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Financial "Whac-a-Mole": Bubbles, Commodity Prices, and Global Imbalances

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Financial "Whac-a-Mole": Bubbles, Commodity Prices and Global Imbalances

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This paper provides theoretical and empirical evidence to support the proposition that three of the major global macroeconomic phenomena of recent years –the persistent global imbalances, the subprime crisis, and the volatile oil prices that followed it– are tightly interconnected. They all stem from a global environment where sound and liquid financial assets are in scarce supply. In this framework, the large recent fluctuations in oil prices are the result of the interplay between a tight oil market and the search for financial assets: Bad news in US financial markets is good news for oil as an asset; conversely, good news in the former is bad news for the latter. Our analysis also indicates that, in the short run, this endogenous response of oil prices has limited the extent of the adjustment in US's external accounts that otherwise would have taken place in response to the US financial crisis.

Figure 1 displays the main patterns of global imbalances since 1990. In particular, it shows the current account of the US, Europe & Japan, emerging Asia and oil producing economies, relative to world GDP.[1] The facts are well-known: starting in 1991, the US current account deficit worsened continuously, reaching 6.4 percent of US GDP in the fourth quarter of 2005, then stabilizing back to 5 percent of GDP by early 2008. The counterpart

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[1] The sample of European countries includes: Austria, Belgium, France, Germany, Ireland, Italy, Netherlands, Spain, Denmark, Iceland, Sweden and Switzerland. Oil producing countries include: Canada, Norway, Mexico, Venezuela, Russia, Saudi Arabia, Iran, Nigeria, Kuwait, Libya, Oman and Bahrain. Asian countries include: South Korea, China, Taiwan, Hong Kong, Indonesia, Malaysia, the Philippines, Singapore and Thailand.

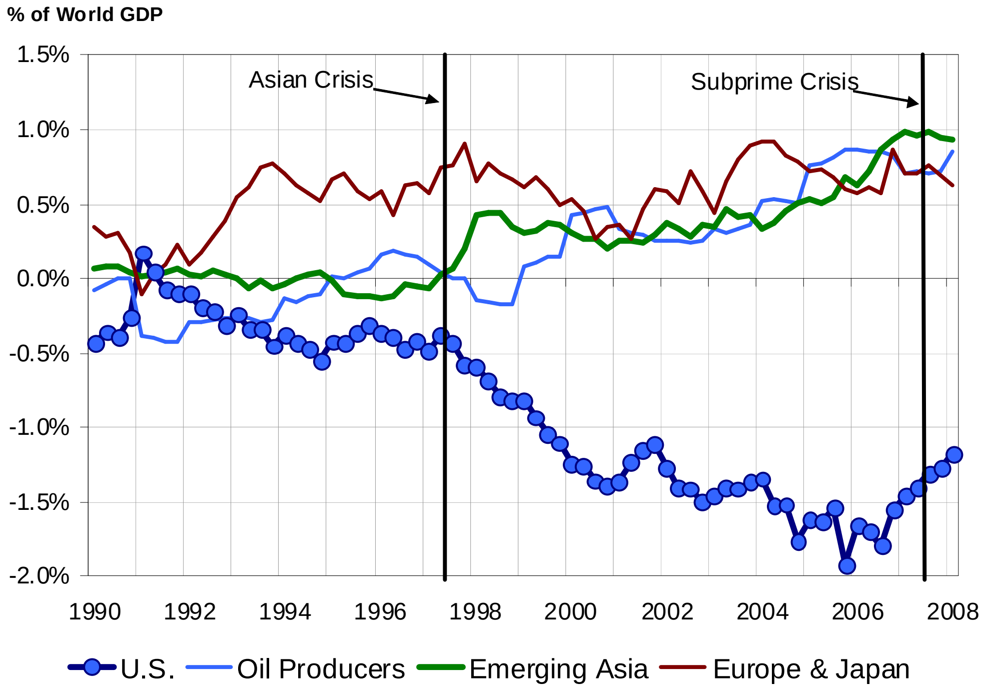

Figure 1: Global Imbalances (as a fraction of World GDP), 1990:1-2008:1. Data Sources: WDI, WEO, IFS & OECD. Author's calculations.

of the US deficits, initially due to Japan and Europe, were bolstered by emerging Asia and commodity producing countries after 1997.

In Caballero, Farhi and Gourinchas (2008) we showed how this build-up in "global imbalances" could be understood as the consequence of asymmetries in financial development and growth prospects across different regions of the world. In particular, we argued that the Emerging Market (EM) crises of the end of the 1990s, the subsequent rapid growth of China and other East Asian economies, and the associated rise in commodity prices in recent years, reoriented capital flows *from* emerging markets *toward* the US. In effect, EMs and commodity producers in need of sound and liquid financial instruments to store their newfound wealth, turned to the US financial markets, perceived as uniquely positioned to provide these financial assets.

As we explained then, a by-product of this reallocation of capital flows was a necessary decline in US and world real interest rates (see figure 2) and a boom in US asset markets. As foretold by then Governor Bernanke in his influential 'Savings Glut' speech (Bernanke (2005)), it is now apparent that this boom was located in no small part in a rise in US housing markets and the related markets for structured credit instruments (see figures 3 and 4). Ex-

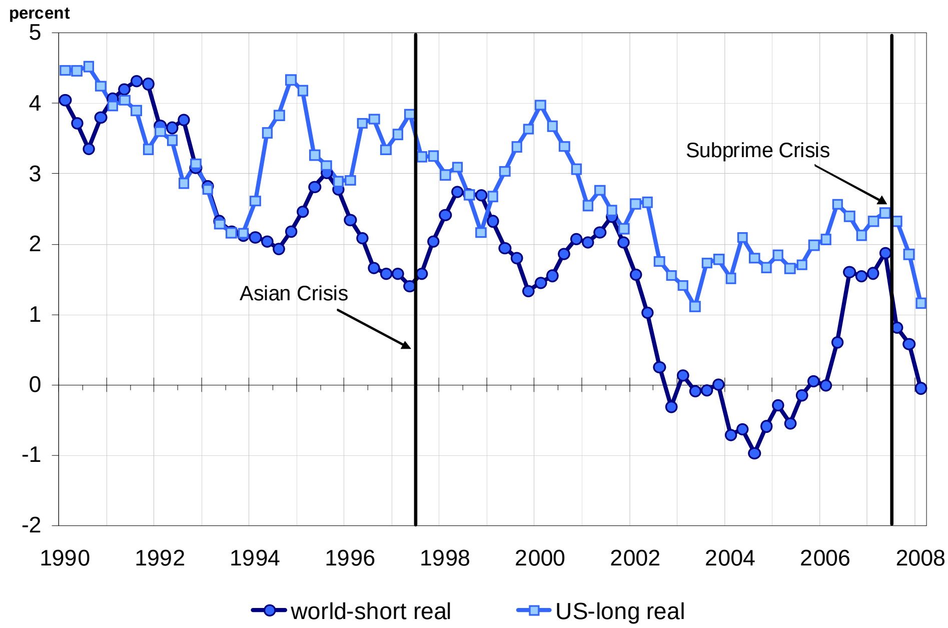

Figure 2: World and US Real Interest Rates, 1990:1-2008:1. Data Sources: IFS, WDI, OECD & SPF. Author's calculations. The world short real rate is a GDP-weighted of the 4-quarter average ex-post 3-months Treasury Bill real rates for the G-7 countries

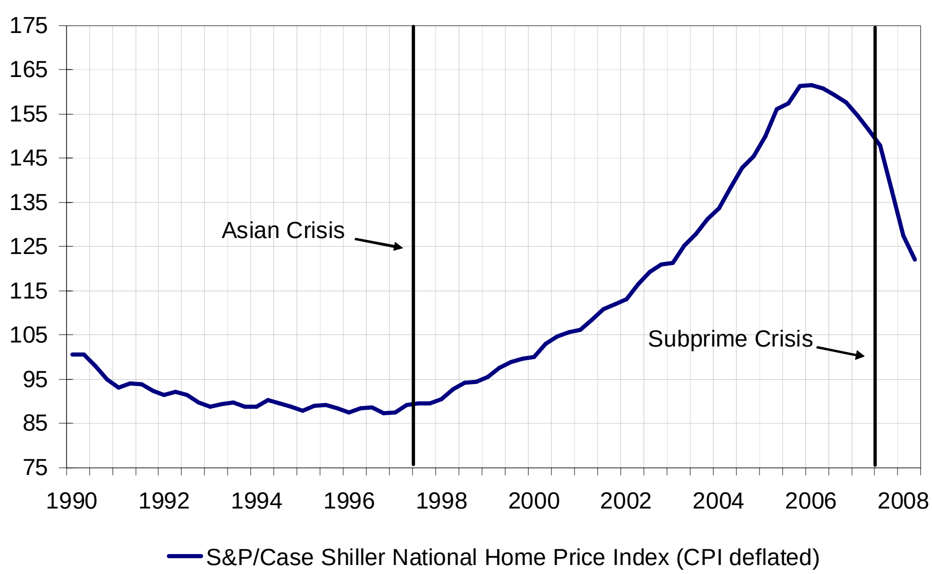

Figure 3: S&P Case Shiller National Home Price Index (CPI-deflated) 1990:1-2008:2. Data Source: Standard and Poors, IFS & Author's Calculations.

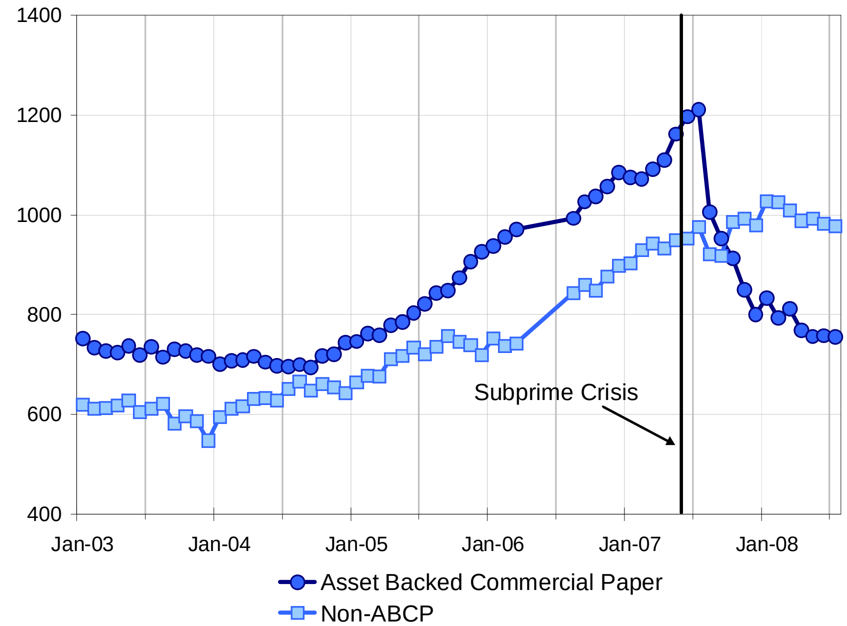

Figure 4: Commercial Paper, Amount Outstanding, billions USD. 2004:1-2008:7. Data Source: Federal Reserve Board.

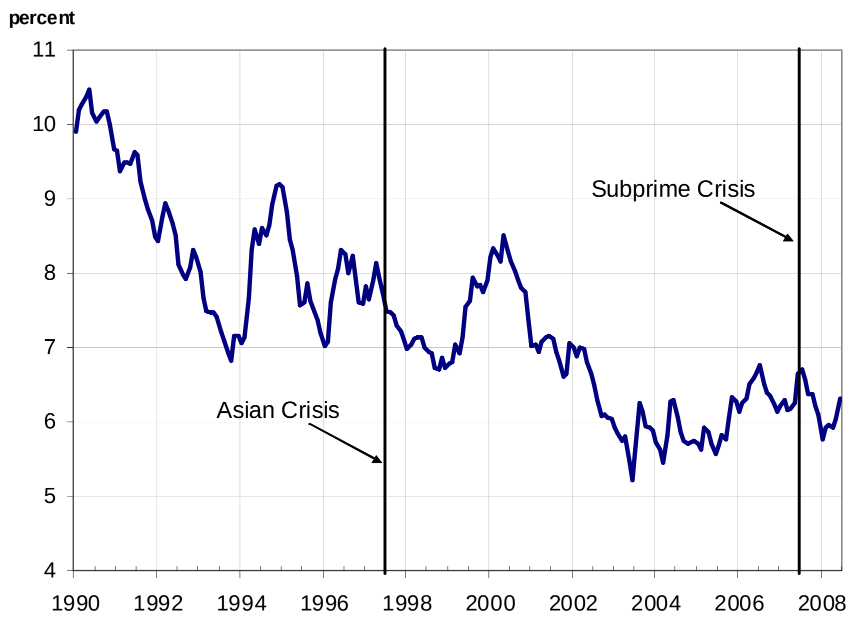

Figure 5: Contract Rate on 30-Year, Fixed-Rate Conventional Home Mortgage Commitments, 1990:1-2008:2. Data Source: Federal Reserve Bank series H15.

ante real interest rates on 10-year U.S. government bonds fell below 2% in 2002 (see figure 2) while the rate on a 30-year fixed rate conventional mortgage reached 5.23 percent in June 2003 (see figure 5), with a 2.9 percent annual inflation. In the context of low real interest rates, US households were encouraged to take on more housing risk than they could bear, risks that *magically* disappeared from the mortgage-backed securities and other structured investment vehicles whose supply exploded over the same period (figure 4). The catastrophic and systemic failures of this originate-to-distribute model are now well-documented.

By sometime in 2006, the appreciation in US real estate prices came to a halt and the US current account deficit began to turn around (see Figures 1 and 3). Starting in earnest in June 2007 with the bailout of two Bear Stearns funds that could not meet their margin calls, the world economy entered, with a certain fracas, into a period of significant global adjustment. Within weeks, funding dried up for entire segments of the U.S. and international banking sector, especially asset-backed commercial paper (see figure 4), leading to major convulsions of credit and money markets, including the dramatic collapse and rescue of several US and European commercial and investment banking institutions. More than 12 months after the onset of the crisis, and despite an accalmy following the Fed-orchestrated bailout of Bear Stearns, financial markets appear nowhere near stabilized.[2]

Most likely, the strong US capital inflows of the last few years contributed to the significant weakening of US credit markets. The eventual recognition of their degraded performance was one of the triggers of the current crisis. However, this weakening is in itself part of the endogenous response of US financial markets to world financial conditions. In effect, US assets became stretched in trying to accommodate the world's excess demand for assets. Therein lies the structural problem. This chronic excess demand for assets derives from financial underdevelopment in emerging markets and most commodity producing economies, rather than from macroeconomic imbalances. Excess asset demand leaves an unmistakable signature in low real interest rates, which in turn provide a fertile ground for bubbles to emerge. Thus an alternative -perhaps metaphoric- interpretation of the sequence of events is that the bubble located in emerging markets during the 1990s migrated toward the US housing and credit markets (and the NASDAQ before that) following the EM crisis and the coming on-line of capitalist China.[3]

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[2]See Brunnermeier (forthcoming 2008) and Greenlaw, Hatzius, Kashyap and Shin (2008) for detailed and recent accounts of the subprime crisis.

[3]See Caballero and Krishnamurthy (2006) for a model of bubbles and capital flows in emerging markets based on financial underdevelopment.

With the US financial crisis, that bubble collapsed too. The excess asset demand that produced it did not. Emerging Markets and commodity producers are more than ever in search of investment opportunities. Witness the long list of sovereign wealth funds and the financial means at their disposal. According to Deutsche Bank (2007), Sovereign Wealth Funds managed $3 trillion USD as of September 2007, and are expected to manage an additional $7 trillion within 10 years. Another bubble could easily appear as the endogenous response of a world economy that tries to increase the global supply of financial assets. And so, we are engaged in a global game of '*whac-a-mole*', after the popular arcade game where players wait for moles to appear before 'whacking' them back into their holes. Like the players of the game, we are waiting for bubble conditions to emerge somewhere else in the world economy. This time, we did not have to wait too long!

According to Caballero et al. (2008), the sharp contraction in US asset supply caused by the subprime crisis should lower equilibrium interest rates and trigger a rebalancing away from now 'toxic' US assets.[4] The resulting decline in US wealth reduces domestic consumption, improves the trade balance and the current account. This is in line with what we observed since June 2007: US long real rates fell from 2.3 percent to 1.2 percent by March 2008 (figure 2). The current account (trade) deficit improved from 5.6 (5.2) percent of GDP in June 2007 to 5.0 (5.0) percent in June 2008.

While qualitatively right, the response of the trade balance and the current account has been muted relative to what our basic view implies. That is, if the relative financial appeal of the US is what is behind the initial imbalances, how come the subprime crisis has not led to a sharper turn around in the US current account? We argue here that the answer lies in the *endogenous* response of commodity prices. Essentially, the decline in equilibrium real interest rates makes it privately worthwhile to transform commodities into an asset (or even a new bubble). The mechanism is related to that in Hotelling (1931): sufficiently low real interest rates make inventory accumulation profitable and drive up the price of exhaustible resources. Figure 6 reports the real price of a barrel of West Texas Intermediate (WTI) in 2008 US dollars. Between June 2007 and June 2008, real crude prices increased by almost 100 percent, before retrenching significantly in recent weeks.[5]

However, because commodity inventories are very low, a by-product of the strong demand

[4]The model predicts a simultaneous move towards 'safe' US assets. This flight-to-quality is an important feature of our analysis.

