Valuation Effects with Transitory and Trend Output Shocks

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Abstract

This paper investigates the role of valuation effects on a country’s net foreign asset position. It shows that following transitory output shocks, valuation effects are stabilizing; they counteract current account movements and mitigate the impact of the current account on a country’s net foreign assets. Following trend shocks, valuation effects are amplifying; they move in the same direction as the current account and reinforce the impact of the current account on net foreign assets. The results are illustrated by the external imbalances between the U.S. and other industrialized countries since the 1990s.

Keywords: Valuation Effects, Current Account, External Imbalances, Net Foreign Assets, Portfolio Choice.

JEL Classifications: F32, F41, G11, C68

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1 Introduction

1.1 Contributions

In traditional models of the balance of payment, the evolution of a country’s net foreign asset (NFA) position is fully determined by the current account. For example, countries that run a current account deficit experience a parallel reduction in their NFA position. These models were based on the assumption that countries only traded a single bond of constant real value. However, in the past decades cross country portfolio holdings of a large variety of assets have risen sharply (see Figure 1). This has created a potentially important role for changes in asset prices, or “valuation effects”, in affecting a country’s NFA position (Lane and Milesi-Ferretti (2007)). Valuation effects are changes in the value of a country’s gross external assets and liabilities due to asset price and exchange rate fluctuations. Positive valuation effects arise when the capital gains on foreign assets held by domestic agents are larger than those on domestic assets held by foreign agents. Ceteris paribus positive valuation effects enhance a country’s external financial wealth and improve its NFA position. Following this argument, Gourinchas and Rey (2007) point out that large, persistent current account deficits of a country such as the U.S. do not necessarily lead to a sharp deterioration in the NFA position if the country experiences positive valuation effects. In such a situation, current account deficits can be much more sustainable than was previously thought and valuation effects exert a stabilizing role – they offset part of the current account deficit and mitigate the decline in the country’s NFA position.

Figure 1: U.S. and other G7 countries’ gross external assets and liabilities, 1970-2004
This paper investigates analytically if valuation effects do move to offset the current account and stabilize a country’s NFA position. It shows that the impact of valuation effects depends critically on the nature of underlying output shocks. In response to transitory shocks, valuation effects are stabilizing; they counteract current account movements and help to soften the impact of the current account on a country’s NFA position. In response to trend shocks, valuation effects are amplifying; they move in the same direction as the current account, and reinforce, or “amplify” the impact of the current account on the NFA position. The theoretical predictions are illustrated by the evolution of the NFA position between the U.S. and other industrialized countries since the 1990s.

The mechanism of valuation effects works as follows: in response to a positive output shock, either trend or transitory, domestic asset prices appreciate relative to foreign asset prices. This is because asset prices are forward looking and agents incorporate expected domestic output growth into domestic asset prices. In both cases, the appreciation of domestic asset prices creates a negative valuation effect.

However, the role of the valuation effect in the two scenarios is very different. Following a positive transitory shock on home output, agents smooth consumption and save; the domestic country runs a current account surplus. The valuation effect, which is negative, then partly offsets the current account surplus. As a result, the increase in the NFAs is smaller than the current account surplus. In other words, valuation effects have a stabilizing property on NFAs as they counteract the fluctuations of the current account.

On the other hand, after a positive trend output shock, the role of the valuation effect is amplifying. A positive trend output shock implies that growth is sustained, i.e. higher output today will be followed by even higher output tomorrow. Put differently, the increase in current income is lower than the increase in permanent income. Consumption smoothing incentive implies that consumption rises more than output, and the domestic country runs a current account deficit. The negative valuation effect then moves in the same direction with the current account, and reinforces the current account deficit. As a result, the decrease in the NFAs is now more than the current account deficit, which means valuation effects are amplifying. Simulation results indicate sizable valuation effects, especially in response to trend shocks because asset price appreciations are more dramatic in this case.

The empirical literature has sought to identify if the U.S.’s valuation effects are
stabilizing. Gourinchas and Rey (2007) impute net foreign asset returns from 1952 to 2004, and interpret these as the “valuation channel” of changes in NFAs. They find that the valuation channel is stabilizing and accounts for 27 percent of the U.S.’s cyclical external adjustments. However, Curcuru, Dvorak, and Warnock (2008), after correcting for measurement errors, find that the average return differential of U.S. claims over U.S. liabilities is essentially zero during the period from 1994 to 2006.

Although net foreign asset returns and return differentials are the focus of the empirical literature, they are not precise measures of valuation effects. While total returns include asset price changes and dividend yields or interest payment, valuation effects are associated with asset price changes only (dividend payments and interest payments are capture in the current account). Large valuation effects, therefore, can exist even if Curcuru, Dvorak, and Warnock (2008) find a small average return differential\(^1\),\(^2\). Empirical evidence suggests that valuation effects are indeed sizable. For example, in 2006, non-FDI valuation effects account for 4% of GDP\(^3\).

On the theoretical front, Devereux and Sutherland (2008) investigate valuation effects in a two-country dynamic model. However, similar to the empirical literature, they restrict valuation effects to return differentials, which, as discussed above, are not a precise measure of valuation effects. My paper considers asset prices and associates valuation effects with changes in asset prices only. Ghironi, Lee, and Rebucci (2007) also explicitly consider asset prices. They illustrate the working of valuation effects, and show that the quantitative importance of valuation effects depends on features of the international transmission mechanism such as the size of financial frictions, substitutability across goods, and the persistence of the shocks. However, they do not focus on the role of valuation effects on NFAs. One key contribution of my paper is that it identifies contrasting role of valuation effects as stabilizing after transitory shocks, and amplifying after trend shocks. Unlike the conventional wisdom that valuation effects are generally stabilizing, as showcased in empirical findings of

\(^1\)Thanks to Fabio Ghironi for pointing this out.
\(^2\)In addition, sharply increasing cross country portfolio holdings over the past decades imply that taking average return differentials can be potentially inadequate. To see why, we can consider a hypothetical example. Suppose U.S. equities’ return is higher than European equities’ in early 1990s, but is lower than European equities’ after 2000. Average return differential over the period could be close to zero, but since the cross border portfolio holdings are much larger after 2001, U.S. investors could end up making much larger gains from European equities than European investors did from U.S. equities in early 1990s.
\(^3\)Data are from Bertaut and Tryon (2007). Please see Figure 6 in Appendix B for more details.
Gourinchas and Rey (2007), and also implicit in Devereux and Sutherland (2008) and Ghironi, Lee, and Rebuffi (2007)’s theoretical results, this paper shows that valuation effects can be amplifying too. This situation is clearly illustrated by the evolution of NFA position between the U.S. and other industrialized countries during the 1990s, which I will discuss shortly below.

Benigno (2006) follows a different line of inquiry by focusing on the exchange rate valuation channel. He argues that when there are small frictions in the price mechanism, valuation effects play a minor role. However, these theoretical predictions are at odds with empirical evidence. The fact that valuation effects are significant implies substantial movements of asset prices and exchange rates. My paper shows that asset price changes and valuation effects can be large in response to output shocks.

About methodology, my paper uses the approach of Tille and van Wincoop (2007) (henceforth referred to as TV) and Devereux and Sutherland (2007) (henceforth DS) to solve for portfolio choice, and the approach of Aguiar and Gopinath (2007) to introduce trend shocks. TV and DS develop an approximation method to characterize time-varying equilibrium portfolios in a two-country dynamic general equilibrium model, where financial markets are incomplete. In my paper, market incompleteness, along with home bias in portfolio holdings, is assumed, by the presence of an exogenous cost of investing in foreign equities.

The paper is related to a large literature on global imbalances. Also using expected higher growth of the U.S. compared to those of other industrialized countries, Engel and Rogers (2006) explain the U.S.’s current account deficit, although they do not examine valuation effects. Caballero, Farhi, and Gourinchas (2006) use the growth gap between U.S. and Continental Europe in the 1990s to explain U.S.-Europe capital flow.

The paper’s theoretical results have some important implications for the U.S.’s external imbalances. The U.S. has experienced persistently higher economic and productivity growth than other industrialized countries in the 1990s. The average annual growth rate of U.S. PPP GDP during 1990-2000 was 1.94%, compared to 1.47% for

For different solution methods, see Evans and Hnatkovska (2007); Heathcote and Perri (2007); Pavlova and Rigobon (2008).

For papers that seek to explain home bias in portfolio holdings, see Kollmann (2006); Engel and Matsumoto (2006); Heathcote and Perri (2007); Benigno (2007); Kollmann, Coeurdacier, and Martin (2008).

