The Declining Equity Premium: What Role Does Macroeconomic Risk Play?*

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Abstract

Aggregate stock prices, relative to virtually any indicator of fundamental value, soared to unprecedented levels in the 1990s. Even today, after the market declines since 2000, they remain well above historical norms. Why? We consider one particular explanation: a fall in \textit{macroeconomic risk}, or the volatility of the aggregate economy. Empirically, we find a strong correlation between low frequency movements in macroeconomic volatility and low frequency movements in the stock market. To model this phenomenon, we estimate a two-state regime switching model for the volatility and mean of consumption growth, and find evidence of a shift to substantially lower consumption volatility at the beginning of the 1990s. We then use these estimates from post-war data to calibrate a rational asset pricing model with regime switches in both the mean and standard deviation of consumption growth. Plausible parameterizations of the model are found to account for a significant portion of the run-up in asset valuation ratios observed in the late 1990s.

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1 Introduction

It is difficult to imagine a single issue capable of eliciting near unanimous agreement among the many opposing cadres of economic thought. Yet if those who study financial markets are in accord on any one point, it is this: the close of the 20th century marked the culmination of the greatest surge in equity values ever recorded in U.S. history. Aggregate stock prices, relative to virtually any indicator of fundamental value, soared to unprecedented levels. At their peak, equity valuations were so extreme that even today, after the broad market declines since 2000, aggregate price-dividend and price-earnings ratios remain well above their historical norms. More formally, the recent run-up in stock prices relative to economic fundamentals is sufficiently extreme that econometric tests for structural change (discussed below) provide overwhelming evidence of a structural break in the mean price-dividend ratio around the middle of the last decade.

How can such persistently high stock market valuations be justified? One possible explanation is that the equity premium has declined (e.g., Blanchard (1993); Jagannathan, McGrattan, and Scherbina (2000); Fama and French (2002)). Thus, stock prices are high because future returns on stocks are expected to be lower. These authors focus less on the question of why the equity premium has declined, but other researchers have pointed to reductions in the costs of stock market participation and diversification (Heaton and Lucas (1999); Siegel (1999); Calvet, Gonzalez-Eiras, and Sodini (2003)).

In this paper, we consider an alternative explanation for the declining equity premium and persistently high stock market valuations: a fall in macroeconomic risk, or the volatility of the aggregate economy. It is convenient to illustrate how macroeconomic risk can affect asset prices by using a simple model, in which the stochastic discount factor, or pricing kernel, is equal to the intertemporal marginal rate of substitution in aggregate consumption, $C_t$. A classic specification assumes there is a representative agent who maximizes a time-separable power utility function given by $u(C_t) = C_t^{1-\gamma}/(1-\gamma)$, $\gamma > 0$. With this specification, the

\footnote{Some have suggested that shifts in corporate payout policies may have contributed to the dramatic run-up in price dividend ratios. This explanation seems unlikely to explain the full increase in financial valuation ratios, for two reasons. First, the price-earnings ratios remain unusually high. Second, although the number of dividend paying firms has decreased in recent years, large firms actually increased real cash dividend payouts over the same period; as a consequence, aggregate payout ratios exhibit no downward trend over the last two decades (DeAngelo, DeAngelo, and Skinner (2002); Fama and French (2001); Campbell and Shiller (2003)).}
Sharpe ratio, $SR_t$, may be written, to a first order approximation, as

$$SR_t \equiv \max_{\text{all assets}} \frac{E_t [R_{t+1} - R_{f,t+1}]}{\sigma_t (R_{t+1})} \approx \gamma \sigma_t (\Delta \log C_{t+1})$$

where $R_{f,t+1}$ is a riskless return known at time $t$, and $\sigma_t (\cdot)$ denotes the conditional standard deviation. This expression shows that macroeconomic risk plays a direct role in determining the equity premium: fixing $\sigma_t (R_{t+1})$, lower consumption volatility, $\sigma_t (\Delta \log C_{t+1})$, implies a lower equity premium and a lower Sharpe ratio. Of course, this stylized model has important limitations, but its very simplicity serves to illustrate the crucial point: macroeconomic risk plays an important role in determining asset values. Below, we investigate these issues using a more complete asset pricing model.

The idea that changing volatility of consumption or aggregate cash-flows can affect asset prices and equity premia has a long-standing place in the asset pricing literature. Early work investigating this volatility channel includes Barsky (1986), Abel (1988), Giovannini (1989), Kandel and Stambaugh (1989, 1990) and Gennette and Marsh (1992). More recently, Bansal and Yaron (2004) have taken this idea to a model of recursive preferences of the type explored by Epstein and Zin (1989, 1991) and Weil (1989), showing that a reduction in consumption volatility can raise asset prices if the intertemporal elasticity of substitution is greater than unity. They model conditional volatility of monthly consumption growth as a GARCH process and use it to explain predictability observed in one- to five-year excess stock market returns. Bansal and Lundblad (2002), Bansal, Khatchatrian, and Yaron (2005), and Duffee (2005) further explore theoretical and empirical links between second moments of consumption growth, equity valuation ratios, and returns.

In this paper, we follow Bansal and Yaron (2004) in using Epstein-Zin-Weil preferences with the intertemporal elasticity of substitution in consumption greater than one to study the influence of a decline in macroeconomic risk on aggregate stock prices. But, we differ from this and previous studies in the focus of our investigation. Rather than using changing volatility to explain stationary fluctuations in risk premia that occur over periods ranging from a month to a few years, we focus on the apparent nonstationary regime change, or structural break, in asset prices relative to measures of fundamental value that occurred in the late 1990s. To this end, we depart from the previous literature in the way we model changing consumption volatility, moving away from specifications in which all volatility observations are generated from a single distribution with stationary variance, toward a specification in which volatility is drawn from a mixture of possibly very different distributions with constant variances. In short, to explain a regime change in asset valuations, this paper appeals to a
regime change in macroeconomic risk.

Our model also differs from the previous literature in that we emphasize learning. We adopt a model similar to that of Veronesi (1999)—who studies learning about the mean of asset returns—and show that allowing for learning about macroeconomic volatility can explain both the speed of the run-up in asset prices during the 1990s, as well as the fact that stock market volatility over this period has risen rather than declined.

In modeling macroeconomic risk in this manner, we draw on extensive body of work in the macroeconomic literature that finds evidence of a regime shift to lower volatility of real macroeconomic activity occurring in the last 15 years of the 20th century (Kim and Nelson (1999), McConnell and Perez-Quiros (2000), Kim and Nelson (1999), Blanchard and Simon (2001), Stock and Watson (2002)). Stock and Watson (2002) conclude that the decline in volatility has occurred broadly across sectors of the aggregate economy. It appears in employment growth, consumption growth, inflation and sectoral output growth, as well as in GDP growth in domestic and international data.\(^2\) It is large and it is persistent. Reductions in standard deviations are on the order of 60 to 70 percent relative to the 1970s and 1980s, and the marked change seems to be better described as a structural break, or regime shift, than a gradual, trending decline. The macroeconomic literature is currently involved in an active debate over the cause of this sustained volatility decline.\(^3\)

The subject of this paper is not the cause of the volatility decline, but its possible consequences for the U.S. aggregate stock market. Indeed, it would be surprising if asset prices were not affected by this fundamental change in the structure of the macroeconomy.

The empirical part of this paper follows much of the macroeconomic literature and characterizes the decline in volatility by estimating a regime switching model for the standard deviation and mean of consumption growth. The estimation produces evidence of a shift to substantially lower consumption volatility at the beginning of the 1990s. The theoretical part of our study investigates a learning model with regime switches in both the mean and standard deviation of consumption growth, calibrated to match our estimates from post-war data.\(^4\) We assume that agents cannot observe the regime but must infer it from consump-

\(^2\)Measurement techniques vary both by series and country, so it is unlikely that a reduction in measurement error has caused the decline in volatility.

\(^3\)See Stock and Watson (2002) for a survey of this debate in the literature.

\(^4\)A number of existing papers use theoretical techniques related to those employed here to investigate a range of asset pricing questions. One group of papers investigates asset pricing when there is a discrete-state Markov switching process in the conditional mean of the endowment process; for example, Cecchetti, Lam, and Mark (1990); Kandel and Stambaugh (1991); Cecchetti, Lam, and Mark (1993); Abel (1994); Abel
tion data; this learning aspect is an important feature of the model, discussed further below. Feeding in the (estimated) historical posterior probabilities of being in low and high volatility and mean states, we find plausible parameterizations of the model that can account for an important fraction of the run-up in price-dividend ratios observed in the late 1990s. The model’s predicted valuation ratios move higher in the 1990s because the long-run equity premium declines, a direct consequence of the persistent decline in macroeconomic risk in the early part of the decade. Finally, although the volatility of consumption declines in the 1990s, the model predicts that the volatility of equilibrium stock returns does not–consistent with actual experience.5

The rest of this paper is organized as follows. In the next section we present empirical results documenting regime changes in the mean and volatility of measured consumption growth. We then explore their statistical relation with movements in measures of the price-dividend ratio for the aggregate stock market. Next, we turn to an investigation of whether the observed behavior of the stock market at the end of the last century can be generated from rational, forward looking behavior, as a result of the decline in macroeconomic risk. Section 3 presents an asset pricing model that incorporates shifts in regime, and evaluates how well it performs in explaining the run-up in stock prices during the 1990s. Here we emphasize that the fraction of the 1990s equity boom that can be rationalized by declining macroeconomic volatility depends on the perceived persistence of the volatility decline. Section 4 concludes.

