



THE WORLD BANK



# **Session IV**

# **Instrumental Variables**

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January 2008

# An example to start off with...

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- Say we wish to evaluate a voluntary job training program
  - Any unemployed person is eligible
  - Some people choose to register (“Treatments”)
  - Other people choose not to register (“Comparisons”)
  
- Some simple (but not-so-good) ways to evaluate the program:
  - Compare before and after situation in the treatment group
  - Compare situation of treatments and comparisons after the intervention
  - Compare situation of treatments and comparisons before and after

# Voluntary job training program

Say we decide to compare outcomes for those who participate to the outcomes of those who do not participate:

a simple model to do this:

$$y = \alpha + \beta_1 T + \beta_2 x + \varepsilon$$

Where  $T = 1$  if person participates in training

$T = 0$  if person does not participate in training

$x$  = control variables ( exogenous & observable)

Why is this not working properly?

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Why is this not working properly? 2 problems:

- ▷ Variables that we omit (for various reasons) but that are important
- ▷ Decision to participate in training is endogenous

# Problem #1: Omitted Variables

Even in a well-thought model, we'll miss

- ▷ "forgotten" characteristics: we didn't know they mattered
- ▷ characteristics that are too complicated to measure

*Examples :*

- ▷ Varying talent and levels of motivation
- ▷ Different levels of information
- ▷ Varying opportunity cost of participation
- ▷ Varying level of access to services

The full "correct" model is:  $y = \alpha + \gamma_1 T + \gamma_2 x + \gamma_3 D + \eta$

The model we use:  $y = \alpha + \beta_1 T + \beta_2 x + \varepsilon$

## Problem #2: Endogenous Decision to Participate

Participation is a decision variable ? It's endogenous!

(I.e. it depends on the participants themselves.)

$$y = \alpha + \beta_1 T + \beta_2 x + \varepsilon$$

$$T = \pi + \pi_2 D + \xi$$

## Problem #2: Endogenous Decision to Participate

Participation is a decision variable ? it's endogenous

$$y = \alpha + \beta_1 T + \beta_2 x + \varepsilon$$

$$T = \pi + \pi_2 D + \xi$$

$$\Rightarrow y = \alpha + \beta_1(\pi + \pi_2 D + \xi) + \beta_2 x + \varepsilon$$

$$\Rightarrow y = \alpha + \beta_1 \pi + \beta_2 x + \beta_1 \pi_2 D + \beta_1 \xi + \varepsilon$$

Note: in the two cases: we're missing the  $D$  term, which we need to estimate the correct model, but which we cannot measure properly.

□ The “correct model is:  $y = \alpha + \gamma_1 T + \gamma_2 x + \gamma_3 D + \eta$

□ Simplified model:  $y = \alpha + \beta_1 T + \beta_2 x + \varepsilon$

- Say we estimate the treatment effect  $\gamma_1$  with  $\beta_{1,OLS}$
- If D is correlated with T, and we don't include D in the simplified model, then the estimator of the parameter on T will pick up part of the effect of D. This will happen to the extent that D and T are correlated.
- Thus: our OLS estimator  $\beta_{1,OLS}$  of the treatment effect  $\gamma_1$  captures the effect of other characteristics (D) in addition to the treatment effect.
- This means that there is a difference between  $E(\beta_{1,OLS})$  y  $\gamma_1$ 
  - the expected value of the OLS estimator  $\beta_1$  isn't  $\gamma_1$ , the real treatment effect
  - $\beta_{1,OLS}$  is a biased estimator of the treatment effect  $\gamma_1$ .

CV4

CV4

$$\text{corr}(T, \varepsilon) = \text{corr}(T, \gamma^3 D + \eta) = \gamma^3 \text{corr}(T, D)$$

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□ Simplified model:  $y = \alpha + \beta_1 T + \beta_2 x + \varepsilon$

- This means that there is a difference between  $E(\beta_{1,OLS})$  y  $\gamma_1$ 
  - the expected value of the OLS estimator  $\beta_1$  isn't  $\gamma_1$ , the real treatment effect
  - $\beta_{1,OLS}$  is a biased estimator of the treatment effect  $\gamma_1$ .
  
