilibria - one in which the VCs invest (and they forecast other VCs will invest) and another in which VCs do not invest (and they forecast other VCs will not invest).

**Proof.** See Appendix A.ii. ■

The intuition of the proof is straightforward. If the transition probability, \( \phi \), is one, then the No-Invest equilibrium, once entered, will last forever, and therefore the conditions for the No-Invest equilibrium to be an equilibrium are the same as in Proposition 1. Thus, for \( \phi \) that is

\( ^{28} \)\( \hat{\Pi}^A \) and \( \hat{\Pi}^B \) are unaffected by the signal transition matrix because we have assumed the buyer does not face financing risk. The expected cost is reduced when \( \theta < 1 \) because the probability of the firm reaching fruition is reduced.
\( \epsilon \) less than one the No-Invest state is still an equilibrium. Likewise, if the transition probability to the No-Invest state, \((1 - \theta)\) is zero, then the Invest equilibrium, once entered, will last forever. Therefore, for a \((1 - \theta)\) that is \(\epsilon\) greater than zero, the Invest state is still an equilibrium. The equilibrium actions eventually break down as \(\phi\) and \((1 - \theta)\) become large enough, because a high enough probability of a transition to a state effectively causes participants to behave as if the state occurs today.

It would seem that with a probability that the No-Invest period ends, a wealthier VC or a syndicate of VCs could now overcome the No-Invest equilibrium for some projects. However, we show again that this is not the case. To see this, note that the very last $x_j$ that the VC has will only be spent if the equilibrium has jumped back to the Invest equilibrium. The VC knows this in the period before the last period and also knows that if the industry is still in the No-Invest equilibrium in the period just before this last period, this means that \(1 - \phi\) is not large enough to make investing the second to last $x_j$ a good idea (if it were large enough, it would cause VCs to behave as if the state occurs today and hence have caused the equilibrium to flip). Therefore, in the period just before this last period, the VC understands that the last $x_j$ will only be spent if the equilibrium changes. So the second to last $x_j$ is not invested either. Continuing this backward induction eventually brings us back to the first $x_j$. Thus (in the absence of commitment), until an investor or a syndicate has more than $x_j/r$ (that can generate $x_j$ per period) more money will not break the No-Invest equilibrium. However, we will see the importance of commitment in the next section.

### III. The Benefits and Costs of Commitment

Increasing the dollars held by one investor or forming a syndicate does not help the company get over the No-Invest equilibrium because in each period the investment decision is made rationally and so a syndicate or even one investor with more money makes no decision differently than the market (until they have enough money that they never need the market again). After all, sunk costs are sunk. Therefore, if the market is rationally in the No-Invest equilibrium, then any investor would make the same decision as the market.

However, we show that commitment to invest through a No-Invest equilibrium can change
this result. We now allow an investor to commit to invest in the next period regardless of the equilibrium established by other investors. This increases the offer the firm will get in a sale during the No-Invest equilibrium due to the increase in bargaining power provided by the funding cushion.

Initially we will assume that contracts are complete and that there are no information asymmetries – so that the investor who has committed to invest in the second period does not invest if the project turns unviable (probability $\gamma_j^f$), but will invest if the project is viable and the equilibrium has jumped to the No-Invest equilibrium. Alternatively, an equivalent contract is a state contingent contract where investors give a project $\$2x_j$ or more in a period and the project commits to return any unused funds if the project becomes unviable but not if the state transitions to the No-Invest equilibrium.

Commitment trades off the potential increase in sale price with the potential loss from having to invest during the bad equilibrium. If an investor only invests a single $\$x_j$ then we know from above that the expected project NPV is either equation (18) or (19).

If instead an investor or syndicate commits to invest in both the first and the second period then we will refer to the project NPV as $\Pi_j^I|_{I=2}$ where the $n = 2$ indicates two periods of commitment (one can think of all the NPVs above as having an implicit $n = 1$, although from here forward we will explicitly indicate the number of periods of commitment). The extra period of commitment ensures that the project will receive an investment in the next period even if the state, $I$, has changed to $I = 0$. This, in turn, alters the bargaining outcome of any sale so the negotiated outcomes will now be written with an $n$ superscript to indicate the number of periods of future commitment, such as $\Omega_j^I|_{I=1}^{n=0}$ if there is one more period of money committed to the company.\(^{29}\)

Therefore, the expected project NPV when an investor commits to fund the project for two

\(^{29}\)Where $\Omega_j^I|_{I=1}^{n=1}$ is the same as $\Omega_j^I|_{I=1}$ because the extra period of funding would have occurred even without the commitment, but as we will see $\Omega_j^I|_{I=0}^{n=1}$ is altered by the extra commitment.
where $Z = I\theta + (1 - I)(1 - \phi)$ and $Y = I(1 - \theta) + (1 - I)\phi$. Appendix A.iii. solves for the profits for any level of commitment $n = N$.

These equations differ from equation (18) and (19) in two ways. First, the bargaining outcome changes if the state transitions to $I = 0$ because funding is certain. Second, the investor has agreed to provide financing in the bad state. Therefore, if the project doesn’t sell and the bad state occurs, the investor makes an expected loss since $\Pi^A |_{I=0} < 0$.

Thus, the question of whether it is better to commit to a second round of investment is a question of whether profits with commitment are bigger than profits without. Subtracting the two profit equations, the question is reduced to whether $\Pi^A |_{I=2} - \Pi^A |_{I=1} > 0$ or

$$\Pi^A |_{I=2} - \Pi^A |_{I=1} = \frac{1 - \gamma_f - \gamma_s}{1 + r} [Y(1 - \alpha)\Pi^A |_{t=1} |_{I=0}^n + Y(1 - \alpha)\Pi^A |_{t=1} |_{I=0}^1] + Z\alpha\Omega^A |_{I=1} + Y\alpha\Omega^A |_{I=0}^1 - x^A$$

$$\Pi^B |_{I=2} - \Pi^B |_{I=1} = \frac{1 - \gamma_f - \gamma_s}{1 + r} [Z(1 - \alpha)\Pi^B |_{t=1} |_{I=0}^n + Y(1 - \alpha)\Pi^B |_{t=1} |_{I=0}^1] + Z\alpha\Omega^B |_{I=1} + Y\alpha\Omega^B |_{I=0}^1 - V - x^B$$

where $Z = I\theta + (1 - I)(1 - \phi)$ and $Y = I(1 - \theta) + (1 - I)\phi$. Appendix A.iii. solves for the profits for any level of commitment $n = N$.

