

ANNEXES

Chapter 1

Annex 1: A Numerical Example of Computing the HOI

To help explain the computation of the HOI, we use the example presented in Tables A1.1a-1i (below), in which the overall population is divided in 16 circumstance groups, defined by gender, race and location. We assume that all groups are the same size: 100 persons, leading to a total population of 1600. Table A1.1a presents the number of people in each circumstance group that have access to a specific basic service (for example, clean water, electricity or vaccinations). Overall, 400 people have access to the service. Since the total population is 1600, the overall coverage rate is 25 percent. Under equality of opportunity, 25 percent of each circumstance group should be covered. Coverage rates, however, vary substantially, from 0 percent in some groups to 75 percent in others (Table A1.1b).

The computation of the penalty is done in three steps. First, we identify all circumstance groups with coverage rate below the average rate (25 percent, in our example); we refer to them as the opportunity-vulnerable groups. There are 10 of these groups in our example, marked in red in Table A1.1b. Second, we compute the gap between the number of people in each vulnerable group that should have access to the service for it to reach the average coverage rate, and the actual number of people with access to

the service in that group (Table A1.1c). Third, the penalty is obtained by dividing the sum of the opportunity gaps of all vulnerable groups (called the overall opportunity gap) by the total population. In our example the penalty would be equal to 10 percent, since the sum of all opportunity gaps equals 160 (Table A1.1c) and the overall population is 1600. The overall coverage rate of 25 percent minus the penalty due unequal allocation of 10 percent leads to an HOI of 15 percent ($C - P = O$; $25 - 10 = 15$). In this example, only 90 individuals from opportunity-vulnerable groups are covered, out of 250 that should have been covered had equality of opportunity prevailed. Hence, 160 people among these vulnerable groups should receive the service for their coverage rate to rise at least to the average. This total opportunity gap, 160, equals 10 percent of the total population.

Since services are available for 400 people, and 160 people in non-vulnerable groups receive services in excess of what is needed for equality of opportunity to prevail, only 240 people received services that were allocated equitably. Hence, as a proportion of the total population, the number of people who receive the service according to the principle of equality of opportunities is 15 percent ($240/1600$). This is what the HOI measures.

Table A1.1

Table A1.1a: Distribution of a service					Table A1.1d: Distribution of the population					Table A1.1g: Improperly allocated services							
		North		South				North		South				North		South	
		Urban	Rural	Urban	Rural			Urban	Rural	Urban	Rural			Urban	Rural	Urban	Rural
Whites		75	60	45	20	Whites		100	100	100	100	Whites		50	35	20	0
Men	Blacks	50	50	30	20	Men	Blacks	100	100	100	100	Men	Blacks	25	25	5	0
Whites		15	10	5	0	Whites		100	100	100	100	Whites		0	0	0	0
Women	Blacks	15	5	0	0	Women	Blacks	100	100	100	100	Women	Blacks	0	0	0	0

Table A1.1b: Group-Specific Coverage Rates					Table A1.1e: Opportunity gaps as a proportion of the total population					Table A1.1h: Improperly allocated services as a proportion of the total population							
		North		South				North		South				North		South	
		Urban	Rural	Urban	Rural			Urban	Rural	Urban	Rural			Urban	Rural	Urban	Rural
Whites		75%	60%	45%	20%	Whites		0%	0%	0%	0%	Whites		3%	2%	1%	0%
Men	Blacks	50%	50%	30%	20%	Men	Blacks	0%	0%	0%	0%	Men	Blacks	2%	2%	0%	0%
Whites		15%	10%	5%	0%	Whites		1%	1%	1%	2%	Whites		0%	0%	0%	0%
Women	Blacks	15%	5%	0%	0%	Women	Blacks	1%	1%	2%	2%	Women	Blacks	0%	0%	0%	0%

Table A1.1c: Opportunity Gaps (for vulnerable groups)					Table A1.1f: Opportunity gaps as a proportion of the population covered					Table A1.1i: Improperly allocated services as a proportion of the population covered							
		North		South				North		South				North		South	
		Urban	Rural	Urban	Rural			Urban	Rural	Urban	Rural			Urban	Rural	Urban	Rural
Whites		0	0	0	5	Whites		0%	0%	0%	1%	Whites		13%	9%	5%	0%
Men	Blacks	0	0	0	5	Men	Blacks	0%	0%	0%	1%	Men	Blacks	6%	6%	1%	0%
Whites		10	15	20	25	Whites		3%	4%	5%	6%	Whites		0%	0%	0%	0%
Women	Blacks	10	20	25	25	Women	Blacks	3%	5%	6%	6%	Women	Blacks	0%	0%	0%	0%