[5]The increase in real crude prices between June 2007 and end of August 2008 is still in excess of 50 percent.

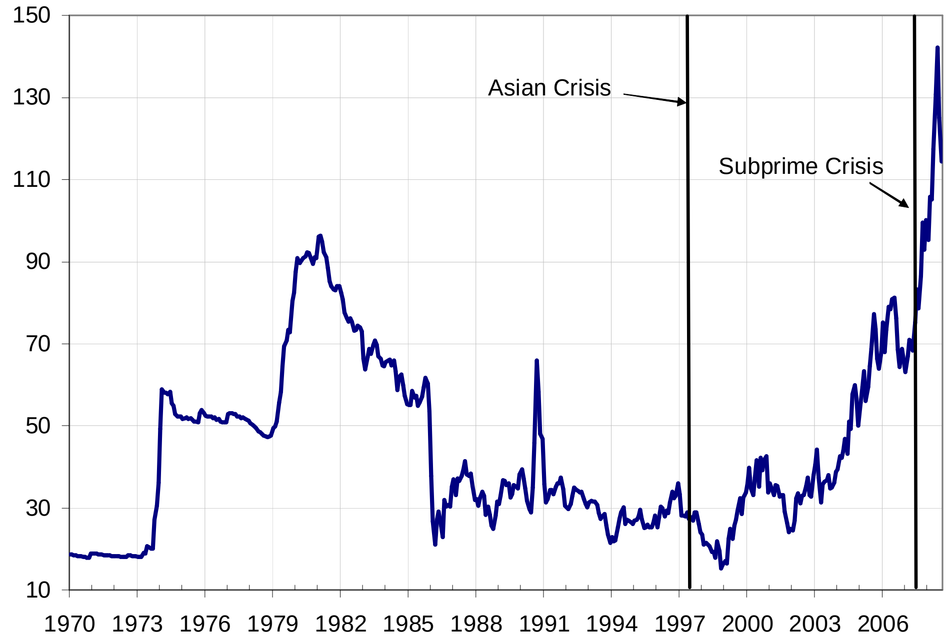

Figure 6: Real Price of West Texas Intermediate barrel of oil, 2008 dollars. Jan-1970 to Aug-2008. Data Sources: Global Financial Database & IFS; Author's calculations.

arising from the robust growth of Emerging Economies, the net asset creation from this mechanism is initially small. In contrast, the strong impact of the price rise on the income of commodity-producing economies leads to a sharp rise in their demand for store of values, which further depresses real interest rates and stabilizes capital outflows to the US in the short run.[6] According to our framework, this "petrodollar" effect is central for understanding the macroeconomic impact of the subprime crisis.

To establish the connection between all three developments more formally, we present a model based on Caballero et al. (2008) with commodities. The model has two regions U and M. We interpret U as the US and M as the rest, with an emphasis on emerging market economies and commodity producers. The model features two goods: a non-storable good X produced by both countries and a storable commodity Z that is only produced by M. The supply of X grows exogenously while the supply of Z is constant. This is meant to capture the growing demand pressures on commodities that arise from robust world economic growth. We set-up the model so that a bubble develops initially in U. As discussed above, we interpret

[6]The reader may wonder why the rise in the price of oil is not simply a transfer of income from oil consumers to producers and hence has no impact on asset demand. The answer is in our choice of numeraire, which is the non-commodity good. This will be clearer once we describe the model but, as with all normalizations, it has no substantive implications.

this bubble metaphorically as the extent to which asset markets in U are stretched to provide financial assets to the world. With the bubble, the US runs a larger current account deficit (the external imbalances) and world interest rates are low.

The original event in our model is the US financial crisis: the bubble bursts at $t = 0$, leaving savers scrambling for alternative stores of values. The resulting decline in real interest rates has two effects. First, it puts a floor under the value of 'good' U assets. This translates into a flight-to-quality, from the bubble to the 'good' U asset. Second, and more importantly, it triggers the commodity markets into action. As speculative hoarding takes place, the price of commodities jumps. Per se, this increase in commodity prices results in a significant wealth transfer from U to M. But M needs good stores of value. Thus a significant portion of that newfound wealth finds its way back into U. The resulting capital inflows further boost the value of the domestic asset and cushion the impact of the bubble's burst. Eventually, and gradually, the increase in asset supply due to commodity inventories pushes up interest rates, which forces rebalancing in U... unless a new bubble emerges along the way in U.

Before turning into the body of the paper, it is worth clarifying two modeling subtleties that are important in interpreting our formal discussion. First, while the commodity side of our model shares some of Hotelling (1931)'s seminal insights, ours does not rely on Hotelling's key stock constraint (exhaustible resource). Instead, in our context there is a flow-extraction constraint which is insufficient to meet demand growth. This mismatch is the main factor behind the structural trend in commodity prices. In this context, the subprime collapse superimposes on the previous trend a speculative reason (in the precise sense of a rational bubble) for price increases. Second, this speculative factor raises the effective opportunity cost of resource extraction for producers since there is now an asset-opportunity-cost, as in Hotelling's model, which reduces extraction incentives for commodity producers. The latter response means that, in equilibrium, there need not be any rise in measured inventories, and hence our allusion to inventories throughout includes previously unextracted commodities.

Section 1 describes the basic mechanism connecting the financial crisis to commodity prices and global imbalances. Section 2 focuses on calibration and dynamic implications. Section 3 describes the connection between inventories and speculation, while Section 4 presents evidence supporting the speculative nature of the rise in oil prices following the subprime crisis, and of the recent drop in these prices. Section 5 concludes and is followed by appendices on our choice of numeraire and an extension to show how declining inventories can be consistent with speculation even if no inventories are held underground.

1 The Basic Mechanism and Some Back-of-the-Envelope Calculations

This section contains our basic framework. It serves to corroborate the quantitative relevance of the mechanism we propose. In the first part of this section we discuss the impact of the subprime crisis on global equilibrium real interest rates and commodity prices. In the second part we study the implications of this shock for global imbalances. We conclude the section with back-of-the-envelope calculations.

1.1 Global Capital and Commodity Markets

1.1.1 The model for the world economy

Time evolves continuously. Infinitesimal agents (traders) are born at a rate θ per unit of time and die at the same rate; population mass is constant and equal to one. Agents receive some endowment at birth, which, for simplicity, they save in its entirety until they die.[7] Denote by W_t the savings accumulated by households at date t. In every period, aggregate consumption C_t is then a constant fraction θ of these accumulated savings:

$$C_t = \theta W_t. \tag{1}$$

Households consume a basket of two goods: an X-good (the numeraire) and a Z-good. Intratemporal preferences over these two goods are of the constant-elasticity-of-substitution (CES) type, with an elasticity of substitution σ:

$$C_t = \left[C_{X,t}^{(\sigma-1)/\sigma} + \alpha^{\frac{1}{\sigma}} C_{Z,t}^{(\sigma-1)/\sigma} \right]^{\sigma/(\sigma-1)}. \tag{2}$$

where $\alpha > 0$ controls the equilibrium share of expenditures on the Z-good.

Given a relative price p_t of the Z-good, households split their consumption between both goods as follows:

$$C_{X,t} = \frac{\theta W_t}{1 + \alpha p_t^{1-\sigma}} \text{ and } C_{Z,t} = \frac{\alpha p_t^{-\sigma} \theta W_t}{1 + \alpha p_t^{1-\sigma}}. \tag{3}$$

[7] As Caballero et al. (2008) show, this can be interpreted equivalently as log-preferences over consumption streams.

The X-good is a conventional non-storable good, while the Z-good is a storable commodity. Denote by $I_t \geq 0$ the outstanding inventories of the Z-good. Storing the commodity imposes an iceberg storage cost $d \geq 0$ per unit of time and good stored. Denote r_t the instantaneous interest rate (in terms of the X-good). By arbitrage, p_t must satisfy:

$$\frac{\dot{p}_t}{p_t} \leq r_t + d, \tag{4}$$

with equality if $I_t > 0$ or $\dot{I}_t > 0$. This arbitrage equation is central to the analysis of storable commodities, such as Hotelling (1931). It states that the capital gains on commodities cannot exceed the interest rate, net of any convenience yield or carrying cost.

The endowment of the X-good, denoted X_t, grows at rate $g > 0$ over time. By contrast, we assume that the endowment of the Z-good is constant through time: $Z_t = Z$, which allows us to capture the idea that demand pressures on commodities are growing over time.[8]

The Z-good is assumed to be noncapitalizable unless it is transformed into inventories (below or above the ground). In contrast, a fraction δ of the X-good is capitalizable. We capture this feature as follows. At every point in time, there is a number X_t of identical trees with an aggregate market value of V_t. Each tree yields δ units of X-good per period to their current owners. Since the number of trees grows at rate g, the total value of new trees is gV_t per unit of time. The fraction of the output that is not capitalized is distributed to newborns, as are the new trees. Hence, the total endowment received by newborns per unit of time is composed of $(1-\delta)X_t$ units of the X-good, Z units of the Z-good and gX_t new trees. The value of this endowment is $(1-\delta)X_t + p_t Z + gV_t$.

The return on existing trees is the dividend price ratio $\delta X_t/V_t$ plus the capital gain $\dot{V}_t/V_t - g$ which, in equilibrium, must be equal to the instantaneous interest rate in the economy, r_t:

$$r_t V_t = \delta X_t + \dot{V}_t - gV_t. \tag{5}$$

In addition to the tree asset, some of our equilibria will exhibit rational bubbles, B_t,

[8] Note that our model differs from Hotelling (1931)'s in that we replace his stock constraint with a flow constraint on commodity production. This has important implications later on since it allows us to separate more cleanly the asset-aspect of commodities from their goods-aspect. Moreover, in our framework, macroeconomic conditions determine whether one aspect or the other dominates in price-determination. See Jovanovic (2007) for a Hotelling based model of bubbles in exhaustible resources.

which must satisfy the arbitrage condition:

$$\dot{B}_t = (r_t + \lambda)B_t, \tag{6}$$

where $\lambda > 0$ is the hazard that the bubble bursts in the next instant. For simplicity we analyze the limit case as λ goes to zero and $d > \lambda$. The former assumption allows us to approximate the solution with the perfect foresight case, while the latter reduces the number of subcases we need to discuss.

Savings decrease with withdrawals (deaths), and increase with the endowment allocated to new generations and the return on accumulated savings:

$$\dot{W}_t = -\theta W_t + (1-\delta)X_t + p_t Z + gV_t + r_t W_t. \tag{7}$$

In equilibrium, savings must be equal to the value of all the assets in the economy:

$$W_t = V_t + p_t I_t + B_t. \tag{8}$$

Using (3) and imposing market clearing in the X-good market, we obtain:

$$\frac{\theta W_t}{1+\alpha p_t^{1-\sigma}} = X_t \text{ and } \frac{\alpha p_t^{-\sigma}\theta W_t}{1+\alpha p_t^{1-\sigma}} = Z - \dot{I}_t - dI_t. \tag{9}$$

In equilibrium, replacing (9) back into (7), we obtain that the equilibrium interest rate is (for the case with inventories, i.e. $\max\left\langle I_t, \dot{I}_t \right\rangle > 0$):

$$r_t = \frac{\theta\delta + \theta g \frac{B_t + p_t I_t}{X_t} - \theta\left(\frac{p_t Z}{X_t} - \alpha p_t^{1-\sigma}\right) + \alpha d(1-\sigma)p_t^{1-\sigma}}{1+\alpha\sigma p_t^{1-\sigma}}. \tag{10}$$

Conditional on X_t, Z and I_t, the interest rate rises with δ because this increases the share of income that is capitalizable and hence it raises the supply of assets. The interest rate increases also with the 'endogenous' increase in asset supply $B_t + p_t I_t$. However, an increase in p_t also raises asset demand. Later on we will see that for plausible parameter values the latter effect dominates and the interest rate is decreasing with respect to commodity prices. Since commodity prices are also decreasing with respect to the interest rate, this interaction gives rise to potentially large feedbacks.

1.1.2 The $\sigma = 1$ case

Although in practice the short run elasticity of demand for the Z-good is significantly smaller than one, it is useful to start with $\sigma = 1$ since it allows us to characterize explicitly the main mechanisms at work. We simplify things further by studying the case where d converges to zero (while preserving the assumption $d > \lambda$).

Assume momentarily that the equilibrium has neither inventories nor bubbles, then replacing (9) back into (7) yields a reference interest rate, $r^{ref} = \theta\delta/(1+\alpha)$. Henceforth we shall assume that in this reference case financial assets are sufficiently scarce (δ low) for the economy to be dynamically inefficient ($r^{ref} < g$):

Assumption 1 $\delta < g(1+\alpha)/\theta$

Bubbleless equilibrium. Suppose for now that there are no bubbles, then the equilibrium *must have inventories*. To see this, note that if there are no inventories $r = r^{ref} < g$. But in this case equation (9) requires $p = \alpha X/Z$, so the price of commodities grows at a rate g which exceeds the equilibrium interest rate. Thus, there is a clear incentive to accumulate inventories, which is a contradiction of the no-inventories premise.

From (4) and (9), the dynamics of the economy can be summarized in a simple phase-diagram with state variables I_t and $q_t \equiv p_t/X_t$:

$$\begin{cases} \dot{I}_t = Z - \alpha q_t^{-1} \\ \dot{q}_t = (r_t - g)\, q_t \end{cases}, \tag{11}$$

where r_t satisfies (from equation (10)):

$$r_t = \theta \frac{\delta + \alpha - q_t\,(Z - gI_t)}{1+\alpha}. \tag{12}$$

Asymptotically, the level of inventories stabilizes at a strictly positive level which is proportional to the degree of dynamic inefficiency in the economy:

$$\lim_{t\to\infty} I_t = \bar{I} = \frac{1+\alpha}{\alpha\theta g}\left(g - r^{ref}\right) Z > 0. \tag{13}$$

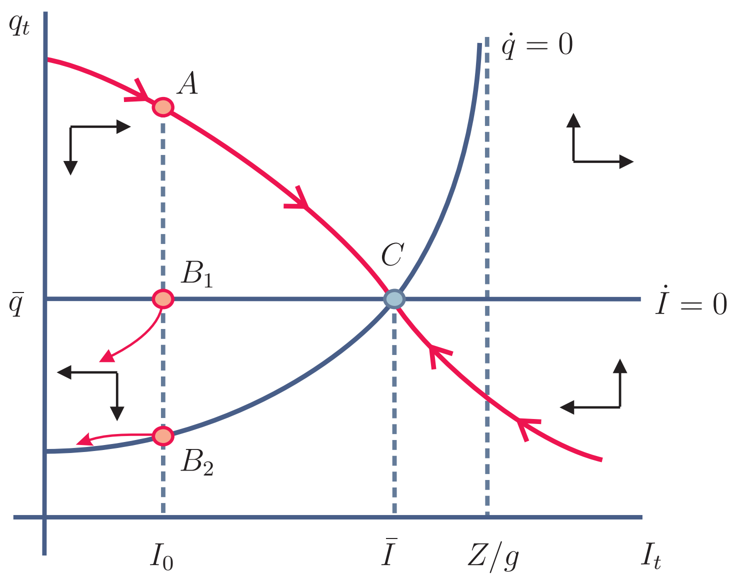

Figure 7: The Model with Inventories when $\sigma = 1$.

The price of the Z-good grows at rate g :

$$p_t \underset{t\to\infty}{\sim} \alpha \frac{X_t}{Z},$$

which implies that interest rates converges to the growth rate of the X-good:

$$\lim_{t\to\infty} r_t = g.$$

The value of accumulated savings also grows at rate g as does the value of the trees:

$$W_t = \frac{1+\alpha}{\theta} X_t; \quad V_t \underset{t\to\infty}{\sim} \frac{\delta}{g} X_t.$$

Figure 7 reports the phase diagram. The dynamic system (11) exhibits the saddle-path property.[9] This saddle-path is downward sloping: when inventories are low ($I_t < \bar{I}$), the price of commodities is high ($q_t > \bar{q} \equiv \alpha/Z$) and decreasing ($r_t < g$). Conversely, when inventories are abundant ($I_t > \bar{I}$), the price of commodities is low ($q_t < \bar{q}$) and increasing ($r_t > g$).

[9]Figure 7 is drawn for the case where $\theta\delta/ (1+\alpha) < g < \theta (\delta + \alpha) / (1+\alpha)$, where the first inequality is a consequence of assumption 1. The case where $g > \theta (\delta + \alpha) / (1+\alpha)$ is similar and features also a downward sloping saddle-path, but the $\dot{q} = 0$ is downward sloping.

A key element of our model lies in the slope of this saddle-path. To understand why it is downward sloping, consider an initial inventory position $I_0 < \bar{I}$ and suppose that the price is such that the commodity market is initially in equilibrium at that inventory level ($\dot{I}_0 = 0$,or $q_0 = \bar{q}$). This is point B_1 on figure 7. It is immediate that the interest rate r_0 that clears the asset markets at point B_1 must be *below* the growth rate g.[10] This is because the locus $r = g$ defines the $\dot{q} = 0$ curve, and we know from (12) that an increase in q above that locus lowers interest rates. The economic intuition is that at $q = \bar{q}$, too few assets are created through inventories (whose equilibrium value is $\bar{q}I_0$). Equilibrium on global asset markets then requires a low interest rate. But when $r_0 < g$, the (normalized) price of commodities declines over time, which increases demand for commodities and reduces inventories.