Most notable papers include Obstfeld and Rogoff (2005); Caballero, Farhi, and Gourinchas (2006); Engel and Rogers (2006); Mendoza, Quadrini, and Rios-Rull (2007)...

\footnote{4For different solution methods, see Evans and Hnatkovska (2007); Heathcote and Perri (2007); Pavlova and Rigobon (2008).}
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other G7 countries (henceforth referred to as G6). At the same time, U.S.’s relative stock prices have been in an upward trend, while the U.S.-G6 current account balance has continued to worsen (Figures 3,4). 

Figure 2: U.S.-G6 output ratio, 1990-2007

Figure 3: Log of normalized stock price index ratios, after incorporating changes in exchange rates (Jan-1990=0), 1990-2007

The theoretical results imply that if the U.S indeed had a positive trend output shock relative to other industrialized countries, the valuation effects between the U.S. and these countries were negative and they worsened the impact of the current account deficit on the U.S.’s NFA position.

Figure 4 confirms that from 1994 to 2001, U.S.-G6 valuation effects were negative (except in 1999), and they exacerbated the impact of the current account deficit on

7For sources of data used in the figures, please see the Data Appendix.
Figure 4: U.S.-G6 current account and non-FDI valuation effects, 1994-2006

the NFAs\(^8\). After 2002, valuation effects became positive, reflecting a slowdown of the U.S. economy and the decline of the U.S. stock market. The large sizes of the valuation effects after 2002 are partly due to the increase in cross country portfolio holdings.

1.2 The framework

The framework is a simple stationary symmetric one-good two-country DSGE model. Output has a transitory and a trend component, both of which are subject to AR(1) shocks. There are two assets, each is a claim on a fraction of one country’s output, as in Lucas (1982). Agents observe output and choose their consumption, as well as the two assets in their portfolios.

In the model, financial assets serve two purposes: for inter-temporal smoothing, and for the purpose of risk sharing. Economic agents would like to insure themselves against the risks of undiversifiable labor income and domestic equity holdings. Ideally, in a frictionless asset market, agents would hold 50\% of domestic endowment and 50\% of foreign endowment to completely insure themselves against any country specific shocks (Lucas (1982)). In this case domestic and foreign agents would have exactly the same consumption and wealth in all states.

However in reality residents of most countries exhibit home bias in their portfolio holdings (French and Poterba (1991); Tesar and Werner (1995)). A number of explanations for the home bias puzzle have been presented; in this paper I assume

\(^8\)Unfortunately data on valuation effects are only available after May 1994 (the 1994 position in the graph only covers the last seven months of the year).
that there is a small cost of investing abroad (as in TV, Heathcote and Perri (2004), Coeurdacier and Guibaud (2006)). These costs reflect a lack of market knowledge, market access and information, as well as cultural and language barriers. Such costs make investing abroad less attractive and create home bias in portfolio holdings.

The portfolio home bias is also important to generate non-trivial current account. In the model, without the home bias, current account would be always zero because all agents are effectively insured (they would optimally hold 50% of home endowment and 50% of foreign endowment).

Note that there is only one good in the model, thus no room for real exchange rate. This is for the paper to clearly show the impact of the output shocks on the current account and valuation effects. Having two goods would not change the results of the model. In that case, exchange rate movements would be equilibrium responses and only partly offset output shocks. The role of valuation effects then would remain the same (that is, stabilizing in response to transitory shocks, and amplifying in response to trend shocks).

Tightly connected to valuation effects is the role of return differentials, which are attracting a lot of attention in the literature (e.g. Curcuru, Dvorak, and Warnock (2008)). Return differentials are the differences between returns on foreign assets and those on domestic assets. In this paper, domestic returns increase after positive shocks to domestic output. For the case of a positive transitory shock, most of the return differential is due to the increase of dividend yields. On the other hand, for a positive trend shock, the return differential is driven mostly by changes in asset prices.

Recent literature has also paid attention to the role of expected valuation effects (e.g. Gourinchas and Rey (2007) and TV). Expected valuation effects are predictable movements of valuation effects during the transitional phase after an output shock. As pointed out by Gourinchas and Rey (2007), expected valuation effects, along with expected current account, will potentially play a role in the adjustment process to external balance. My paper can gain insights about “expected” and “unexpected” components of valuation effects. After positive transitory shocks, expected valuation effects are positive because the relative price of domestic assets is expected to fall gradually (the shock dies out). Put differently, the expected capital gains of domestic investors holding foreign assets are larger than the those of foreign investors holding

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9 Except in the special case of unit elasticity of substitution, as studied in Cole and Obstfeld (1991), real exchange rate movements would completely offset output shocks.
domestic assets, leading to positive expected valuation effects for the domestic country. However after positive trend shocks, expected valuation effects are negative, because domestic asset prices keep rising and bring about more capital gains to foreign investors. Numerical results indicate quantitative importance of both expected and unexpected valuation effects. This is a major departure from Devereux and Sutherland (2008)’s results, where anticipated valuation effects are of second order\textsuperscript{10}.

2 Setup of the model

I consider a two-country symmetric model. Both countries produce an identical perishable good. Transitory shocks and trend shocks can occur in both countries.

2.1 Production

Production of the home country takes the form of an endowment process:

\[ Y_t = z_t \Gamma_t \]

We abstract from investment and labor for simplicity. However, a constant fraction \(1 - \alpha\) of the endowment is considered as labor income. The rest can be considered as capital rent.

As in Aguiar and Gopinath (2007), \(z_t\) and \(\Gamma_t\) represent two productivity processes. The two processes are characterized by different stochastic properties. Specifically, \(z_t\) follows an AR(1) process:

\[ \log(z_t) = \rho_z \log(z_{t-1}) + \varepsilon^z_t \quad (1) \]

where \(0 < \rho_z < 1\) and \(\varepsilon^z_t\) represents iid draws from a normal distribution with zero mean and standard deviation \(\sigma_z\).

The parameter \(\Gamma_t\) represents a combination of a cumulative product of the growth shocks (as in Aguiar and Gopinath (2007)) and a convergence process. In particular:

\[ \Gamma_t = g_t \Gamma_{t-1} \left( \frac{\Gamma^*_t}{\Gamma_{t-1}} \right)^\lambda \quad (2) \]

\[ \log(g_t) = (1 - \rho_g) \log(\bar{g}) + \rho_g \log(g_{t-1}) + \varepsilon^g_t \quad (3) \]

\textsuperscript{10}In Devereux and Sutherland (2008), expected valuation effects are connected to expected return differentials, which are zero up to the first order approximation.
where $\Gamma_{t-1}^*$ is the permanent component of the foreign country, $\rho_g$ and $\lambda$ are between 0 and 1. $\epsilon_t^z$ is iid normal with zero mean and standard deviation $\sigma_g$. $g^* > 1$ is the long run mean growth rate.

A one time shock to $g$ changes the growth rate and has a permanent impact on the economy. The $\epsilon_t^z$ is considered as trend shocks. Following a trend shock, agents will expect the economy to grow faster than its long run growth rate. This generates spending incentives in expectation of even higher output in the future. On the other hand, the $\epsilon_t^z$ shocks are temporary, and hence are called transitory shocks.

The permanent component $\Gamma_t$ is also affected by the output ratio of the two countries. All else equal, a lower home-foreign output ratio increases growth of the home country’s output, reflecting a convergence process. Eventually in the long run, the output ratio goes to one, and the two countries grow at the same long run growth rate $\bar{g}$. This assumption is to generate long run output stationarity, which allows us to pin down a unique deterministic steady state and solve the model numerically.

The assumption is not unrealistic, particularly among countries and regions with similar institutional levels (for example, see Barro and Sala-i Martin (2003), chapter 1 for different states of the U.S., and Dowrick and Nguyen (1989) or Madsen (2007) for OECD countries). Eaton and Kortum (1999) record that technology diffusions among G-7 countries are pervasive. Having said that, it is important to note that the main results of the paper do not depend on the assumption. In this paper, we firstly set $\lambda$ very close to zero (implying a very long convergence), and later redo the numerical exercise with a larger value for $\lambda$ (a faster convergence).

Similarly, production of the foreign country takes the form:

$$Y_t^* = z_t^* \Gamma_t^*$$

The fraction $1 - \alpha$ of the endowment comes as labor income. The fraction $\alpha$ of the endowment is capital rent.

$z_t^*$ also follows an AR(1) process:

$$\log(z_t^*) = \rho_z \log(z_{t-1}^*) + \epsilon_t^{z*}$$

where $\epsilon_t^{z*}$ is iid $\sim N(0,\sigma_z)$.