(1999); Veronesi (1999) (also discussed below); Whitelaw (2000); Wachter (2003), or in technology shocks (Cagetti, Hansen, Sargent, and Williams (2002)). Bonomo and Garcia (1994, 1996) and Drifil and Sola (1998) allow for regime changes in the variance of macroeconomic fundamentals, but their sample ends in 1985 and therefore excludes the regime switch in macroeconomic volatility in the 1990s that is the focus of this study. Otk, Ravikumar, and Whiteman (2002) study the temporal distribution of consumption variance and its implications for habit-based asset pricing models. The study here, by contrast, focuses on low-frequency shifts in the overall level of volatility, rather than on shifts in its temporal composition.

5The literature has offered other possible explanations for the persistently high stock market valuations observed in the 1990s, including “irrational exuberance” (Shiller (2000)), higher intangible investment in the 1990s (Hall (2000)), changes in the effective tax rate on corporate distributions (McGrattan and Prescott (2002)), the attainment of peak saving years during the 1990s by the baby boom generation (Abel (2003)), and a redistribution of rents away from factors of production towards the owners of capital (Jovanovic and Rousseau (2003)). We view the story presented here as but one of several possible contributing factors to the stock market boom of the 1990s, and leave aside these alternative explanations in order to isolate the possible influence of declining macroeconomic volatility.
2 Macroeconomic Volatility and Asset Prices: Empirical Linkages

In this section we document the decline in volatility for consumer expenditure growth. We investigate the volatility decline in total per capita personal consumer expenditures (PCE). The series is in 1996 chain-weighted dollars. As has been argued elsewhere (e.g., Cecchetti, Lam, and Mark (1990)), the equilibrium model studied below—in which consumption equals output—is somewhat ambiguous as to the appropriate time-series for calibrating the endowment process. We use the broad PCE measure of consumption to calibrate the model, since it exhibits lower volatility at the beginning of the 1990s, by which time the vast majority of other macroeconomic time-series also exhibited a volatility decline (Stock and Watson (2002)). This is important because individual series will be an imperfect measure of the relevant theoretical concept provided by our model, and we are interested in when agents could have plausibly inferred that macroeconomic volatility reached a new, lower regime. The Appendix at the end of this paper gives a complete description of the data and our sources. Our data are quarterly and span the period 1952:1 to 2002:4. We focus our primary analysis on postwar data because prewar data on consumption and output are known to be measured with significantly greater error that exaggerates the size of cyclical fluctuations in the prewar period (Romer (1989)).

We begin by looking at simple measures of the historical volatility of this series. Figure 1 provides graphical evidence of the decline in volatility. The growth rates of this series is plotted over time along with (plus or minus) two standard deviation error bands in each estimated volatility “regime,” where a regime is defined by the estimated two-state Markov switching process described below. (For the purposes of this figure, a low volatility regime is defined to be a period during which the posterior probability of being in a low volatility state is greater than 50 percent.) The figure clearly shows that volatility is lower in the 1990s than previously.

Another way to see the low frequency fluctuations in macroeconomic volatility is to look at volatility estimates for non-overlapping five-year periods. Figure 2 (top panel) plots the standard deviation of consumption growth for non-overlapping five-year periods. There is a significant decline in volatility in the five-year window beginning in 1992, relative to the immediately preceding five-year window. In particular, the series is about one-half as volatile in the 1990s as it is in the whole sample. To illustrate how these movements in volatility are related to the stock market, this panel also plots the mean value of the log dividend-price
ratio in each five year period. Our measure of the log dividend-price ratio for the aggregate stock market is the corresponding series on the CRSP value-weighted stock market index. The bottom panel of Figure 2 plots the same, but with the log earnings-price ratio in place of the log dividend-price ratio. The data for the price-earnings ratio is taken from Robert Shiller’s Yale web site. The figure shows how these low frequency shifts in macroeconomic volatility are related to low frequency movements in the stock market.

Figure 2 exhibits a striking correlation between the low frequency movements in macroeconomic risk and the stock market: both volatility and the log dividend-price ratio (denoted $d_t - p_t$) are high in the early 1950s, low in the 1960s, high again in the 1970s, and then begin falling to their present low values in the 1980s. The correlation between consumption volatility and $d_t - p_t$ presented in this figure is 72 percent. A similar picture holds for the price-earnings ratio (bottom panel).

In previous work, Bansal, Khatchatrian, and Yaron (2005) investigate higher frequency, quarterly price-dividend ratios and find that they are predicted by quarterly GARCH volatility measures, for the U.S., U.K., Germany, and Japan. Analogously, we find here that low frequency correlations between high asset valuations and low volatility are present in countries other than the U.S. These results are reported in the working paper version of this paper (Lettau, Ludvigson, and Wachter (2005)), which plots the volatility estimates for non-overlapping five-year periods, along with the mean value of the log dividend-price ratio in each five year period, for ten countries: Australia, Canada, France, Germany, Italy, Japan, the Netherlands, Sweden, Switzerland, and the United Kingdom. The international data display a striking correlation between the low frequency movements in macroeconomic risk and the national stock market for the respective country, similar to that obtained for the U.S. For the vast majority of countries, the 1990s were a period of record-low macroeconomic volatility and record-high asset prices.

Moving back to U.S. data, Figure 3 shows that the strong correlation between macroeconomic volatility and the stock market is also present in prewar data. Although consistently constructed consumption data going back to the 1800s are not available, we do have access to quarterly GDP data from the first quarter of 1877 to the third quarter of 2002. The data are taken from Ray Fair’s web site, which provides an updated version of the GDP series constructed in Balke and Gordon (1989). Figure 3 plots estimates of the standard

\[ \frac{6}{\text{Replacing the mean with mid-point or end-points of } d_t - p_t \text{ in each five year period produces a similar picture.}} \]

\[ \frac{7}{\text{http://aida.econ.yale.edu/~shiller/data.htm}} \]

\[ \frac{8}{\text{http://fairemodel.econ.yale.edu/RAYFAIR/PDF/2002DTBL.HTM}} \]
deviation of GDP growth for non-overlapping ten year periods along with the mean value of
the log dividend-price ratio in each ten year period, for whole decades from 1880 to 2000.
The absolute value of GDP volatility in prewar data must be viewed with caution. We focus
our primary analysis on postwar data in this paper because the quality of pre-war macro
data is low, tending to overstate volatility in the real series. In addition, consistent data
collection methodologies were not in place until the postwar period. While these factors cer-
tainly affect the overall magnitude of measured volatility in prewar data, they are unlikely
to have an important influence on measured correlations. From this perspective, Figure 3
is informative: we see that the strong correlation between macroeconomic volatility and the
stock market is not merely a feature of postwar data or of a single episode in the 1990s.
Rather, it present in over a century of data spanning the period since 1880.

To characterize the decline in macroeconomic volatility more formally, the macroeconomic
literature has generally taken two approaches: (i) tests for structural breaks in the variance
at an unknown date, and (ii) estimates from a regime switching model.\textsuperscript{9} We follow both
of these approaches here. Table 1 provides the results of undertaking structural break tests
for the volatility of each consumption measure described above, and for the mean of the
price-dividend ratio on the CRSP value-weighted index.\textsuperscript{10} Notice that these tests test the
hypothesis of a permanent shift in the volatility or mean of the series in question. The top
panel of Table 1 shows the results of a test for the break in the variance of consumption
growth using the Quandt (1960) likelihood ratio (QLR) statistic employed by Stock and

\textsuperscript{9}As noted, previous work has modeled changes in volatility using a GARCH process. Such processes are
useful for describing higher frequency, stationary fluctuations in variance, but are inappropiate for describing
very infrequent, prolonged shifts to a period of moderated volatility like that observed at the end of the last
century. For example, GARCH models do not generate the observed magnitude of volatility decline during
this period. Intuitively, the GARCH model does a reasonable job of modelling changes in volatility \textit{within}
regimes, once those have been identified by other procedures, but does not adequately capture infrequent
movements in volatility across regimes.

GARCH affects in consumption have been investigated in correlations as well as variances. Duffee (2005),
finds that the conditional correlation between stock returns and consumption growth fluctuates over time
and reached a peak at the end of 2000. It is important to note that these findings–of interest in their own
right–do not necessarily contradict the conclusions of this paper. Separately adding high-frequency changes
in conditional correlations and/or volatility to the model explored below would complicate our analysis but
would not change our basic result as these high-frequency changes would still be dominated by the large,
low frequency shift in volatility that occurred at the end of the sample.

\textsuperscript{10}See Lettau and Van Nieuwerburgh (2005) for a recent study of the affects of structural breaks on the
forecasting power of the price-dividend ratio for excess returns.
The null hypothesis of no break is tested against the alternative of one. The null hypothesis of no break in the variance is rejected at the 1% significance level for consumption. The break date is estimated to be 1992:Q1, with 67% confidence intervals equal to 1991Q3-1994Q4.\textsuperscript{12} Note that these tests, unlike estimates from the regime switching model discussed below, are ex post dating tests that use the whole sample and are therefore not appropriate for inferring the precise timing of when agents would most likely have assigned a high probability of being in a new, low volatility regime. Nevertheless, they provide evidence of a persistent shift down in macroeconomic volatility in our sample and give us a sense of when that break may have actually occurred.