Suppose instead that q_0 is such that $r_0 = g$, i.e. $\dot{q}_0 = 0$. This is point B_2 in figure 7. While asset markets are in equilibrium, demand for commodities is high, so that inventories must decline: $\dot{I}_0 < 0$. This decline in inventories reduces asset supply, which pushes down interest rates below g, leading to a subsequent decline in (detrended) commodity prices.

The equilibrium requires that the price of commodities is initially sufficiently high to depress the demand for commodities, and allow inventory accumulation ($\dot{I}_0 > 0$). Equivalently, the price of commodities needs to rise sufficiently to depress equilibrium interest rates and make inventory accumulation profitable. This is represented by point A in the figure. This high initial price depresses interest rates *below* r_0. Over time, since $r_t < g$, (normalized) commodity prices decrease, which increases demand for commodities and slows down inventory accumulation. The steady state is reached at point C.

The price of commodities performs a dual role in the model with inventories: it influences the demand for Z-good on the spot market; and it influences the global supply of assets in the economy $(V_t + p_t I_t)$. As in traditional models of portfolio balance, it is the tension between these two functions that generates interesting dynamics.[11]

Bubbles. Now let us turn to the opposite extreme, where bubbles exist and do not vanish asymptotically relative the size of the economy. In the limit, since we assumed $d > \lambda$, there are no inventories:

$$\lim_{t \to \infty} I_t = 0.$$

[10] One can check that $r_0 = [\theta\delta + \theta\alpha g I_0/Z] / (1 + \alpha) < g$.

[11] See Kouri (1982) and more recently Blanchard, Giavazzi and Sa (2005) for examples of the portfolio balance models.

Without inventories, the Z-good is for consumption only, and its price grows at rate g :

$$p_t \underset{t\to\infty}{\sim} \bar{q}X_t.$$

From the equilibrium on the condition for the X-good the value of accumulated savings grows at rate g :

$$W_t = \frac{1+\alpha}{\theta} X_t. \tag{14}$$

Substituting into the savings equation (7), one finds that the interest rate converges to g

$$\lim_{t\to\infty} r_t = g,$$

while the value of the trees converges to:

$$V_t \underset{t\to\infty}{\sim} \frac{\delta}{g} X_t. \tag{15}$$

Taking the difference between (14) and (15), the bubble converges to:

$$B_t \underset{t\to\infty}{\sim} \frac{1+\alpha}{\theta g} \left(g - r^{ref}\right) X_t. \tag{16}$$

Note that the size of the asymptotic bubble in (16) is the same as the size of the asymptotic equilibrium inventories pI in the bubbleless equilibrium (13).

The No-Inventory Economy (a benchmark). In our model, the price of the Z-good is both a relative price, and, when inventories are non-zero, an asset price. In order to illustrate the importance of this dual role, we provide a benchmark-economy where the inventory-channel is turned off. That is, we assume that storage costs are prohibitive (i.e., d is very large) so the Z-good cannot be stored.

This benchmark economy has two long run steady-states: a bubbly one and a bubbleless one. The bubbly steady-state is exactly as above with the same equilibrium prices and quantities. However, the bubbleless equilibrium is different since inventories cannot be accumulated. In the bubbleless equilibrium, market clearing for the Z-good implies that p_t grows at rate g :

$$p_t = \bar{q}X_t;$$

while equilibrium on asset markets implies that the interest rate is equal to

$$r_t = r^{ref} < g;$$

Finally, the value of accumulated savings (which equals the value of the tree) grows at rate g as before:

$$W_t = V_t = \frac{1+\alpha}{\theta} X_t.$$

Note that Assumption 1 implies that the interest rate r^{ref} in the bubbleless equilibrium of the no-inventory economy is smaller than the interest rate g of the bubbleless equilibrium of the economy with inventories. The reason for this difference is that total asset supply is smaller in the economy without inventories. Note also that p_t is the *same* in the bubbly equilibrium and in the bubbleless equilibrium without inventories. That is, the price is entirely determined by the relative endowments of the X-good and the Z-good and is completely decoupled from the asset market.

1.1.3 The Financial Crash and Commodity Boom

Suppose now that a 'subprime' shock takes place. This can be interpreted as the realization that financial instruments are less sound than they were once perceived to be. It could result from, *inter alia*, the realization that corporate governance is less benign than once thought (excessive risk-taking and poor risk management by investment banks, or the realization that securitization and certification by rating agencies involve important agency problems); a significant loss of informed and intermediation capital (deleveraging of commercial and investment banks hit by losses) and so on. All of these factors -and more- have been mentioned in the events surrounding the recent subprime crisis.[12] We assume that this shock is completely unanticipated, but this is not crucial to our analysis as long as there is some degree of market incompleteness, preventing agents from fully hedging away these shocks.

In the model we capture this shock with a crash in the bubble B at date t_0. The dynamics that follow are described by those in the bubbleless system and are illustrated in Figure 8. Right before the shock, the economy is at point A, with $q = \bar{q}$ and no inventories ($I_{t_0} = 0$). When the crisis erupts, the price of commodities jumps to point B, on the saddle-path. With decreased demand on the spot market, the economy immediately begins to build inventories

[12] See Greenlaw et al. (2008) and Brunnermeier (forthcoming 2008).

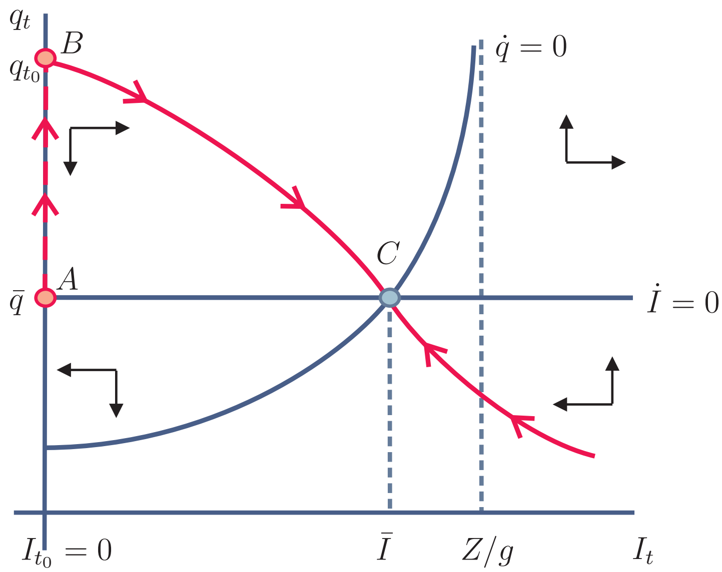

Figure 8: A 'Subprime Crisis' when $\sigma = 1$.

(which could be kept under the ground). The price of commodities remains high until the economy converges to the new steady state (point C).

The collapse of the bubble reduces asset supply and leads to a drop in the interest rate. By effectively reducing the discount rate, lower interest rates make more attractive the strategy of storing the Z-good to sell it in the future, which validates the build-up in inventories. Higher commodity prices along the transition are required to lower demand and restore equilibrium in the Z-good market. The commodity price jumps at $t = t_0$, and asymptotes from above to the same path as in the pre-crash economy.

The interest rates initially drops by:

$$r_{t_0^+} - r_{t_0^-} = -g\frac{B_{t_0^-}}{W_{t_0^-}} - \frac{\theta\alpha}{1+\alpha}\left(\frac{p_{t_0^+}}{p_{t_0^-}} - 1\right) < 0 \tag{17}$$

and then converges smoothly back to the asymptotic level g. There are two terms on the right hand side of (17). The first "bubble-burst" term $-gB_{t_0^-}/W_{t_0^-}$ is directly due to the collapse of the bubble. The second "commodity-price-jump" term follows from the increase in the price of the Z-good, which raises the rate of wealth accumulation. Since inventories are only gradually accumulated, an additional gap opens between asset supply and asset demand, which requires a further decline in interest rates.

In the benchmark no-inventory economy, the normalized price of commodities stays constant and equal to $\bar{q}$, so the economy remains indefinitely at point A in figure 8. Since the price of commodities does not jump, the second term in equation (17) would equal 0, and the interest rate drop would be entirely given by the bubble-burst term.[13] We conclude inventory accumulation, through its effect on p, further depresses interest rates immediately following the shock.

Note that there is a strong *flight-to-quality* feature in the model since both the value of accumulated savings and the *total* value of assets are continuous at $t = t_0$:

$$W_{t_0^-} = (V_{t_0^-} + B_{t_0^-}) = W_{t_0^+} = V_{t_0^+} = \frac{1+\alpha}{\theta} X_{t_0}.$$

This means that the decline in interest rates raises the value of the trees (the 'good' asset) enough to fully offset the loss in value due to the crash in the bubble. Later on we will show that when $\sigma < 1$, the decline in interest rate is more pronounced that in the $\sigma = 1$ case, which further raises the value of the remaining good assets. It is important to highlight at this time that if we were to use a true CPI (rather than the price of good X) to deflate quantities, then wealth would always drop in real terms after a crash. This alternative numeraire formulation, which we develop in the appendix, modifies the "language" but none of our substantive conclusions.

1.2 Global Imbalances

Let us now study global equilibrium in a world with two large regions, $i = \{U, M\}$. We interpret region U as the US, with initially good but perhaps fragile financial conditions, and region M as the set of emerging and commodity producing economies that offset US external imbalances. We distinguish the long run and the short run. In the long run, the presence of commodities leads to a larger global rebalancing in response to a subprime shock in the US (the U-region). In the short run, we show that the opposite holds when $\sigma < 1$. That is, in the short run, the endogenous rise in commodity prices *dampens* the equilibrium response of the US current account to a subprime shock.

Each of the regions is described by the same setup as the world economy, with an instantaneous return from hoarding a unit of either tree, r_t, which is common across both regions and satisfies:

$$r_t V_t^i = \delta X_t^i + \dot{V}_t^i - g\dot{V}_t^i \qquad (18)$$

[13] We know from the previous analysis that the interest rate would drop to $r^{ref} = \delta\theta/(1+\alpha)$.

where V_t^i is the value of country i's tree at time t. We assume throughout that both regions have common parameters g, δ and θ, but that the initial bubble is concentrated in the U region. The latter captures the idea that the U region has more attractive assets than the M region. Moreover, we assume that the Z-good is produced only in the M economy and that the potential inventories are held in this region (perhaps under the ground, see later discussion). These two features are all that differentiates the two regions, aside from scale.

Let W_t^i denote the savings accumulated by agents in country i at date t. By analogy with (7):

$$\dot{W}_t^i = -\theta W_t^i + (1-\delta)X_t^i + gV_t^i + r_t W_t^i + 1_{\{i=M\}} p_t Z_t, \tag{19}$$

where $1_{\{i=M\}}$ is an indicator for country M. Adding (18) and (19) for U and M, the world economy is exactly the one described in Section 1.1 with:

$$W_t = W_t^U + W_t^M; \quad V_t = V_t^U + V_t^M; \quad X_t = X_t^U + X_t^M;$$

The current account CA_t^U of country U represents the net accumulation of assets by country U and is given by

$$CA_t^U = \dot{W}_t^U - \dot{V}_t^U - \dot{B}_t^U. \tag{20}$$

As before, the subprime shock takes place at $t = t_0$.

1.2.1 The Long run

Pre-crash equilibrium. Let us start from the steady-state with bubbles described in Section 1.1.2. Since the world interest rate is $r = g$, we obtain:

$$W_{t_{0-}}^U = \frac{1}{\theta} X_{t_0}^U \; ; \quad V_{t_{0-}}^U = \frac{\delta}{g} X_{t_0}^U; \quad B_{t_{0-}} = \frac{1+\alpha}{\theta g}\left(g - r^{ref}\right) X_{t_0} \tag{21}$$

The bubble increases the supply of US assets from $V_{t_0}^U = \frac{\delta}{g} X_{t_0}^U$ to $V_{t_0}^U + B_{t_0} = \frac{\delta}{g} X_{t_0}^U + \left(\frac{1+\alpha}{\theta} - \frac{\delta}{g}\right) X_{t_0}$. To interpret this increase in asset supply, observe that it is equivalent to an increase in the 'perceived' financial development index δ of the U region from δ to $\hat{\delta}^U$ that satisfies:

$$\hat{\delta}^U = \delta + \frac{1+\alpha}{\theta}\left(g - r^{ref}\right) \frac{X_{t_0}}{X_{t_0}^U}.$$

This increase in the 'perceived' capacity to capitalize assets in the U-region is just sufficient to ensure that the 'perceived' world capitalization index

$$\hat{\delta}^W = \frac{X_{t_0}^U}{X_{t_0}}\hat{\delta}^U + \left(1 - \frac{X_{t_0}^U}{X_{t_0}}\right)\delta = \frac{1+\alpha}{\theta}g,$$

is high enough to eliminate the dynamic inefficiency in the world economy ($r = g = \hat{\delta}^W \theta / (1+\alpha)$). Hence, the bubble in U is the endogenous response to the scarcity of good financial assets.

From (21), it follows that the current account satisfies:

$$\frac{CA_{t_{0-}}^U}{X_{t_0}^U} = \frac{g - r^{ref}}{\theta/(1+\alpha)}\left(1 - \frac{X_{t_0}}{X_{t_0}^U}\right) - \frac{g\alpha}{\theta} < 0 \tag{22}$$

under Assumption 1. The increase in perceived asset supply from δ to $\hat{\delta}^U$ allows U to run a larger persistent current account deficit (the so-called global imbalances). The contribution of the bubble to global imbalances can be assessed by comparing the current account deficit in (22) to that of an economy without bubbles or inventories (with $r = r^{ref}$).[14] In this economy, one obtains:

$$CA_{t_{0-}}^{U \text{ no bubble}} = -\frac{\alpha g}{\theta + g - r^{ref}},$$

from which we conclude that:

$$CA_{t_{0-}}^{U \text{ no-bubble}} - CA_{t_{0-}}^U = \frac{g - r^{ref}}{\theta/(1+\alpha)}\left[\frac{\alpha g}{(1+\alpha)(\theta + g - r^{ref})} + \frac{X_{t_0}}{X_{t_0}^U} - 1\right] > 0 \tag{23}$$

under Assumption 1. The reason for this larger current account deficit is that a disproportionate share of M's income is non-capitalizable (because their commodity income, pZ, is noncapitalizable unless it is transformed into inventories), while U produces a disproportionate share of the global assets.

Figure 9 represents graphically the external equilibrium in U in a 'Metzler-diagram' (Metzler (1960)). The curve labeled V^U/X^U represents the long run value of the U-tree, relative to output, as a decreasing function of the interest rate r. It is equal to δ/r. The curve labeled W^U/X^U represents the long run value of the savings to output ratio, as a function of the equilibrium interest rate. It is equal to $\left(1 - \delta + \frac{g\delta}{r}\right)/(\theta + g - r)$. It decreases, then increases with r.[15]

[14] Note that the economy without bubbles or inventories coincides with the benchmark no-inventory economy *after* the collapse of the bubble.

[15] While the asset demand schedule W^U/X^U can be downward sloping, the gap between W^U/X^U and

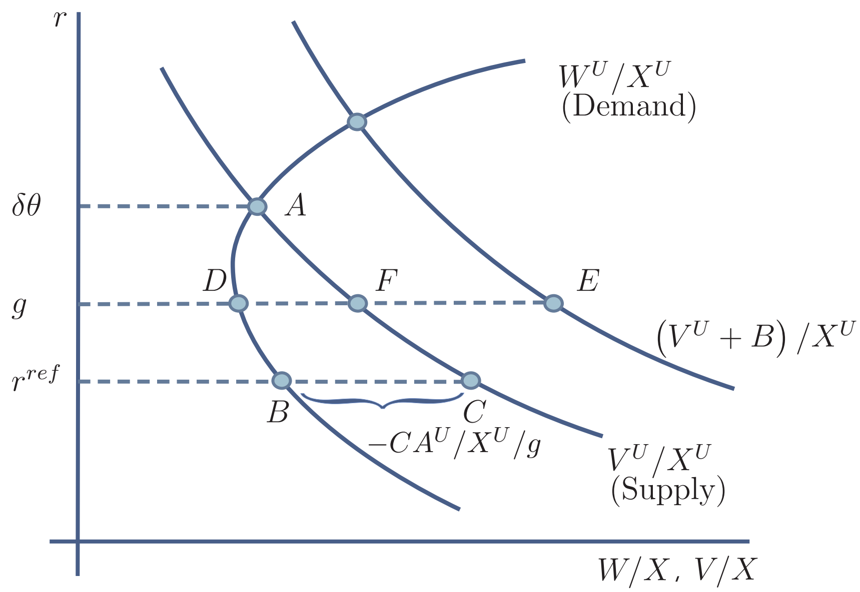

Figure 9: The Metzler Diagram when $\sigma = 1$.