$\Gamma_t^*$ also contains an exogenous permanent component and a convergence component:

$$\Gamma_t^* = g_t \Gamma_{t-1}^* \left( \frac{\Gamma_{t-1}}{\Gamma_{t-1}} \right)^{-\lambda}$$

$$\log(g_t^*) = (1 - \rho_g) \log(\bar{g}) + \rho_g \log(g_{t-1}^*) + \epsilon_t^{g*}$$
where $\varepsilon^g_t$ is iid $\sim N(0,\sigma_g)$.

For clarity, (2) and (5) can be rewritten as follow:

$$\frac{\Gamma^*_t}{\Gamma_t} = \frac{g^*_t}{g_t} \left( \frac{\Gamma^*_{t-1}}{\Gamma_{t-1}} \right)^{1-2\lambda}$$

(7)

If the system is in the long run equilibrium (i.e. $\frac{\Gamma^*_t}{\Gamma_t} = 1$ and $g_t = \bar{g}$), $\Gamma_t$ and $\Gamma^*_t$ will grow at the long run rate $\bar{g}$. In disequilibrium, the gap between $log(\Gamma_t)$ and $log(\Gamma^*_t)$ slowly narrows. The speed of convergence is dictated by $\lambda$. In the long run, $\Gamma_t$ and $\Gamma^*_t$ converge in ratio (i.e. $\frac{\Gamma^*_t}{\Gamma_t} \rightarrow 1$, or $log(\Gamma_t) - log(\Gamma^*_t) \rightarrow 0$).

Figures 5 shows two examples of the convergence process following a positive growth shock of 0.2% to the home country’s growth for $\lambda = 0.017$ (a fast convergence) and $\lambda = 0.001$ (a very slow convergence). Other parameters are $\bar{g} = 1.02; \rho_g = 0.963; \varepsilon_g = 0.002$.

![Figure 5: Home-Foreign Output ratio after a 0.2% trend shock](image)

2.2 Assets

There are two assets: a claim on the Home capital stock and a claim on the Foreign capital stock, I refer to these as Home and Foreign equities (or assets). These two terms will be used interchangeably below. The price at time $t$ of a unit of Home equity carried into the next period is denoted $Q_{t+1}$, measured in terms of the consumption good. The holder of this claim gets a dividend in period $t$ which is a share $\alpha$ of output, and can sell the claim for price $Q_{t+1}$. The overall return to the Home equity, in terms of the common good is:

$$R_t = \frac{Q_{t+1}}{Q_t} + \frac{\alpha Y_t}{Q_t}$$

(8)
Equation (8) states that the return to investment in domestic equity comprises of a dividend yield and an appreciation of the domestic equity.

Similarly, the price at time $t$ of a unit of Foreign equity that is carried into the next period is denoted $Q_{t+1}^*$ expressed in terms of the good. The return to Foreign equity is:

$$R_t^* = \frac{Q_{t+1}^*}{Q_t^*} + \frac{\alpha Y_t^*}{Q_t^*}$$

(9)

2.3 Households

An infinitely-live representative household maximizes its expected discounted utility. I assume an endogenous discount factor in the utility, as in Schmitt-Grohe and Uribe (2003). This is a simplest technical device to induce uniqueness of the deterministic steady state and stationary responses to temporary shocks. Specifically, the endogenous discount factor decreases with the consumption-output ratio. Intuitively, this means that an agent whose consumption is growing relative to output has a larger discount rate for his future consumption. Note that with this specification, the endogenous discount factor is stationary, and consistent with long run growth.

$$U = E_0 \sum_{t=0}^{\infty} e^{-\phi \sum_{\tau=0}^{t-1} \log \left( \frac{C_\tau}{Y_\tau} \right)} \beta^t \frac{C_t^{1-\sigma}}{1 - \sigma}$$

(10)

I assume a credit market friction. In particular, agents investing abroad receive the gross return times an “local expert” cost $e^{-\tau}$, as in TV. The cost captures expenses paid to local experts for local market access and information, as well as expenses spent to overcome cultural and language barriers. This friction generates a home-bias in portfolio holdings and market incompleteness. Follow TV, $\tau$ is second order (i.e. proportional to the variances of the shocks) so that the portfolio holding is well-behaved. This assumption implies that when the shock variances go to zero, the cost $\tau$ will also go to zero. The “local expert” cost is paid in the host country; for instance, the cost could represent payments to experts in the local economy.

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11 A well-known problem in open macroeconomics with incomplete markets is that transitory shocks to output have permanent effects on wealth. Without any mechanism to induce stationarity, long run wealth will be non-stationary (as in Evans and Hnatkovska (2007)). To obtain a stationary long run wealth distribution, TV assume agents die with a constant probability and consume all his wealth, and new agents are born at the same rate. Ghironi, Lee, and Rebucci (2007) and Heathcote and Perri (2007) assume a convex cost of holding portfolios. DS avoid this problem altogether by assuming zero wealth.
Denote $\theta_t$ as the fraction of domestic wealth invested in domestic equity carried from the last period to the current period, and $\theta^*_t$ the fraction of foreign wealth held in foreign equity. Domestic wealth in terms of the consumption good evolves according to the following law of motion:

$$W_{t+1} = \theta_t W_t R_t + (1 - \theta_t) W_t R^*_t e^{-\tau} + (1 - \alpha)Y_t - C_t + (1 - \theta^*_t) W^*_t R_t (1 - e^{-\tau})$$ (11)

where $\theta_t W_t R_t + (1 - \theta_t) W_t R^*_t e^{-\tau}$ is income from equities, $(1 - \alpha)Y_t$ is the labor income, and $(1 - \theta^*_t) W^*_t R_t (1 - e^{-\tau})$ is the local expert cost that foreign investors have to pay to domestic agents.

The timing of the agent’s problem is as follows: A representative agent enters the period knowing his wealth, his domestic and foreign equity holdings, and the domestic and foreign equity prices. Output is then observed. The agent then chooses consumption and portfolio holdings for the next period, taking the returns as given. However in equilibrium, the returns are affected by the agent’s portfolio choice.

Similarly, the budget constraint faced by foreign agents is:

$$W^*_{t+1} = (1 - \theta^*_t) W^*_t R_t e^{-\tau} + \theta^*_t W^*_t R^*_t + (1 - \alpha)Y^*_t - C^*_t + (1 - \theta_t) W^*_t R^*_t (1 - e^{-\tau})$$ (12)

Due to Walras law, only one budget constraint is relevant.

### 2.4 Bellman equation

Without loss of generality, I only consider the dynamic programming problem of domestic agents. Denote $d_t \equiv \theta_t W_t$; $f_t \equiv (1 - \theta_t) W_t$ and $f^*_t \equiv (1 - \theta^*_t) W^*_t$ hence $d_{t+1} \equiv \theta_{t+1} W_{t+1}$ and $f_{t+1} \equiv (1 - \theta_{t+1}) W_{t+1}$.

The domestic agent’s Bellman equation is:

$$V(d_t, f_t) = \max_{C_t, d_{t+1}} \frac{C_t^{1-\sigma}}{1 - \sigma} + \beta \left( \frac{C_t}{Y_t} \right)^{-\phi} E_t V(d_{t+1}, d_t R_t + f_t R^*_t e^{-\tau} + (1 - \alpha)Y_t - C_t + f^*_t R (1 - e^{-\tau}) - d_{t+1})$$ (13)

where $\beta \left( \frac{C_t}{Y_t} \right)^{-\phi}$ is the discount factor. Following Schmitt-Grohe and Uribe (2003), I assume that agents do not internalize the discount factor. This can be rationalized by assuming that the discount factor depends not upon the agents own consumption and effort, but rather on the average per capita levels of these variables. If a small $\phi$ is imposed, the short run dynamics of the system will be very close to those of a standard model with a fixed exogenous discount factor, except that now there exists a unique steady state and long run wealth distribution is stationary.
The Euler equations for domestic agents are:

\[ C_t^{\sigma} = \beta \left( \frac{C_t}{Y_t} \right)^{-\phi} E_t[C_{t+1}^{-\sigma} R_{t+1}] \] (14)

\[ E_t[C_{t+1}^{-\sigma} R_{t+1}] = E_t[C_{t+1}^{-\sigma} R_t^*] e^{-\tau} \] (15)

Similarly, for foreign investors:

\[ C_t^{*\sigma} = \beta \left( \frac{C_t^*}{Y_t^*} \right)^{-\phi} E_t[C_{t+1}^{\sigma*} R_{t+1}^*] \] (16)

\[ E_t[C_{t+1}^{\sigma*} R_{t+1}] e^{-\tau} = E_t[C_{t+1}^{\sigma*} R_t^*] \] (17)

(15) and (17) describe optimal portfolio choice. Note that the portfolio shares do not enter these equations directly. They enter indirectly by affecting the portfolio returns, which affect wealth in the next period and hence the asset pricing kernels. The intuition for (15) and (17) is standard. For example (15) states that domestic investors choose their portfolios such that the expected marginal utility gain from investing in domestic equity equals that from investing in foreign equity, after adjusting for the “local expert” cost \( \tau \).