We will see exactly how in a moment.
and
\[
\Pi^B_t|^{n=2} - \Pi^B_t|^{n=1} = \frac{1 - \gamma_f - \gamma_s}{1 + r} Y[(1 - \alpha)\Pi^B_{t+1}|^{n=1} + \alpha\Omega^B|^{n=1} - \alpha\Omega^B|^{n=0}] > 0 \tag{27}
\]
where \(Y = I(1 - \theta) + (1 - I)\phi\). Thus, we can see that the question becomes one of whether or not the expected improvement in negotiating power, \(\alpha\Omega^B|^{n=1} - \alpha\Omega^B|^{n=0}\), is worth the potential negative expected value investment if the state becomes \(I = 0\) (because \(\Pi^B_{t+1}|^{n=1} < 0\)).

When there is no commitment, \(n = 0\), then the negotiated outcome of a sale is as above, equation (11). Using Lemma 1 we can see that the negotiated outcome when \(n = 1\) and \(I = 0\) is \(\Omega^j|^{n=1}_{I=0} = \frac{1}{2}(\Pi^j - \hat{C}^j|^{n=1}_{I=0} + \Pi^j|^{n=1}_{I=0})\), where \(\Pi^j|^{n=1}_{I=0}\) is defined by equations (24) and (25) and
\[
\hat{C}^A|^{n=1}_{I=0} = -\frac{\gamma_s(1 - \phi)\hat{C}^B|^{n=0}_{I=1}}{(1 + r) - \theta(1 - \gamma_f - \gamma_s)} \quad \text{and} \quad \hat{C}^B|^{n=1}_{I=0} = -\frac{\gamma_s\lambda\hat{V} + (1 - \phi)(1 - \gamma_f - \gamma_s)}{(1 + r) - \theta(1 - \gamma_f - \gamma_s)} 
\]
Appendix A.iii. solves for the expected costs for any level of commitment \(n = N\).

In general, the gain from committing more dollars to a project is the higher purchase price if a buyer arrives. The loss from committing more is the negative NPV from investing if no buyer has arrived nor success occurred but the state has transitioned.

The following proposition shows the impact of this trade-off.

**Proposition 3** If investors or syndicates can commit to invest in future periods and contracts are complete then for any project \(\{V, \hat{V}, x^j, \gamma^j_s, \gamma^j_f, \lambda, \alpha, r\}\) at a stage (early or late) in which it faces financing risk, committing enough money increases the project NPV and eliminates the No-Invest equilibrium, i.e. the project no longer suffers from financing risk.

**Proof.** See Appendix A.iii. ■

By committing to one more period of investment the investor essentially ‘puts off’ having to make the negative NPV investment by one period. Simultaneously committing more improves the projects bargaining power for all previous periods of commitment. Thus, eventually by committing to fund the project for enough periods the bargaining improvement outweighs the ever more unlikely negative NPV investment. The negative NPV investment becomes less and less likely because a firm with more money is more likely to succeed or be bought before it runs out of money - and it is only when the project has little money that it becomes negative NPV.
Therefore, large investors and syndicates can actually increase the NPV of the projects they fund by giving them more dollars or implicitly or explicitly committing to fund them for longer. Enough committed dollars make the project NPV positive even in the No-Invest state. That is, if enough investors join together, then a large enough investment in the bad state becomes NPV positive. For these projects, the only equilibrium is the Invest equilibrium and commitment eliminates financing risk.

The logic above would seem to suggest that all projects should get significant up front funding. However, as noted above, we have so far assumed that an investor or syndicate that commits to fund a project can withdraw funding if the project becomes unviable, i.e. the commitment only relates to the state of the world and not to the project quality.

The analogous venture capital contract is a tranched investment, in which the investors have committed to fund a project if certain milestones are reached. These type of contracts provide the investor with a real option, but we believe they are also an attempt to overcome financing risk as they commit the investor to invest if the company has done well even if the world has done poorly. However, they rarely cover more than one future financing, and for many projects (particularly innovative ones), it very difficult to articulate and delineate a clear milestone. Thus, it is unrealistic to assume that complete state-contingent contracts can be written for all future funding dates at the start of a project. The next section explores the trade-offs under the more realistic scenario of incomplete contracts.

A. Incomplete Contracts and the Lost Real Option

Complete contracts are unrealistic as investors cannot contract on every future funding need at the start of a project. In this section, we assume that contracts are incomplete (a la Grossman and Hart (1986); Hart and Moore (1990)). We assume that it is not possible to either write down or verify all future states in which funding should or should not occur. For example, it might be the case that states of nature are observable by the investors by not verifiable by a court. Specifically we define an incomplete contract as follows.

**Definition 1** In an incomplete contract, investors cannot contract on actions that differ between the No-Invest equilibrium, \( I = 0 \) and project becoming unviable, \( \text{Prob} \gamma^f_j \).
We still assume that it is costly for investors to renege on a commitment. Since one way to ‘commit’ to future funding is to provide extra funding today, the assumption that it is costly to renege on a commitment is the same as assuming that it is costly to shut down a project and return any unspent capital to investors. For simplicity we assume it is never optimal for the investor to fail to fund a contract. Effectively, this is the same as assuming commitment is enforceable.