Annex 2: A Numerical Illustration of the Decomposition of the HOI

The HOI can be easily computed from the distribution of the population by circumstance groups (Table A2.1a) and specific coverage rates (Table A2.1b) through four simple steps (Table A2.1c). First, we compute the overall coverage rate as the weighted average of all group-specific coverage rates (35 percent in population A). Second, we identify groups with specific coverage rates below average (marked in red). Third, we obtain the penalty (12.5 percent in population A) by weighting the difference between the overall coverage and each group-specific coverage rate among vulnerable groups: $(35 - 15) * 0.25 + (35 - 5) * 0.25 = 12.5$. Finally, we obtain the HOI (22.5 percent in population A) by subtracting the penalty from the overall coverage.

Using the same procedure and the information for population B in the last row of Table A2.4c, we obtain an HOI of 37 percent for population B. One can think of population A and B as being the same country in two different time periods. Population B has an HOI 14.5 percentage points higher than population A. This difference is due to disparities in both the distribution of the populations among circumstance groups and the pattern of their specific coverage rates. In population B, the north region has a 20 percentage-point higher population share. In terms of coverage rates, those among the vulnerable groups tend to be higher in population B, whereas those among the non-vulnerable groups are higher in population A. Coverage rates are on average higher and less unequal in population B than in population A.

Table A1.2

Table A2.1a: Distribution of the population A

100%	North	South
Men	25%	25%
Women	25%	25%

Table A2.1b: Group-specific coverage rate for population A

35%	North	South
Men	75%	45%
Women	15%	5%

Table A2.1c: Computing the HOI for population A

Overall coverage rate	35.0%
Penalty	12.5%
HOI	22.5%

Table A2.2a: Distribution of the population B

100%	North	South
Men	40%	10%
Women	30%	20%

Table A2.2b: Group-specific coverage rate for population A

40%	North	South
Men	75%	45%
Women	15%	5%

Table A2.2c: Computing the HOI for population B with the coverage rates from population A

Overall coverage rate	40.0%
Penalty	14.5%
HOI	25.5%

Table A2.3a: Distribution of the population B

100%	North	South
Men	40%	10%
Women	30%	20%

Table A2.3b: Group-specific coverage rate (structure from population A, average level from population B)

45%	North	South
Men	84%	51%
Women	17%	6%

Table A2.3c: Computing the HOI for population B with the structure of coverage rates from population A and average level from population B

Overall coverage rate	45.0%
Penalty	16.3%
HOI	28.7%

Table A2.4a: Distribution of the population B

100%	North	South
Men	40%	10%
Women	30%	20%

Table A2.4b: Group-specific coverage rate for population B

45%	North	South
Men	65%	40%
Women	40%	15%

Table A2.4c: Computing the HOI for population B

Overall coverage rate	45.0%
Penalty	8.0%
HOI	37.0%

To isolate the effect on the HOI of differences in the distribution of population among circumstance groups, we estimate the HOI for a combined situation: coverage rates are still those of population A, but the distribution of the population among circumstance groups is now that of population B. The estimated HOI for this hybrid situation is 25.5 percent.

Since this hybrid population shares with population A the same group-specific coverage rate, their difference in HOI (3.0 p.p.) is entirely due to their differences in the distribution of the population among circumstance groups. This is the composition effect. On the other hand, since the hybrid population shares with population B the same distribution of the population among circumstance groups, their difference in HOI (11.5 p.p.) is entirely due to their differences in their specific coverage rates. Hence, this difference is the coverage effect. Hence, the total difference in HOI (14.5 p.p.) between populations A and B is decomposed into the composition effect of 3 p.p. and 11.5 p.p. due to the coverage effect.

The coverage effect can be decomposed further (see Tables A2.3a-c). The group-specific coverage rates in Table A2.3b are constructed to reach the average level in population B, holding the inequality level of population A. This is accomplished by proportionally increasing all group-specific rates from population A. Inequality would be

preserved while the average coverage rate is adjusted. To reach the average coverage rate of population B, all group-specific coverage rates from population A are multiplied by the ratio between the overall coverage for population B and A ($45/40=1.125$).¹

As a consequence, the HOIs in Tables A2.3c and A2.4c share the same distribution of the population among circumstance groups and the same overall coverage rate (45 percent). They only differ with respect to the inequality of their group-specific coverage rates. Since the overall coverage rate is the same in both cases, the difference in inequality is captured by corresponding differences in the size of the penalty: 16.3 percent in Table A2.3c and 8 percent in Table A2.4c. The difference between these two penalties, 8.3 p.p., is a measure of the equalization effect. It is the contribution of the greater equality of opportunities in population B to the difference in HOI between the two populations.