- Why did this happen?
  - One of the basic conditions for OLS to be BLUE was violated:
  
- In other words  $E(\beta_{1,OLS}) \neq \gamma_1$  (biased estimator)
- Even worse..... $\text{plim}(\beta_{1,OLS}) \neq \gamma_1$  (inconsistent estimator)

CV6

CV6

$$\text{corr}(T, \varepsilon) = \text{corr}(T, \gamma^3 D + \eta) = \gamma^3 \text{corr}(T, D)$$

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# What can we do to solve this problem?

$$y = \alpha + \gamma_1 T + \gamma_2 x + \gamma_3 D + \eta$$

$$y = \alpha + \beta_1 T + \beta_2 x + \varepsilon$$

- ❑ Try to clean the correlation between  $T$  and  $\varepsilon$ :
- ❑ Isolate the variation in  $T$  that is not correlated with  $\varepsilon$  through the omitted variable  $D$
- ❑ We can do this using an instrumental variable (IV)

## Basic idea behind IV

$$y = \alpha + \gamma_1 T + \gamma_2 x + \gamma_3 D + \eta$$

$$y = \alpha + \beta_1 T + \beta_2 x + \varepsilon$$

- The basic problem is that  $\text{corr}(T, D) \neq 0$
- Find a variable  $Z$  that satisfies two conditions:
  1. Correlated with  $T$ :  $\text{corr}(Z, T) \neq 0$   
 ----  $Z$  and  $T$  are correlated, or  $Z$  predicts part of  $T$
  2.  $Z$  is not correlated with  $\varepsilon$ :  $\text{corr}(Z, \varepsilon) = 0$   
 ---- By itself,  $Z$  has no influence on  $y$ . The only way it can influence  $y$  is because it influences  $T$ . All of the effect of  $Z$  on  $y$  passes through  $T$ .
- Examples of  $Z$  in the case of voluntary job training program?

# Two-stage least squares (2SLS)

Remember the original model with endogenous T:

$$y = \alpha + \beta_1 T + \beta_2 x + \varepsilon$$

**Step 1:** Regress the endogenous variable T on the instrumental variable(s) Z and other exogenous variables

$$T = \delta_0 + \delta_1 x + \theta_1 Z + \tau$$

- ▷ Calculate the predicted value of T for each observation:  $\hat{T}$
- ▷ Since Z and x are not correlated with  $\varepsilon$ , neither will be  $\hat{T}$ .
- ▷ You will need one instrumental variable for each potentially endogenous regressor

# Two-stage least squares (2SLS)

**Step 2:** Regress  $y$  on the predicted variable  $\hat{T}$  and the other exogenous variables

$$y = \alpha + \beta_1 \hat{T} + \beta_2 x + \varepsilon$$

- ▷ Note: the standard errors of the second stage OLS need to be corrected because  $\hat{T}$  is not a fixed regressor.
- ▷ In practice: use STATA ivreg command, which does the two steps . at once and reports correct standard errors.
- ▷ Intuition: by using  $Z$  for  $T$ , we cleaned  $T$  of its correlation with  $\varepsilon$ .
- ▷ It can be shown that (under certain conditions) IV yields a consistent estimator of  $\gamma_1$  (large sample theory)

# Uses of instrumental variables

- Simultaneity:  $X$  and  $Y$  cause each other
  - instrument  $X$
  
- Omitted Variables:  $X$  is picking up the effect of other variables which are omitted
  - instrument  $X$  with a variable that is not correlated with the omitted variable(s)
  
- Measurement Error:  $X$  is not measured precisely
  - instrument  $X$

# Examples

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- Hoxby (2000) and Angrist (1999)
  - The effect of class size on learning achievements
  
- Chaudhury, Gertler, Vermeersch (work in progress)
  - Estimating the effect of school autonomy on learning in Nepal

# School autonomy in Nepal

## □ Goal is to evaluate

- A. Autonomous school management by communities
- B. School report cards
- C. School information networks

## □ Data

- We could integrate about 1500 schools in the evaluation
- Each community freely chooses to participate or not.
- School report cards and school networks done by NGOs
  - School network can only be done in autonomous schools
  - School report cards can be done in any school.

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  - School report cards and school networks done by NGOs
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    - School report cards can be done in any school.
  - Assume each community has exactly one schools
- Task: design the implementation of the program so it can be evaluated – propose method of evaluation.

# School autonomy in Nepal

- Interventions:
- A. Autonomous school management by communities
  - B. School report cards
  - C. School information networks

	<b>Creation of demand for devolution to the community (A)</b>	<b>Feedback of performance indicators (B)</b>	<b>Network support after devolution to the community (C)</b>	<b>Number of schools in the group</b>
Group O (Control)	No	No	No	200
Group A	Yes	No	No	300
Group B	No	Yes	No	100
Group AB	Yes	Yes	No	300
Group AC	Yes	No	Yes	300
Group ABC	Yes	Yes	Yes	300
<b>Total</b>				<b>1500</b>

# Reminder and a word of caution...

## □ $\text{corr}(Z, \varepsilon) = 0$

- if  $\text{corr}(Z, \varepsilon) \neq 0$  “Bad instrument” ;Problem!
- ;Finding a good instrument is hard!
- ;Use both theory and common sense to find one!
- We can think of designs that yield good instruments.

## □ $\text{corr}(Z, T) \neq 0$

- “Weak instruments”: the correlation between Z and T needs to be sufficiently strong.
- If not, the bias stays large even for large sample sizes.