Commitment was enforceable in the last section as well, but now, without complete contracts, project CEOs and investors are not able to write contracts that release the investor or return capital when bad firm specific information arrives. We assume the CEO gets private benefits of control and is therefore biased toward continuation. Therefore, incomplete contracts create a world in which all dollars given or committed to a firm are spent no matter what information arrives. If all the money given to a firm will be spent, then giving more money to a firm destroys some of the value of the firm’s real option to shut down in the event that intermediate information is not positive. On the other hand, more money better-protects a firm from the No-Invest equilibrium. Thus, it is those firms with more valuable real options for which protection from the No-Invest equilibrium is more costly.

In our model the real option value in a firm depends on the probability that a firm loses viability before it is sold. If $\gamma_f = 0$ the firm is always viable and there is no real option value. However, for higher values of $\gamma_f$ it becomes valuable to give the firm less up front funding (smaller commitment) and wait to learn that it is still a viable firm in the next period.

We can see the effect of incomplete contracts and real options on the profitability of committing extra dollars to a firm. In section III., when we assumed complete contracts, the profit from committing to invest an extra $x^j$ was equations (24) and (25). With complete contracts if the project lost viability the investor would not lose the second committed $x^j$. Now, however, committing $2x^j$ requires the investor to lose the second $x^j$ if the project fails (i.e., it will be

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31The main tradeoff is the same if only a fraction of the committed dollars would be spent after the arrival of bad news.
spent by the CEO). Thus, the expected profit from committing $2x_j^j$ becomes

$$
\Pi^A_t|_{n=2} = \frac{1 - \gamma^A_f - \gamma^A_s}{1 + r} \left[ \frac{Z(1 - \alpha)}{I} \Pi^A_{t+1}|_{I=1} + Y(1 - \alpha) \Pi^A_{t+1}|_{I=0} \right]
+ Z \alpha \Omega^A_t|_{I=1} + Y \alpha \Omega^A_t|_{I=0} \\
+ \frac{\gamma^A_s}{1 + r} \left[ \frac{Z(1 - \alpha)}{I} \Pi^B_{t+1}|_{I=1} + Y(1 - \alpha) \Pi^B_{t+1}|_{I=0} \right]
+ Z \alpha \Omega^B_t|_{I=1} + Y \alpha \Omega^B_t|_{I=0} - x^A(1 + \gamma^A_f)
$$

(29)

$$
\Pi^B_t|_{n=2} = \frac{1 - \gamma^B_f - \gamma^B_s}{1 + r} \left[ \frac{Z(1 - \alpha)}{I} \Pi^B_{t+1}|_{I=1} + Y(1 - \alpha) \Pi^B_{t+1}|_{I=0} \right]
+ Z \alpha \Omega^B_t|_{I=1} + Y \alpha \Omega^B_t|_{I=0} + \frac{\gamma^B_s}{1 + r} V - x^B(1 + \gamma^B_f)
$$

(30)

where $Z = I\theta + (1 - I)(1 - \phi)$ and $Y = I(1 - \theta) + (1 - I)\phi$.

The difference between an profit function with complete contracts equations (24) and (25) and without is the $(1 + \gamma^B_f)$ multiplying the investment of $\$x^B$. This is the additional cost to commitment when the money will not be returned if the firm fails. This leads directly to our next proposition

**Proposition 4** Incomplete contracts reduce the value of committing more money and the reduction in value is larger for more innovative firms (firms with more real option value).

**Proof.** See Appendix A.iv. ■

The central insight comes from comparing the profit equations with and without complete contracts. Note that if $\gamma^B_f = 0$ there is no real option value and no difference between the profit functions. With $\gamma^B_f = 0$ there is no chance the project will fail so commitment only effects the No-Invest state of the world. Thus, just like in the last section, with $\gamma^B_f = 0$ commitment trades off the cost of investing during the No-Invest equilibrium with the potential increase is sale price from doing so. However, the larger $\gamma^B_f$ becomes the more valuable it becomes to give the project only one period of funding to see if it fails. Thus, commitment becomes more and more costly.

Investors who give a firm enough funding to get over the No-Invest equilibria lose the option to give the firm a little funding and wait to see how it performs to give it more. Therefore, it is more costly to overcome the No-Invest equilibrium for innovative projects with high real option
value. The less innovative firms can be given a larger amount of up-front financing in order to avoid the No-Invest equilibrium. But the innovative firms cannot be given significant funding up front or the loss of the real options may change it to an NPV negative project. Therefore, more innovative firms should receive less funding up front and are more exposed to financing risk.

Thus, in a world with incomplete contracts, less innovative firms are not hurt as much by the prospect of financing risk. Instead it is the innovative end of the economy that is most impacted by waves of investor interest and disinterest in the sector. This does not require any behavioral explanation, although the effect could certainly be magnified by behavioral considerations. Rational investors know they face financing risk. They rationally try to mitigate that risk by forming syndicates and providing larger sums of money up-front. But for more innovative firms providing more money reduces the option value of the investment. Thus, innovative projects must be left exposed to the whims of the financial market.

IV. Implications

A. Innovation Bubbles and Project Mix and Funding Levels

A key implication of our model is that we should see bubbles of innovative activity that are endogenously driven by self fulfilling fluctuations in the capital markets. Investors attempt to protect their firm from this effect by committing more money to a firm up front but the more innovative the firm the more likely early failure may occur and the more costly it is to provide significant money up front.

The likelihood that the world enters the ‘bad’ equilibrium depends on investors beliefs about other investors. We have assumed that some economic signal shifts investor beliefs. The likelihood of this shift, \((1 - \theta)\) in the model, could vary with time and by industry.

Any increase in financing risk\(^{32}\) lowers the NPV of all firms that suffer from financing risk. If this occurs some firms will become NPV negative with their current level of commitment. At which point some of these firms will be unable to get funding while other firms will find it value enhancing to raise more money and thereby reduce the value of some of their real options but

\(^{32}\)A decrease in \(\theta\) or an increase in \(\phi\).
defend better against the potential No-Invest equilibrium. Therefore, firms that can do so will raise more money while firms which cannot raise enough money to defend against the No-Invest equilibria will be unable to raise any money.

