



THE WORLD BANK



# **Technical Track**

## **Session IV**

# **Instrumental Variables**

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# An example to start off with...

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- Say we wish to evaluate a voluntary job training program
  - Any unemployed person is eligible
  - Some people choose to register (“Treatments”)
  - Other people choose not to register (“Comparisons”)
  
- Some simple (but not-so-good) ways to evaluate the program:
  - Compare before and after situation in the treatment group
  - Compare situation of treatments and comparisons after the intervention
  - Compare situation of treatments and comparisons before and after

# Voluntary job training program

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Say we decide to compare outcomes for those who participate to the outcomes of those who do not participate:

a simple model to do this:

$$y = \alpha + \beta_1 D + \beta_2 x + \varepsilon$$

Where  $D = 1$  if person participates in training

$D = 0$  if person does not participate in training

$x =$  control variables ( exogenous & observable)

Why is this not working properly? 2 problems:

- ▷ Variables that we omit (for various reasons) but that are important
- ▷ Decision to participate in training is endogenous

# Problem #1: Omitted Variables

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Even in a well-thought model, we'll miss

- ▷ "forgotten" characteristics: we didn't know they mattered
- ▷ characteristics that are too complicated to measure

*Examples :*

- ▷ Varying talent and levels of motivation
- ▷ Different levels of information
- ▷ Varying opportunity cost of participation
- ▷ Varying level of access to services

The full "correct" model is:  $y = \alpha + \gamma_1 D + \gamma_2 x + \gamma_3 M + \eta$

The model we use:  $y = \alpha + \beta_1 D + \beta_2 x + \varepsilon$

## Problem #2: Endogenous Decision to Participate

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- Participation is a decision variable? → endogenous!
- (I.e. it depends on the participants themselves)

$$y = \alpha + \beta_1 D + \beta_2 x + \varepsilon$$

$$D = \pi + \pi_2 M + \xi$$

$$\Rightarrow y = \alpha + \beta_1 (\pi + \pi_2 M + \xi) + \beta_2 x + \varepsilon$$

$$\Rightarrow y = \alpha + \beta_1 \pi + \beta_2 x + \beta_1 \pi_2 M + \beta_1 \xi + \varepsilon$$

- So: in the two cases: we're missing the M term, which we need to estimate the correct model, but which we cannot measure properly.

□ The “correct model is:  $y = \alpha + \gamma_1 D + \gamma_2 x + \gamma_3 M + \eta$

□ Simplified model:  $y = \alpha + \beta_1 D + \beta_2 x + \varepsilon$

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- Say we estimate the treatment effect  $\gamma_1$  with  $\beta_{1,OLS}$
- If  $M$  is correlated with  $D$ , and we don't include  $M$  in the simplified model, then the estimator of the parameter on  $D$  will pick up part of the effect of  $M$ . This will happen to the extent that  $M$  and  $D$  are correlated.
- Thus: our OLS estimator  $\beta_{1,OLS}$  of the treatment effect  $\gamma_1$  captures the effect of other characteristics ( $M$ ) in addition to the treatment effect.
- This means that there is a difference between  $E(\beta_{1,OLS})$  and  $\gamma_1$ 
  - the expected value of the OLS estimator  $\beta_1$  isn't  $\gamma_1$ , the real treatment effect
  - $\beta_{1,OLS}$  is a biased estimator of the treatment effect  $\gamma_1$ .

□ The “correct model is:  $y = \alpha + \gamma_1 T + \gamma_2 x + \gamma_3 D + \eta$

□ Simplified model:  $y = \alpha + \beta_1 T + \beta_2 x + \varepsilon$

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□ This means that there is a difference between  $E(\beta_{1,OLS})$  and  $\gamma_1$   
→ the expected value of the OLS estimator  $\beta_1$  isn't  $\gamma_1$ , the real treatment effect

→  $\beta_{1,OLS}$  is a biased estimator of the treatment effect  $\gamma_1$ .

□ Why did this happen?