### 2.5 Equilibrium conditions:

The goods market clearing condition is:

\[ Y_t + Y_t^* = C_t + C_t^* \] (18)

while asset market clearing conditions are:

\[ Q_{t+1} = \theta_{t+1} W_{t+1} + (1 - \theta_{t+1}^*) W_{t+1}^* \] (19)

\[ Q_{t+1}^* = (1 - \theta_{t+1}) W_{t+1} + \theta_{t+1}^* W_{t+1}^* \] (20)

(19) and (20) state that asset prices equate asset demand and asset supply (which are fixed at one unit). Adding up (19) and (20) yields:

\[ Q_{t+1} + Q_{t+1}^* = W_{t+1} + W_{t+1}^* \] (21)

### 2.6 Valuation effects

In standard inter-temporal models, the change in the net foreign asset position equals the current account. In this model, however, the change in NFAs needs not equal the
current account, because the model explicitly considers capital gains/losses arising from changes in domestic and foreign asset prices. This is referred to as “valuation effects”. The valuation effects refer to changes in real value of international asset holdings due to changes in asset prices, or to changes in exchange rates. In the model, the valuation effects for the home country are:

\[ V E_t = (1 - \theta_t)W_t \left( \frac{Q^{*}_{t+1}}{Q_t} e^{-\tau} - 1 \right) - (1 - \theta^*_t)W^*_t \left( \frac{Q^{*}_{t+1}}{Q_t} e^{-\tau} - 1 \right) \quad (22) \]

where \((1 - \theta_t)W_t \left( \frac{Q^{*}_{t+1}}{Q_t} e^{-\tau} - 1 \right)\) are home country’s capital gains from foreign asset holdings, after adjusting for the “local expert” costs, and \((1 - \theta^*_t)W^*_t \left( \frac{Q^{*}_{t+1}}{Q_t} e^{-\tau} - 1 \right)\) are the foreign investors’ capital gain from holding domestic equity.

The valuation effects can be split into “expected” components and “unexpected” components. The expected component is:

\[ EV E_t = (1 - \theta_t)W_t \left( \frac{E_tQ^{*}_{t+1}}{Q_t} e^{-\tau} - 1 \right) - (1 - \theta^*_t)W^*_t \left( \frac{E_tQ^{*}_{t+1}}{Q_t} e^{-\tau} - 1 \right) \]

while the unexpected components is:

\[ UVE_t = (1 - \theta_t)W_t \frac{Q^{*}_{t+1} - E_tQ^{*}_{t+1}}{Q_t} e^{-\tau} - (1 - \theta^*_t)W^*_t \frac{Q^{*}_{t+1} - E_tQ^{*}_{t+1}}{Q_t} e^{-\tau} \]

The current account consists of the trade balance and net factor income:

\[ CA_t = Y_t - C_t + (1 - \theta_t)W_t \frac{\alpha Y^{*}_t}{Q_t} e^{-\tau} - (1 - \theta^*_t)W^*_t \frac{\alpha Y^*_t}{Q_t} e^{-\tau} \quad (23) \]

where \(\frac{\alpha Y^*_t}{Q_t}\) and \(\frac{\alpha Y_t}{Q_t}\) are foreign and home dividend yields respectively.

Net assets at time \(t\) equal gross assets minus gross liabilities: \((1 - \theta_t)W_t - (1 - \theta^*_t)W^*_t\). The change in NFAs hence equals:

\[ \Delta NFA_t = [(1 - \theta_{t+1})W_{t+1} - (1 - \theta^*_{t+1})W^*_{t+1}] - [(1 - \theta_t)W_t - (1 - \theta^*_t)W^*_t] \quad (24) \]

The change in NFAs equals the current account plus the valuation effects:

\[ \Delta NFA_t = CA_t + VE_t \quad (25) \]

To see this, substituting (22),(23) and (24) into (25), and use (8) and (9), equation (25) can be expressed as:

\[ Y_t - C_t + (1 - \theta_t)W_t R^*_t e^{-\tau} - (1 - \theta^*_t)W^*_t R_t e^{-\tau} = [(1 - \theta_{t+1})W_{t+1} - (1 - \theta^*_{t+1})W^*_{t+1}] \quad (26) \]
Subtracting (26) from the budget constraint (11) yields:

\[- \alpha Y_t + \theta_t W_t R_t + (1 - \theta_t^*) W_t^* R_t = \theta_{t+1} W_{t+1} + (1 - \theta_{t+1}^*) W_{t+1}^* \]  

(27)

Using the asset market clearing condition (19), (27) can be expressed as:

\[- \alpha Y_t + Q_t R_t = Q_{t+1} \]  

(28)

which is true because \( R_t = \frac{Q_{t+1}}{Q_t} + \frac{\alpha Y_t}{Q_t} \). Therefore, (25) holds.

### 2.7 De-trending the system

Given that a realization of \( g \) permanently affects \( \Gamma \), output is non-stationary with a stochastic trend. For any home variable \( X \), following Aguiar and Gopinath (2007), I introduce a lower-case \( x \) to denote its detrended counterpart.

\[ x_t = \frac{X_t}{\Gamma_{t-1}} \]

For any foreign variable \( X^* \), I also use introduce \( x^* \):

\[ x^*_t = \frac{X^*_t}{\Gamma_{t-1}} \]

This insures that if \( X_t \) and \( X^*_t \) are in the information set of time \( t \), so are \( x_t \) and \( x^*_t \).

The system in terms of detrended variables is presented in Appendix C. Note that there is now a new variable, \( \pi_t = \frac{\Gamma_{t-1}}{\Gamma_{t-1}} \), which is the ratio of the two trend processes. \( \pi_t \) is a state variable and converges to one in the steady state.

### 3 Solution of the model

It is well-known that up to a first order approximation, the values of the portfolio choice \( \theta_t \) and \( \theta_t^* \) are indeterminate, because at this level of approximation the two assets are perfect substitutes. Previous literature usually relies on perfect market structures that make portfolio choice irrelevant.

Following the approach of TV and DS, I solve for the first order accurate solution of the detrended system above (including portfolio choice decisions). I do this in two steps. In the first step I solve for the long run steady state portfolio choice. This involves taking a first order approximation of the system, and solving for first order
approximations of the non-portfolio choice variables (conditional on the long run steady state portfolio choice). Subsequently, the conditional solution is substituted into the second order approximations of the portfolio choice equations to determine the values of the long run portfolio choice. It can be also that the current account, changes in NFAs and valuation effects can be first order approximated.

In the second step, having the values of the long run portfolio choice, the first order accurate solution for portfolio choice can be solved based on the second order accurate solution of other non-portfolio variables. In this step I will follow DS closely.

After solving for the detrended variables, level variables are recovered. Note that interest rate and portfolio choice decisions are invariant to this conversion.

The solution for the steady state equilibrium can be solved:

\[
\begin{align*}
    y &= y^* = \bar{y} \\
    c &= c^* = \bar{y} \\
    R &= R^* = g^\sigma \\
    w &= w^* = q = q^* = \frac{\alpha y}{R - \bar{y}} \\
    \theta &= \theta^* 
\end{align*}
\]

(29)

### 3.1 Step 1: Long run steady state portfolio choice

The first step is to pin down the values of the long run portfolio choice. First, I take first order approximations of the model’s Euler equations and equilibrium conditions. The 16 equations are numbered (C1) to (C17), except (C10). Note that (C10) is redundant as the two portfolio choice Euler equations (C8) and (C10) yield the same first order approximations. I derive a linear system of 16 equations and 16 variables:

\[
\begin{align*}
    \hat{w}_t + 1, \hat{w}_t^* + 1, \hat{q}_t + 1, \hat{q}_t^* + 1, \hat{\pi}_t + 1, \hat{\pi}_t^* + 1, \hat{\theta}_t + 1, \hat{\theta}_t^* + 1, \hat{\pi}_t + 1, \hat{\pi}_t^* + 1, \hat{\theta}_t + 1, \hat{\theta}_t^* + 1, \text{conditional on}
\end{align*}
\]

\[
\begin{align*}
    \hat{w}_t, \hat{w}_t^*, \hat{q}_t, \hat{q}_t^*, \hat{\pi}_t, \hat{\pi}_t^*, \hat{\pi}_t - 1, \hat{\pi}_t^* - 1, \hat{\theta}_t - 1, \hat{\theta}_t^* - 1, \theta \text{ and } \theta^* \text{ (note that in the steady state } \theta = \theta^* \text{).} \n\end{align*}
\]

For all variables except for \( \hat{\theta}_t \) and \( \hat{\theta}_t^* \), \( \hat{x} \) indicates the log-deviation of \( x \) from the steady state (\( \hat{\theta}_t \) and \( \hat{\theta}_t^* \) are deviation in levels from the long run steady state portfolio choice).