Thus, the most innovative firms will be the firms that cannot raise significant up-front financing because too much value is destroyed in the loss of their real options. This inability to raise an extra $x^j$ will matter more when financing risk is larger. For example, as financing risk went to zero no firm would raise extra funding in order to maximize the value of their real options. Therefore, as financing risk rises not only should fewer firms be financed but the mix of financed firms should become less innovative.

Furthermore, among the firms that do get financed, a variation in the financing risk should vary the amount of funding they receive.

As financing risk rises, if contracts were complete then all firms would create more value by raising extra $x^j$. But with incomplete contracts the more innovative firms find raising more money more costly because it destroys more real options and thus they may not be able to find investors. As financing risk falls eventually no project will find it valuable to raise an extra $x^j$. Thus, the additional firms that are financed when financing risk is low should receive less funding (relative to their burn rate) than the average firm financed when financing risk was higher.

Note the the above point is a statement relative to burn rate since it would also be logical that when financing risk was high firms slowed down their burn rate.

This also implies that if the world does jump to the No-Invest equilibrium when the No-Invest equilibrium seems remote, firms will not have much of a cash cushion. With less of a cushion to survive the No-Invest equilibrium these firms will go out of business, and this will always be more true for more innovative firms. Therefore, any surprising shock to the economy that results in a funding void is likely to lead to the destruction of the most innovative firms in the economy first.

B. Stage and Financing Risk

While a firm at any stage can face financing risk there are still some implications relating to the life cycle of a project.

It is likely that for most projects the earlier the stage of the project the more likely it is to
fail (i.e. $\gamma_A > \gamma_B$). So it is the early stage firms, on average, that are less able to defend against financing risk and are more impacted by it. Therefore, early stage firms should get less funding (relative to burn rate) than later stage firms and should have to go back for financing sooner. Furthermore, as financing risk changes through time we should see a larger fluctuation in the birth of early stage firms and funding for early stage firms relative to later stage projects.

We can imagine projects with more than two stages. Projects with stages A, B, C, D, etc, face more financing risk than a project with only one stage (holding other parameters constant) and has more chances to fail. Essentially projects with only one stage can be more certain they have the money they need when they start relative to a project with many hurdles to jump. This suggests that projects that require more money spent over longer time frames face greater financing risk.

C. Investor Mix

Lower financing risk lowers the amount of capital firms need and should therefore also allow smaller investors with more limited capital to invest. Our model therefore suggests that the mix of investors should shift towards smaller and more early stage investors in times when financing risk is low. When financing risk is high, a large investor might be able to give a firm more support and break them out of the No-Invest equilibrium but small investors don’t even have this option and must therefore stop investing.

In making this point we are implicitly assuming some sort of coordination costs that prevent myriad little investors from simply joining to be a large investors. But this seems like a reasonable assumption.

Thus, in the low investing times small angel investors should virtually disappear from the market as the coordination costs to bring together enough of them is too high. Further, the only firms we should see getting funded should be funded by larger investors and actually given a larger fraction of total money needed. So while less total money will enter the sector and fewer firms will get funded, the few firms that get funded will be well funded relative to burn rate. Note we are not suggesting that firms will get more funding in bad times, only that those that get funded should have more funding relative to their burn rate so they can better survive the funding drought.
D. ‘Herd Behavior’ in Innovative Investments

Conventional wisdom suggests that contrarian strategies might be good because following the crowd leads to a flood of capital in a sector and lowers returns. Our model implies that this is not true in every case. In our model, fully rational investors who only make NPV positive investments are optimally entering the market when prices are high (because the financing risk is low) and everyone else is also in the market. When financing risk is low, giving a firm less money and seeing how it does makes sense. Smaller investors who face greater hurdles to forming large pools of money can find valuable investments in high real option companies that need only a little money but only during good times. Making this same investment when financing risk is high or during the No-Invest equilibrium is NPV negative. Thus for innovative projects with high real option value, it may actually make sense to invest with the crowd.

The corollary to this view also provides a more positive interpretation to the bubbles of activity that are associated with the initial diffusion of very radical new technologies, such as railways, motor cars, internet or clean technologies. Our model implies that such technologies may in fact need ‘hot’ financial markets, where financing risk is extremely low and lots of investors are in the market, to help with the initial diffusion of such technologies. Related to this, our model provides an understanding as to why asset prices in such times can steadily rise and then precipitously fall, even when the fundamentals of the firms have changed little. Since expectations of a low probability of a No-Invest equilibrium lead to high NPVs and hence high asset prices, a sudden change in the equilibrium will lead many firms to become negative NPV and lead asset prices to fall commensurately.

V. Conclusion

Startups have been associated with the initial diffusion of several technological revolutions (railway, semiconductors and computers, internet, motor cars, clean technology) and there is increasing evidence of the important role of startup firms in driving aggregate productivity growth in the economy (Foster et al. (2008)). This paper builds on the emerging research examining the role of the capital markets in driving innovation in the real economy (Kortum and Lerner (2000), Mollica and Zingales (2007), Samila and Sorenson (2010)) and provides a mechanism for why
innovation in new firms might occur in waves of activity.

We argue that a particular feature of innovative startups is that they don’t know how much investment will be required to get to the ‘finish line’. Intermediate results may be equivocal, or additional investments may be required to get to cash flow positive. Any investor in such startups with limited resources must therefore also rely on other investors to bring innovative firms to fruition.

Because of this, such startups face two risks - fundamental risk (that the project gets an investment but turns out not to be viable) and financing risk (that the project needs more money to proceed but cannot get the financing even if it is fundamentally sound). Financing risk is typically ignored in the finance literature because all firms with positive fundamental NPV are assumed to get funded. This ignores the fact that investing requires coordination across time between investors with limited resources. Investors must, therefore, forecast the probability that other investors will be there to fund the firm in the future.