The bottom panel of Table 1 presents results from considering a sup\textsuperscript{F} type test (Bai and Perron (2003)) of no structural break versus one break in the mean of the price-dividend ratio.\textsuperscript{13} The sup\textsuperscript{F} test statistic is highly significant (with a \textit{p}-value less than 1%), implying structural change in the price-dividend ratio. The break date is estimated to be 1995:Q1, with a 90 percent confidence interval of 1994:Q1 to 1999:Q3. The mean price-dividend ratio before the break is estimated to be 28.22; after the break, the mean is estimated to be 66.69, an over two-fold increase. It is interesting that the break date is estimated to occur after the estimated break dates for consumption volatility, consistent with the learning model we present below.

Next, we follow Hamilton (1989) and much of the macroeconomic literature in using our postwar data set to estimate a regime-switching model based on a discrete-state Markov process.\textsuperscript{14} This approach has at least two advantages over the structural break approach for our application. First, the structural break approach assumes that regime shifts are literally

\textsuperscript{11}This test also allows for shifts in the conditional mean, by estimating an autoregression that allows for a break in the autoregressive parameters at an unknown date.

\textsuperscript{12}As Stock and Watson point out, the break estimator has a non-normal, heavy-tailed distribution that renders 95\% confidence intervals so wide as to be uninformative. Thus, we follow Stock and Watson (2002) and report the 67\% confidence intervals for this test.

\textsuperscript{13}The linear regression model has one break and two regimes:

\[ y_t = z_t \tau_j + u_t \quad t = T_{j-1} + 1, ..., T_j, \]

for \( j = 1, 2 \), where \( y_t \) denotes the price-dividend ratio here, \( z_t \) is a vector of ones and the convention \( T_0 = 0 \) and \( T_{m+1} = T \) has been used. The procedure of Bai and Perron (2003) is robust potential serial correlation and heteroskedasticity both in constructing the confidence intervals for break dates, as well as in constructing critical values for the sup\textsuperscript{F} statistic for the test of the null of no structural change.

\textsuperscript{14}We focus on the larger U.S. dataset for this procedure, as it is known to require a large number of data points to produce stable results.
permanent; by contrast, the regime switching model provides a quantitative estimate of how long changes in regime are expected to last, through estimates of transition probabilities. Second, unlike the structural break estimates, the regime switching model allows one to treat the underlying state as latent, and provides an estimate of the posterior probability of being in each state at each time $t$, formed using only observable data available at time $t$. The estimates from this regime-switching model will serve as a basis for calibrating the asset pricing model we explore in the next section.

Consider a time-series of observations on some variable $C_t/C_{t-1}$ and let lowercase letters denote log variables, i.e., $\Delta c_t \equiv \log C_t/C_{t-1}$. A common empirical specification takes the form

$$\Delta c_t = \mu(S_t) + \epsilon_t$$ (1)

$$\epsilon_t \sim N \left(0, \sigma^2(V_t) \right),$$

where $S_t$ and $V_t$ are latent state variables for the states of mean and variance and $\Delta c_t$ denotes the log difference of consumption. We assume that the probability of changing mean states is independent of the probability of changing volatility states, and vice versa. To model the volatility reduction we follow the approach taken in the macroeconomic literature (e.g., Kim and Nelson (1999), McConnell and Perez-Quiros (2000)), by allowing the mean and variance of each series to follow independent, two-state Markov switching processes. It follows that there are two mean states, $\mu_l \equiv \mu(S_l) \in \{\mu_l, \mu_h\}$ and two volatility states, $\sigma_l \equiv \sigma(V_l) \in \{\sigma_l, \sigma_h\}$, where $l$ denotes the low state and $h$ the high state.\footnote{Although a greater number of states could be entertained in principle, there are important practical reasons for following the existing macro literature in a two-state process. On the empirical side, more regimes mean more parameters and fewer observations within each regime, increasing the burden on a finite sample to deliver consistent parameter estimates. On the theory/implementation side, we use these empirical estimates to calibrate our regime switching model discussed below. The two-state model already takes several days to solve on a work-station computer; a three-state model would more than double the state space and would be computationally infeasible.} Note that independent regimes do not imply that the mean and volatility of consumption growth are themselves independent. Even with a single volatility regime, the volatility of consumption growth would be higher in recessions than in booms, because the probability of switching regimes is higher in the low mean state than in the high mean state. Note also that the posterior regime probabilities inferred by theoretical agents observing data, as well as by the econometrician, are not independent.
We denote the transition probabilities of the Markov chains
\[
\begin{align*}
P(\mu_t = \mu_h | \mu_{t-1} = \mu_h) &= p_{hh}^\mu \\
P(\mu_t = \mu_l | \mu_{t-1} = \mu_l) &= p_{ll}^\mu
\end{align*}
\]
and
\[
\begin{align*}
P(\sigma_t = \sigma_h | \sigma_{t-1} = \sigma_h) &= p_{hh}^\sigma \\
P(\sigma_t = \sigma_l | \sigma_{t-1} = \sigma_l) &= p_{ll}^\sigma
\end{align*}
\]
where the probabilities of transitioning between states are denoted \(p_{hl}^\mu = 1 - p_{lh}^\mu\) and \(p_{hh}^\mu = 1 - p_{hl}^\mu\) for the mean state, and \(p_{ll}^\sigma = 1 - p_{lh}^\sigma\) and \(p_{hh}^\sigma = 1 - p_{hl}^\sigma\) for the volatility state. Denote the transition probability matrices
\[
P^\mu = \begin{bmatrix} p_{hh}^\mu & p_{hl}^\mu \\ p_{lh}^\mu & p_{ll}^\mu \end{bmatrix}, \\
P^\sigma = \begin{bmatrix} p_{hh}^\sigma & p_{hl}^\sigma \\ p_{lh}^\sigma & p_{ll}^\sigma \end{bmatrix}.
\]
The parameters \(\Theta = \{\mu_h, \mu_l, \sigma_h, \sigma_l, P^\mu, P^\sigma\}\) are estimated using maximum likelihood, subject to the constraints \(p_{ij}^k \geq 0\) for \(i = l, h, j = l, h\) and \(k = \{\mu, \sigma\}\).

Let lower case \(s_t\) represent a state variable that takes on one of \(2^2 = 4\) different values representing the four possible combinations for \(S_t\) and \(V_t\). Equation (1) may be written as a function of the single state variable \(s_t\).

Since the state variable, \(s_t\), is latent, information about the unobserved regime must be inferred from observations on \(x_t\). Such inference is provided by estimating the posterior probability of being in state \(s_t\), conditional on estimates of the model parameters \(\Theta\) and observations on \(\Delta c_t\). Let \(Y_t = \{\Delta c_0, \Delta c_1, \ldots, \Delta c_t\}\) denote observations in a sample of size \(T\) based on data available through time \(t\). We call the posterior probability \(P\{s_t = j | Y_t, \hat{\Theta}\}\), where \(\hat{\Theta}\) is the maximum likelihood estimate of \(\Theta\), the state probability for short.

The estimation results are reported in Table 2. The regime represented by \(\mu(S_t) = \mu_h\) has average consumption growth equal to 0.623% per quarter, whereas the regime represented by \(\mu(S_t) = \mu_l\), has an average growth rate of -0.323% per quarter. Thus, the high growth regime is an expansion state and the low growth regime a contraction state. These fluctuations in the conditional mean growth rate of consumption mirror cyclical variation in the macroeconomy.

The volatility estimates give a sense of the degree to which macroeconomic risk varies across regimes. For example, the high volatility regime represented by \(\sigma(V_t) = \sigma_h\), has
residual variance of 0.556 per quarter, whereas the low volatility regime represented by 
$\sigma(V_t) = \sigma_l$ has the much smaller residual variance of 0.163 per quarter. This corresponds 
to a 46 percent reduction in the standard deviation of consumer expenditure growth. The 
results for GDP growth (not reported) qualitatively similar.

How persistent are these regimes? The probability that high mean growth will be followed 
by another high mean growth state is 0.97, implying that the high mean state is expected to 
last on average about 33 quarters. The volatility states are more persistent than the mean 
states. The probability that a low volatility state will be followed by another low volatility 
state is 0.991, while the probability that a high volatility state will be followed by another 
high volatility state is 0.994. This implies that the low volatility state reached in the 1990s 
is expected to last about 125 quarters, over 30 years. In fact, a 95% confidence interval 
includes unity for these values, so we cannot rule out the possibility empirically that the low 
macroeconomic volatility regime is an absorbing state, i.e., expected to last forever. This 
characterization is consistent with that in the macroeconomic literature, which has generally 
viewed the shift toward lower volatility as a very persistent, if not permanent, break.

Figure 4 shows time-series plots of the smoothed and unsmoothed posterior probabilities 
of being in a low volatility state, $P(\sigma_t = \sigma_l)$, along with the smoothed and unsmoothed 
probabilities of being in a high mean state, $P(\mu_t = \mu_h)$.\footnote{$P(\sigma_t = \sigma_l)$ is calculated by summing the joint probabilities of all states $s_t$ associated with being in a low volatility state. $P(\mu_t = \mu_h)$ is calculated by summing the joint probabilities of all states $s_t$ associated with being in a high mean growth state.} consumption exhibits a sharp 
increase in the probability of being in a low volatility state at the beginning of the 1990s. 
Over a period of roughly six years, the probability of being in a low volatility state switches 
from essentially zero, where it resided for most of the post-war period prior to 1991, to unity, 
where it remains for the rest of the decade. Thus, the series shows a marked decrease in 
volatility in the 1990s relative to previous periods.