Two aspects of portfolio decisions that enter the first order system is firstly, the steady state portfolio \( \theta \), and secondly, the term \( \hat{\theta}_t + 1 - \hat{\theta}_t^* + 1 \). They enter the system only through the first order approximations of the budget constraint equation (C12)
and the market clearing condition (C16):

\[ gw(\hat{w}_{t+1} + \hat{g}_t + \lambda \hat{\pi}_t) + c\hat{c}_t - (1 - \alpha)yy_t = \theta wR \hat{R}_t + (1 - \theta)wR \hat{R}_t^* + wR\hat{w}_t \] (30)

\[ q\hat{q}_{t+1} - \theta w\hat{w}_{t+1} - (1 - \theta^*)w\hat{w}_{t+1}^* = w(\hat{\theta}_{t+1} - \hat{\theta}_{t+1}^*) \] (31)

(31) shows that \( \hat{\theta}_{t+1} - \hat{\theta}_{t+1}^* \) enters the system as a choice variable and hence has no impact on other state variables. This information however will be useful to pin down the first order accurate solution of portfolio choice \( \hat{\theta}_{t+1} \) and \( \hat{\theta}_{t+1}^* \) in step 2.

As discussed above, the two portfolio choice Euler equations (C8) and (C10) have the same first order approximations, written below:

\[ E_t[\hat{R}_{t+1} - \hat{R}_{t+1}^*] = 0 \] (32)

(32) indicates that to a first order approximation, the expected excess return is zero.

The system (conditional on long run values of portfolio choice \( \theta, \theta^* \)) can be solved by any standard solution method for linear rational expectations models with \( \hat{w}_{t+1}, \hat{w}_{t+1}^*, \hat{q}_{t+1}, \hat{q}_{t+1}^*, \hat{\pi}_t \) as the five endogenous state variables and \( \hat{y}_t, \hat{y}_t^*, \hat{c}_t, \hat{c}_t^*, \hat{R}_t, \hat{R}_t^* \) and \( (\hat{\theta}_t - \hat{\theta}_t^*) \) as the seven choice variables.

Next, the second order approximations of the two portfolio choice Euler equations are derived:

\[ E_t[\hat{R}_{t+1}] + \frac{1}{2} E_t[\hat{R}_{t+1}^2 - 2\sigma \hat{c}_{t+1} \hat{R}_{t+1}] = E_t[\hat{R}_{t+1}^*] + \frac{1}{2} E_t[\hat{R}_{t+1}^* - 2\sigma \hat{c}_{t+1} \hat{R}_{t+1}^* - \tau] \] (33)

\[ E_t[\hat{R}_{t+1}] + \frac{1}{2} E_t[\hat{R}_{t+1}^2 - 2\sigma \hat{c}_{t+1} \hat{R}_{t+1} - \tau] = E_t[\hat{R}_{t+1}^*] + \frac{1}{2} E_t[\hat{R}_{t+1}^* - 2\sigma \hat{c}_{t+1} \hat{R}_{t+1}^*] \] (34)

Note that since the local expert cost \( \tau \) is of second order, it does not appear in the system of first order approximation, but does appear in (33) and (34). Subtracting (34) from (33), we obtain:

\[ E_t[(\hat{c}_{t+1} - \hat{c}_{t+1}^*)(\hat{R}_{t+1} - \hat{R}_{t+1}^*)] = \frac{\tau}{\sigma} \] (35)

Equation (35) states that long run portfolio shares are chosen such that the covariance (approximated to second order) between the difference in consumption and the excess return is proportional to the local expert cost. Note that up to second order, the covariance is time-invariant. If the “local expert” cost \( \tau \) is zero, the covariance is zero because domestic and foreign agents will have the same level of consumption regardless of the interest rate difference. In other words, both domestic and foreign
investors are completely insured against country-specific risk (i.e. the market is effectively complete). A positive $\tau$ makes foreign investment less attractive, thereby creating home biased portfolios and thus market incompleteness. As a result, the difference in consumption is positively correlated with the realized excess return because a country whose assets yield a higher return can afford to consume more. Note also that when agents are more risk averse (i.e. larger $\sigma$), the covariance is lower, implying more balanced portfolios and a higher degree of risk sharing.

More precisely, one can rewrite (35) as follow:

$$E_t \left[ \left( \left( -\sigma \hat{c}_{t+1} \right) - \left( -\sigma \hat{c}^*_t \right) \right) \left( \hat{R}_{t+1} - \hat{R}^*_t \right) \right] = -\tau \quad (36)$$

Equation (36) can be interpreted as follows: the covariance between the difference in marginal utilities and the excess return equals minus the ice-berg cost. The lower the covariance, the more home-biased the portfolio holdings.

I solve for $\theta$ by substituting the conditional result of the system into (35). The value of the foreign agents’ long run portfolio choice of is simply $\theta^* = \theta$.

Denote the normalized current account, NFAs and Valuation Effects as $ca_t \equiv \frac{CA_t}{\Gamma_{t-1}}$; $nfa_t \equiv \frac{NFA_t}{\Gamma_{t-1}}$; and $ve_t \equiv \frac{VE_t}{\Gamma_{t-1}}$. Furthermore, we can define normalized Expected Valuation Effects and Unexpected Valuation Effects as $eve_t \equiv \frac{EVE_t}{\Gamma_{t-1}}$ and $uve_t \equiv \frac{UVE_t}{\Gamma_{t-1}}$.

Note that first-order approximations of all the economic variables of interest can be expressed in first order terms.

$$\hat{ca}_t = y\hat{c}_t - c\hat{c}_t + \alpha y(1 - \theta)(\hat{w}_t - \hat{w}^*_t + \hat{y}_t - \hat{y}^*_t - \hat{q}_t - \hat{q}^*_t) - \alpha y(\hat{\theta}_t - \hat{\theta}^*_t)$$

$$\hat{nfa}_t = (1 - \theta)w(\hat{w}_t - \hat{w}^*_t) - w(\hat{\theta}_t - \hat{\theta}^*_t)$$

$$\hat{ve}_t = (1 - \theta)w\beta(\hat{q}^*_t - \hat{q}_t + \hat{w}_t - \hat{w}^*_t + \hat{q}_t - \hat{q}^*_t) + (1 - \theta)w(\hat{w}^*_t - \hat{w}_t) + w(1 - \beta)(\hat{\theta}_t - \hat{\theta}^*_t)$$

$$\hat{eve}_t = (1 - \theta)w\beta(E_t[\hat{q}^*_t - \hat{q}_t + \hat{w}_t - \hat{w}^*_t + \hat{q}_t - \hat{q}^*_t] + (1 - \theta)w(\hat{w}^*_t - \hat{w}_t) + w(1 - \beta)(\hat{\theta}_t - \hat{\theta}^*_t)$$

$$\hat{uve}_t = \hat{ve}_t - \hat{eve}_t \quad (37)$$

Note that $(\hat{\theta}_t - \hat{\theta}^*_t)$, which can be interpreted as the relative portfolio choice, is a first order term and solved in the first order system. The dynamics of the current account, changes in NFAs and valuation effects can be recovered and analyzed with the first order system. Readers who are not interested in portfolio choice of individual agents can skip Step 2.
3.2 Step 2: First order accurate solution of portfolio choice

The purpose of the second step is to pin down the first order accurate solution of the portfolio choice (i.e. $\hat{\theta}_t$ and $\hat{\theta}^*_t$). The process is as follows:

First, the second order approximation of the system is derived. The first order component of portfolio choice enters the system only through the second order component of the budget constraint (C12):

$$w\bar{g}(\hat{w}^2_{t+1} + \hat{g}^2_t + \lambda^2\hat{\pi}^2_t + 2\hat{w}_{t+1}\hat{g}_t + 2\lambda\hat{w}_{t+1}\hat{\pi}_t + 2\lambda\hat{g}_t\hat{\pi}_t) = 2\theta wR\hat{w}_t\hat{R}_t + \theta wR\hat{R}^2_t + 2(1 - \theta)wR\hat{w}_t\hat{R}^*_t + (1 - \theta)wR\hat{R}^2_t + wR\hat{w}_t + 2wR\hat{\theta}_{t-1}(\hat{R}_t - \hat{R}^*_t) + (1 - \alpha)y\hat{g}^2_t - cc^2_t(38)$$

First note that the values of long run portfolio choice $\theta$ found in step 1 will be used here. In addition, as in DS I realize that $\hat{\theta}_{t-1}(\hat{R}_t - \hat{R}^*_t)$ is an i.i.d random variable with zero mean (since $E_t[R_{t+1} - R^*_{t+1}] = 0$). Denote $\xi_t \equiv \hat{\theta}_{t-1}(\hat{R}_t - \hat{R}^*_t)$. $\xi_t$ is considered as an exogenous i.i.d. zero mean random variable.