The impact of financing risk on a firm can be reduced by giving the firm more funding. However, this comes at a cost. A firm with more funding may spend some or all of the money even in the event of disappointing intermediate information. This cost is much more important for highly innovative firms where outcomes are uncertain and the real option to shut down the firm is valuable. The more valuable the real option to shut down a firm, the less funding the firm should receive at a given time. Firms that receive less funding are more affected by a jump to the No-Invest equilibrium. Thus early round investors investing in innovative firms face an important trade-off between lowering financing risk and increasing real option value. The most innovative firms are thus most susceptible to financing risk as they are least able to acquire a ‘war chest’ to survive a down turn.

We therefore argue that the most innovative firms or the early period of a technology adoption may need ‘hot’ financing environments to help with their initial financing or diffusion. By driving investment waves in innovative sectors financing risk may play a key role in magnifying bubbles of innovation and technological revolutions.
References


A. Appendix

i. Proof of Proposition 1.

If the VCs forecast that other VCs will invest then the project NPV becomes

\[ \Pi^A|_{I=1} = \frac{(1 - \gamma_f^A - \gamma_s^A)\frac{\alpha}{2}(\hat{\Pi}^A - \hat{C}^A) + \gamma_s^A[(1 - \alpha/2)\Pi^B|_{I=1} + \frac{\alpha}{2}(\hat{\Pi}^B - \hat{C}^B)] - (1 + r)x^A}{(1 + r) - (1 - \gamma_f^A - \gamma_s^A)(1 - \alpha/2)} \]  
(A-1)

\[ \Pi^B|_{I=1} = \frac{(1 - \gamma_f^B - \gamma_s^B)\frac{\alpha}{2}(\hat{\Pi}^B - \hat{C}^B) + \gamma_s^B V - (1 + r)x^B}{(1 + r) - (1 - \gamma_f^B - \gamma_s^B)(1 - \alpha/2)} \]  
(A-2)

where \(\hat{\Pi}^A - \hat{C}^A\) is defined in equation (14) and \(\hat{\Pi}^B - \hat{C}^B\) is defined in equation (15), each with \(I\) set equal to 1. It is only rational for a VC to forecast that a future VC will invest even if the project has not improved if \(\Pi^A|_{I=1} \geq 0\) and \(\Pi^B|_{I=1} \geq 0\).

If, on the other hand, VCs forecast that other VCs will not invest then the project NPV becomes

\[ \Pi^A|_{I=0} = \frac{(1 - \gamma_f^A - \gamma_s^A)\frac{\alpha}{2}\hat{\Pi}^A + \gamma_s^A\frac{\alpha}{2}\hat{\Pi}^B}{(1 + r)} - x^A \]  
(A-3)

\[ \Pi^B|_{I=0} = \frac{(1 - \gamma_f^B - \gamma_s^B)\frac{\alpha}{2}\hat{\Pi}^B + \gamma_s^B V}{(1 + r)} - x^B \]  
(A-4)

But it is only rational to forecast other VCs will not invest if \(\Pi^A|_{I=0} < 0\) and \(\Pi^B|_{I=0} < 0\).

It is, of course, possible that for some parameters \(\Pi^A|_{I=1} \geq 0\) and \(\Pi^B|_{I=1} \geq 0\), while for others \(\Pi^A|_{I=0} < 0\) and \(\Pi^B|_{I=0} < 0\). However, for both equilibria to simultaneously hold for a project, after the project has crossed the first hurdle, it must be that case that

\[ (1 - \gamma_f^B - \gamma_s^B)\frac{\alpha}{2}\hat{C}^B \leq (1 - \gamma_f^B - \gamma_s^B)\frac{\alpha}{2}\hat{\Pi}^B + \gamma_s^B V - x^B(1 + r) < 0 \]  
(A-5)

and if the following condition holds then there will be two equilibria before the project has crossed the first hurdle

\[ (1 - \gamma_f^A - \gamma_s^A)\frac{\alpha}{2}\hat{C}^A + \gamma_s^A\alpha\frac{\alpha}{2}\hat{\Pi}^B - \gamma_s^A(1 - \alpha/2)\Pi^B|_{I=1} \leq (1 - \gamma_f^A - \gamma_s^A)\frac{\alpha}{2}\hat{\Pi}^A + \gamma_s^A\frac{\alpha}{2}\hat{\Pi}^B - (1 + r)x^A < 0 \]  
(A-6)

It is clearly possible for all inequalities to hold. When both hold the project’s funding depends on the signal \(I\). It is also possible that the first inequality in both equations (A-5) and (A-6) does not hold but the second inequality in both does hold. In this case the project never receives funding as it is always NPV negative. It is also possible that the second inequality in both equations does not hold.\(^{33}\) In which case the project always gets funding as it is always NPV positive so a ‘no-invest’ forecast is not rational.

Finally, it could be the case that two equilibria are possible before the project has crossed the first hurdle but not after, i.e., the project only faces financing risk in its early stages. In

\(^{33}\)In which case the first inequality in each does hold by definition.
which case

$$\Pi^A|_{I=0} = \frac{(1 - \gamma_f^A - \gamma_s^A)\frac{\alpha}{2} \hat{\Pi}^A + \gamma_s^A[(1 - \alpha/2)\Pi^B|_{I=1} + \frac{\alpha}{2}(\hat{\Pi}^B - \hat{C}^B)]}{(1 + r)} - x^A \quad (A-7)$$

And for this to be an equilibrium

$$(1 - \gamma_f^A - \gamma_s^A)\frac{\alpha}{2} \hat{\Pi}^A + \gamma_s^A[(1 - \alpha/2)\Pi^B|_{I=1} + \frac{\alpha}{2}(\hat{\Pi}^B - \hat{C}^B)] - x^A(1 + r) < 0. \quad (A-8)$$

And it could also be possible for the project to only face financing risk in the later stages. For this to be an equilibrium inequality (A-5) would have to hold and

$$(1 - \gamma_f^A - \gamma_s^A)\frac{\alpha}{2} \hat{\Pi}^A + \gamma_s^A\frac{\alpha}{2} \hat{\Pi}^B - (1 + r)x^A > 0. \quad (A-9)$$

This might be a company for which a little money could initially make a lot of progress on the hope of a sale but $x^B$ was large enough that financing risk became a problem at later stages.