Under autarky, without bubble or commodities, the long-run equilibrium would be at point A with $r = \delta\theta$. Under financial integration, but without bubble or inventories, the interest rate is lower, at $r^{ref} = \delta\theta/\left(1+\alpha\right)$, and the current account deficit is proportional to the distance between points B and C on figure 9. The reason for the lower equilibrium interest rate is that a larger fraction of global output is not capitalized when there are commodities. The lower interest rate allows U to supply more assets to M and to run both a current account and trade deficit.

In the presence of the bubble, the supply of assets increase from V^U/X^U to $\left(V^U+B\right)/X^U$. The increase has to be such that the world equilibrium interest rate increases from $r^{ref}$to g. As shown in (23), the current account deficit in the bubble equilibrium (proportional to the distance between points D and E) is always larger than in the no-bubble & no inventories case (the distance between points B and C).

Post-crash (asymptotic) equilibrium Since the asymptotic interest rate in the absence of bubbles is still $r = g$, the asymptotic current account deficit of the U region following the

V^U/X^U, equal to $\left(1-\delta\theta/r\right)/\left(\theta+g-r\right)$, is always increasing with the interest rate. The downward sloping part of the W^U/X^U curve comes from the impact of interest rates on asset demand through the new trees gV^U. When $g < \delta\theta$, the W^U/X^U curve has the shape shown in figure 9. When $g > \delta\theta$, the asset and demand curves cross on the downward sloping part of the asset demand curve W^U/X^U.

collapse of the bubble is smaller by *exactly* the size of the bubble:

$$\frac{CA_t^U}{X_t^U} \underset{t\to\infty}{\sim} g\left[\frac{1}{\theta}-\frac{\delta}{g}\right]$$

This asymptotic current account will be in deficit if the degree of dynamic inefficiency in the global economy is not too severe ($\delta\theta > g$), as is assumed in Figure 9. Otherwise, the buildup in inventories is significant, which increases the supply of assets in the M region and reduces its need to buy foreign assets for store of value.

In fact, this buildup of inventories implies that endogenous commodity prices lead to *more* rebalancing in the long-run. The reason is that inventories contribute to increasing asset supply in the M region, and hence endogenously reduce the effective asymmetry between the two regions in the long-run. In terms of figure 9, the current account deficit contracts from $D-E$ to $D-F$ as the bubble collapses and inventories are accumulated. Therefore, the inventory channel unambiguously leads to more rebalancing in the long run.

In the short run, however, this result can be overturned. We turn to this issue next.

1.2.2 The Short Run

The behavior of the current account in the short run depends on the initial portfolios, the degree of home portfolio bias, and the degree of substitution between the commodity and the general consumption good. Following Caballero et al. (2008), we assume an extreme form of home bias: at $t=t_0$ all the assets held by the agents in the U region are U assets. Moreover, we assume that domestic residents' portfolios are proportional to the relative value of trees and bubbles. The assumption of extreme portfolio home bias is a good approximation. As of 2005, Sercu and Vanpee (2007) find that the degree of home equity bias varies between 0.5 for Norway and 0.90 for Japan.[16] The assumption that domestic residents' portfolios are proportional to the relative value of trees and bubbles implies that M has a significant exposure to U's bubble asset. Again, this is a reasonable assumption. The onset of the subprime crisis was marked by the failure of a small German bank, IKB, and a few months later by the collapse of Northern Rock, a U.K. bank.[17]

[16] The degree of home equity bias is defined as one minus the ratio of the share of foreign equities in the domestic and world portfolios. It varies between zero (when the weight on foreign equities is given by their relative market capitalization) and one (when investors hold no foreign equities). It has declined in recent years, but remains very high for most countries.

[17] According to Beltran, Pounder and Thomas (2008), table 6, foreigners hold 39 percent –$2.3 trillion out of $6 trillion– of outstanding ABS backed by U.S. assets, and about 16 percent of all U.S. credit market

Under these assumptions the degree of rebalancing on impact, $CA^U_{t_0^+} - CA^U_{t_0^-}$, is obtained by differentiating (20) and given by:

$$\begin{aligned} & \theta\mu_{t_{0-}} \left(V^U_{t_0^-} + B_{t_{0-}} - V^U_{t_0^+}\right) \\ & + r_{t_0^-}\left(1-\mu_{t_{0-}}\right)\left(V^U_{t_0^-} + B_{t_{0-}} - V^U_{t_0^+}\right) \\ & + \left(r_{t_0^-} - r_{t_0^+}\right)\left(1-\mu_{t_{0-}}\right) V^U_{t_0^+} \end{aligned} \tag{24}$$

where $\mu_{t_{0-}} = W^U_{t_0^-} / \left(V^U_{t_0^-} + B_{t_{0-}}\right)$ represents the share invested in the domestic tree and the domestic bubble before the crash, and $\mu_{t_{0-}} < 1$ when U is a net debtor at time 0.

There are three terms in equation (24). The first and most important term is always positive and corresponds to the adjustment in the trade balance $X^U_t - \theta W^U_t$:

$$TB^U_{t_0^+} - TB^U_{t_0^-} = -\theta\left(W^U_{t_0^+} - W^U_{t_0^-}\right) = \theta\mu_{t_{0-}}\left(B_{t_0^-} + V^U_{t_0^-} - V^U_{t_0^+}\right). \tag{25}$$

The drop in $W^U_{t_0^-} - W^U_{t_0^+}$ is exactly proportional to the change in the value of the U assets, $V^U_{t_0^-} + B_{t_{0-}} - V^U_{t_0^+}$. At impact, the direct effect of the bubble crash is a reduction in wealth $W^U_{t_0}$, which lowers consumption and improves the trade balance.[18] Note that there is always less trade rebalancing in our economy than in the benchmark no-inventory economy.[19]

The second and third terms correspond the change in interest payments on external debt, though valuation and change in interest rate effects, respectively. Both are positive if U is a net external debtor. In practice, when the external debt is small, $\mu_{t_{0-}}$ is close to 1 and these two terms are swamped by the adjustment in the trade balance. The latter is directly proportional to the decline in the value of U's assets. Thus we focus our attention on the extent of the decline in the value of U's assets in order to determine the extent of rebalancing implied by the financial crisis. We develop these calculations in the simulations section, but before doing so we sketch the main ingredients behind them.

As a starting point, note that when $\sigma = 1$, the decline in asset prices is exactly the same

instruments.

[18] However, in equilibrium, the drop in wealth is dampened because interest rates plummet, raising the value of non-bubbly assets and making up for part of the drop in wealth.

[19] In the appendix, we show that the difference in trade rebalancing between these two economies is strictly less than the direct effect of the change in terms of trade resulting from speculation in commodities, holding imports and exports constant. In other words, imported and exported quanditities adjust by more in the endogenous commodity price economy than in the no-inventory economy.

in the economy with and without endogenous commodity prices:

$$V^U_{t_0^-} + B_{t_{0^-}} - V^U_{t_0^+} = B_{t_{0^-}} \left(1 - x^U_{t_0}\right) \tag{26}$$

where $x^U_{t_0} = X^U_{t_0}/X_{t_0}$ is the output share of U. This result is particular to the case $\sigma = 1$ because the share of X-goods in consumption is invariant to the price p_t of Z-goods. In contrast, in the more realistic $\sigma < 1$ case, the fall in the price of the U-assets in response to the subprime shock, $V^U_{t_0^-} - V^U_{t_0^+}$, is less severe when commodity prices are endogenous than when they are not. The reason for the smaller drop in the value of U assets is that the share of X-goods in value-added decreases, which raises asset demand relative to asset supply. As a result of this gap, asset supply has to increase by more in equilibrium. This "petrodollar" channel will prove crucial later on in our quantitative exercises.

Formally, recall that in equilibrium $\theta W_t = \left(1 + \alpha p_t^{1-\sigma}\right) X_t$. Because $I_{t_0} = 0$, $W_{t_0^+} = V_{t_0^+}$ and therefore $V_{t_0^+} - \left(V_{t_0^-} + B_{t_0^-}\right) > 0$ when $\sigma < 1$. This increase in global asset value (when measured in units of X) mitigates the fall in the value of U-assets. In fact the change in wealth can be expressed as

$$B_{t_0^-} + V^U_{t_0^-} - V^U_{t_0^+} = B_{t_{0^-}} \left(1 - x^U_{t_0}\right) - \frac{\alpha}{\theta} X^U_{t_0} \left(p^{1-\sigma}_{t_0^+} - p^{1-\sigma}_{t_0^-}\right). \tag{27}$$

The first term on the right hand side of this expression is positive and is exactly the same as in equation (26). It represents the direct effect of the bubble-burst and also corresponds to the drop in $U's$ wealth that would occur in the benchmark no-inventory economy. The second term is negative in our economy but vanishes in the benchmark no-inventory economy. It represents the drop in the share of the X good in total consumption, and mitigates the fall in the value of U's assets. As we argued earlier, this "petrodollar" term plays a key role in limiting the extent of short-run rebalancing.

1.3 Back-of-the-Envelope Calculations

In this section we gauge the order of magnitude of the effects we discuss. We focus on the impact effect of the financial crisis, which we can develop analytically, and postpone to the next section simulations of the full dynamics and general equilibrium. We find from this "short-run" exercise that our model can explain much of the observed decline in real rates and rise in the price of oil following the subprime crisis. The model also goes a long way in explaining why the US current account adjustment has been only minor.

We begin with the impact of the crisis on interest rates. According to Figure 2, real interest rates declined by about 1.4 percent between September 2006, when house prices start to decline and the current account turns around, and March 2008 (see Figures 1 and 3).[20] With a unit elasticity, the change in interest rates is given by (17); when the elasticity of substitution σ is smaller than 1, the drop in interest rates can be expressed as:

$$r_{t_0^+} - r_{t_0^-} = -g\frac{B_{t_0^-}}{W_{t_{0-}}} - \left[\delta\theta\left(1 - s_{zt_0^-}\right) + \frac{g(1-\sigma)}{\sigma}s_{zt_0^-} - \left(\frac{\delta\theta\left(1 - s_{zt_0^-}\right) + \theta\left(\left(\frac{p_{t_{0+}}}{p_{t_{0-}}}\right)^{1-\sigma} - \frac{p_{t_{0+}}}{p_{t_{0-}}}\right)s_{zt_0^-}}{1 + s_{zt_0^-}\left(\sigma\left(\frac{p_{t_{0+}}}{p_{t_{0-}}}\right)^{1-\sigma} - 1\right)}\right)\right] \tag{28}$$

where s_{zt} is the expenditure share of commodities. As in the $\sigma = 1$ case, there are two terms on the right hand side of (28). The first one corresponds to the bubble-burst term while the second (bracketed) one corresponds to the commodity-price-jump term. The bubble-burst term reflects the direct impact of the collapse of the bubble on asset supply. The commodity-price-jump term reflects the impact of the decline in the price of commodities on global asset supply and demand.

The starting point in assessing the role of the two terms is an estimate of the size of the losses generated by the financial crisis, in relation to the world's financial wealth: $B_{t_0^-}/W_{t_0^-}$. Estimates of the size of the initial collapse of the bubble are difficult to come by, and necessarily imprecise. A key issue is that the endogenous response of interest rates offsets the impact of the crash in B_t on global wealth.[21] Empirically, this means that the estimates of the size of the initial bubble that we obtain are likely to be downward biased.

Direct losses in U.S. mortgage markets alone are estimated in the vicinity of $500 billion (Greenlaw et al. (2008)). In the 2008 Global Financial Stability Report, the International Monetary Fund reaches a similar estimate of the aggregate losses in the U.S. residential mortgage market (IMF (2008)). Adding the potential losses to broader credit markets, the IMF calculates aggregate losses of about $945 billion. To these losses, one needs to add the declines in assets values generated by the broad process of deleveraging and the associated contraction in lending across markets. For instance, Greenlaw et al. (2008) estimate an overall $2.3 trillion contraction in intermediary balance sheets. Moreover, mortgage market

[20] The world short real rate drops from 1.6 percent to 0. The U.S. long real rate drops from 2.4 percent to 1.2 percent.

[21] For instance, we have seen that in the case of a unit elasticity $\sigma = 1$, aggregate wealth remains unchanged.

losses only reflect the increased rate of foreclosure (as well as the declining value of repossessed houses). To this, one needs to add the decline in housing wealth for residential borrowers that remain in good standing on their mortgage. Estimates of the latter are significantly larger than the direct losses in mortgage markets. For instance, the Federal Reserve estimates household's housing wealth at $19.6 trillion dollars as of June 2006.[22] According to the Case-Shiller U.S. National Home Price index, U.S. housing prices declined 18 percent in nominal terms from 188.99 in September 2006 to 155.32 in June 2008 (see figure 3). Assuming that this decline is across the board would wipe out at least an additional $3.5 trillion in U.S. housing wealth alone.[23]

Adding these estimates yields a total loss in housing wealth and mortgage markets in the U.S. in the range of $2 to $4 trillion. What is relevant in our calculation is the ratio of these initial losses to the world's financial wealth $W_{t_0^-}$. We construct a crude estimate of the world's financial wealth at the onset of the crisis as the sum of the US household net worth of $51.7 trillion at the end of 2005, and an estimate of the financial wealth of the rest of the world of $80.7 trillion.[24] This would give an initial size of the bubble between 1.5 and 3.0 percent of the world's financial wealth. In what follows, we assume an initial bubble equal to 2 percent of the world's financial wealth.

From (28), it is immediate that the bubble-burst term is relatively small. With a growth rate around 3 percent, the bubble-burst term is equal to –0.03*0.02, or -0.06 percent. On the other hand, the commodity-price jump term can be substantial. To see this, Table 1 reports estimates of the decline in r for different values of the elasticity of substitution σ and different estimates of the increase in commodity prices. In the table, we assume that $s_{zt_0} = 4$ percent, which corresponds to the average share of oil in world GDP in 2005 and 2006.[25] The remaining parameters will be discussed in more detail in Section 2.3.

[22]See table B.100 of the June 2008 release of the Flow of Funds.

[23]This is calculated under the extreme assumption that all mortgage market losses are housing market losses. Of course, foreclosures and reposessions generate additional losses beyond the decline in housing values.

[24]See Table B.100 of the June 2008 issue of the Federal Reserve's Flow of Funds Accounts for the US figure. To obtain the estimate of the financial wealth of the rest of the world in 2006, we calculate the ratio of output to financial wealth for the United States, the European Union and Japan between 1982 and 2004. We find a GDP-weighted average of 2.48 (see Caballero et al. (2008) for additional details). We apply this ratio to the GDP of the rest of the world in 2005 and obtain $80.7 trillion. To the extent that many countries are less financially developed than the U.S., Europe or Japan, we are likely to overstate the world's financial wealth. This would also bias downward our estimate of B_{t_0}/W_{t_0}.

[25]According to the Energy Information Administration (Table 2.4, World Petroleum Demand), world demand for oil in 2005 was 83,646 thousand barrels per day. With a WTI price of the barrel equal to $56.64 dollars, this corresponds to $1.7 trillion, or 3.79 percent of world GDP. In 2006, the share of oil in expenditures was 4.16 percent.

Change in interest rates, $r_{t_0^+} - r_{t_0^-}$:

		$p_{t_0^+}/p_{t_0^-}$:				
		1	1.2	1.5	*2*	3
	0.05	-2.22	-2.23	-2.25	-2.28	-2.37
	0.1	-1.02	-1.04	-1.08	**-1.16**	-1.33
σ :	0.2	-0.44	-0.48	-0.55	-0.69	-1.00
	0.5	-0.12	-0.21	-0.37	-0.66	-1.27
	1	-0.06	-0.23	-0.50	-0.93	-1.80

Table 1: Decline in world interest rates (percent). The table reports the initial drop in interest rates $r_{t_0^+} - r_{t_0^-}$. Source: Authors Calculations.

Between September 2006 and June 2008 the real price of a barrel of West Texas Intermediate increased from \$68.5 to \$142.3 in constant 2008 prices (see Figure 6). Interpreting this surge as the direct effects of the crisis yields $p_{t_0^+}/p_{t_0^-} = 2.08$. The associated decrease in real interest rates in table 1 is consistent with what we see in the data. For a realistically low level of the short-term price elasticity of demand $\sigma = 0.1$, we find a decline in interest rates of 1.16 percent, smaller but close to the 1.4 percent observed over that period, but much larger than the 0.06 percent associated with the direct effect of the crash. Most of the decline in interest rates comes from the indirect effect of higher commodity prices, hinting that the endogenous response of commodity prices to the subprime crisis is critical in understanding the current global macroeconomic environment.