On the other hand, Tables A2.2c and A2.3c share the same distribution of the population and the same inequality among group-specific coverage rates as measured by the ratio between the penalty and the overall coverage rate. This ratio is equal to 0.3625 in the two cases. Hence, the difference between HOIs in these tables (3.2 p.p.) is entirely due to their corresponding differences in overall coverage rates. It represents the scale effect.

¹ Notice that the relevant average for population A is not the original average but that using the population weights from population B (see Table A2.2c).

Annex 3: The Algebra of Decomposing the Human Opportunity Index

Consider two populations A and B. Let w_k^A denote the population share of circumstance-group k and C_k^A its specific coverage rate in population A, with w_k^B and C_k^B denoting the corresponding characteristics in population B. In this case, HOI^A can be expressed as

$$HOI^A = C^A - \sum_{k \in V^A} w_k^A (C^A - C_k^A)$$

where

$$C^A = \sum_k w_k^A C_k^A$$

and

$$V^A = \{k : C_k^A < C^A\}$$

with similar expressions holding for HOI^B .

To obtain the decomposition we begin with a hybrid HOI^{A^B} combining the group-specific coverage rates of population A with the composition of population B.

$$HOI^{A^B} = C^{A^B} - \sum_{k \in V^{A^B}} w_k^B (C^{A^B} - C_k^A)$$

where

$$C^{A^B} = \sum_k w_k^B C_k^A$$

and

$$V^{A^B} = \{k : C_k^A < C^{A^B}\}$$

Since HOI^A and HOI^{A^B} share the same group-specific coverage rates, $\Delta_c = HOI^{A^B} - HOI^A$ measures the composition effect, i.e., the impact of differences in the distributions of the populations A and B among circumstance groups. On the other hand, since HOI^B and HOI^{A^B} have the same population shares, $\Delta_r = HOI^B - HOI^{A^B}$ measures the coverage effect, i.e., the impact of the differences in group-specific coverage rates between populations A and B. Notice that the total difference is the sum of the coverage and composition effect: $\Delta = HOI^B - HOI^A = \Delta_r + \Delta_c$.

To further decompose the coverage effect in an equalization effect and a scale effect, we construct the following hybrid group-specific coverage rate,

$$C_k^{A^B} = C_k^A \frac{C^B}{C^{A^B}}$$

Based on these hybrid group-specific coverage rates and noticing that

$$C^* = \sum_k w_k^B C_k^{A^B} = C^B$$

we estimate a new index via

$$HOI^* = C^B - \sum_{k \in V^*} w_k^B (C^B - C_k^{A^B})$$

where

$$V^* = \{k: C_k^{A^B} < C^B\}$$

Since HOI^* and HOI^{A^B} have the same population shares and level of inequality among group-specific coverage rates, $\Delta_s = HOI^* - HOI^{A^B}$ measures the scale effect, i.e., the impact of the differences in the level of the coverage rates between populations A and B. On the other hand, since HOI^B and HOI^* have the same population shares and overall coverage rate, $\Delta_e = HOI^B - HOI^*$ measures the equalization effect, i.e., the impact of the differences in the degree of inequality among group-specific coverage rates between populations A and B. Notice that the coverage effect is the sum of the scale and equalization effects: $\Delta_r = HOI^B - HOI^{A^B} = \Delta_s + \Delta_e$.

Annex 4: Definitions of Indicators

Adequate access to water	Children 0 to 10 year: old	This variable takes the value of one if the household has access to running water within the dwelling. Thus access includes public network connections and all water pumped into the dwelling even if it is not from the public network.
Access to electricity	Children 0 to 10 year: old	This variable takes the value of one if the dwelling has access to electricity from any source. Thus sources can range from the electrical grid system to solar panels.
Adequate access to sanitation	Children 0 to 10 year: old	This variable takes the value of one if the dwelling has access to a flush toilet (either inside the dwelling or inside the property) that is connected to any mechanism whereby household waste is allowed to flow away from the dwelling.
School attendance rate	Children 10 to 14 years old	This is the measured as children aged 10-14 attending to school, independent of grade. This variable measures the gross attendance rate.
Probability of completing sixth grade on time	Children 12 to 16 years old	This is measured by computing the probability of having ended sixth grade on time for all children ages 12 to 16. In most countries of the region, this means having completed primary education. Given that on average children start school at the age of 7, by age 13, students that have survived in the system without repetition should have completed six years of basic education.