Thus at each stage in a companies life there are only two possible equilibria - the ‘invest’ equilibria in which each investor forecasts that the future VCs will invest or the ‘no-invest’ equilibria in which each investor forecasts that the future VCs will not invest. Q.E.D.

ii. Proof of Proposition 2:

It is only rational for a VC to forecast that a future VC will invest even if the project has not improved if $\Pi^A|_{I=1} \geq 0$ and $\Pi^B|_{I=1} \geq 0$.

$$\Pi^A|_{I=1} = \frac{(1 - \gamma_f^A - \gamma_s^A)\frac{\alpha}{2} (\hat{\Pi}^A - \theta \hat{C}^A) + \gamma_s^A[\theta(1 - \alpha/2)\Pi^B|_{I=1} + \frac{\alpha}{2}(\hat{\Pi}^B - \theta \hat{C}^B)]}{(1 + r) - (1 - \gamma_f^A - \gamma_s^A)\theta(1 - \alpha/2)} \quad (A-10)$$

and

$$\Pi^B|_{I=1} = \frac{(1 - \gamma_f^B - \gamma_s^B)[\theta(1 - \alpha/2)\Pi^B|_{I=1} + \frac{\alpha}{2}(\hat{\Pi}^B - \theta \hat{C}^B)] + \gamma_s^B(1 + r)x^B}{(1 + r) - (1 - \gamma_f^B - \gamma_s^B)\theta(1 - \alpha/2)} \quad (A-11)$$

If, on the other hand, VCs forecast that other VCs will not invest then the project NPV becomes

$$\Pi^A|_{I=0} = \frac{(1 - \gamma_f^A - \gamma_s^A)\frac{\alpha}{2}(\hat{\Pi}^A - (1 - \phi)\hat{C}^A)}{(1 + r) - (1 - \gamma_f^A - \gamma_s^A)(1 - \phi)(1 - \alpha/2)} + \frac{\gamma_s^A[(1 - \theta)(1 - \alpha/2)\Pi^B|_{I=0} + \frac{\alpha}{2}(\hat{\Pi}^B - (1 - \phi)\hat{C}^B)]}{(1 + r) - (1 - \gamma_f^A - \gamma_s^A)(1 - \phi)(1 - \alpha/2)} - x^A \quad (A-12)$$

and

$$\Pi^B|_{I=0} = \frac{(1 - \gamma_f^B - \gamma_s^B)[\theta(1 - \alpha/2)\Pi^B|_{I=0} + \frac{\alpha}{2}(\hat{\Pi}^B - (1 - \phi)\hat{C}^B)] + \gamma_s^B(1 + r)x^B}{(1 + r) - (1 - \gamma_f^B - \gamma_s^B)(1 - \phi)(1 - \alpha/2)} \quad (A-13)$$

But it is only rational to forecast other VCs will not invest if $\Pi^A|_{I=0} < 0$ and $\Pi^B|_{I=0} < 0$. 39
In the limit as $\theta \to 1$ and $\phi \to 1$ equation (A-10) goes to equation (A-1), and equation (A-11) goes to equation (A-2), and equation (A-12) goes to equation (A-3), and equation (A-13) goes to equation (A-4). Thus, the same parameters that produce two equilibria in proposition 1 (when $\theta = 1$ and $\phi = 1$) continue to produce two equilibria as long as $\theta$ and $\phi$ are close enough to 1.

Q.E.D.

iii. Proof of Proposition 3:

We begin by solving for the profit functions for early and late stage firms for any level of commitment. This can be done using an iterative expansion process or by simply multiplying each potential outcome by the probability it occurs. For a late stage investment with $N$ periods of committed capital, $n = N$, there could be success, failure, a sale or the commitment could run out and the project would continue only if the state was $I = 1$ at the time the money ran out. So for any $N \geq 1$

$$\Pi^B \bigg|_{I^n} = \left( \frac{\gamma^B_V (1 - \gamma^B_f - \gamma^B_s)(1 - \frac{\alpha}{2})}{1 + r} \right) \sum_{i=0}^{N-1} \frac{(1 - \gamma^B_f - \gamma^B_s)^i (1 - \frac{\alpha}{2})^i}{(1 + r)^i} + \Pi^B \bigg|_{I^0} \frac{(1 - \gamma^B_f - \gamma^B_s)^N (1 - \frac{\alpha}{2})^N}{(1 + r)^N} \left[ Z \ Y \right] S^{N-1} \left[ \begin{array}{c} 1 \\ 0 \end{array} \right]$$

$$+ \sum_{i=1}^{N} \left( \hat{\Pi}^B - \left[ Z \ Y \right] S^{i-1} \left[ \hat{C}^B \bigg|_{I^{N-i}} \right] \right) \frac{\alpha (1 - \gamma^B_f - \gamma^B_s)^i (1 - \frac{\alpha}{2})^{i-1}}{(1 + r)^i}$$

where $Z = I \theta + (1 - I)(1 - \phi)$ and $Y = I (1 - \theta) + (1 - I) \phi$. The first term accounts for the possibility that the project might succeed in any period and investors must pay $x^B$ until it succeeds, is sold, or fails. The second term accounts for the possibility that after $N$ periods the firm has neither progressed nor been bought or failed. If the state is $I = 0$ then the project is worth zero at that point but if $I = 1$ then the project is worth $\Pi^B \bigg|_{I=1}$, which must be multiplied by the probability that the state is $I = 1$ which depends on the initial state. And the final term is the value that comes from a negotiation. This depends on when the negotiation happens because as the commitment runs out $\hat{C}^B$ falls and the acquirer is willing to pay less for a less well funded project.