The entire system can be summarized as:

$$A_1\begin{pmatrix} ss_{t+1} \\ E_t[cc_{t+1}] \end{pmatrix} = A_2\begin{pmatrix} ss_t \\ cc_t \end{pmatrix} + A_3x_t + A_4\Lambda_t + B\xi_t$$

$$x_t = Nx_{t-1} + \varepsilon_t$$

where

$$\Lambda_t = \begin{pmatrix} x_t \\ ss_t \\ cc_t \end{pmatrix}$$

and $ss_t = [\hat{w}_t, \hat{w}^*_t, \hat{g}_t, \hat{g}^*_t, \hat{\pi}_t]'$ is a column vector of endogenous state variables, $cc_t = [\hat{g}_t, \hat{g}^*_t, \hat{\pi}_t, \hat{R}_t, \hat{R}^*_t]'$ is a column vector of choice variables, $x_t = [z_t, z^*_t, g_t, g^*_t]'$ is a column vector of exogenous state variables. $\varepsilon_t = [\varepsilon^x_t, \varepsilon^{z}_t, \varepsilon^g_t, \varepsilon^{g^*}_t]'$ is a vector of exogenous shocks. $B$ is a column vector with a coefficient of $wR$ in the row corresponding to the equation of evolution of net wealth (38) and zero in all other rows. $\xi_t$ is considered an exogenous i.i.d. variable. Note that the system does not contain first order approximations of portfolio choice $\hat{\theta}_{t+1}$ or $\hat{\theta}^*_t$.

The system (39)-(40) can be solved by various available second order solution methods for rational expectation models. I use the method of Schmitt-Grohe and Uribe (2004). By extracting the appropriate rows and columns from the state-space solution it is possible to write expressions for the second order behavior of $(\hat{c}_t - \hat{c}^*_t)$ and $(\hat{R}_t - \hat{R}^*_t)$ in

20
the following form:

\[ \hat{c}_t - \hat{c}_t^* = D_0 + D_1 \xi_t + D_2 \varepsilon_t + D_3 \hat{\nu}_{t-1} + \varepsilon'_t D_4 \varepsilon_t + \hat{\nu}'_{t-1} D_5 \varepsilon_t + \hat{\nu}'_{t-1} D_6 \hat{\nu}_{t-1} \]  
\[ \hat{R}_t - \hat{R}_t^* = R_0 + R_1 \xi_t + R_2 \varepsilon_t + R_3 \hat{\nu}_{t-1} + \varepsilon'_t R_4 \varepsilon_t + \hat{\nu}'_{t-1} R_5 \varepsilon_t + \hat{\nu}'_{t-1} R_6 \hat{\nu}_{t-1} \]

(41)
\(\text{and (42)}\)

where \(\hat{\nu}_{t-1} = [\hat{\omega}_{t-1}, \hat{\omega}_{t-1}, \hat{\theta}_{t-1}, \hat{\theta}_{t-1}, \hat{\mu}_{t-1}, \hat{\mu}_{t-1}]'\)

In addition, the solution method also requires expressions for the first order behavior of home and foreign consumption and the two asset returns:

\[ \hat{c}_t = C^H \varepsilon_t + C^H \hat{\nu}_{t-1} \]  
\[ \hat{c}_t^* = C^F \varepsilon_t + C^F \hat{\nu}_{t-1} \]
\[ \hat{R}_t = R^1 \varepsilon_t + R^1 \hat{\nu}_{t-1} \]  
\[ \hat{R}_t^* = R^2 \varepsilon_t + R^2 \hat{\nu}_{t-1} \]

(43)
\(\text{and (44)}\)

(45)
\(\text{and (46)}\)

Next, the third order approximations of the two portfolio choice Euler equations are derived below:

\[ E_{t-1}[\hat{R}_t - \hat{R}_t^*] + \frac{1}{2} E_{t-1}[\hat{R}_t^2 - \hat{R}_t^*2 - 2\sigma \hat{c}_t(\hat{R}_t - \hat{R}_t^*)] + \frac{1}{6} E_{t-1}[\hat{R}_t^3 - \hat{R}_t^*3] \]
\[ + \frac{1}{6} E_{t-1}[3\sigma^2 \hat{c}_t^2(\hat{R}_t - \hat{R}_t^*) - 3\sigma \hat{c}_t(\hat{R}_t^2 - \hat{R}_t^*2)] = -\frac{1}{2} \tau - \frac{1}{6} \tau E_{t-1}[-\sigma \hat{c}_t + \hat{R}_t^*] \]  
\[ E_{t-1}[\hat{R} - \hat{R}_t^*] + \frac{1}{2} E_{t-1}[\hat{R}_t^2 - \hat{R}_t^*2 - 2\sigma \hat{c}_t^* (\hat{R}_t - \hat{R}_t^*)] + \frac{1}{6} E_{t-1}[\hat{R}_t^3 - \hat{R}_t^*3] \]
\[ + \frac{1}{6} E_{t-1}[3\sigma^2 \hat{c}_t^2(\hat{R}_t - \hat{R}_t^*) - 3\sigma \hat{c}_t^* (\hat{R}_t^2 - \hat{R}_t^*2)] = \frac{1}{2} \tau + \frac{1}{6} \tau E_{t-1}[-\sigma \hat{c}_t^* + \hat{R}_t] \]  
\[ \text{Subtracting (48) from (47), we have:} \]

\[ E_{t-1} \left[ -(\sigma \hat{c}_t - \sigma \hat{c}_t^*)(\hat{R}_t - \hat{R}_t^*) + \frac{1}{2}(\sigma^2 \hat{c}_t^2 - \sigma^2 \hat{c}_t^{*2})(\hat{R}_t - \hat{R}_t^*) - \frac{1}{2}(\sigma \hat{c}_t - \sigma \hat{c}_t^*)(\hat{R}_t^2 - \hat{R}_t^{*2}) \right] \]
\[ = -\tau - \frac{1}{6} \tau E_{t-1}[-\sigma \hat{c}_t + \hat{R}_t] + (-\sigma \hat{c}_t^* + \hat{R}_t)] \]  
\[ \text{The left hand side of equation (49) is the time varying, third order approximation of the covariance between the difference in marginal utilities and the excess return (i.e.} \]
\[ E_{t-1}[(c_t - c_t^*)(R_t - R_t^*))]. \text{A lower, more negative covariance implies a higher level of home bias. If the ice-berg cost } \tau \text{ is zero (i.e. no financial frictions), the time varying covariance is zero at all time, which indicates perfect cross-country consumption smoothing and complete risk sharing. For a positive } \tau, \text{ the covariance is no longer zero. The first term in the right hand side of equation (49) represents the long run home-bias. The second term represents the time-varying impact of the ice-berg cost on the covariance. In particular, the covariance will be lower (i.e. more home-biased)} \]
when expected marginal utility losses from investing abroad are higher. In other words, domestic agents invest more in domestic equity when they see a higher potential loss from investing abroad.