We now turn to the effect of the crash on commodity prices. From (27), we can express the decline in $U's$ financial wealth as:

$$\begin{aligned} W^U_{t_0^+} - W^U_{t_0^-} &= \mu_{t_0^-}\left(V^U_{t_0^+} - V^U_{t_0^+} - B_{t_0^-}\right) \\ &= \mu_{t_0^-}\left(\left(x^U_{t_0} - 1\right) B_{t_0^-} + \frac{\alpha}{\theta} X^U_{t_0}\left(p^{1-\sigma}_{t_0^+} - p^{1-\sigma}_{t_0^-}\right)\right). \end{aligned}$$

This expression yields:

$$\frac{W^U_{t_0^+} - W^U_{t_0^-}}{X^U_{t_0}} = \frac{\mu_{t_0^-}}{\theta\left(1 - s_{zt_0^-}\right)}\left[\left(1 - \frac{1}{x^U_{t_0}}\right)\frac{B_{t_0^-}}{W_{t_0^-}} + s_{zt_0^-}\left(\left(\frac{p_{t_0^+}}{p_{t_0^-}}\right)^{1-\sigma} - 1\right)\right],$$

which can be used to find an expression for the jump in commodity prices as a function of

Change in commodity prices, $p_{t_0^+}/p_{t_0^-}$:

		$B_{t_0^-}/W_{t_0^-}$ (percent)				
		1	*2*	3	4	5
	0.05	1.11	1.91	2.73	3.57	4.42
σ :	*0.1*	1.11	**1.98**	2.89	3.83	4.80
	0.2	1.13	2.16	3.30	4.53	5.83
	0.5	1.21	3.42	6.76	11.22	16.81

Table 2: The effect of the subprime crisis on commodity prices. The table reports the initial increase in the price of commodities $p_{t_0^+}/p_{t_0^-}$. Source: Authors Calculations.

the decline in $U's$ wealth and the size of the original collapse of the bubble:

$$\frac{p_{t_0^+}}{p_{t_0^-}} = \left[1 + \frac{W_{t_0^+}^U - W_{t_0^-}^U}{X_{t_0}^U}\frac{\theta}{\mu_{t_0^-}}\frac{1 - s_{zt_0^-}}{s_{zt_0^-}} + \frac{1}{s_{zt_0^-}}\left(\frac{1}{x_{t_0}^U} - 1\right)\frac{B_{t_0^-}}{W_{t_0^-}}\right]^{1/(1-\sigma)}. \tag{29}$$

We already have an estimate for $B_{t_0^-}/W_{t_0^-}$ and $s_{zt_0^-}$. We estimate the decline in U.S. financial wealth $W_{t_0^+}^U - W_{t_0^-}^U$ from the Federal Reserve Flow of Funds Accounts. Between June 2007 and March 2008, U.S. households financial wealth declined \$1.65 trillion, or 12 percent of output.[26] Next, we construct an estimate of $\mu_{t_0^-}$. In 2005, the net foreign liabilities of the U.S. amounted to \$1.85 trillion, or 15 percent of US GDP.[27] This corresponds to $\left(W_{t_0^-}^U - V_{t_0^-}^U - B_{t_0^-}\right)/X_{t_0}^U$. Substituting the expression for $\mu_{t_0^-}$, and using the fact that $W_{t_0^-}^U/X_{t_0^-}^U = 4.16$, we obtain $\mu_{t_0^-} = 0.96$.[28] Finally, we set the output ratio in 2005 at approximately 0.25.[29] Table 2 reports estimates of the increase in commodity prices as a function of the elasticity σ and the size of the initial bubble collapse $B_{t_0^-}/W_{t_0^-}$.

The results in Table 2 support our view that the collapse in the U.S. housing market and the contraction in credit markets played a significant role in explaining the surge in commodity prices that followed the subprime crisis. We find that, for our benchmark estimate of the size of the bubble of 2 percent, commodity prices increase by 98 percent, when the short run elasticity of substitution equals 0.1, which is very close to the 108 percent observed

[26]See Table B.100 of the June 2008 Federal Reserve Flow of Funds estimates. Households net worth was \$57.6 trillion in June 2007 at the onset of the crisis and only \$55.9 trillion in March 2008.

[27]From Table 2 of the BEA's International Investment Position. The net asset position is estimated at market value.

[28]This represent an overestimate of the share of U.S. assets held by U.S. investors since we assume an extreme form of home bias.

[29]US GDP in 2005 was \$12.4 trillion, while the world's GDP was about \$45 trillion. While the theoretical model refers only to U and M, in this back of the envelope exercise and the simulations that follow, it is natural to include other countries as part of M.

Change in the Trade Balance, $\left(TB^U_{t_0^+} - TB^U_{t_0^-}\right)/X^U_{t_0}$ (percent):						
		$B_{t_0^-}/W_{t_0^-}$ (percent):				
		1	*2*	3	4	5
	1	3.01	6.02	9.04	12.05	15.06
	1.2	2.30	5.31	8.32	11.33	14.34
$p_{t_0^+}/p_{t_0^-}$:	1.5	1.24	4.26	7.27	10.28	13.29
	2	-0.47	**2.55**	5.56	8.57	11.58
	3	-3.77	-0.75	2.26	5.27	8.28

Table 3: The effect of the subprime crisis on the trade balance (percent). The table reports the initial change in the trade balance relative to output, $\left(TB^U_{t_0^+} - TB^U_{t_0^-}\right)/X^U_{t_0}$. Source: Authors Calculations.

in the data. Recall that without an asset-channel (no-inventories benchmark), commodity prices would not jump when the crisis occurs.

Turning to the external accounts, between September 2006 and June 2008, the US trade deficit on goods and services improved from \$793 billion to \$710 billion (annualized), an improvement that represents 1.02 percent of US GDP.[30] Can the model explain this very limited rebalancing? We answer this question by rewriting the trade balance equation (25) as:

$$\frac{TB^U_{t_0^+} - TB^U_{t_0^-}}{X^U_{t_0}} = \frac{\mu_{t_0^-}}{1 - s_{zt_0^-}} \left(\frac{B_{t_0^-}}{W_{t_0^-}} \left(\frac{1}{x^U_{t_0}} - 1 \right) - s_{zt_0^-} \left(\left(\frac{p_{t_0^+}}{p_{t_0^-}} \right)^{1-\sigma} - 1 \right) \right)$$

The first term inside the brackets represents the direct impact of the collapse of the bubble on the trade balance. It contributes positively to global rebalancing. The second term reflects the contribution of commodity prices. Table 3 reports the sum of the direct and indirect impacts of the subprime crisis on the trade balance as a function of the commodity price surge $p_{t_0^+}/p_{t_0^-}$ and the size of the initial bubble $B_{t_0^-}/W_{t_0^-}$ for an elasticity of substitution σ equal to 0.1.[31]

The first line of the table reports the change in the trade balance in the benchmark no-

[30] See the Bureau of Economic Analysis's National Income Accounts, Table 4.1.

[31] It is easy to evaluate the effect of the elasticity of substitution. In the special case where $1-\mu \approx s_z = 0.04$, with $x_{t_0} = 0.25$ one obtains in the limit $\sigma = 0$:

$$\frac{TB^U_{t_0^+} - TB^U_{t_0^-}}{X^U_{t_0}} = 3\frac{B_{t_0^-}}{W_{t_0^-}} - 0.04 \left(\frac{p_{t_0^+}}{p_{t_0^-}} - 1 \right)$$

so with $B_{t_0^-}/W_{t_0^-} = 0.02$ and $\left(\frac{p_{t_0^+}}{p_{t_0^-}} - 1 \right) = 1$, the lower bound on trade rebalancing is 2 percent.

Change in the Current Account, $\left(CA^U_{t_0^+} - CA^U_{t_0^-}\right)/X^U_{t_0}$ (percent):						
		$B_{t_0^-}/W_{t_0^-}$ (percent):				
		1	*2*	3	4	5
	1	3.17	6.20	9.23	12.26	15.29
	1.2	2.46	5.49	8.52	11.54	14.57
$p_{t_0^+}/p_{t_0^-}$:	1.5	1.42	4.45	7.47	10.50	13.53
	2	-0.23	**2.80**	5.83	8.86	11.81
	3	-3.35	-0.33	2.70	5.73	8.76

Table 4: The effect of the subprime crisis on the current account (percent). The table reports the initial change in the current account relative to output, $\left(CA^U_{t_0^+} - CA^U_{t_0^-}\right)/X^U_{t_0}$. Source: Authors Calculations.

inventory economy (which coincides with the direct effect). We find a large and implausible improvement in the trade balance. For instance, for a bubble equal to 2 percent of world financial wealth, the no-inventory economy predicts a 6.02 percent improvement in the trade balance relative to output. This is a far cry from the 1.02 percent observed in the data. Again, once we introduce the "petrodollar" channel, the required rebalancing drops significantly. For instance, the trade balance improves 'only' by 2.55 percent, instead of 6.02 percent when commodity prices double. If instead, we consider a tripling of commodity prices, or a smaller initial bubble collapse, it is possible for the trade balance to worsen on impact. While our preferred numbers are on the high side (2.5 percent against 1.02), it is apparent that the model has the capacity to rationalize the very limited global rebalancing that we are witnessing.

Finally, turning to the current account, we write (24) as

$$\begin{aligned}\frac{CA_{t_0^+} - CA_{t_0^-}}{X^U_{t_0}} &= \left(r_{t_0^+} - r_{t_0^-}\right)\left(\mu_{t_0^-} - 1\right)\frac{V^U_{t_0^+}}{X^U_{t_0}} \\ &+ \left[\frac{\mu_{t_0^-} + \left(1 - \mu_{t_0^-}\right) r_{t_0^-}/\theta}{1 - s_{zt_0^-}}\right]\left(\frac{B_{t_0^-}}{W_{t_0^-}}\left(\frac{1}{x^U_{t_0}} - 1\right) - s_{zt_0^-}\left(\left(\frac{p_{t_0^+}}{p_{t_0^-}}\right)^{1-\sigma} - 1\right)\right)\end{aligned}$$

As argued earlier, our estimate of $\mu_{t_0^-}$ is sufficiently close to 1 that the change in interest payments remain small. Table 4 reports estimates of the change in the current account for the same value of $B_{t_0^-}/W_{t_0^-}$ and $p_{t_0^+}/p_{t_0^-}$ as Table 3.

The qualitative results for the current account are very similar to the trade balance in

Table 3. Without the endogenous response of commodity prices, the rebalancing is massive, around 6.20 percent of GDP. However, the increase in commodity prices attenuates the decline in $U's$ financial wealth, limiting the extent of rebalancing to 2.80 percent of GDP. In the data, the current account deficit improved from $825.6 billion in September 2006 to 709 billion in June 2008, or 1.28 percent of GDP.

All in all, we conclude that our model is in the right ballpark. We turn next to the analysis of full general equilibrium dynamic simulations.

2 Calibration and Dynamics

2.1 Short and Long Run Elasticity

A key parameter of our model is the elasticity of substitution. Nordhaus (1980) finds low 'apparent' short run price elasticities of demand, around 0.3, at the time of the 1973 oil price shock. Long run elasticities are typically higher since it takes time to substitute away from energy-intensive technology. Nordhaus (1980) notes that for many components of the physical capital stock, energy-substitution is only possible when the existing capital is scrapped. In the transportation sector for instance, in which energy consumption depends in large part on the fuel efficiency of the outstanding stock of vehicles, energy consumption responds gradually as old vehicles are slowly replaced with more fuel efficient ones. Similarly, in the case of electricity generation, there is almost no possibility for substitution in the short run. In the long run, on the other hand, electricity generation can switch to other methods of production such as nuclear or wind power generation.

More recent studies confirm the 'crude' estimates in Nordhaus (1980) for the short run while finding higher long run estimates.[32] The typical estimates for short run price elasticities vary between 0.1 and 0.3 while long run estimates vary between 0.4 and 1.[33]

Table 5 provides an update on Nordhaus (1980)'s apparent price elasticity estimates around the recent increase in oil prices. It reports recent data on U.S. petroleum consumption and prices before and after 2003, where the break in oil prices is apparent in figure 6. Between 2003 and 2007 petroleum prices increased by 8.55 percent per year, a sharp break from the 1.44 percent annual increase between 1988 and 2003. Nevertheless, the growth in demand

[32] See Roy, Sanstad, Sathaye and Khaddaria (2006) or Dahl and Sterner (1991) for an older survey.
[33] See Hamilton (2008) for a recent discussion of crude oil prices, and the references therein.

	1988-2003	2003-07
Growth in real GDP (percent)	2.83	2.90
Growth in real petroleum product prices (percent)	1.44	8.55
Growth in real petroleum product demand (percent)	0.84	0.61
Apparent price elasticity	0.04	

Table 5: U.S. Petroleum Demand and Apparent Price Elasticity. Source: Annual Energy Review, EIA, Tables 1.5, 3.1, 3.4, Monthly Energy Review, EIA, Tables 2.1-2.6. Consumer Price Estimate for Petroleum before 2005; Fossil Fuel Composite Price after 2005. All growth rates are compound annual averages. The apparent elasticity is calculated under the assumption of a unit sectoral elasticity with respect to GDP.

for petroleum products slowed only from 0.84 percent to 0.61 percent. The 'apparent' price elasticity is calculated as in Nordhaus (1980), under the assumption of a unit elasticity of petroleum product demand to GDP, as the (opposite of the) percentage slowdown in energy demand corrected for the percentage change in real output growth, divided by the percentage acceleration in prices.[34] We obtain an estimate of 0.04, on the low end of available empirical estimates.[35] This is consistent with recent empirical estimates that find an even smaller short run price elasticity now than in the 1970s.[36]

A simple way of capturing this time variation in σ is to assume that the elasticity of substitution remains significantly smaller than one until the share of expenditures on the Z-good reaches a certain exogenous level $\bar{s}_z$. When that level is reached, we assume that the elasticity of substitution becomes equal to 1. This transition is fully anticipated by economic agents.

Continuity of the demand schedule also requires that α differ as we transition from $\sigma < 1$ to $\sigma = 1$. To see this, suppose that the transition occurs at some time T. Aggregate demand for good X right before T is given by $W_{T^-} = \left(1 + \alpha p_T^{1-\sigma}\right) X_T$. Right after the switch, it is equal to $W_{T^+} = (1 + \alpha') X_T$. Continuity of commodity prices and wealth requires that

[34] The income elasticity of petroleum demand is largely irrelevant in these calculations since output growth was essentially the same over both subperiods. The income elasticity of demand of industrial countries has declined significantly since the oil price shocks of the 1970s and is now closer to 0.5. However, the income elasticity of emerging and oil producing countries appears to be much closer or above unity. See Gately and Huntington (2002) and the discussion in Hamilton (2008).

[35] Interestingly, the same (unreported) calculations for residential demand for petroleum products yields a much larger apparent elasticity, at 0.78. The price elasticity is lowest for the industrial and transport sectors, close to 0.

[36] Hughes, Knittel and Sperling (2008) find a short run price elasticity between 0.03 and 0.08 between 2001 and 2006.

$\alpha p_T^{1-\sigma} = \alpha'$. Since $\alpha' / (1+\alpha') = \bar{s}_z$ is determined exogenously, the preceding relation imposes a relationship between the price of commodities at the time the transition occurs and α.

2.2 The Dynamic System and the Shock

2.2.1 The bubble economy: $t < t_0$.

We start the economy in the bubble equilibrium with $\sigma < 1$, for $t < t_0$. The economy is characterized by the following equations:

$$\begin{aligned} \dot{B}_t &= r_t B_t \\ r_t &= \frac{\delta\theta + \theta g B_t / X_t + \alpha \hat{q}^{1-\sigma} X_t^{1/\sigma - 1} g \left(1/\sigma - 1\right)}{1 + \alpha \hat{q}^{1-\sigma} X_t^{1/\sigma - 1}} \end{aligned}$$

where $\hat{q} \equiv p_t / X_t^{1/\sigma} = (\alpha / Z)^{1/\sigma}$ is constant. This is a differential system in B_t with a forcing term X_t. It requires a terminal condition. To find this terminal condition, we need to characterize the path of the economy, in the event that the bubble does not collapse. Observe that $p_t = \hat{q} X_t^{1/\sigma}$, hence the share of consumption expenditures on the Z-good, $s_{zt} = \alpha p_t^{1-\sigma} / \left(1 + \alpha p_t^{1-\sigma}\right)$, is increasing without bounds. The transition to $\sigma = 1$ must occur at some time T_1 such that $\alpha \hat{q}^{1-\sigma} X_{T_1}^{1/\sigma - 1} = \alpha'$. From T_1 onwards, the elasticity of substitution is equal to one. The bubble economy with $\sigma = 1$ reaches its steady state instantly with:

$$\begin{aligned} r_t &= g\,; \quad q_t = \alpha' / Z \\ \frac{W_t}{X_t} &= \frac{1+\alpha'}{\theta}\,; \quad \frac{V_t}{X_t} = \frac{V_t^U}{X_t^U} = \frac{\delta}{g} \\ B_t &= \left[\frac{1+\alpha'}{\theta} - \frac{\delta}{g}\right] X_t \end{aligned} \tag{30}$$

This provides the terminal condition for the value of the bubble at time $T_{1^-} : B_{T_{1^-}} = \left[\frac{(1+\alpha')}{\theta} - \frac{\delta}{g}\right] X_{T_1}$. Solving backwards from $t = T_1$, we can then characterize the entire path $\left\{B_t, W_t, V_t, V_t^U, p_t\right\}$ that is expected to occur in the absence of a collapse of the bubble.

This global system is consistent with any initial net foreign asset position at $t = t_0$. Assume that we want to start the economy with an external debt $NA_{t_0}^U = \eta X_{t_0}^U$. Under the assumption that the bubble is initially located in U, this implies that U's savings are equal to $W_{t_0}^U = V_{t_0}^U + B_{t_0} + \eta X_{t_0}^U$. One can then solve for the path of domestic savings from the asset accumulation equation. In turn, this pins down net foreign assets $NA_t^U = W_t^U - V_t^U - B_t$, the current account $CA_t^U = \dot{W}_t^U - \dot{V}_t^U - \dot{B}_t$ and the trade balance $TB_t^U = X_t^U - \theta W_t^U$ in all previous periods.