3 An Asset Pricing Model With Shifts in Macroeconomic Risk

The results in the previous section show that the shift toward lower macroeconomic risk co-
cincides with a sharp increase in the stock market in the 1990s. We now investigate whether 
such a relation can be generated in a model of rational, forward-looking agents. To do so, our 
primary analysis considers an asset pricing model augmented to account for regime switches
in both the mean and standard deviation of consumption growth, with the shifts in regime calibrated to match our estimates from post-war data. Modelling such shifts as changes in regime is an appealing device for addressing the potential impact of declining macroeconomic risk on asset prices, for several reasons. First, the macroeconomic literature has characterized the moderation in volatility as a sharp break rather than a gradual downward trend, a phenomenon that is straightforward to capture in a regime-switching framework (e.g., McConnell and Perez-Quiros (2000); Stock and Watson (2002)). Second, changes in regime can be incorporated into a rational, forward-looking model of behavior without regarding them as purely forecastable, deterministic events, by explicitly modelling the underlying probability law governing the transition from one regime to another. The probability law can be calibrated from our previous estimates of post-war consumption data. Third, the regime switching model provides a way of modelling how beliefs about an unobserved state evolve over time, by incorporating Bayesian updating. Finally, notice that the regime switching framework encompasses a structural break model as a special case, since the model is free to estimate transition probabilities that are absorbing states.

Consider a representative agent who maximizes utility defined over aggregate consumption. To model utility, we use the more flexible version of the power utility model developed by Epstein and Zin (1989, 1991) and Weil (1989). Let $C_t$ denote consumption and $R_{w,t}$ denote the simple gross return on the portfolio of all invested wealth. The Epstein-Zin-Weil objective function is defined recursively as

$$U_t = \left\{ (1 - \delta) C_t^{\frac{1-\gamma}{\alpha}} + \delta (E_t U_{t+1}^{1-\gamma})^{\frac{1}{\gamma}} \right\}^{\frac{\gamma}{1-\gamma}}, \quad (2)$$

where $\alpha \equiv (1 - \gamma) / (1 - 1/\psi)$, $\psi$ is the intertemporal elasticity of substitution in consumption (IES), $\gamma$ is the coefficient of relative risk aversion. We follow Campbell (1986) and Abel (1999), and assume that the dividend on equity, $D_t$, equals aggregate consumption raised to a power $\lambda$: \footnote{The main findings of this paper are robust to modeling consumption and dividends as cointegrated processes. The working paper version of this paper (Lettau, Ludvigson, and Wachter (2005)) provides results for a cointegrated model of consumption and dividends.}

$$D_t = C_t^\lambda. \quad (3)$$

When $\lambda > 1$, dividends and the return to equity are more variable than consumption and the return to aggregate wealth, respectively. Abel (1999) shows that $\lambda > 1$ can be interpreted as a measure of leverage. We refer to the dividend claim interchangeably as the levered
consumption claim. In what follows, we use lower case letters to denote log variables, e.g., \( \log(C_t) \equiv c_t \).

The specification (3) implies that the decline in the standard deviation of consumption growth in the 1990s should be met with a proportional decline in the volatility of dividend growth, \( \sigma(\Delta c_t) = \lambda \sigma(\Delta d_t) \). In fact, such a proportional decline is present in cash-flow data. The standard deviation of consumption growth declined of 43% from the period 1952:Q1 to 1989:Q4 to 1990:Q1 to 2002:Q4. In comparison, the standard deviation of Standard and Poor 500 dividend growth declined 58%,\(^{18}\) the standard deviation of NIPA dividends declined 42% and the standard deviation of NIPA Net Cash Flow declined 40%. We calibrate the model based on estimates of the consumption process, and model dividends as a scale transformation of consumption. This practice has an important advantage: we do not need to empirically model the short-run dynamics of cash-flows, which were especially affected in the 1990s by pronounced shifts in accounting practices, corporate payout policies, and in the accounting treatment of executive compensation.

To incorporate regime shifts in the mean and volatility of consumption growth, we impose the same model for the first difference of log consumption used in the estimation on historical consumption data:

\[
\Delta c_t = \mu(s_t) + \sigma(s_t) \epsilon_t, \tag{4}
\]

where \( \epsilon_t \sim N(0, 1) \) and \( s_t \) again represents a state variable that takes on one of \( N \) different values representing the possible combinations for the mean state \( S_t \) and the volatility state \( V_t \).

An important feature of our model is captured by the assumption that agents cannot observe the underlying state, but instead must infer it from observable consumption data. This learning aspect is also a feature of previous work, including Veronesi (1999), who studies an equilibrium model in which the mean of the endowment follows a latent two-state regime switching process. In our framework, learning is important because it implies that agents only gradually discover over time the very low frequency changes in volatility that occur in the data. As we shall see below, this assumption permits the framework to deliver a sustained rise in equilibrium asset prices in response to a low frequency reduction in volatility, rather than implying an abrupt, one-time jump in the stock market.\(^{19}\)

\(^{18}\)The data for Standard and Poor dividend growth are monthly from Robert Shiller’s web site. These data are not appropriate for calibrating the level of dividend volatility because the monthly numbers are smoothed by interpolation from annual data. But they can be used to compare changes in volatility across subsamples of the data, as we do here.

\(^{19}\)Our model should be contrasted with models in which there is learning, but a constant regime. In
When agents cannot observe the underlying state, inferences about the underlying state are captured by the posterior probability of being in each state based on data available through date $t$, given knowledge of the population parameters. Define the $N \times 1$ vector $\hat{\xi}_{t+1|t}$ of unsmoothed posterior probabilities in the following manner, where its $j$th element is given by

$$\hat{\xi}_{t+1|t}(j) = P \{ s_{t+1} = j \mid \mathbf{Y}_t; \Theta \} .$$

$\mathbf{Y}_t$ denotes a vector of all the data up to time $t$ and $\Theta$ contains all the parameters of the model. Throughout it will be assumed that a representative agent knows $\Theta$, which consequently will be dropped from conditioning statements unless essential for clarity.

Bayes’ Law implies that the posterior probability $\hat{\xi}_{t+1|t}$ evolves according to

$$\hat{\xi}_{t+1|t} = \mathbf{P} \frac{(\hat{\xi}_{t|t-1} \odot \eta_t)}{1'(\hat{\xi}_{t|t-1} \odot \eta_t)} ,$$

where $\odot$ denotes element-by-element multiplication, $\mathbf{1}$ denotes an $(N \times 1)$ vector of ones, $\mathbf{P}$ is the $N \times N$ matrix of transition probabilities and

$$\eta_t = \begin{bmatrix}
f(\Delta c_t \mid s_t = 1, \mathbf{Y}_{t-1}) \\
\vdots \\
f(\Delta c_t \mid s_t = N, \mathbf{Y}_{t-1})
\end{bmatrix}
$$

is the vector of Gaussian likelihood functions conditional on the state.

As in the econometric model, we assume that the mean and variance of consumption growth follow two-state Markov switching processes, implying that $s_t$ takes on one of four different values representing the $2^2 = 4$ possible combinations for the mean state $S_t$, and the variance state $V_t$.

As above, let $\mathbf{P}^\sigma$ be the $2 \times 2$ transition matrix for the variance and $\mathbf{P}^\mu$ be the $2 \times 2$ transition matrix for the means. Then the full $4 \times 4$ transition matrix is given by

$$\mathbf{P} = \begin{bmatrix}
p^\mu_{hh} \mathbf{P}^\sigma & p^\mu_{hl} \mathbf{P}^\sigma \\
p^\mu_{lh} \mathbf{P}^\sigma & p^\mu_{ll} \mathbf{P}^\sigma
\end{bmatrix} .$$

The elements of the four-state transition matrix can be calculated from the two-state transition matrices $\mathbf{P}^\mu$ and $\mathbf{P}^\sigma$. The theoretical model can therefore be calibrated to match such models, the agent eventually learns the state given enough data. By contrast, in our model the mean and volatility of consumption growth can each switch in every period between two values with non-zero probability. In fact, the mean state switches relatively frequently given our empirical estimates. The agent’s belief about what state she is in does not converge to zero or one because the probability of the state does not converge to zero or one.
our estimates of $P, \hat{\xi}_{t+1|t}$ and $\Theta$ from the regime switching model for aggregate consumption data, and closed as an general equilibrium exchange economy in which a representative agent receives the endowment stream given by the consumption process (4).