Lastly, recognizing that $\xi_t = \hat{\theta}_{t-1}(\hat{R}_t - \hat{R}^*_t)$ where $\hat{\theta}_{t-1} = \gamma \hat{v}_{t-1}$, we need to solve for $\gamma$. Substitute from (41) to (46) and $\hat{\theta}_{t-1} = \gamma \hat{v}_{t-1}$ into (49) and deleting terms of order higher than three, we can solve for $\gamma$:

$$- \gamma \sigma D_1 R_2 \Sigma R'_2 - \sigma R_2 \Sigma D'_5 - \sigma D_2 \Sigma R'_5 - \frac{1}{2} \tau (-\sigma C'_{\delta} + R^H_{\delta} + (-\sigma C'_{F} + R^F_{F}))$$

$$= -\frac{1}{6} \tau (-\sigma C'_{\delta} + R^H_{\delta} + (-\sigma C'_{F} + R^F_{F}))$$  (50)

Equation (50) is equation (49) written in matrix form. The right hand side of (50) is still the expected marginal utility losses of investing abroad. The left hand side of (50) is the third order approximation of the covariance between the difference in marginal utilities and the excess return. The variable of interest is $\gamma$ in the first term. $\gamma$ represents the fraction of domestic equity in domestic investors' wealth. An increase in $\gamma$ implies more home bias and therefore lowers the covariance (note that $D_1 R_2 \Sigma R'_2$ is a positive scalar). $D5$ and $R5$ capture the impact of state variables on the response of the difference in consumption and of the excess return to stochastic shocks, which leads to a predictable change in the covariance between consumption difference and the excess return. The change in the covariance requires a change in the optimal portfolio. In addition, the last term in the left hand side indicates that increases in marginal utilities lower the covariance and as a result, domestic equity holding has to fall to offset for that change in the covariance. This effect quantitatively dominates the effect caused by expected marginal utility losses (in the right hand side).

$\gamma$ can be solved:

$$\gamma = \frac{-R_2 \Sigma D'_5 - D_2 \Sigma R'_5}{D_1 R_2 \Sigma R'_2} - \frac{1}{3} \frac{\tau (-\sigma C'_{\delta} + R^H_{\delta} + (-\sigma C'_{F} + R^F_{F}))}{D_1 R_2 \Sigma R'_2}$$  (51)

where $\Sigma = \begin{pmatrix} \sigma^2_{\delta} & 0 & 0 & 0 \\ 0 & \sigma^2_{\delta} & 0 & 0 \\ 0 & 0 & \sigma^2_g & 0 \\ 0 & 0 & 0 & \sigma^2_g \end{pmatrix}$

Equation (51) presents the first order accurate solution for domestic agents’ time varying portfolio choice. The denominators are of second-order, where $D_1 R_2 \Sigma R'_2$ is the

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12See Appendix D for detailed algebra.
variance of the excess return $R_t - R_t^*$. The numerators are of third-order. The first term represent the risk sharing component of portfolio choice where $D5$ and $R5$ are just explained above. The second term can be called the home bias component of the portfolio choice. It represents the impact of the “local expert” cost. In particular, the fraction of domestic equity held by domestic agents decreases with expected marginal utility gains, approximated to first order.

With the first order accurate solutions for $\theta_{t+1}$, and $\theta_{t+1} - \theta^*_{t+1}$ obtained from step 1, I can work out the solution for foreign agents’ portfolio choice $\theta^*_{t+1}$.

4 A numerical exercise

4.1 Calibrations

The coefficient of risk aversion, discount factor and labor share are set as standards. To estimate parameters pertaining to the shocks, I filter annual log of real PPP GDP per capita of the U.S. and of G6 from 1870-2006 \(^{13}\), by a Hodrick Prescott (HP) filter with a smoothing parameter of 100 to recover the trend and transitory components of the series. For the U.S., the estimated parameters of the transitory component are $\rho_z = 0.663; \sigma_z = 0.045$. For G6, the estimated parameters are $\rho_z = 0.725; \sigma_z = 0.037$. Since the parameters are similar for the two countries, I use those of the U.S. for the exercise.

For the parameters of the permanent component, I jointly estimate the two permanent components of G6 and of the U.S. The estimated parameters are $\rho_g = 0.962; \sigma_g = 0.002$ and $\overline{g} = 1.0195$ (we use $\overline{g} = 1.02$ in the exercise).

$\phi$ is set at an arbitrarily small value of 0.001. Recall the role of $\phi$ is to induce stationarity of long run wealth distribution. Statistics of simulated series are robust to changes in $\phi$, as long as $\phi$ remains small.

$\lambda$ is also set at 0.001. I later repeat the exercise for a larger value $\lambda = 0.017$. At this value of $\lambda$, the gap of log output between the two countries in the model is reduced by half after 20 years. This is to correspond to an estimate of Eaton and Kortum (1999) that among G7 countries, about half of new domestic patents will be adopted overseas.

\(^{13}\)Historical PPP GDP data are obtained from The Groningen Growth and Development Centre. Since GDP values are expressed in the same unit for all countries, I derive per capita GDP for the G6 by summing GDP across the G6 countries and divide the sum by the total population of the G6 countries.
<table>
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<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
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<tr>
<td>$\phi$</td>
<td>Stationarity inducing coefficient</td>
<td>0.001</td>
</tr>
</tbody>
</table>

Table 1: Values for parameters

after 20 years or less.

$\tau$ is calibrated to be 0.002, that is the cost of overseas investment is 0.2 percent of the total return. $\tau$ is set so that the long run domestic equity holding $\theta$ is about 87 percent of an agent’s total portfolio. In other words, people holds 87 percent of their wealth in domestic equity (which implies that long run gross external assets are about 82.4 percent percent of output, which is about the level of U.S. in 2000). Overseas investing, although more costly, serves as an insurance mechanism against domestic income shocks (including labor income and equity income).

4.2 Impulse Responses

4.2.1 A transitory shock of one standard deviation (4.5%)

The left columns of figures 7 to 12 in Appendix E present impulse responses to a 4.5 percent transitory shock to home output. The shock decays quickly due to a small $\rho_z = 0.663$. The shock, together with the long run growth of $\Gamma_t$ implies that the home economy will grow at a rate of $1.05 \times 1.02 = 1.071$ or 7.1 percent right after the shock, but quickly return to its long run growth path of 2 percent after about ten years.

Since the shock is entirely transitory, home agents smooth consumption, and save for future consumption when output falls. As a result, trade and current account surpluses follow the shock in After that, trade and current account turn into deficits. Trade balance will eventually get balance thanks to the mechanism to induce stationarity ($\phi > 0$). Without this mechanism, the transitory shock would have a permanent
impact on wealth distribution. The home country would be forever richer than the foreign country and could afford to run trade deficit forever.

Home equity prices increase more than foreign asset prices. The asset price ratio jumps close to 1.004, about 10% the magnitude of the shock, since the shock is transitory. This has an implication on the magnitude of the valuation effects. Valuation effects after transitory shocks are modest, since domestic asset price appreciations are small. Following the initial jump, asset prices quickly converge and go back to the steady state equilibrium ratio. This implies an expected relative decline in the prices of domestic assets, beginning in the period following the shock.

The increase in domestic asset prices coupled with the current account surplus raises the home country’s wealth, only modestly however, as the domestic price appreciation is small. Wealth ratio goes back to 1 in the long run. This is because of the stationary inducing mechanism.

In terms of valuation effects, the transitory shock causes an immediate negative effect of about about 0.4 percent of the GDP, or about a quarter of the size of the current account surplus. The “unexpected valuation effect” partly offsets the current account surplus. However beginning period 2, the expected valuation effects become positive, since relative domestic asset prices are expected to fall.

In the case of a transitory shock, most of the portfolio movements are due to the foreign investment costs, because expected marginal utilities decrease. As a result, I see substantially more home biased portfolios following a positive transitory shock. Domestic agents decide to hold more domestic equity and less foreign equity; whereas foreign agents hold more foreign equity and less domestic equity. Therefore, gross assets and gross liabilities of the home country decrease after the shock.

After the initial shock, gross assets bounce back faster than gross liabilities, making net assets quickly rise to more than 5 % of home output. This is mostly due to the savings of home investor and partly to the relative increase in the prices of foreign assets.

4.2.2 A growth shock of one standard deviation (0.2%)

The impulse responses to a 0.2% trend (growth) shock are shown in the right column of figures 7 through 12. Following the shock, the output ratio grows for 60 years and peaks at 1.046 before converging to unity. Note that this convergence is assumed (as $\lambda > 0$). Without the assumption, output ratio would not go back to 1. In that sense,
the shock is truly permanent.

Since the trend shock implies that the relative growth is sustained for a long time, domestic agents smooth consumption and runs both trade and current account deficits. The trade deficit lasts for 14 years, while the current account deficit lasts 60 years. This causes net assets to decrease. After that, agents anticipate a potential catch-up and start to save. In the long run, as the output ratio slowly decreases, the current account goes to surplus and gets balanced. Net assets therefore become positive and slowly converge to zero in the long run.

The asset price ratio jumps even larger to 1.024, twelve times larger than the magnitude of the shock. The trend shock produces a hump-shaped response in relative asset prices, reflecting sustained relative output growth. The reason is because asset prices are forward looking, they incorporate the future relative output growth. Domestic asset prices keep increase for more than 60 years after the shock.