Source: The World Bank and Universidad Nacional de La Plata (CEDLAS) Socioeconomic Database for Latin America and the Caribbean.

Annex 5: Choosing the Aggregation Sequence

To create an overall HOI, one needs to aggregate both the different dimensions of each person's opportunities (that is, the indicators used to proxy a basic opportunity) and also aggregate the opportunities of different persons to obtain a societal aggregate. Ideally, one first should aggregate the opportunities of each person first, to fully consider the interdependence among the dimensions.

This interdependence among the dimensions has two features worth noting. First, the dimensions could be complements or substitutes. For instance, should attendance and progression in school be considered complements or substitutes? Second, the access to different goods and services could be concomitant or alternative. For instance, an overall 50 percent coverage rate for water and electricity could still lead to very distinct distributions. In one extreme, it may be the case that those having access to water are the same people having access to electricity. In this case, the accesses to these two services are perfectly correlated. On the other extreme, it may be that either one has access to water or to electricity, never to both. In this case, access to these two services is inversely related. If the two services are substitutes, an inverse relation is preferable. If they are complements, a positive association is preferable.

Hence, to properly evaluate opportunities

one needs to know the degree of complementarity or substitutability between goods and services as well as the distribution of a population's access opportunities. Ultimately, aggregation takes into consideration the complementarities and substitutability among the many dimensions.

It may be useful to consider the extreme case in which the access opportunity is either 0 percent or 100 percent. Suppose I_1 indicates whether everyone has ($I_1=1$) or not ($I_1=0$) proper access to sanitation and I_2 indicates whether everyone has ($I_2=1$) or not ($I_2=0$) adequate access to electricity. Three alternative aggregation strategies are possible in this example. The most demanding alternative would consider that one has an opportunity when he/she has access to both sanitation and electricity. In this case, the aggregated opportunity index would be: $I=I_1 \times I_2$ and $I=1$ if and only if $I_1=1$ and $I_2=1$. This is the intersection approach. In the other extreme, the least demanding alternative would consider the two opportunities as substitutes. In this case having one of them is enough. Access to a second would not lead to any significant improvement. In this case, the aggregated opportunity index would be: $I=1-(1-I_1) \times (1-I_2)$ and $I=1$ if and only if $I_1=1$ or $I_2=1$. This is the union approach. An intermediate alternative is to relax the requirement that both are essential and, instead of going to the extreme that just one is enough, one may consider each one as an independent advantage. In this case, the aggregated opportunity index would be: $I=I_1+I_2$. This is the counting approach.

To properly take the distribution of access opportunity and degree of complementarity/substitutability into consideration, the many dimensions of opportunities faced by each person must be aggregated before aggregating among people. Once the aggregation across persons of each dimension is conducted, how positively or negatively correlated are the accesses to the many relevant key goods and services is lost.

A comprehensive consideration of the inter-relation among dimensions also demands that information on the many dimensions be available for each person. To properly consider inter-dependence, for each person in the analysis one needs simultaneous information on his/her access to all key goods and services being considered. Since many opportunities are age-specific, like attending school at age 10 and completing 6th grade at age 13, a longitudinal survey or a survey with considerable amount of retrospective information would be required.

Given the lack of such information among Latin American countries and for simplicity, we opted for reversing the order of aggregation. We first aggregate

each dimension (like the opportunity of access to water) across people, and later aggregate dimensions. Certainly, we could aggregate first a proxy of the five indicators consider in this study, and we plan to do this exercise in the very near future. However, one of the main advantages of the HOI is its flexibility to track progress in a variety of basic goods and services. In country specific works we can combine information from different sources such as health surveys, living standard measurements surveys, student assessment test scores, among others. For keeping this flexibility, we would like to have as our base procedure a simple methodology that will allow us to combine information from different sources in building an overall HOI.

If the indicators are continuous, a variety of other alternative aggregation procedures would be available. These alternatives would include weighted versions of the three approaches introduced above as well as completely different function forms. For instance, an expression mimicking a constant elasticity of substitution function could be used.