For an early stage project there are more possible endings but the logic is the same. The firm can fail, succeed and become late stage, or continue on, or be bought. If it becomes late stage then the same outcomes as noted above are possible. Thus, each potential outcome must be multiplied by the probability it occurs so for any $N \geq 2$
\[
\Pi^A|_{I}^{n=N} = \frac{\gamma^A}{1+r} \left( \frac{\gamma^B V}{1+r} - x^B \right) \sum_{i=0}^{N-2} \left( \frac{1 - \gamma^A - \gamma^A_i(1 + \frac{\theta}{2})^i}{(1+r)^i} \sum_{j=0}^{N-2-i} \frac{1 - \gamma^B - \gamma^B_j(1 + \frac{\theta}{2})^j}{(1+r)^j} \right) \\
+ \gamma^A \Pi^B|_{I}^{n=0} \sum_{i=0}^{N-1} \frac{(1 - \gamma^A - \gamma^A_i(1 + \frac{\theta}{2})^i)}{(1+r)^i} \frac{(1 - \gamma^B - \gamma^B_j(1 + \frac{\theta}{2})^j)^N(1 + \frac{\theta}{2})^{N-i}}{(1+r)^j} [Z \ Y] S^{N-1} \left[ \begin{array}{c} 1 \\ 0 \end{array} \right] \\
+ \Pi^A|_{I}^{n=0} (1 - \gamma^A - \gamma^A_N(1 + \frac{\theta}{2})^N) (1 + \frac{\theta}{2})^{N-i} [Z \ Y] S^{N-1} \left[ \begin{array}{c} 1 \\ 0 \end{array} \right] - x^A \sum_{i=0}^{N} \frac{(1 - \gamma^A - \gamma^A_i(1 + \frac{\theta}{2})^i)}{(1+r)^i} (1 + \frac{\theta}{2})^{N-i} \right) \\
+ \sum_{i=1}^{N} \left( \hat{\Pi} - [Z \ Y] S_i \left[ \frac{\hat{C}^A|\theta=1_{N-1-i}}{\hat{C}^A|\theta=0_{i=0}} \right] \right) (1 + \frac{\theta}{2})^{i-1} N \right) (1 + \frac{\theta}{2})^{i} \
\right] \right)
\] (A-15)

where \( Z = I \theta + (1 - I)(1 - \phi) \) and \( Y = I(1 - \theta) + (1 - I) \phi \).

Furthermore, in both \( \Pi^A|_{I}^{n=N} \) and \( \Pi^B|_{I}^{n=N} \) for \( n = N \geq 1 \),

\[
\hat{C}^B|_{I}^{n=N} = -\frac{\gamma^B V \hat{\Phi}}{1+r} \sum_{i=0}^{N-1} \frac{(1 - \gamma^B - \gamma^B_i)}{(1+r)^i} + \hat{C}^B|_{I=1}^{n=0} (1 - \gamma^B - \gamma^B_N) \frac{(1 + \frac{\theta}{2})^{N-1}}{(1+r)^N} [Z \ Y] S^{N-1} \left[ \begin{array}{c} 1 \\ 0 \end{array} \right] \] (A-16)

and for \( n \geq 2 \)

\[
\hat{C}^A|_{I}^{n=N} = -\frac{\gamma^A \gamma^B V \hat{\Phi}}{(1+r)^2} \sum_{i=0}^{N-2} \frac{(1 - \gamma^A - \gamma^A_i)(1 + \frac{\theta}{2})^i}{(1+r)^i} \sum_{j=0}^{N-2-i} \frac{(1 - \gamma^B - \gamma^B_j)(1 + \frac{\theta}{2})^j}{(1+r)^j} + \gamma^A \hat{C}^B|_{I=1}^{n=0} \sum_{i=0}^{N-1} \frac{(1 - \gamma^A - \gamma^A_i)(1 + \frac{\theta}{2})^i(1 - \gamma^B - \gamma^B_j)^N(1 + \frac{\theta}{2})^{N-i}}{(1+r)^N} [Z \ Y] S^{N-1} \left[ \begin{array}{c} 1 \\ 0 \end{array} \right] \\
+ \hat{C}^A|_{I=1}^{n=0} \frac{(1 - \gamma^A - \gamma^A_N)}{(1+r)^N} [Z \ Y] S^{N-1} \left[ \begin{array}{c} 1 \\ 0 \end{array} \right] \] (A-17)

where \( \hat{C}^A|_{I=1}^{n=0} \) and \( \hat{C}^B|_{I=1}^{n=0} \) are defined in equations (22) and (23) respectively. And \( \hat{C}^A|_{I=1}^{n=1} \) is the same as the general \( \hat{C}^A|_{I=1}^{n=N \geq 2} \) just with the first term removed.

A project faces financing risk even with a commitment of \( n = N \) as long as \( \Pi^A|_{I=0}^{n=N} < 0 \) and/or \( \Pi^B|_{I=0}^{n=N} < 0 \). Any particular level of commitment may not make the project NPV
positive. However, the limit of $\Pi^B|_{I=0}^{n=N}$ as $N \to \infty$ is

$$\Pi^B|_{I=0}^{n=\infty} = \frac{(1 - \gamma_f^B - \gamma_s^B) [\frac{\tilde{\Pi} B - \tilde{C} B}{2}] + \gamma_s^B V - (1 + r) x^B}{(1 + r) - (1 - \gamma_f^B - \gamma_s^B)(1 - \alpha/2)}$$

(A-18)

which is just the value of the project without financing risk, and thus clearly NPV positive. Therefore, there is some $N < \infty$ such that $\Pi^A|_{I=0}^{n=N} > 0$. So investors will make an investment of $N x^B$ regardless of the signal $I$. So the project no longer has financing risk.