3.1 Pricing the Consumption and Dividend Claims

Let $P^D_t$ denote the ex-dividend price of a claim to the dividend stream measured at the end of time $t$. From the first-order condition for optimal consumption choice and the definition of returns

$$E_t [M_{t+1} R_{t+1}] = 1, \quad R_{t+1} = \frac{P^D_{t+1} + D_{t+1}}{P^D_t}$$

where $M_{t+1}$ is the stochastic discount factor, given under Epstein-Zin-Weil utility as

$$M_{t+1} = \left( \delta \left( \frac{C_{t+1}}{C_t} \right)^{-\frac{1}{\psi}} \right)^{\alpha} R^\alpha_{w,t+1}.$$  

Again, $R_{w,t+1}$ is the simple gross return on the aggregate wealth portfolio, which pays a dividend equal to aggregate consumption, $C_t$. The return on a risk-free asset whose value is known with certainty at time $t$ is given by

$$R^f_{t+1} \equiv (E_t [M_{t+1}])^{-1}.$$  

In contrast to Cecchetti et. al (1990, 2000) and Bonomo and Garcia (1994, 1996), we assume that investors do not observe the state $s_t$ directly, but must instead infer it from observable consumption data. Because innovations to consumption growth are i.i.d. conditional on state, and because agents cannot observe the underlying state, the posterior probabilities $\hat{\xi}_{t+1|t}$ summarize the information upon which conditional expectations are based. The price-dividend ratio for either claim may be computed by summing the discounted value of future expected dividends across states, weighted by the posterior probabilities of being in each state, and the price-dividend ratio for both the consumption and dividend claims are functions only of $\hat{\xi}_{t+1|t}$. An appendix available on the authors’ web sites explains how we solve for these functional equations numerically on a grid of values for the state variables $\hat{\xi}_{t+1|t}$.  

\footnote{In a model without learning, the work of Hung (1994) could be employed to check the numerical accuracy of our solution procedure. This procedure cannot be directly applied in our learning environment. However, when consumption and dividend growth are i.i.d., the price-consumption and price-dividend ratios have an analytical solution. In this case, the analytical solution gives the same answer as the numerical solution when}
Given the price-dividend ratio as a function of the state $\hat{\xi}_{t+1|t}$, we calculate the model’s predicted price-dividend ratio over time by feeding in our time-series estimates of $\hat{\xi}_{t+1|t}$ presented above. We also compute an estimate of the $L$ year equity premium (the difference between the equity return and the risk-free rate over an $L$-year period) as a function of time $t$ information. For $L$ large, this “long-run” equity premium is analogous to what Fama and French (2002) call the unconditional equity premium, as of time $t$.

3.2 Choosing Model Parameters

We calibrate the model above at a quarterly frequency. The rate of time-preference is set to $\delta = 0.9925$. The parameters of the consumption process, (4), are set to match the empirical estimates reported in Table 2. Other key parameters for our investigation are the leverage parameter, $\lambda$, the coefficient of relative risk aversion, $\gamma$, the IES, $\psi$, and the transition probabilities of staying in a high or low volatility state. We discuss these in turn.

To calibrate the transition probabilities, we use the empirical estimates from consumption data. The probability of remaining in the same volatility state next period is quite high and exceeds 0.99 regardless of whether the volatility state is high or low. Moreover, values as high as one for this parameter are equally plausible empirically: a 95% confidence interval for these estimates includes unity. Thus, the point estimates in Table 2 are statistically indistinguishable from those that would imply the low volatility regime reached in the 1990s is an absorbing state, and they coincide with evidence from the macroeconomic literature that the shift to lower macroeconomic volatility is well described as an extremely persistent, if not permanent, break (Kim and Nelson (1999), McConnell and Perez-Quiros (2000), Stock and Watson (2002)). Indeed, the reduction in volatility in the last decade has been dubbed “the great moderation,” by Stock and Watson (2002), consistent with a common perception that this is evidence not of a transitory decline in volatility, but as a structural change in the economy as a whole.

In order to capture a very persistent decline in macroeconomic volatility, we set $p_{hh}^\sigma = p_{ll}^\sigma = 0.9999$ for the baseline results, but we also examine the sensitivity of these results to alternative values in Table 3, discussed below. The transition probabilities for the mean state, $p_{hh}^\mu$ and $p_{ll}^\mu$ are set to their samples estimates for consumption.

To calibrate $\lambda$, following Abel (1999) we are guided by the sample standard deviation each of the four combinations of mean and volatility are absorbing states. We check our results by setting the probabilities of remaining in the mean state and volatility states to be very close to one and verifying that the numerical algorithm replicates the analytical results.
of dividend growth relative to that of consumption growth, $\lambda = \frac{\sigma(\Delta \ln D_t)}{\sigma(\Delta \ln C_t)}$. As discussed, this specification has support in the data in that the volatility of dividend growth has decreased by about the same proportion as that of consumption growth. As reported in Lettau and Ludvigson (2005), the percent standard deviation of real, per capita dividend growth constructed from CRSP index returns is 12.2 at an annual rate in post-war data, about 8 times as high as that of real, per capita consumption growth, equal to 1.52 percent.\textsuperscript{21} For our benchmark results we apply a more conservative estimate for this parameter, given by $\lambda = 4.5$, in order to help account for idiosyncratic variation in dividend growth not captured by our model.

To study the financial effects of a secular decline in macroeconomic risk, it is essential that the model economy we expose to such a shift be consistent with the average levels of the stock market and the equity premium. Therefore, we calibrate the coefficient of relative risk aversion, $\gamma$, in order to insure that our model is able to roughly match the mean equity premium, level of dividend-price ratio, and risk-free rate in post-war data. To do so, the model presented above requires moderately high risk aversion, around $\gamma = 30$. We use this value for the baseline results reported in this paper.

Finally, to choose parameter values for the IES, $\psi$, we consider how macroeconomic volatility influences the behavior of the equilibrium price-dividend ratio in the model presented above. A change in macroeconomic volatility has three affects on the equilibrium price-dividend ratio. First, regardless of the IES, lower macroeconomic volatility reduces the long-run equity premium because it lowers consumption risk; this effect drives up the price-dividend ratio. Second, lower macroeconomic volatility reduces the precautionary motive for saving, increasing the desire to borrow and therefore the equilibrium risk-free rate; this effect drives down the price-dividend ratio. Third, because we model a regime shift in the volatility of log changes, lower macroeconomic volatility reduces the mean of dividend growth in levels, $D_t/\ln D_{t-1}$, because of a Jensen’s inequality effect; this drives down the price-dividend ratio. The magnitude of the latter two effects relative to the first depends on the value of $\psi$ and $\gamma$. If $\gamma > 1$, the first effect will dominate the last two only if $\psi > 1$.

\textsuperscript{21}Abel (1999) calculated a smaller value for $\lambda$ (approximately 3), by calibrating his model to the 1889-1978 sample used in Mehra and Prescott (1985). The reason he obtained a smaller number is that this sample includes prewar consumption data, which is over three times as volatile, relative to dividends, as is postwar consumption data. But the greater volatility of prewar consumption data relative to postwar data has been attributed, not to greater volatility of economic fundamentals in the prewar period, but to greater measurement error in the prewar consumption and output series (Romer (1989)). For this reason, we calibrate our model to postwar consumption data.
The requirement that $\psi$ must be greater than one for a decline in volatility to raise asset prices has been pointed out in previous work by Bansal and Yaron (2004). Empirical estimates of $\psi$ using aggregate consumption data often suggest that the IES is relatively small, and in many cases statistically indistinguishable from zero (e.g. Campbell and Mankiw (1989), Ludvigson (1999), Campbell (2003)). But there are several reasons to think that the IES may be larger than estimates from aggregate data suggest. First, other researchers have found higher values for $\psi$ using cohort level data (Attanasio and Weber (1993), Beaudry and van Wincoop (1996)), or when the analysis is restricted to asset market participants using household-level data (Vissing-Jorgensen (2002)). More recently, Vissing-Jorgensen and Attanasio (2003) estimate the IES using the same Epstein-Zin framework employed in this study and find that this parameter for stockholders is typically above 1 (depending on the specification), with the most common values ranging from 1.17 to 1.75. Second, Bansal and Yaron (2004) suggest that estimates of $\psi$ based on aggregate data will be biased down if the usual assumption that consumption growth and asset returns are homoskedastic is relaxed. Third, Guvenen (2003) points out that macroeconomic models with limited stock market participation imply that properties of aggregate variables directly linked to asset wealth are almost entirely determined by stockholders who have empirically higher values for $\psi$. For the results reported below, we follow Bansal and Yaron (2004) and set $\psi = 1.5$, in the mid-range of the estimates reported by Vissing-Jorgensen and Attanasio (2003).

### 3.3 Model Results

In this section we present results from solving the model numerically. We focus on how stock prices are influenced by the break in macroeconomic volatility documented in the empirical macroeconomic literature. To this end, we characterize the behavior of the equilibrium price-dividend ratio of a claim to the dividend stream by plotting the model’s solution for this quantity as a function of the posterior probabilities, and by feeding the model the historical values of $\hat{\xi}_{t+1}$, estimated as discussed above.

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22 Vissing-Jorgensen emphasizes that even though estimates of the IES for non-asset holders are lower than those of asset holders, the difference should not be interpreted as evidence of heterogeneity in the IES across households. The reason is that estimates of the IES are based on Euler equations. Since the Euler equation for a given asset return cannot be expected to hold for households who do not have a position in the asset, IES estimates for non-asset holders will be inconsistent estimates of the IES for those households, and may be substantially biased down.

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3.3.1 Model Intuition

Figure 5 (left plot) presents the model’s predicted log price-dividend ratio of the dividend claim as a function of the posterior probability of being in a low volatility state, $\sigma = \sigma_l$, when the mean state is high either with probability one or with probability zero. The price-dividend ratio increases with the posterior probability of being in a low volatility state, regardless of whether the mean state is high or low. The increasing function is not linear, but is instead a convex function of investor’s posterior probability of being in the low volatility state.