■ One of the basic conditions for OLS to be BLUE was violated:

□ In other words  $E(\beta_{1,OLS}) \neq \gamma_1$  (biased estimator)

□ Even worse..... $plim(\beta_{1,OLS}) \neq \gamma_1$  (inconsistent estimator)

# What can we do to solve this problem?

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$$y = \alpha + \gamma_1 D + \gamma_2 x + \gamma_3 M + \eta$$

$$y = \alpha + \beta_1 D + \beta_2 x + \varepsilon$$

- Try to clean the correlation between  $D$  and  $\varepsilon$ :
- Isolate the variation in  $D$  that is not correlated with  $\varepsilon$  through the omitted variable  $M$
- We can do this using an instrumental variable ( $IV$ )

# Basic idea behind IV

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$$y = \alpha + \gamma_1 D + \gamma_2 x + \gamma_3 M + \eta$$

$$y = \alpha + \beta_1 D + \beta_2 x + \varepsilon$$

- The basic problem is that  $\text{corr}(D, M) \neq 0$
- Find a variable  $Z$  that satisfies two conditions:
  1. Correlated with  $D$ :  $\text{corr}(Z, D) \neq 0$ 
    - $Z$  and  $D$  are correlated, or  $Z$  predicts part of  $D$
  2.  $Z$  is not correlated with  $\varepsilon$ :  $\text{corr}(Z, \varepsilon) = 0$ 
    - By itself,  $Z$  has no influence on  $y$ . The only way it can influence  $y$  is because it influences  $D$ . All of the effect of  $Z$  on  $y$  passes through  $D$ .
- Examples of  $Z$  in the case of voluntary job training program?

# Two-stage least squares (2SLS)

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Remember the original model with endogenous  $D$ :

$$y = \alpha + \beta_1 D + \beta_2 x + \varepsilon$$

**Step 1:** Regress the endogenous variable  $D$  on the instrumental variable(s)  $Z$  and other exogenous variables

$$D = \delta_0 + \delta_1 x + \theta_1 Z + \tau$$

- ▷ Calculate the predicted value of  $D$  for each observation:  $\hat{D}$
- ▷ Since  $Z$  and  $x$  are not correlated with  $\varepsilon$ , neither will be  $\hat{D}$ .
- ▷ You will need one instrumental variable for each potentially endogenous regressor

# Two-stage least squares (2SLS)

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**Step 2:** Regress  $y$  on the predicted variable  $\hat{D}$  and the other exogenous variables

$$y = \alpha + \beta_1 \hat{D} + \beta_2 x + \varepsilon$$

- ▷ Note: the standard errors of the second stage OLS need to be corrected because  $\hat{D}$  is not a fixed regressor.
- ▷ In practice: use STATA `ivreg` command, which does the two steps at once and reports correct standard errors.
- ▷ Intuition: by using  $Z$  for  $D$ , we cleaned  $D$  of its correlation with  $\varepsilon$ .
- ▷ It can be shown that (under certain conditions)  $IV$  yields a consistent estimator of  $\gamma_1$  (large sample theory)

# Uses of instrumental variables

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- Simultaneity: X and Y cause each other
  - ▣ instrument X
- Omitted Variables: X is picking up the effect of other variables which are omitted
  - ▣ instrument X with a variable that is not correlated with the omitted variable(s)
- Measurement Error: X is not measured precisely
  - ▣ instrument X