The limit of $\Pi^A|_{I=0}^{n=\infty}$ as $N \to \infty$ is

$$\Pi^A|_{I=0}^{n=\infty} = \frac{(1 - \gamma_f^A - \gamma_s^A) [\frac{\tilde{\Pi} A - \tilde{C} A}{2}] + \gamma_s^A [(1 - \alpha/2) \Pi^B|_{I=0}^{n=\infty} + \frac{\tilde{\Pi} B - \tilde{C} B}{2}] - (1 + r) x^A}{(1 + r) - (1 - \gamma_f^A - \gamma_s^A)(1 - \alpha/2)}$$

(A-19)

or just

$$\Pi^A|_{I=0}^{n=\infty} = \frac{(1 - \gamma_f^A - \gamma_s^A) [\frac{\tilde{\Pi} A - \tilde{C} A}{2}] + \gamma_s^A [(1 - \alpha/2) \Pi^B|_{I=0}^{n=\infty} + \frac{\tilde{\Pi} B - \tilde{C} B}{2}] - (1 + r) x^A}{(1 + r) - (1 - \gamma_f^A - \gamma_s^A)(1 - \alpha/2)}$$

(A-20)

which is just the value of the project without financing risk, and thus clearly NPV positive. Q.E.D.

iv. Proof of Proposition 4:

With complete contracts we know from Proposition 3 that the value of the project with commitment is either equations (A-14) or (A-15).

With incomplete contracts the profit functions with commitment for $n = N$ is

$$\Pi^B|_{I=1}^{n=N} = \frac{(\gamma_s^B V}{1 + r} - x^B) \sum_{i=0}^{N-1} \frac{(1 - \gamma_f^B - \gamma_s^B)^i (1 - \alpha/2)^i}{(1 + r)^i}$$

(A-21)

$$+ \Pi^B|_{I=1}^{n=0} \frac{(1 - \gamma_f^B - \gamma_s^B)^N (1 - \alpha/2)^N}{(1 + r)^N} [Z \ Y] S^{N-1} \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$+ \sum_{i=1}^{N} \left( \tilde{\Pi}^B - [Z \ Y] S^{i-1} \begin{bmatrix} \tilde{C}^B|_{I=1}^{N-1} \\ \tilde{C}^B|_{I=0}^{i-1} \end{bmatrix} \frac{\alpha}{2} (1 - \gamma_f^B - \gamma_s^B)^i (1 - \alpha/2)^{i-1} (1 + r)^i \right)$$

$$- \frac{x^B \gamma_f^B}{1 + r} \sum_{i=0}^{N-2} \frac{(N - 1 - i) (1 - \gamma_f^B - \gamma_s^B)^i (1 - \alpha/2)^i}{(1 + r)^i}.$$
and for \( n \geq 3 \)

\[
\Pi^n_{A\mid r} = \gamma^A(1 - \frac{\alpha}{2}) \frac{(1 - r^N)}{1 + r} \left( \frac{\gamma^B V}{1 + r} - x^B \right) - \sum_{i=0}^{N-2} \left( \frac{(1 - \gamma^f - \gamma^A)^{(1 + \frac{\alpha}{2})^i}}{(1 + r)^i} \sum_{j=0}^{N-2-i} \left( \frac{(1 - \gamma^f - \gamma^A)^{(1 + \frac{\alpha}{2})^j}}{(1 + r)^j} \right) \right) \\
+ \gamma^A \Pi^n_{B\mid r} = \sum_{i=0}^{N-1} \left( \frac{(1 - \gamma^f - \gamma^A)^{(1 + \frac{\alpha}{2})^i}}{(1 + r)^i} \right) - x^A \sum_{i=0}^{N-1} \left( \frac{(1 - \gamma^f - \gamma^A)^{(1 + \frac{\alpha}{2})^i}}{(1 + r)^i} \right)
\]

\[
\frac{\gamma^A}{1 + r} \left[ (1 - \gamma^f - \gamma^A)^{(1 + \frac{\alpha}{2})} \right] (1 + r)^i \sum_{j=0}^{N-1-i} \left( \frac{(1 - \gamma^f - \gamma^A)^{(1 + \frac{\alpha}{2})^j}}{(1 + r)^j} \right)
\]

\[
\sum_{i=1}^{N} \left( \frac{(1 - \gamma^f - \gamma^A)^{(1 + \frac{\alpha}{2})^i}}{(1 + r)^i} \right) \left( \frac{(1 - \gamma^f - \gamma^A)^{(1 + \frac{\alpha}{2})^j}}{(1 + r)^j} \right)
\]

where \( Z = I(1 - \theta) + (1 - I)(1 - \phi) \) and \( Y = I(1 - \theta) + (1 - I)\phi \). For \( n = 2 \) the last term in equation (A-22) must be dropped. And for \( n < 2 \) there is no difference between complete and incomplete contracts.

Thus, the difference between complete and incomplete contracts is (A-21) - (A-14)

\[
-x^B \gamma^B \sum_{i=0}^{N-2} \left( \frac{(1 - \gamma^f - \gamma^A)^{(1 + \frac{\alpha}{2})^i}}{(1 + r)^i} \right)
\]

and (A-22) - (A-15)

\[
x^A \gamma^A \sum_{i=0}^{N-1} \left( \frac{(1 - \gamma^f - \gamma^A)^{(1 + \frac{\alpha}{2})^i}}{(1 + r)^i} \right)
\]

\[
-x^B \gamma^B \sum_{i=0}^{N-3} \left( \frac{(1 - \gamma^f - \gamma^A)^{(1 + \frac{\alpha}{2})^i}}{(1 + r)^i} \right) \sum_{j=0}^{N-3-i} \left( \frac{(1 - \gamma^f - \gamma^A)^{(1 + \frac{\alpha}{2})^j}}{(1 + r)^j} \right)
\]
Thus, the value of committing and extra $x^j$ with incomplete contracts is less than with complete contracts. Furthermore, the derivative of both equation (A-21) with respect to $\gamma^B_f$ and equation (A-22) with respect to $\gamma^B_f$ or $\gamma^A_f$ is negative. Therefore, the reduction in value from incomplete contracts is larger for projects with larger $\gamma^B_f$. Q.E.D.