The intuition for this convexity is similar to that given in Veronesi (1999) for an asset pricing model with regime shifts in the mean of the endowment process. Suppose the probability of being in a low consumption volatility state is initially zero. News that causes an increase in the posterior probability of being in a low volatility state has two effects on the price-dividend ratio. First, because investors believe that the probability of being in a low volatility state has risen, consumption risk is perceived to be lower, which works to decrease the equilibrium risk-premium and raise the price-dividend ratio. Second, because the probability of being in a low volatility state is farther from zero, investors are more uncertain about which volatility regime the economy is in, which works to lower the equilibrium price-dividend ratio. The two effects are offsetting. Consequently, as the posterior probability of being in a low volatility state increases from zero, the price-dividend ratio rises only modestly.

Conversely, suppose the probability of being in a low consumption volatility state is initially at unity. News that causes a decrease in this posterior probability again has two effects on the price-dividend ratio. First, consumption risk is perceived to be higher, which works to increase the equilibrium risk-premium and lower the price-dividend ratio. Second, because the probability of being in a low volatility state is now farther from unity, investors are more uncertain about which volatility regime the economy is in, which works to further lower the equilibrium price-dividend ratio. In this case, the two effects are reinforcing rather than offsetting. Consequently, as the posterior probability of being in a low volatility state declines from unity, the price-dividend ratio falls dramatically. This explains why the equilibrium price-dividend ratio is a convex function of the posterior probabilities. But the degree of convexity is affected by risk-aversion. The more risk-averse agents are, the higher the posterior probability of being in a low volatility state must be before it has a noticeable affect on the equilibrium price-dividend ratio.

Figure 5 (right plot) displays the log price-dividend ratio of the dividend claim as a
function of the posterior probability of being in a high mean growth state, \( \mu = \mu_h \), when the volatility state is high either with probability one or with probability zero. The price-dividend ratio increases with the posterior probability of being in a high mean state. For reasons similar to those just given, the function is again convex in the investor’s posterior probability of being in the high mean state, but is substantially less convex than the function plotted against the low volatility probability. The effect of a change in mean probability on the price-dividend ratio is also much smaller than the effect of a change in the volatility probability on the price-dividend ratio. These differences appear to be attributable to the lower persistence of the mean regimes compared to the volatility regimes. For example, the probability that a low mean (contraction) state will be followed by another period of contraction is 0.8 for consumption growth, so that this regime will persist on average for only 5 quarters. The estimated high mean, or expansion, regime is more persistent, but is still only expected to last 33 quarters on average. By contrast, the volatility regimes we estimate are far more persistent; thus asset prices can rise dramatically as investors become increasingly certain that a low macroeconomic volatility state has been reached.

3.3.2 Valuation Ratios in the 1990s

How well does this model capture the run-up in asset prices observed in the late 1990s? To address this question, we feed the model historical values of \( \xi_{t+1|t} \) for our post-war sample, 1951:Q4 to 2002:Q4. Figure 6 presents the actual log price-dividend ratio on the CRSP value-weighted index, along with the post-war history of the price-dividend ratio on the dividend claim implied by the model. The figure displays plots of the model’s prediction for \( p_t - d_t \) using the estimated unsmoothed posterior probabilities. Note that the “model” line in each graph is produced using only the posterior probabilities estimated from consumption data; no asset market data are used.

Using the historical values of the unsmoothed probabilities, Figure 6 shows that the benchmark model provides a remarkable account of the longer-term tendencies in stock prices over the period 1990-2002. In particular, it captures virtually all of the boom in equity values that began in the early 1990s and continued through the end of the millennium. In fact, the model’s predicted price-dividend ratio is almost identical to the actual price-dividend ratio reached at the end of 2002. Moreover, the increase in valuation ratios predicted by the model is not well described by a sudden jump upward, but instead occurs gradually over several years, as in the data. This is a result of the learning built in to the model by the assumption that agents cannot observe the underlying state directly. Thus, the model produces about
the right average value for stock returns during the 1990s.\textsuperscript{23}

What drives up the price-dividend ratio in the 1990s in this model? Although the shift to a higher mean growth state during this period generates a small part of the increase, the vast majority of the boom is caused by the shift to reduced macroeconomic volatility. This can be seen in Figure 5. The right panel shows that, fixing the volatility state, variation in the equilibrium price-dividend ratio across mean states is quite modest. For example, fixing the probability of being in a low volatility state at one, the log price-dividend ratio ranges between 3.24 (when the probability of being in a high mean state is zero), to 3.57 (when the probability of being in a high mean state is one). Thus, the maximum possible variation in \( p_t - d_t \) across mean states is about 10 percent. Fixing the probability of being in a low volatility state at zero, the maximum possible variation in \( p_t - d_t \) across mean states is even smaller, about 8 percent. This variation should be contrasted with the results for variation across volatility states, shown in the left panel. Fixing the probability of being in a high mean state at one, the log price-dividend ratio ranges between 3.57 (when the probability of being in a low volatility state is zero), to 4.34 (when the probability of being in a low volatility state is one), a range of variation of over 22 percent. Fixing the probability of being in a high mean state at zero, the maximum possible variation in \( p_t - d_t \) across volatility states is about 24 percent. In short, large swings in the price-dividend ratio in this model are generated not by shifts in the mean of the endowment process, but by changes in the posterior probability of being exposed to a less volatile endowment process.

\textbf{3.3.3 The Long-run Equity Premium}

To understand what happens to the equity premium in the model, we plot the \( L = 100 \)-year equity premium implied by the model, computed recursively from the one period equity premium. We refer to these values as measure of the “long-run,” or “unconditional” equity premium. Given that we calibrate the model at quarterly frequency, the \( L \)-year equity premium

\textsuperscript{23}The model we investigate is not designed to capture the higher frequency fluctuations observed in the log price-dividend ratio prior to 1990, or the degree to which the price-dividend ratio overshot its value at the end of our sample. One framework that is better able to capture these shorter-term, cyclical fluctuations in equity values is the model explored by Campbell and Cochrane (1999). A shortcoming of that model, however, lies with its inability to capture the extraordinary stock market boom in the 1990s and the low frequency movements in the price-dividend ratio that dominate its behavior at the close of the last century. The work by Bansal and Yaron (2004) suggests that our model could be augmented to better capture cyclical fluctuations in expected returns by combining GARCH effects in consumption volatility within regimes, with the low frequency regime switching affects we explore here.
premium is computed as the expectation as of year \( t \) of the annual compound rate of return from investing in the dividend claim from years \( t \) to \( t + L \), less the annual compound return from investing in the risk-free rate over years \( t \) to \( t + L \). (The technical Appendix provides details about how this quantity is computed numerically.) The model predicts that the equity premium declines as the probability of being in a low volatility state increases, and drops off sharply once that probability exceeds 90 percent. This results in a drop in the equity premium in the middle of the 1990s. Figure 7 plots the post-war history of the log annual (100-year) equity premium on the dividend claim implied by the model, obtained feeding in the history of estimated posterior probabilities. The model equity premium is relatively flat for most of the post-war period, but begins to decline in the early 1990s. For the benchmark specification, premium declines by a little under two percentage points from peak to trough. We should not be surprised that the percentage decline is not greater: even small changes in the equity premium can have a large impact on asset values if they are sufficiently persistent.

3.3.4 Additional Implications of the Model

We emphasize an additional aspect of this model: although the volatility of consumption declines in the 1990s, the volatility of equilibrium stock returns does not—consistent with actual experience. In fact, in the data, stock market volatility appears to be, if anything, slightly higher in the late 1990s than in much of the rest of the postwar sample.\(^2^4\) Figure 8 plots the post-war history of the conditional quarterly standard deviation of the log stock market return implied by the model at benchmark parameter values, reported at an annual rate. The figure shows that stock market volatility in the model is no lower in the 1990s than previously in the sample, despite the lower macroeconomic volatility. This result is attributable to the increased uncertainty about which volatility regime the economy was in during the transition from a high to low macroeconomic volatility state.

Three further aspects of the results above are worthy of note. First, the model’s predictions for the risk-free rate are reasonable. If we feed the model historical values of \( \hat{\xi}_{t+1|t} \) we may compute the post-war history of the risk-free rate predicted by the model. Using the baseline parameters discussed above and used to create the results in Figure 8, this rate has a has a mean of 1.44 percent per annum and a standard deviation of 0.35 percent per annum, in line with actual values for an estimate of the real rate of return on a short-term

\(^{24}\)Updated plots of volatility of aggregate stock market indexes are provided by G. William Schwert at his University of Rochester web site: http://schwert.ssb.rochester.edu/volatility.htm.
Second, the consumption-wealth ratio is far less affected than the price-dividend ratio by the shift to lower consumption volatility, because the price of an unlevered consumption claim is much less sensitive to swings in consumption risk than is the price of a levered claim. Results (not shown) show no appreciable structural change in the consumption-wealth ratio as a result of the low frequency shift toward lower consumption volatility.

Third, the term premium on real bonds in the benchmark model is small and its change, given the decline in volatility, is also small. We find small negative values for the term premium on the real bond as in Bansal and Yaron (2004); The term premium fluctuates around -1.0 percent per annum prior to the regime shift in volatility, and around -1.25 percent per annum after. This shows that neither the large equity premium in our model, nor its shift downward in the 1990s, are driven by term structure effects.