2.2.2 Collapse of the bubble: the short run.

Consider now what happens at time $t = t_0$ when the bubble collapses. As long as $\sigma < 1$, the economy satisfies the following equations:

$$\begin{aligned}
\dot{I}_t &= Z - \alpha \hat{q}_t^{-\sigma} \\
\dot{\hat{q}}_t &= \left(r_t - \frac{g}{\sigma}\right) \hat{q}_t \\
r_t &= \frac{X_t^{1-1/\sigma}\theta\delta + \theta g \hat{q}_t I_t - \theta \hat{q}_t \left(Z - \alpha \hat{q}_t^{-\sigma}\right)}{X_t^{1-1/\sigma} + \alpha\sigma \hat{q}_t^{1-\sigma}}
\end{aligned}$$

This is a dynamic system in I and $\hat{q}$ with a forcing term X_t. We have one initial condition: $I_{t_0} = 0$, by assumption. We need a terminal condition on $\hat{q}_t$. To find it, consider what happens to the share of commodities in expenditures, $\alpha p_t^{1-\sigma} / \left(1 + \alpha p_t^{1-\sigma}\right)$ over time. From the second equation above, the growth rate of $p_t^{1-\sigma} = \hat{q}_t^{1-\sigma} X_t^{1/\sigma - 1}$ is $(1-\sigma)(r_t - g/\sigma) + g(1/\sigma - 1) = (1-\sigma) r_t$, which must be positive eventually (since the interest rate converges to g/σ). Hence the expenditure share must eventually reach $\bar{s}_z$, at which point the elasticity of substitution becomes unity. Let's denote T_2 the time at which this happens, and $\hat{q}_{T_{2-}}$, $I_{T_{2-}}$ the values of the system at that time. Note that $\hat{q}_{T_2}$ and T_2 are also linked by $\alpha \hat{q}_{T_2}^{1-\sigma} X_{T_2}^{1/\sigma - 1} = \alpha'$. Thus, we can parameterize potential equilibrium paths by T_2.

2.2.3 Collapse of the bubble: the long run.

When $t \geq T_2$, the economy is now in the unitary elasticity inventory model described in the previous section. The system follows a saddle path dynamics with:

$$\begin{aligned}
\dot{I}_t &= Z - \alpha' q_t^{-1} \\
\dot{q}_t &= (r_t - g)\, q_t \\
r_t &= \theta \frac{\delta + \alpha' - q_t (Z - g I_t)}{1 + \alpha'}
\end{aligned}$$

where $q_t = p_t / X_t$. The boundary conditions for that system are $I_{T_2} = I_{T_{2-}}$ and $\lim_{t \to \infty} q_t =$

α'/Z. Solving the saddle-path dynamics provides the unique initial value $q_{T_{2+}}$ that is consistent with the equilibrium (see figure 7).

Lastly, continuity of the price at T_2 requires that we select T_2 such that $\hat{q}_{T_{2-}} = q_{T_{2+}} X_{T_2}^{(1-1/\sigma)}$. This completes the characterization of the economy.

2.3 Calibration and Results.

We need to provide values for the following parameters: the capitalization ratio δ, the growth rate of the economy g, the relative size of U and M; the elasticity of substitution σ, the propensity to consume out of financial wealth θ as well as the share of commodity expenditures $\bar{s}_z$ when the elasticity of substitution becomes unitary. We adopt a mixed approach, setting the value of some parameters based on plausible values, and calibrating others so as to reproduce key features of the data.

We start by setting the growth rate of the X-good to $g = 0.03$, which is close to the real output growth rate in the U.S. between 1950 and 2007 (3.28%). We assume that U represents a quarter of the world's output. We set the elasticity of substitution to $\sigma = 0.3$. This is significantly larger than the apparent elasticity estimate Table 5. Nonetheless, as argued above, it is well within the range of estimates in the empirical literature. Furthermore, this value of σ produces realistic levels of adjustment in commodity prices in the model. It also implies that the price of commodities increases initially at $g/\sigma = 10\%$ in the equilibrium with bubble.

We set $\bar{s}_z = 0.1$, so that the long run model kicks in when the expenditure share of commodities reaches 10 percent. This seems a reasonable value since the share of oil in world output reached 4.16 percent in 2006, up from 1.29 percent in 1998. This yields $\alpha' = 11.11\%$. Finally, we set the value of the world capitalization index δ to 0.15, which corresponds to about half of the share of capital in national accounts. As discussed in Caballero et al. (2008), δ should be substantially lower than the capital share since many forms of capital do not generate capitalizable streams of revenue.[37] We then calibrate each countries' δ so as to stabilize global imbalances before the crisis erupts. We obtain $\delta^U = 0.144$ and $\delta^M = 0.1520$.[38]

The two remaining parameters to calibrate are θ and α. We set their value so as to control

[37] Caballero et al. (2008) assumed $\delta = 0.12$. The results are largely unchanged with $\delta = 0.12$.

[38] While the calibration sets δ^U slightly lower than δ^M, the 'perceived' capitalization index in U is much larger, equal to $\hat{\delta}^U = \left(\hat{\delta}^W - \left(1 - x_{t_0}^U\right)\delta^M\right)/x_{t_0}^U = 0.156$. In that sense, the calibration is 'extreme' in that it assumes that U does not have any fundamental advantage in supplying stores of value.

Main Parameters

Parameter	g	σ	X^u/X	$\bar{s}_z$	δ	β_0	β_1	η
Value	0.03	0.3	0.25	0.1	0.15	2%	2.02%	-0.15

Inferred Parameters

Parameter	α	θ	δ^U	δ^M
Value	0.40	0.22	0.144	0.152

Table 6: Main Parameters. See text for description.

(a) the size of the initial bubble relative to aggregate wealth at $t = 0$, $B_0/W_0 = \beta_0$; and (b) the limit size of the bubble that would emerge in the bubbly equilibrium under $\sigma = 1$, $\lim_{t\to\infty} B_t/W_t$. From equation (30), we obtain:

$$\frac{B_t}{W_t} \underset{t\to\infty}{\sim} 1 - \frac{\delta\theta}{g\left(1+\alpha'\right)} = \beta_1$$

For given values of β_0 and β_1, we infer back the corresponding values of θ and α (or equivalently, T_1).

In practice, we set $\beta_0 = 0.02$, so that the collapse of the bubble represents roughly 2% of world's wealth, as estimated in the previous section. We set $\beta_1 = 1.01\beta_0$, so that the economy is not far from its long run steady state when the bubble collapses (this ensures that the share of commodities in expenditures is not too small). We obtain $\theta = 0.22$ and $\alpha = 0.40$. We view these values as plausible. As a point of reference, Caballero et al. (2008) computed a value of $\theta = 0.25$, using data on US household sector net worth and US GDP (θ can be interpreted as the output to financial wealth ratio). These values imply that the economy is dynamically inefficient since $\delta\theta/\left(1+\alpha'\right) = 2.94\% < g = 3\%$. Finally, we set $\eta = -0.15$, in line with estimates of the US net external debt position in 2006. Table 6 summarizes all the parameter values.

Figure 10 reports the simulation obtained with these parameter values. Before the crisis, the real interest rate is slightly above 3.4% and increasing, commodity prices (normalized) are equal to their steady state value $\hat{q}$, while both the trade balance and current account are in deficit and improving (-3.5% and -4% of output, respectively).

At $t = 0$, the crisis hits, wiping out 2% of aggregate financial wealth. The response of interest rates is quite stark (panel A): they drop from about 3.5 percent to 2.7 percent and take 12 years to climb back to their long run equilibrium value. This decline in interest rates is much larger than the one obtained in the benchmark economy (the dashed line in panel A), where the decline is a mere 6 basis points. The fall in interest rates in our economy is strong enough to trigger inventory accumulation. As panel B shows, the price of commodities jumps 2.3 times and gradually converges back over the next 12 years (by contrast, in the benchmark economy, the (normalized) price of commodities remains unchanged and equal to $\hat{q}$).[39] The jump in prices lowers the demand for commodities and allows inventory accumulation. We find that, starting from $I_{t_0} = 0$, inventories rise relatively slowly: it takes 12 years before their market value pI peaks at 3.2% of world financial wealth. In the initial periods after the shock, in particular, inventories remain very low, not contributing much to the global supply of assets.

Panel D reports the current account relative to output in our economy and the benchmark economy. In both cases, the current account improves as a result of the collapse of the bubble. However, as conjectured in the previous section, the rebalancing is much smaller in the economy with inventories. In the benchmark economy, the current account jumps from -4% to 2%, an instant rebalancing of 6% of GDP. This is not surprising, given the fact that the bubble is located in the US: the reduction in asset supply leads agents to move part of their financial investments to M. By contrast, in our economy, the rebalancing is 'only' from -4% to -1.4%. As we discussed earlier, this is larger than the rebalancing observed in the data, but of a similar order of magnitude.

Eventually, the rebalancing must become larger in our economy, with a long run current account deficit of -0.6% against -0.9% in the benchmark no-inventory economy. Nevertheless, the role of commodities is to stabilize capital outflows for the first four years after the initial shock. Panel E shows that the implications for the trade balance are similar. Panel F further decomposes the trade balance into its non-commodity component $\left(X_t^U - \theta W_t^U / \left(1 + \alpha p_t^{1-\sigma}\right)\right)$ and its commodity component $\left(-\alpha p_t^{1-\sigma} \theta W_t^U / \left(1 + \alpha p_t^{1-\sigma}\right)\right)$. Underlying the muted response of the trade balance, both the commodity and non-commodity balances adjust sharply. The commodity balance goes from -4.6% to -7.9% of output, while the non-commodity balance jumps from 1.0% to 6.9% of output.

This asymmetric response of the commodity and non-commodity component of the trade balance is consistent with the evidence. Table 7 reports the change in the US trade balance

[39] This still implies that in the benchmark economy p_t increases at the rate g/σ, faster than the rate of economic growth.

percent of GDP	1980-1987	1987-1989	1999-2006	2006-2008
Change in the trade balance:	-2.5	1.5	-2.8	0.8
of which:				
change in non-oil BGS:	-4.4	1.5	-1.4	1.5
change in oil balance:	1.9	0.0	-1.4	-0.7

Table 7: Change in the US Trade Balance, 1980-1989 and 1999-2008. Source: Bureau of Economic Analysis.

during the last two rebalancing episodes: 1987-1989 and 2006-2008.[40]

The table shows that the recent improvement in the US trade balance comes entirely from the non-oil balance, that improved more than 1.5 percent of GDP. By contrast, the oil balance worsened by 0.7 percent of GDP. Comparing this rebalancing episode with the previous episode, centered around 1987, it is striking to note that oil prices played no role in attenuating the external rebalancing: the deficit of the oil balance did not change between 1987 and 1989. In fact, low oil prices attenuated the build up in imbalances between 1980 and 1987, contributing 1.9 percent of GDP.

Finally, panel G reports the share of expenditures on commodities. We see that starting from 4.4%, the share jumps to 7.8%. This is also qualitatively in line with the change in expenditure share reported for the US.

3 Inventories and Speculation

3.1 Inventories

One objection to stories like ours, where asset-demand for oil plays an important role in price determination, is that measured oil inventory levels did not rise during the recent price spike. Table 8 reports measured inventories of petroleum products for the US and OECD, between January 2000 and April 2008.

Over that period, OECD petroleum inventories increased from 3743 million barrels (mb) to 4082mb between 2000 and 2008. Yet, as table 8 reports, this increase mostly occured between 2000 and 2006. After the onset of the subprime crisis, OECD petroleum inventories declined from 4248mb to 4082mb and the decline in inventories is across the board. In the US, petroleum and crude oil stocks increased between 2000 and 2006 by 307mb and 168mb

[40] Milesi-Ferretti (2008) presents additional evidence on the contrast between the two episodes.

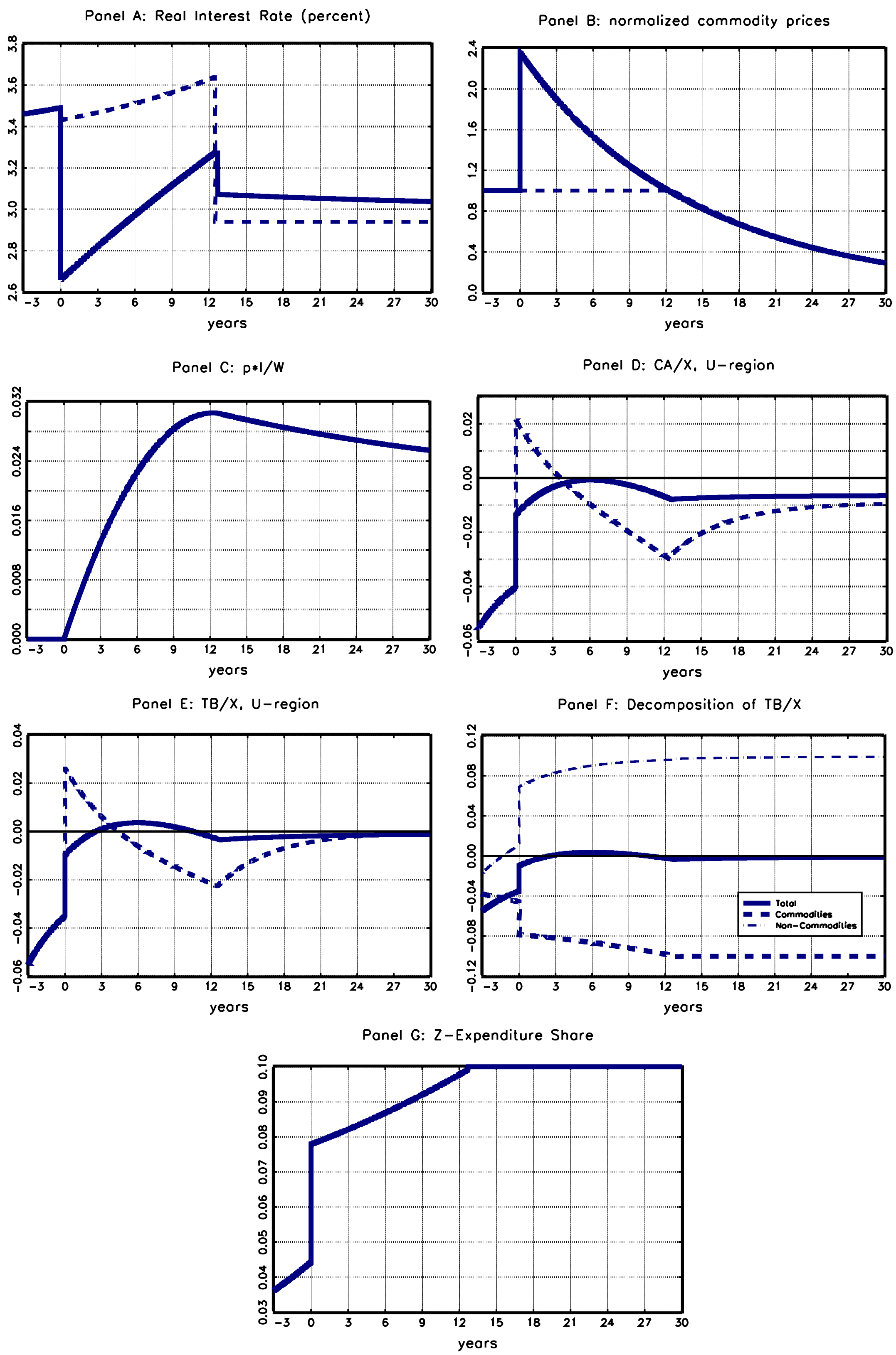

Figure 10: A Subprime Crisis. Short Run and Long Run Responses. See text for description. Author's Calculations.

Petroleum Stocks (million barrels)			
	2000:1	2006:9	2008:4
United States	1477	1785	1665
of which:			
Crude oil stocks	852	1021	1020
-Strategic Petroleum Reserve	568	688	701
-non SPR	284	333	319
Other (distillate, jet fuel, LPG...)	625	764	645
OECD-Europe	1266	1359	1362
Canada	139	185	202
Japan	622	649	609
South Korea	128	160	141
Other OECD	110	109	102
Total OECD	3743	4248	4082

Table 8: OECD Petroleum Stocks, 2000-2008. Petroleum inventories include crude oil, distillate fuel oil, jet fuel, propane/propylene, LPG and Motor Gasoline. Source: Energy Information Administration, Monthly Energy Review, Tables 3.4 and 11.3.

respectively. After the crisis, US petroleum stocks decline from 1785mb to 1665mb, while crude oil inventories remain constant at 1020mb. A closer look, however, reveals that non strategic reserves decreased by 20mb. The only component that increased since 2006 is the Strategic Petroleum Reserve (SPR).[41]

Yet, there are at least two reasons why the absence of a rise in measured inventories need not be a serious concern. First, observed inventories are the result of two opposing forces: the asset-market force which leads to an increase in inventories and a demand force which does the opposite. In the appendix we show that if the long-run elasticity of substitution exceeds one, then inventories follow a non-monotonic path —rising first and declining afterwards— in response to speculation.