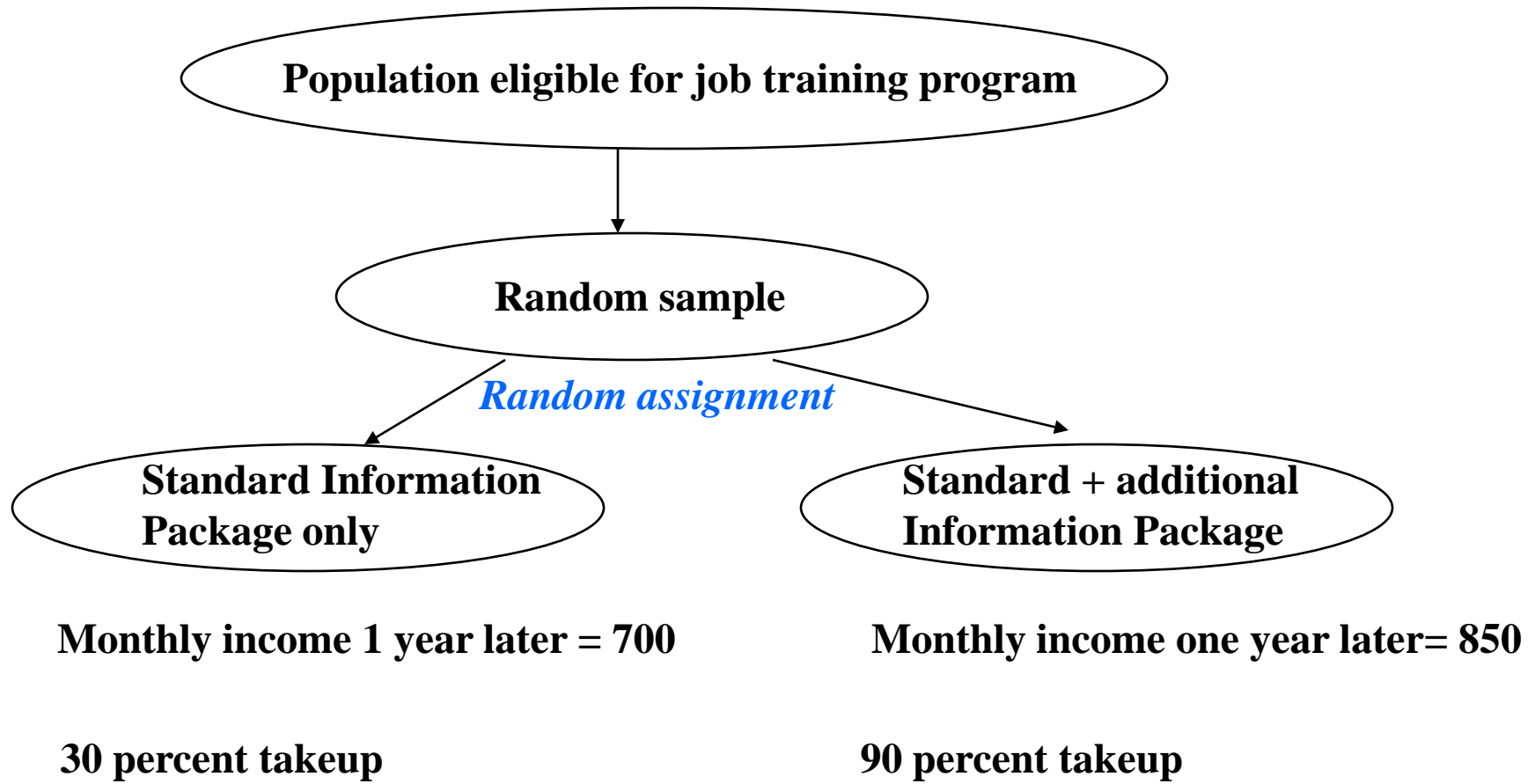
# Where do we find instrumental variables?

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- Searching for one – hard!
- Creating one with information
  - If everyone is eligible to participate in treatment
  - But some have more information than others
    - Who has more information will be more likely to participate
  - Provision of “additional information” on a random basis

# Example 1: voluntary job training program

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**Question: what is the impact of the job training program?**

**Standard Information  
Package only**

**Standard + additional  
Information Package**

Monthly income 1 year later = 700

Monthly income one year later = 850

30 percent takeup

90 percent takeup

**Question: what is the impact of the job training program?**

• Difference between the “well informed” and “not well informed” group

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• Corrected for the differential take-up rate

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• Practically: Impact = .....

# Link back to the estimation formula

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## □ Stage 1:

- Regress the participation on training on a dummy for whether person received additional information package (linear model)
- Compute predicted value of participation

## □ Stage 2:

- Regress wages on the predicted value of participation

# Example 2: School autonomy in Nepal

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- Goal is to evaluate
  - A. Autonomous school management by communities
  - B. School report cards
- Data
  - You can include 1000 schools in the evaluation
  - Each community freely chooses to participate or not.
  - School report cards done by NGOs
  - Each community has exactly one school.
- Task: design the implementation of the program so it can be evaluated – propose method of evaluation.

# School autonomy in Nepal

		Intervention B: School report card intervention by NGO		
		Yes	No	<i>Total</i>
Instrumental Variable for Intervention A: NGO visits community to inform on procedures for transfer of the school to community management	Yes	300	300	<i>600</i>
	No	200	200	<i>400</i>
	<i>Total</i>	<i>500</i>	<i>500</i>	<i>1000</i>

# Reminder and a word of caution....

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□  $corr(Z, \varepsilon) = 0$

- if  $corr(Z, \varepsilon) \neq 0$  “Bad instrument” ; Problem!
- ; Finding a good instrument is hard!
- ; Use both theory and common sense to find one!
- We can think of designs that yield good instruments.

□  $corr(Z, D) \neq 0$

- “Weak instruments”: the correlation between  $Z$  and  $D$  needs to be sufficiently strong.
- If not, the bias stays large even for large sample sizes.