### 3.3.5 Persistence of Volatility Regime

We now explore how the model’s predictions change when we depart from the benchmark parameter values for $\lambda$ and, more importantly, the posterior probabilities $p_{jj}$, which denote the agent’s inference that next period’s volatility state will be $j$ given that this period’s volatility state is $j$. The results of permuting these parameters to other values within two-standard errors of the point estimate are summarized in Table 3, which exhibits the model’s predictions for the price-dividend ratio and the long-run (100-year) equity premium in 1990:Q1 (before the estimated volatility shift) and in 2002:Q4 (after the volatility shift), computed as before by feeding the model the historical values of the posterior probabilities. Many researchers have interpreted the shift toward lower macro volatility as a pure structural break, which in our model corresponds to making the new lower volatility state an absorbing one. Thus, the first row of Table 3 presents results for this case, when $p_{jj}$ is unity. The second row of Table 3 presents the results from our benchmark parameter values, used to generate the results reported in Figure 6. Subsequent rows show how those results are changed when we depart from the benchmark parameter configuration by assigning the values indicated in the first four columns of Table 3. Given that each of these values lie within a 95 percent confidence interval of the empirical estimates, a case can be made that they are all equally plausible.

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25 Empirical estimates of volatility of the risk-free rate are typically based on the annualized sample standard deviation of the ex post real return on US Treasury bills—about 2% per annum in postwar data. This figure likely overstates the true volatility of the ex ante real interest rate, however, since much of the volatility of these returns is due to unanticipated inflation.
Several notable aspects of the model are exhibited in Table 3. First, observe that the price-dividend ratio in the data rises from about 30 to 58.44 over the period 1990:Q1 to 2002:Q4, an increase of 28.44 (Figure 1). Row 1 of Table 3 shows that, if we assume a structural break, the model predicts an increase of 33 in the price-dividend ratio over this same period, larger than that observed. Thus this parameterization can more than explain the surge in asset values over the period, explaining 118 percent of the boom when $\lambda = 4.5$ and 104 percent when $\lambda = 3$. At benchmark parameter values, the model predicts a slightly lower value for the price-dividend ratio at the end of our sample than what actually occurred (51.48 versus 58.44), but explains 89 percent of the run-up in asset values. Row 4 shows what happens when the transition probability of remaining in the same volatility state next period is lowered to the statistically indistinguishable level of $p_{hh} = p_{ll} = 0.9999$; now the model predicts a run-up in stock prices over the period 1990:Q1 to 2002:Q4 equal to roughly 77 percent of that observed. For this parameterization, the equilibrium price-dividend ratio rises from 33 to 49. The sixth row shows what happens when we use the exact point estimates for these transition probabilities, which are also statistically indistinguishable from the benchmark values. In this case the model explains about 20 percent of the total run-up, with the equilibrium price-dividend ratio rising from 35 to 40. The result is essentially the same if we set $p_{hh} = p_{ll} = 0.99$, slightly lower than the point estimates (row 4). These findings illustrate the importance of the perceived permanence of the volatility decline in determining the magnitude of the rise in the equilibrium price-dividend ratio. Even a modest decrease in macroeconomic volatility can cause a dramatic boom in stock prices when the decrease is perceived to be sufficiently permanent.\textsuperscript{26}

In summary, if the volatility moderation is perceived to be very persistent–lasting many decades–a large fraction of the run-up in stock prices can be explained. If the volatility decline is expected to be more transitory, less of the run-up can be rationalized through this mechanism. Similar conclusions have been reached in previous work that has modeled quarterly changes in consumption volatility (e.g., Bansal and Yaron (2004) and Bansal and Lundblad (2002)). Given the extraordinary behavior of equity valuation ratios in the 1990s, any rational explanation of the stock market during this period must rest on an extremely persistent shift in some underlying fundamental. For macroeconomic volatility, we have

\textsuperscript{26}For all parameter-value combinations, price-dividend ratios rise over the period 1990:Q1 to 2002:Q4, not because the long-run risk-free rate falls, but because the long-run equity premium falls. In fact, results (not reported) show that the long-run risk-free rate actually rises modestly in each case, but not by enough to offset the decline in the equity premium and cause an increase in the total rate of return.
independent evidence about this persistence, and the point estimates in Table 2 imply that the low volatility regime will persist on average for more than 40 years. How likely is persistence of 80, 100 or even 1000 years? Figure 9 plots the log likelihood of our empirical model (1), as a function of $p_{ll}$, the probability of remaining in a low volatility state next period given a low volatility state this period. The likelihood has a clear peak at the point estimate, 0.994, but is virtually as high at unity as it is at the point estimate. Thus values for $p_{ll}$ that imply the low volatility regime will persist indefinitely are just as empirically defensible, statistically, as those that suggest it will persist for 40 to 80 years. In the model, the difference between 40 years and indefinitely is not inconsequential for equilibrium asset prices, but even the low end of the empirically plausible range implies extreme persistence. This means that regardless of what value for $p_{ll}$ one favors, results in Table 3 suggest that the decline in volatility plays some role in the rise of equity values since 1990. One view of these theoretical results is that the stock market appears to be very informative about the expected persistence of the volatility moderation. These estimates are obtained without using any stock market data. Had we included data on the stock market in our estimation such estimates of the persistence would likely have been pushed to the very high end of the range obtained from pure macroeconomic data.

4 Conclusions

This paper considers the low frequency behavior of post-war equity values relative to measures of fundamental value. Such longer-term movements are dominated by the stock market boom of the 1990s, an extraordinary episode in which price-dividend ratios on aggregate stock market indices increased three-fold over a period of five years. Indeed, Figure 1 shows this period to be the defining episode of postwar financial markets. As Campbell (1999) notes, the relationship between stock prices and fundamentals in the 1990s appears to have changed. A growing body of literature is now working to understand this phenomenon, and explanations run the gamut from declining costs of equity market participation and diversification, to irrational exuberance, to changes in technology and demography.

In this paper, we consider a different explanation for why the relationship between stock prices and fundamentals appears to have changed. We ask whether the phenomenal surge in asset values that dominated the close of the 20th century can be plausibly described as a rational response to macroeconomic factors, namely the sharp and sustained decline

\footnote{We thank Lars Hansen for suggesting this plot.}
in macroeconomic risk. We find that, in large part, it can. There is a strong correlation between the low frequency movements in macroeconomic volatility and asset prices in post-war data, both in the US and internationally. We show that, when such a shift toward decreased consumption risk is perceived to be sufficiently persistent, an otherwise standard asset pricing model can explain a large fraction of the surge in equity valuation ratios observed in U.S. data in the 1990s. In the model economy, a boom in stock prices occurs because the decline in macroeconomic risk leads to a fall in expected future stock returns, or the equity risk-premium. An implication of these findings is that multiples of price to earnings or dividends may remain above previous historical norms into the indefinite future.
5 Data Appendix

The sources and description of each data series we use are listed below.

GDP
GDP is gross domestic product, measured in 1996 chain-weighted dollars. Our source is the U.S. Department of Commerce, Bureau of Economic Analysis.

CONSUMPTION
Consumption is measured as total personal consumption expenditures. The quarterly data are seasonally adjusted at annual rates, in billions of chain-weighted 1996 dollars. Our source is the U.S. Department of Commerce, Bureau of Economic Analysis.

POPULATION
A measure of population is created by dividing real total disposable income by real per capita disposable income. Consumption, wealth, labor income, and dividends are in per capita terms. Our source is the Bureau of Economic Analysis.

PRICE DEFLATOR
Real asset returns are deflated by the implicit chain-type price deflator (1996=100) given for the consumption measure described above. Our source is the U.S. Department of Commerce, Bureau of Economic Analysis.

PRICE-DIVIDEND RATIO
The price-dividend ratio is that of the CRSP value-weighted index, constructed as in Campbell (2003). Our source is the Center for Research in Security Prices, University of Chicago.
References


Table 1: Tests for Structural Breaks

<table>
<thead>
<tr>
<th>Stock-Watson Test for Break in Variance</th>
<th>QLR statistic</th>
<th>p-value</th>
<th>Break Date</th>
<th>67% Confidence Interval</th>
</tr>
</thead>
<tbody>
<tr>
<td>∆c</td>
<td>14.34</td>
<td>0.0034</td>
<td>1992Q1</td>
<td>1991Q3 1994Q3</td>
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</table>

<table>
<thead>
<tr>
<th>Bai-Perron Test for Break in Mean</th>
<th>supF Test</th>
<th>p-value</th>
<th>Break Date</th>
<th>90% Confidence Interval</th>
</tr>
</thead>
<tbody>
<tr>
<td>p − d</td>
<td>33.85</td>
<td>&lt; 0.01</td>
<td>1995Q1</td>
<td>1994Q1 1999Q3</td>
</tr>
</tbody>
</table>

Notes: This table reports results from structural break tests. The Quandt Likelihood Ratio test is described in detail in Appendix 1 of Stock and Watson (2002). The bottom panel reports Bai and Perron’s (2003) supF test statistic for a break in the mean of the log CRSP-VW price-dividend ratio. Both tests test the null hypothesis of no structural break against the alternative of a single structural break. The data are quarterly and span the period from 1952 to 2002.
### Table 2: A Markov-Switching Model