Second, as argued by Frankel (2006) and others, producers are the most efficient inventory-holders since they do not need to extract the oil to do so. In our model this amounts to assuming $d^{oilproducer} < d$, which implies that *all* inventories are held underground.

Our model is amenable to several interpretations where inventories are just oil in the ground. Suppose, for example, that new reserves of the $Z-$ good are discovered every period. More precisely, the stock of discovered reserves increases by Z per unit of time. The economy

[41] This is a consequence of the Energy Policy Act of 2005, mandating a gradual increase of the SPR from 700mb to 1000mb. In May 2008, in response to the rapid increase in oil prices, Congress voted to stop depositing oil in the SPR.

cannot consume resources that have not been discovered yet. In addition, suppose that the rights to these new reserves are not capitalized, either because they accrue to new entrants, because they are likely to be expropriated, or because they embody unmodelled uncertainty. This economy would be exactly equivalent to our economy. Under this interpretation, there are no physical inventories of $Z-$ goods. Inventories just reflect discovered and not yet consumed reserves of the Z-good.

Another, more abstract, interpretation is that of a social contract in M countries. This (implicit) contract specifies that each generation is entitled to an endowment Z of Z-goods. They can decide when to sell it. If they do not sell it immediately, they can store it. Moreover, they can trade rights to future consumption claims on their endowment, that is they can sell the Z-good forward. They can also acquire the Z good from agents of the same or different generation in M : they can then treat these newly acquired goods exactly as their own endowment. This economy is once again completely isomorphic to our economy. Inventories are just goods in the ground.

As we will see next, the underground inventory holding view has important implications for the effectiveness of recent proposals to tax speculative transactions in commodities.

3.2 Speculation and Policy

3.2.1 Futures

Let us start by introducing (fully collateralized) future contracts on the Z-good. We make two simple and related points: (i) first, the payoff of the strategy that consists in buying the Z-good and storing it can be replicated by simple futures positions; (ii) second, in our model, the introduction of futures market has no impact on the equilibrium.

By covered interest parity, the forward rate f_{t+s} is equal to:

$$f_{t+s} = p_t \exp^{\int_t^{t+s}(r_u+d)du} .$$

Consider the strategy of buying a forward contract at t with maturity $t+s$ and reselling it at date $t+s' < t+s$. The payoff at $t+s'$ is

$$p_{t+s'} - p_t \exp^{\int_t^{t+s}(r_u+d)du} \exp^{-\int_{t+s'}^{t+s}(r_u+d)du} = p_{t+s'} - p_t \exp^{\int_t^{t+s'}(r_u+d)du}$$

which, in net present value, is exactly the same payoff as that of buying one unit of the Z-good at t, storing it until $t + s'$ and then selling it on the spot market at $t + s'$. To the extent that there is heterogeneity in the storage cost that agents need to pay to store the Z-good, all the inventories will be held by the agents with the lowest storage costs – typically the producers, who can at least partly leave the Z-good in the ground. Agents with higher storage costs will prefer to buy future contracts from the producers.

3.2.2 Equilibrium and policy

However, the introduction of futures contracts has absolutely no effect on the equilibrium of our economy. Futures do not increase asset supply: every long position is offset by a corresponding short position, and there are no agents with biased beliefs deviating from the prefect foresight price-path. As a result, the imposition of a tax on futures trading, or their prohibition, would have absolutely no moderating effect on commodity prices.

In order for a tax to have any consequence in our model, it must affect the agent with the smallest storage costs, which hold the inventories. Thus, let us consider the effect of taxing producers for holding inventories. While in practice this is extremely hard to do since producers are likely to hold most of their inventories underground, it is a useful positive exercise to gauge the potential impact of these type of policies.

It turns out that taxes on the value of inventory holdings are almost isomorphic with the holding cost parameter d, except that that under the tax interpretation, the proceeds can be rebated lump sum to the agents at no resource cost. We take the latter route here and let τ denote the tax rate per unit of value of inventories (that is, the tax per unit of inventory is τp_t

We maintain the assumption that $\sigma = 1$ but strengthen the dynamic inefficiency Assumption 1 to:

Assumption 2 $g - \tau > \frac{\theta\delta}{1+\alpha}$.

Under Assumption 2, the bubbleless steady state of the economy is now such that the interest rate is given by

$$r_t = g - \tau,$$

the price grows at rate g

$$p_t = \alpha \frac{X_t}{Z}$$

and long run inventories are constant

$$I_t = \frac{1+\alpha}{\alpha} \frac{1}{\theta (g-\tau)} \left[g - \tau - r^{ref} \right] Z.$$

The long run level inventory function $I(\tau)$ is decreasing with respect to τ while the price of oil is unaffected since the relative consumption of X and Z goods is unchanged in the long run. However, in the short run, the imposition of a tax τ lowers inventories and the price of oil which is equal to $p_t = \alpha X_t / \left(Z - \dot{I}_t \right)$.

In summary, a tax on inventories reduces the level of these and succeeds in temporarily depressing the price of the Z good but it does not affect it in the long run. Note also that the tax reduces the equilibrium interest rate and aggravates dynamic inefficiency in the bubbleless equilibrium, at a cost for the economy. The intuition is transparent. A dynamically inefficient economy is characterized by a scarcity of assets. A tax on inventories discourages the accumulation of inventories and hence reduces asset supply. The interest rate has to adjust downward to clear the asset market.[42]

4 The Empirical Link Between Oil prices and Asset Supply

The asset-role of oil suggests a negative correlation between oil prices and the value of assets negatively affected by financial shocks. As a starting point in testing this component of our view we run the simple OLS regression:

$$\Delta p_t = \alpha + \beta \Delta s_t + \epsilon_t \tag{31}$$

where lower case variables are in log, p_t denotes the spot price of crude oil and s_t is the Standard & Poors 500 index. In the model, a decline in the value of bubble assets that lowers stock prices leads to a reallocation towards commodities, so we expect to find $\beta < 0$. Table 9 reports the results of the simple OLS regression in (31) for the period 1984-2008.

[42] Note that the bubbly equilibrium of the economy is not affected by τ. Moreover, if Assumption 2 is violated, then the bubbleless equilibrium becomes identical to that of the benchmark no-inventory economy.

	(1) daily	(2) weekly	(3) monthly	(4) annual	(5) daily	(6) weekly	(7) monthly	(8) annual
SP500	-0.11	-0.13	-0.26	-0.54	-0.11	-0.14	-0.26	-0.59
	(2.76)	(1.53)	(1.87)	(1.27)	(2.76)	(1.63)	(1.84)	(1.33)
Lagged SP500					0.02	-0.13	-0.22	0.30
					(0.52)	(1.75)	(1.27)	(0.68)
Constant	0.0003	0.0015	0.0071	0.1068	0.0003	0.0017	0.0087	0.0808
	(0.96)	(1.05)	(1.31)	(1.51)	(0.94)	(1.21)	(1.48)	(0.97)
Observations	6153	1279	293	23	6152	1278	292	22
R-squared	0.002	0.003	0.015	0.062	0.002	0.006	0.024	0.081

Table 9: Oil-Stock Market Regressions. Sample period is 1984-2008. Dependent variable is change in log of the spot price of crude; independent variable is change in log S&P 500 . Changes are taken over the end-of-period value for each interval. Robust t-statistics in parentheses.

We find a negative and significant link between oil prices and stock market performance in the daily returns. At lower frequencies, the coefficient remains negative, but is no longer significant.

There is an obvious reverse causality concern with these OLS results. For instance, an exogenous increase in oil prices could increase the chance of a recession, leading to a decline in stock returns, or push up inflation rates, leading to a tightening of monetary policy, that would also send equities tumbling. To control for this, we use two instruments: the price of gold, as well as the performance of financial stocks, relative to the broader market. Increases in the price of gold are often associated with flight to quality episodes. In fact, since gold provides per se little services or yield, it is the perfect example of an asset held for speculative reasons. The relative performance of financials captures the fact that the financial crises impact more directly the financial sector, while there is no reason for oil shocks to affect this service sector more than, e.g. energy-intensive transportation or manufacturing sectors.

The top panel of Table 10 presents the IV estimates. We notice that the coefficient is both much larger, and also strongly significant at the daily, weekly and monthly frequencies. The elasticity is in excess of one. The bottom panel reports the first stage of the IV regression. We find that both gold and financial relative performance have the right impact on equilibrium assets values: bad news in US financial markets is good news for oil as an asset. Conversely, good news in US financial markets is bad news for oil.

	1986-2008				
	(1)	(2)	(3)	(4)	(5)
	daily	weekly	monthly	quarterly	annual
SP500	-1.55	-1.85	-2.75	-3.03	-1.07
	(6.12)	(4.57)	(3.43)	(3.48)	(1.07)
Constant	0.00	0.00	0.03	0.08	0.15
	(2.07)	(2.33)	(2.23)	(2.74)	(1.36)
First Stage regressions (dependent variable S&P 500)					
Financials	0.24	0.25	0.26	0.26	-0.21
	(7.45)	(5.28)	(2.92)	(2.40)	(0.79)
gold	-0.09	-0.08	-0.16	-0.19	-0.50
	(3.99)	(2.45)	(2.58)	(2.71)	(1.85)
Constant	0.00	0.00	0.00	0.00	0.289
	(2.70)	(2.88)	(3.19)	(4.13)	(2.65)
R-squared	0.04	0.05	0.07		
Observations	6015	1279	293	291	282

Table 10: Instrumented Oil-Stock Market Regressions.Sample period is 1984-2008. Dependent variable is change in log of the spot price of crude; independent variable is change in log S&P 500 . Instruments are the log change in the price of S&P 500 financials less the S&P 500, and the log change in the price of gold. Changes are taken over the end-of-period value for each interval. Quarterly and annual regressions are run on overlapping monthly data. Eicker-White robust t-statistics are reported in parentheses for the daily, weekly and monthly regressions; Newey-West t-statistics are reported for the quarterly and annual regressions, with windows of 2 and 11 months, respectively.

5 Final Discussion

The prevailing view is one of central banks excesses and mistakes leading to excess liquidity, speculative bubbles, and unavoidable crises. This seems overstated: central banks, when reasonable, are not nearly as powerful. In this paper we take a contrarian view and provide an entirely private-sector account of the main facts, without any role for monetary factors. Reality is probably in between.

Our framework builds on the idea that the world economy has a chronic excess demand for financial assets, where the subprime market development may just have been a (failed) market attempt to bridge this gap. Within this perspective, we argue that the sharp rise in oil prices following the subprime crisis — nearly 100 percent in just a matter of months and on the face of recessionary shocks— was the result of a speculative response to the financial crisis itself, in an attempt to rebuild asset supply. That is, the global economy was subject to one shock with multiple implications rather than to two separate shocks (financial and oil). It follows as a corollary that if the US financial crisis subsides, the price of oil and the widespread inflationary pressure it has caused should attenuate. In fact, we have already seen the latter pattern develop over the last few weeks following the GSEs policy package announcement.

This does not mean that our framework predicts that the price of oil will decline permanently. Quite the opposite, the main driving force behind price increases in the long run is the rapid increase in oil demand relative to supply. The speculative process we identify in this paper only adds to and anticipates this ultimately dominant trend. By the same token, an improvement in the current conditions in US financial markets would only postpone this speculative price rise but it would reinforce the price pressure stemming from fast global growth.

The speculative rise of oil prices has also prevented the need for a rapid rebalancing of US external accounts in response to the financial crisis. The reason is that the effect of oil price rises on asset demand (petrodollars) is larger than its effect on asset supply given the low oil inventory levels, even when adding to these any cutback that oil producers may have implemented. This exacerbation of the shortage of financial assets also has put downward pressure on (low risk) real interest rates, a phenomenon which is likely to persist for an extended period of time.

While our framework puts speculation a the center of the recent oil price volatility, taxing speculation per-se is ineffective, unless inventory accumulation (including production cutbacks) could be taxed. And even if the latter were feasible, it is not clear at all that it would be desirable, as it would exacerbate the shortage of assets problem in the long run.

To conclude, we note that our message is mixed in terms of its implications for global macroeconomic health. On one hand, it is an optimistic one as its most likely scenario is one where the (endogenous) supply shocks subside and equilibrium real interest rates remain low. On the other, the starting point of our framework is one where a chronic (good quality) financial asset shortage remains prevalent and hence a source of bouts of financial speculation and temporary financial instability. This problem is not remedied by market specific regulatory responses. The real problem is more macroeconomic in nature and unlikely to go away until the world economy's ability to generate sound store of value (rather than pure diversification) financial assets catches up with its rapid income growth. That is, as so much else on these days, it depends largely on developments within China and other emerging markets.

References

Beltran, Daniel O., Laurie Pounder, and Charles Thomas, "Foreign Exposure to Asset-Backed Securities of U.S. Origin," International Finance Discussion Papers 939, Board of Governors of the Federal Reserve System (U.S.) 2008.

Bernanke, Ben, "The Global Saving Glut and the U.S. Current Account Deficit," Sandridge Lecture, Virginia Association of Economics, Richmond, Virginia, Federal Reserve Board March 2005.

Blanchard, Olivier, Francesco Giavazzi, and Filipa Sa, "International Investors, the U.S. Current Account, and the Dollar," in "Brookings Papers on Economic Activity" Spring 2005, pp. 1–65.

Brunnermeier, Markus K., "Deciphering the 2007-08 Liduidity and Credit Crunch," *Journal of Economic Perspectives*, forthcoming 2008.

Caballero, Ricardo J. and Arvind Krishnamurthy, "Bubbles and Capital Flow Volatility: Causes and Risk Management," *Journal of Monetary Economics*, January 2006, *53:1.*

_ , Emmanuel Farhi, and Pierre-Olivier Gourinchas, "An Equilibrium Model of "Global Imbalances" and Low Interest Rates," *American Economic Review*, March 2008, *98* (1), 358–93.

Dahl, Carol and Thomas Sterner, "Analysing gasoline demand elasticities: a survey," *Energy Economics*, July 1991, *13* (3), 203–210.

Deutsche Bank, "Sovereign Wealth Funds - State Investments on the Rise," Current Issues September 2007.

Frankel, Jeffrey A., "The Effect of Monetary Policy on Real Commodity Prices," NBER Working Papers 12713 December 2006.

Gately, Dermot and Hillard G. Huntington, "The Asymmetric Effects of Changes in Price and Income on Energy and Oil Demand," *Energy Journal*, 2002, *23* (1), 19–56.

Greenlaw, David, Jan Hatzius, Anil K Kashyap, and Hyun Song Shin, "Leveraged Losses: Lessons from the Mortgage Market Meltdown," U.S. Monetary Policy Forum Report No. 2 2008.

Hamilton, James, "Understanding Crude Oil Prices," mimeo, UC San Diego May 2008.

Hotelling, Harold, "The Economics of Exhaustible Resources," *Journal of Political Economy*, August 1931, *39* (2), 137–175.

Hughes, Jonathan E., Christopher R. Knittel, and Daniel Sperling, "Evidence of a Shift in the Short-Run Price Elasticity of Gasoline Demand," *Energy Economics*, 2008, *29* (1), 93–114.

IMF, *Containing Systemic Risk and Restoring Financial Soundness: Global Financial Stability Report*, International Monetary Fund, 2008.

Jovanovic, Boyan, "Bubbles in Prices of Exhaustible Resources," NBER Working Papers 13320 August 2007.

Kouri, Pentti, "Balance of Payment and the Foreign Exchange Market: A Dynamic Partial Equilibrium Model," in Jagdeep Bhandari and Bluford Putnam, eds., *Economic Interdependance and Flexible Exchange Rates*, MIT Press, 1982, pp. 116–156.

Metzler, Lloyd A., "The process of International Adjustment under Conditions of Full Employment. A Keynesian View," in Richard Caves and Harry G. Johnson, eds., *Readings in International Economics*, Richard D. Irwin, Homewood, IL 1960, pp. 465–486.

Milesi-Ferretti, Gian Maria, "Fundamentals at Odds? The Dollar and The U.S. Current Account Deficit," mimeo prepared for the June 27-28 Siena conference on The Impact of Global Imbalances September 2008.

Nordhaus, William D, "Oil and Economic Performance in Industrial Countries," *Brookings Papers on Economic Activity*, 1980, *1980* (2), 341–399.

Roy, Joyashree, Alan H. Sanstad, Jayant A. Sathaye, and Raman Khaddaria, "Substitution and price elasticity estimates using inter-country pooled data in a translog cost model," *Energy Economics*, November 2006, *28* (5-6), 706–719.

Sercu, Piet M. and Rosanne Vanpee, "Home Bias in International Equity Portfolios: A Review," KU Leuven mimeo 2007.

A Appendix: The Numeraire

Throughout we have chosen the X-good as the numeraire. This appendix confirms that the main substantive results do not depend on this particular choice of numeraire.

The price index corresponding to the composite consumption good is given by $\left(1+\alpha p_t^{1-\sigma}\right)^{1/(1-\sigma)}$ if $\sigma \neq 1$, and $p_t^{\frac{\alpha}{1+\alpha}}$ if $\sigma = 1$. We term the corresponding numeraire the composite numeraire. We denote with a tilde the variable expressed in the composite numeraire. The interest rate in the composite numeraire $\tilde{r}_t$ can be computed from r_t as follows:

$$\tilde{r}_t = r_t - \frac{d \log \left[\left(1+\alpha p_t^{1-\sigma}\right)^{1/(1-\sigma)}\right]}{dt} = r_t - \frac{\alpha p_t^{-\sigma}}{1+\alpha p_t^{1-\sigma}} \dot{p}_t$$

Hence if $\max\{I_t, \dot{I}_t\} > 0$, we have

$$\tilde{r}_t = r_t - \frac{\alpha p_t^{-\sigma}(r_t + d) p_t}{1+\alpha p_t^{1-\sigma}} = \frac{r_t - d\alpha p_t^{1-\sigma}}{1+\alpha p_t^{1-\sigma}}$$

which using (10) we can rewrite as

$$\tilde{r}_t = \frac{\theta\delta + \theta g \frac{B_t}{X_t} - p_t\left(\theta \frac{Z}{X_t} - \theta g \frac{I_t}{X_t} - \alpha\theta p_t^{-\sigma}\right) - \sigma d \alpha p_t^{1-\sigma}\left(1+\alpha p_t^{1-\sigma}\right)}{\left(1+\alpha\sigma p_t^{1-\sigma}\right)\left(1+\alpha p_t^{1-\sigma}\right)}$$

Let's now investigate how this change in numeraire would modify our conclusions concerning the crash and the steady states. Note that when $\sigma = 1$, then we simply have

$$\tilde{r}_t = \frac{r_t}{1+\alpha}.$$

Hence

$$\tilde{r}_{t_0^-} - \tilde{r}_{t_0^+} = \frac{r_{t_0^-} - r_{t_0^+}}{1+\alpha} = \frac{1}{1+\alpha}\left[\frac{\theta g}{1+\alpha} \frac{\tilde{B}^U_{t_0-}}{p_{t_0^-}^{\frac{\alpha}{1+\alpha}} X_{t_0}} + \frac{\theta}{1+\alpha} \frac{Z}{X_{t_0}}\left(p_{t_0^+} - p_{t_0^-}\right)\right].$$

Let's now turn to the drop in wealth and asset value at impact. We have

$$\begin{aligned}\frac{\theta\tilde{W}_{t_0^-}}{1+\alpha} &= p_{t_0^-}^{-\frac{\alpha}{1+\alpha}}X_{t_0} = \tilde{V}^U_{t_0^-}\frac{X_{t_0}}{X^U_{t_0}} + \tilde{B}^U_{t_0-} \\ \frac{\theta\tilde{W}_{t_0^+}}{1+\alpha} &= p_{t_0^+}^{-\frac{\alpha}{1+\alpha}}X_{t_0} = \tilde{V}^U_{t_0^+}\frac{X_{t_0}}{X^U_{t_0}}\end{aligned}$$

Hence we have

$$\tilde{V}^U_{t_0^-} + \tilde{B}^U_{t_0-} - \tilde{V}^U_{t_0^+} = \tilde{B}^U_{t_0-}\left(1 - \frac{X^U_{t_0}}{X_{t_0}}\right) + \left(p_{t_0^-}^{-\frac{\alpha}{1+\alpha}} - p_{t_0^+}^{-\frac{\alpha}{1+\alpha}}\right)\frac{1}{\theta}X^U_{t_0}$$

The amount of rebalancing is given by $\widetilde{CA}^U_{t_0^+} - \widetilde{CA}^U_{t_0^-}$

$$\left[\tilde{r}_{t_0^-}\left(1 - \frac{\tilde{W}^U_{t_0^-}}{\tilde{V}^U_{t_0^-} + \tilde{B}^U_{t_0-}}\right) + \theta\frac{\tilde{W}^U_{t_0^-}}{\tilde{V}^U_{t_0^-} + \tilde{B}^U_{t_0-}}\right]\left(\tilde{V}^U_{t_0^-} + \tilde{B}^U_{t_0-} - \tilde{V}^U_{t_0^+}\right) - X^U_{t_0}\frac{1}{\theta}\left(p_{t_0^-}^{-\frac{\alpha}{1+\alpha}} - p_{t_0^+}^{-\frac{\alpha}{1+\alpha}}\right)$$
$$+\left(\tilde{r}_{t_0^-} - \tilde{r}_{t_0^+}\right)\left(\tilde{V}^U_{t_0^-} + \tilde{B}^U_{t_0-} - \tilde{W}^U_{t_0^-}\right)\frac{\tilde{V}^U_{t_0^+}}{\tilde{V}^U_{t_0^-} + \tilde{B}^U_{t_0-}}$$

The right comparison when use the composite numeraire should be with an economy with exogenous commodity prices. With endogenous commodity prices, the interest rate drops more. The drop in asset value is also more pronounced because the trees pay dividend in X-goods the value of which depreciates at impact. Both effects contribute to more rebalancing with endogenous commodity prices. A counterbalancing effect is that the value of GDP goes down since GDP is composed of X goods, the value of which depreciates at impact. This last effect contributes to less rebalancing.

Another quantity we could look at is the amount of rebalancing as a fraction of GDP $\widetilde{CA}^U_{t_0^+}/\left(p_{t_0^+}^{-\frac{\alpha}{1+\alpha}}X^U_{t_0}\right) - \widetilde{CA}^U_{t_0^-}/\left(p_{t_0^-}^{-\frac{\alpha}{1+\alpha}}X^U_{t_0}\right)$

$$\left[\tilde{r}_{t_0^-}\left(1 - \frac{\tilde{W}^U_{t_0^-}}{\tilde{V}^U_{t_0^-} + \tilde{B}^U_{t_0-}}\right) + \theta\frac{\tilde{W}^U_{t_0^-}}{\tilde{V}^U_{t_0^-} + \tilde{B}^U_{t_0-}}\right]\left(\frac{\tilde{V}^U_{t_0^-} + \tilde{B}^U_{t_0-}}{p_{t_0^-}^{-\frac{\alpha}{1+\alpha}}X^U_{t_0}} - \frac{\tilde{V}^U_{t_0^+}}{p_{t_0^+}^{-\frac{\alpha}{1+\alpha}}X^U_{t_0}}\right)$$
$$+\left(\tilde{r}_{t_0^-} - \tilde{r}_{t_0^+}\right)\frac{\tilde{V}^U_{t_0^-} + \tilde{B}^U_{t_0-} - \tilde{W}^U_{t_0^-}}{p_{t_0^-}^{-\frac{\alpha}{1+\alpha}}X^U_{t_0}}\frac{\tilde{V}^U_{t_0^+}}{\tilde{V}^U_{t_0^-} + \tilde{B}^U_{t_0-}}\frac{p_{t_0^-}^{-\frac{\alpha}{1+\alpha}}}{p_{t_0^+}^{-\frac{\alpha}{1+\alpha}}}$$

which we can rewrite as

$$\left[\tilde{r}_{t_0^-}\left(1-\frac{\tilde{W}^U_{t_0^-}}{\tilde{V}^U_{t_0^-}+\tilde{B}^U_{t_0-}}\right)+\theta\frac{\tilde{W}^U_{t_0^-}}{\tilde{V}^U_{t_0^-}+\tilde{B}^U_{t_0-}}\right]\frac{B^U_{t_0-}}{X^U_{t_0}}\left(1-\frac{X^U_{t_0}}{X_{t_0}}\right)$$
$$+\left(\tilde{r}_{t_0^-}-\tilde{r}_{t_0^+}\right)\left(\tilde{V}^U_{t_0^-}+\tilde{B}^U_{t_0-}-\tilde{W}^U_{t_0^-}\right)\frac{V^U_{t_0^+}}{V^U_{t_0^-}+B^U_{t_0-}}.$$

With this normalization (which is probably the one we should adopt), we get that endogenous commodity prices only contribute to less rebalancing with $\sigma = 1$ because they lead to lower interest rates.

Let's now turn to $\sigma < 1$. In this case we have $\tilde{r}_t = \frac{r_t}{1+\alpha p_t^{1-\sigma}}$ so that

$$\tilde{r}_{t_0^-}-\tilde{r}_{t_0^+}=\frac{r_{t_0^-}-r_{t_0^+}}{1+\alpha p_{t_0^-}^{1-\sigma}}+r_{t_0^+}\left(\frac{1}{1+\alpha p_{t_0^-}^{1-\sigma}}-\frac{1}{1+\alpha p_{t_0^+}^{1-\sigma}}\right).$$

Similarly, we have

$$\frac{\tilde{V}^U_{t_0^-}+\tilde{B}^U_{t_0-}}{\left(1+\alpha p_{t_0^-}^{1-\sigma}\right)^{-1/(1-\sigma)}X^U_{t_0}}-\frac{\tilde{V}^U_{t_0^+}}{\left(1+\alpha p_{t_0^+}^{1-\sigma}\right)^{-1/(1-\sigma)}X^U_{t_0}}=\frac{B^U_{t_0-}}{X^U_{t_0}}\left(1-\frac{X^U_{t_0}}{X_{t_0}}\right)-\alpha\frac{1}{\theta}\left(p_{t_0^+}^{1-\sigma}-p_{t_0^-}^{1-\sigma}\right).$$

And we can compute $\widetilde{CA}^U_{t_0^+}/\left(\left(1+\alpha p_{t_0^+}^{1-\sigma}\right)^{-1/(1-\sigma)}X^U_{t_0}\right)-\widetilde{CA}^U_{t_0^-}/\left(\left(1+\alpha p_{t_0^-}^{1-\sigma}\right)^{-1/(1-\sigma)}X^U_{t_0}\right)$

$$\left[\tilde{r}_{t_0^-}\left(1-\frac{\tilde{W}^U_{t_0^-}}{\tilde{V}^U_{t_0^-}+\tilde{B}^U_{t_0-}}\right)+\theta\frac{\tilde{W}^U_{t_0^-}}{\tilde{V}^U_{t_0^-}+\tilde{B}^U_{t_0-}}\right]\left(\frac{\tilde{V}^U_{t_0^-}+\tilde{B}^U_{t_0-}}{\left(1+\alpha p_{t_0^-}^{1-\sigma}\right)^{-1/(1-\sigma)}X^U_{t_0}}-\frac{\tilde{V}^U_{t_0^+}}{\left(1+\alpha p_{t_0^+}^{1-\sigma}\right)^{-1/(1-\sigma)}X^U_{t_0}}\right)$$
$$+\left(\tilde{r}_{t_0^-}-\tilde{r}_{t_0^+}\right)\left(\frac{\tilde{V}^U_{t_0^-}+\tilde{B}^U_{t_0-}-\tilde{W}^U_{t_0^-}}{\left(1+\alpha p_{t_0^-}^{1-\sigma}\right)^{-1/(1-\sigma)}X^U_{t_0}}\right)\frac{\tilde{V}^U_{t_0^+}}{\tilde{V}^U_{t_0^-}+\tilde{B}^U_{t_0-}}\frac{\left(1+\alpha p_{t_0^-}^{1-\sigma}\right)^{-1/(1-\sigma)}}{\left(1+\alpha p_{t_0^+}^{1-\sigma}\right)^{-1/(1-\sigma)}}$$

Hence when net positions are small compared to gross positions endogenous commodity prices can lead to less rebalancing. Basically, all the qualitative conclusions we had with the other numeraire carry through.

B Appendix: The Trade Balance in the short Run.

In this appendix we expand on Section 1.2.2 and analyze the behavior of the trade balance in the short run both in our economy and in the benchmark no-inventory economy.

We can decompose this difference more finely by studying the trade balance decomposition into exports $X_t^U - \theta W_t^U / \left(1 + \alpha p_t^{1-\sigma}\right)$ and imports $\alpha p_t^{1-\sigma} \theta W_t^U / \left(1 + \alpha p_t^{1-\sigma}\right)$:

$$
\begin{aligned}
TB_{t_0^+}^U - TB_{t_0^-}^U &= \left(\frac{\theta W_{t_0^-}^U}{1 + \alpha p_{t_0^-}^{1-\sigma}} - \frac{\theta W_{t_0^+}^U}{1 + \alpha p_{t_0^+}^{1-\sigma}} \right) - p_{t_0^+} \left(\frac{\alpha p_{t_0^+}^{-\sigma} \theta W_{t_0^+}^U}{1 + \alpha p_{t_0^+}^{1-\sigma}} - \frac{\alpha p_{t_0^-}^{-\sigma} \theta W_{t_0^-}^U}{1 + \alpha p_{t_0^-}^{1-\sigma}} \right) \\
&\quad - (p_{t_0^+} - p_{t_0^-}) \frac{\alpha p_{t_0^-}^{-\sigma} \theta W_{t_0^-}^U}{1 + \alpha p_{t_0^-}^{1-\sigma}}.
\end{aligned}
$$

The three terms on the right hand side have a traditional Marshall-Lerner interpretation: The first one represents the increase in the volume of exports. The second one represents the decrease in the volume of imports times the terms of trade. These two are positive since the volume of exports rises and the volume of imports falls. The third term is negative and represents imports times the change in the terms of trade. Note that the terms of trade effect – the last term– would be absent in the benchmark no-inventory economy. However, we can show that the positive quantity effect – the first two terms – is stronger in our economy than in the benchmark economy, which means that the difference in trade rebalancing between these two economies is strictly less than the direct effect of the change in terms of trade resulting from speculation in commodities.[43]

[43]This can be verified as follows. The claim amounts to showing that

$$
(p_{t_0^+} - p_{t_0^-}) \frac{\alpha p_{t_0^-}^{-\sigma} \theta W_{t_0^-}^U}{1 + \alpha p_{t_0^-}^{1-\sigma}} > \theta \frac{W_{t_0^-}^U}{B_{t_0^-} + V_{t_0^-}^U} \frac{\alpha}{\theta} X_{t_0}^U \left(p_{t_0^+}^{1-\sigma} - p_{t_0^-}^{1-\sigma} \right)
$$

which can be re-arranged into

$$
\frac{\frac{p_{t_0^+}}{p_{t_0^-}} \left(1 - \frac{p_{t_0^+}^{-\sigma}}{p_{t_0^-}^{-\sigma}} \right)}{\frac{p_{t_0^+}^{1-\sigma}}{p_{t_0^-}^{1-\sigma}} - 1} + 1 > \frac{1}{1 + \frac{B_{t_0^-}}{B_{t_0^-} + V_{t_0^-}} \left(1 - \frac{X_{t_0}^U}{X_t^U} \right)}.
$$

C Appendix: Declining Inventories

In most analyses of commodity price and inventory dynamics, the forward curve for commodity prices plays a central role. In our model, after the shock, the world is deterministic and risk-neutral to that at date t, the forward price s periods ahead f_{t+s} is simply the future spot price p_{t+s}. In our case, it will prove more convenient to reason in terms of the log-forward curve which trace $\log f_{t+s} = \log p_{t+s}$ as a function of maturity s.

The decision to accumulate inventories is determined by a comparison of the slope of the log-forward curve $\frac{\dot{p}_t}{p_t}$ of the price for the Z good with the level of the interest rate r_t. The steeper the slope of the log-forward curve for the price of the Z good, the higher the expected growth in the price of the Z good, and the more storage is attractive. Similarly, the lower the level of the interest rate, the more storage is attractive. The elasticity σ of the demand for the Z good is a key parameter governing the slope of the log-forward curve. The higher σ, the flatter the log-forward curve.

Key to our analysis is the basic idea that the short-run elasticity of demand for commodities σ is low in the short run but high in the long run. Let us denote by σ^{short} the short-run elasticity and by σ^{long} the long-run elasticity with $\sigma^{short} < 1 < \sigma^{long}$. The switch from σ^{short} to σ^{long} is typically gradual, and potentially governed by a number of time and state dependent factors. If the long-run elasticity σ^{long} of the demand for the Z good is high enough, then inventories will eventually be decumulated. This is formalized by the following assumption.

Assumption 4 : $\frac{g}{\sigma^{long}} < \delta\theta$

This assumption is more likely to be verified, the higher is σ^{long}. When it is verified, then the long run steady-state of the economy features no inventories. In the long run steady state, the interest rate is $\delta\theta$. The price of the Z-good is given by $(\alpha X_t/Z)^{\frac{1}{\sigma^{long}}} = p_t$ and grows at rate a g/σ^{long} which is too low to make the accumulation of inventories worthwhile. The share of the Z-good in total consumption converges to 0, and the economy effectively behaves as an economy without commodities.

Turning to transitional dynamics, imagine that the economy enters the region with $\sigma = \sigma^{long} > 1$ with positive inventories $I_t > 0$. The presence of inventories affects both the goods market and the asset market: the total intertemporal supply of the Z-good is higher which depresses the price of the Z-good p_t. Asset supply is higher since inventories act as a store of value, resulting in a higher interest rate r_t. The desire to decumulate inventories can be traced back to how those two markets are affected, as the following two equations show

$$\dot{I}_t = Z - \alpha p_t^{-\sigma} X_t \text{ and } \frac{\dot{I}_t}{X_t} - g\frac{I_t}{X_t} = \frac{1 + \alpha\sigma p_t^{1-\sigma}}{p_t\theta}(\theta\delta - r_t).$$

Hence a process of decumulation of inventories is initiated and the economy eventually converges to a steady-state with no-inventories.