<table>
<thead>
<tr>
<th></th>
<th>$\Delta c$</th>
<th>$\mu_h$</th>
<th>$\mu_l$</th>
<th>$\sigma^2_h$</th>
<th>$\sigma^2_l$</th>
<th>$p_{hh}$</th>
<th>$p_{ll}$</th>
<th>$p_{hh}$</th>
<th>$p_{ll}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.623</td>
<td>-0.323</td>
<td>0.556</td>
<td>0.163</td>
<td>0.966</td>
<td>0.794</td>
<td>0.994</td>
<td>0.991</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.064)</td>
<td>(0.335)</td>
<td>(0.091)</td>
<td>(0.050)</td>
<td>(0.022)</td>
<td>(0.109)</td>
<td>(0.008)</td>
<td>(0.012)</td>
<td></td>
</tr>
</tbody>
</table>

Notes: This table reports the maximum likelihood estimates of the model

\[
\Delta x_t = \mu(S_t) + \epsilon_t \\
\epsilon_t \sim N(0, \sigma^2(V_t)).
\]

We allow for two mean states and two volatility states. $\mu_h$ denotes the growth rate in the high mean state, while $\mu_l$ denotes the growth rate in the low mean state. $\sigma^2_h$ denotes the variance of the shock in the high volatility state and $\sigma^2_l$ denotes the variance of the shock in the low volatility state. $S_t$ and $V_t$ are latent variables that are assumed to follow independent Markov chains. The probabilities of transiting to next period’s state $j$ given today’s state $i$ are $p_{ij}^{\mu}$ and $p_{ij}^{\sigma}$, respectively. Standard errors are in parentheses. Standard errors are in parentheses. The data are quarterly and span the period from the first quarter of 1952 to the fourth quarter of 2002.
Table 3: Model Implications in 1990Q1 and 2002Q4

<table>
<thead>
<tr>
<th>row</th>
<th>$\lambda$</th>
<th>$p^{\sigma}_{j,j}$</th>
<th>$P/D_{90}$</th>
<th>$P/D_{02}$</th>
<th>% of boom</th>
<th>$r^{p}_{90}(100)$</th>
<th>$r^{p}_{02}(100)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4.5</td>
<td>struct. break</td>
<td>32.33</td>
<td>65.07</td>
<td>118%</td>
<td>10.05</td>
<td>8.50</td>
</tr>
<tr>
<td>2</td>
<td>3.0</td>
<td>struct. break</td>
<td>39.61</td>
<td>64.06</td>
<td>104%</td>
<td>6.93</td>
<td>5.48</td>
</tr>
<tr>
<td>3</td>
<td>4.5</td>
<td>0.99999</td>
<td>32.72</td>
<td>51.48</td>
<td>89%</td>
<td>10.05</td>
<td>8.82</td>
</tr>
<tr>
<td>4</td>
<td>4.5</td>
<td>0.99990</td>
<td>32.76</td>
<td>49.08</td>
<td>77%</td>
<td>10.08</td>
<td>8.98</td>
</tr>
<tr>
<td>5</td>
<td>4.5</td>
<td>0.99900</td>
<td>33.20</td>
<td>43.40</td>
<td>48%</td>
<td>10.11</td>
<td>9.41</td>
</tr>
<tr>
<td>6</td>
<td>4.5</td>
<td>estimated</td>
<td>35.12</td>
<td>39.71</td>
<td>20%</td>
<td>10.04</td>
<td>9.84</td>
</tr>
<tr>
<td>7</td>
<td>3.0</td>
<td>0.99999</td>
<td>40.03</td>
<td>54.11</td>
<td>54%</td>
<td>6.60</td>
<td>5.72</td>
</tr>
<tr>
<td>8</td>
<td>3.0</td>
<td>0.99990</td>
<td>40.04</td>
<td>52.29</td>
<td>51%</td>
<td>6.62</td>
<td>5.83</td>
</tr>
<tr>
<td>9</td>
<td>3.0</td>
<td>0.99900</td>
<td>40.39</td>
<td>48.00</td>
<td>29%</td>
<td>6.64</td>
<td>6.14</td>
</tr>
<tr>
<td>10</td>
<td>3.0</td>
<td>estimated</td>
<td>42.07</td>
<td>45.42</td>
<td>12%</td>
<td>6.59</td>
<td>6.45</td>
</tr>
</tbody>
</table>

Notes: This table reports the model implications for asset prices using the estimated state probabilities in 1990:Q1 and 2002:Q4. $\lambda$ is the leverage factor, $\delta$ is the discount rate and $p^{\sigma}_{j,j}$ is the probability that next period is a volatility state $j$ given that today’s state is volatility state $j$, $j \in \{l, h\}$. “Struct. break” refers to the special case of a structural break in consumption volatility. For all cases $\gamma = 30$, $\delta = .996$, and $\psi = 1.5$. $P/D$ and $r^{p}(100)$ are the price-dividend ratio and the 100-year risk premium, respectively. The entry for $p^{\sigma}_{j,j}$ labelled "est" show the results when the point estimates for all transition probabilities are used All returns are annualized in percent. The variables with subscript “90” (“02”) report the model’s predictions using historical state probabilities in 1990:Q1(2002:Q4). The columns denoted “% of boom” reports the change of the P/D ratio from 1990:Q1 to 2002:Q4 in the model relative to the change of the CRSP-VW P/D ratio.
Notes: This figure shows the growth rates of personal consumption expenditures. The lines in the plot correspond to the volatility regimes estimated from the Hamilton regime switching model. The data are quarterly and span the period from the first quarter of 1952 to the fourth quarter of 2002.
Figure 2: 5-Year Volatility Estimates and log Price Ratios

Notes: This figure plots the standard deviation of consumption growth and the average CRSP-VW log dividend-price ratio in 5-year windows. All series are demeaned and divided by their standard deviation. The data are quarterly and span the period from the first quarter of 1952 to the fourth quarter of 2002.
Figure 3: GDP volatility and the D/P Ratio - Pre-war Evidence

Notes: This figure plots the standard deviations of GDP growth and the mean D/P ratio by decade starting in 1880 until 2000. Both series are demeaned and divided by their standard deviation. The GDP data are from Ray Fair’s website (http://fairmodel.econ.yale.edu/RAYFAIR/PDF/2002DTBL.HTM) based on Balke and Gordon (1989). The dividend yield data is from Robert Shiller’s website (http://aida.econ.yale.edu/~shiller/data/ie_data.htm).
Notes: This figure plots the time series of estimated state probabilities. \( P(\text{low variance}) \) is the unconditional probability of being in a low consumption volatility state next period (solid line), calculated by summing the probability of being in a low volatility state and high mean state, and the probability of being in a low volatility state and low mean state. \( P(\text{high mean}) \) is calculated analogously (dashed line). The data are quarterly and span the period from the first quarter of 1952 to the fourth quarter of 2002.
Figure 5: The Price-Dividend Ratio

Notes: The figure shows the log price-dividend ratio $p − d$ as a function of the probability that consumption volatility is low (left panel) and the probability that consumption mean is high (right panel). In the left panel, the probability that the consumption mean is high is set to be zero (solid line) or one (dashed line). In the right panel, the probability that consumption volatility is low is set to be zero (solid line) and one (dashed line). The probability of a change in the consumption volatility state is assumed to be .0001; otherwise the parameters of the endowment process are set equal to their maximum likelihood estimates. The rate of time preference $δ = .996$, the elasticity of intertemporal substitution, $ψ = 1.5$, risk aversion $γ = 30$ and leverage $λ = 4.5$. 
Notes: Time series of the log price-dividend ratio from the data and implied by the model. The probability of a change in the consumption volatility state is assumed to be .0001; otherwise the parameters of the endowment process are set equal to their maximum likelihood estimates. The rate of time preference $\delta = .996$, the elasticity of intertemporal substitution, $\psi = 1.5$, risk aversion $\gamma = 30$ and leverage $\lambda = 4.5$. The data are quarterly and span the period from the first quarter of 1952 to the fourth quarter of 2002.
Notes: Time series of the 100-year expected equity return and equity premium implied by the model. The probability of a change in the consumption volatility state is assumed to be .0001; otherwise the parameters of the endowment process are set equal to their maximum likelihood estimates. The rate of time preference $\delta = .996$, the elasticity of intertemporal substitution, $\phi = 1.5$, risk aversion $\gamma = 30$ and leverage $\lambda = 4.5$. The data are quarterly and span the period from the first quarter of 1952 to the fourth quarter of 2002.
Notes: Time series of conditional volatility of one-quarter ahead equity returns. Volatility is annualized, i.e. $2\sigma_t$. The probability of a change in the consumption volatility state is assumed to be .0001; otherwise the parameters of the endowment process are set equal to their maximum likelihood estimates. The rate of time preference $\delta = .996$, the elasticity of intertemporal substitution, $\psi = 1.5$, risk aversion $\gamma = 30$ and leverage $\lambda = 4.5$. The data are quarterly and span the period from the first quarter of 1952 to the fourth quarter of 2002.
Figure 9: The Likelihood Function

Notes: This figure shows the log-likelihood function of the Hamilton regime switching model. The probabilities of remaining in the high (low) volatility state given that today’s volatility state is high (low) are set to the same value, $P_{\text{sig}}(ii)$. The figure plots the log-likelihood as a function of $P_{\text{sig}}(ii)$. All other parameters are set the optimized values reported in Table 3. The data are quarterly and span the period from the first quarter of 1952 to the fourth quarter of 2002.