Thanks to a huge domestic asset price appreciation, despite the current deficit, the domestic country is still a lot richer. The wealth ratio jumps to 1.016, eight times larger than the shock. After that wealth ratio goes down when the home run current account deficits. When domestic agents start to save again, the wealth ratio would picks up and converges to one in the long run, due to the stationarity inducing mechanism.

In term of the “valuation effects”, there is a huge unexpected negative valuation effect after the shock, amounting to about 2% of GDP, larger than the size of the current account deficit itself. This greatly exacerbates the NFAs. Furthermore, the expected valuation effects remain negative for a long period, since domestic asset prices are expected to rise for a long time.

If we increases the value of $\lambda$, the convergence process will be faster and consequently, trade and current account deficits will more short-lived, and domestic asset price appreciation would not be as dramatic. Having said that, every qualitative results of model would hold for a larger value of $\lambda$. Appendix E shows simulation statistics when $\lambda = 0.17$

Most of the portfolio adjustment is due to risk sharing, since expected marginal utilities do not change significantly. After the growth shock, both foreign and home agents increase their holding of home equity, which correspond to the appreciation of the domestic asset prices. However foreign investors move to home equity more aggressively. Net assets fall, reflecting the spending motive of domestic agents after the growth shock. However in the long run, net assets increase and become positive,
and slowly converge to zero in the long run.

### 4.3 Simulations

I run a stylized simulation exercise to investigate the quantitative importance of valuation effects. Parameters will be set to match U.S.-G6 business cycles. Limited data on U.S.-G6 valuation effects, however, prevents meaningful matching of valuation effects’ moments.

In the simulation I generate 100 histories, each of 100 periods. Each period corresponds to one year. I run three separate simulations. First I have both shocks, then I shut off the trend shocks, and finally I shut off the temporary shocks.

Table 2 reports averaged simulated standard deviations of output growth, consumption growth, the trade balance, the current account, valuation effects and changes in NFAs. Numbers in brackets are the standard deviations of the statistics.

<table>
<thead>
<tr>
<th>Variables</th>
<th>Both shocks</th>
<th>Only Transitory</th>
<th>Only Trend</th>
</tr>
</thead>
<tbody>
<tr>
<td>$std \left( \frac{Y_{t+1}}{Y_t} \right)$</td>
<td>Output growth</td>
<td>0.0510</td>
<td>0.0502</td>
</tr>
<tr>
<td>$std \left( \frac{C_{t+1}}{C_t} \right)$</td>
<td>Consumption growth</td>
<td>0.0385</td>
<td>0.0359</td>
</tr>
<tr>
<td>$std \left( \frac{TB_t}{Y_t} \right)$</td>
<td>Trade balance</td>
<td>0.0755</td>
<td>0.0396</td>
</tr>
<tr>
<td>$std \left( \frac{CA_t}{Y_t} \right)$</td>
<td>Current account</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$std \left( \frac{\Delta NFA_t}{Y_t} \right)$</td>
<td>Changes in NFA</td>
<td>0.0538</td>
<td>0.0326</td>
</tr>
<tr>
<td>$corr \left( \frac{VE_t}{Y_t}, \frac{CA_t}{Y_t} \right)$</td>
<td>Corr(Val. Eff,CA)</td>
<td>0.2521</td>
<td>-0.3428</td>
</tr>
<tr>
<td>$std \left( \frac{UE_t}{Y_t} \right)$</td>
<td>Unexpected Val. Eff.</td>
<td>0.0287</td>
<td>0.0047</td>
</tr>
<tr>
<td>$std \left( \frac{EE_t}{Y_t} \right)$</td>
<td>Expected Val. Eff.</td>
<td>0.0243</td>
<td>0.0057</td>
</tr>
<tr>
<td>$std \left( \frac{VE_t}{Y_t} \right)$</td>
<td>Valuation Effects</td>
<td>0.0364</td>
<td>0.0079</td>
</tr>
</tbody>
</table>

Table 2: Volatility of current account, valuation effects and changes in NFAs
If only transitory shocks are present, the consumption growth is less volatile than output growth, since domestic agents only consume a fraction of additional output. The valuation effects are mitigating, they move in opposite directions of the current account and help soften the impact of the current account’s volatility on the country’s NFAs. As a result, changes in NFAs are slightly less volatile than the current account. Valuation effects are said to be small because their average standard deviation is about one seventh of the output shocks’. In this exercise, valuation effects are on average .79% of output.

We can see an entirely different picture with trend shocks. If only trend shocks are present, consumption growth is more volatile than output growth. Valuation effects are positively correlated with the current account; they amplify the impact of the current account on the country’s NFA position. As a result, the changes in net foreign assets are much more volatile than the current account, their average deviation is one and a half time as large as the current account’s. Finally, valuation effects are larger in magnitude with trend shocks, their standard deviation is almost twice as large as the shocks’. Valuation effects in this exercise are on average about 3.4% of output.

When both shocks are present the changes in NFAs are more volatile than the current account, indicating a dominance of growth shocks over transitory shocks. The correlation between valuation effects and the current account is positive, but not statistically significant.

Unlike Devereux and Sutherland (2008), we consider asset prices explicitly and hence can study the quantitative importance of expected and unexpected valuation effects with first order approximation\textsuperscript{14}. From the exercise, in all three cases, expected and unexpected components of valuation effects have more or less equal quantitative importance. This result differs sharply to that of Devereux and Sutherland (2008), where unexpected valuation effects arise only at higher orders of approximation and are generally small.

The qualitative results regarding the role of the valuation effects are not dependent on \( \lambda \). Appendix F shows statistics of the simulations when \( \lambda = 0.017 \).

\textsuperscript{14}Ghironi, Lee, and Rebucci (2007) also follow the same approach.
5 Conclusion

This paper investigates the role of valuation effects in a two country DSGE model with both transitory output shocks and trend shocks. Valuation effects, the changes in the value of gross assets and liabilities due to asset change and exchange rate movements, are generally believed to offset the current account and help stabilize a country’s NFAs (particularly in the case of the U.S.). This paper shows that whether valuation effects are indeed stabilizing depends on the nature of underlying output shocks. In response to transitory shocks, valuation effects are stabilizing; they counteract current account movements and partly offset the current account. In response to trend shocks, valuation effects are amplifying, they move in the same direction with the current account and reinforce the impact of the current account on changes in net foreign asset position. Unlike conventional wisdom that valuation effects tend to be stabilizing, the paper shows that valuation effects can be amplifying too. This is clearly illustrated by the evolution of NFA position between the U.S. and other industrialized countries in the 1990s, when the U.S. experienced persistently higher economic growth. During the period, the U.S. had current account deficits and negative valuation effects with other G7 countries.

This paper also decomposes valuation effects into unexpected and expected components. Unexpected valuation effects are surprise valuation effects due to output shocks; expected valuation effects are predictable movements in the transitional phases after the shocks. This paper find that both expected and unexpected components are important quantitatively.

The theoretical results of this paper provides guidance for future empirical studies about the U.S.’s valuation effects. Firstly, valuation effects based on asset price and exchange rate movements, not measured returns and return differentials, should be the focus of the literature. In addition, it will be more informative to study the U.S.’s valuation effects with other industrialized countries, and with emerging markets separately, because the U.S. were growing faster then the former group, at least in the 1990s, but slower than the latter group. The U.S.’s valuation effects with other industrialized countries are expected to be negative in the 1990s, while those with emerging markets are likely positive.
References


Engel, C. and A. Matsumoto (2006, May). Portfolio choice in a monetary open-


Madsen, J. B. (2007). Technology spillover through trade and tfp convergence:


## A Data Appendix

In this appendix I provide the details about the data and methodology used to construct aggregates for the figures.

In Figure 1, gross foreign assets, gross liabilities and nominal GDPs are from the paper of Lane and Milesi-Ferretti (2007).

In Figures 2, the GDP series for the G7 countries from 1990-2007 are Real Purchasing Power Parity (PPP) GDPs measured at year 2000 international dollars, extracted from the World Bank’s World Development Indicator. Since GDP values are expressed in the same unit for all countries, I derive per capita GDP for the G6 by summing GDP across the G6 countries and divide the sum by the total population of the G6 countries.

In Figure 3, the historical data of S & P 500 index are obtained Professor Robert Shiller’s website http://www.econ.yale.edu/ shiller/data.htm. The historical data of

In Figure 4, the series of the U.S.-G6 current account are from the Bureau of Economics Analysis (International Transactions). Since bilateral current account and financial account data for France, Germany and Italy are available only from 1986, following Ferrero (2007) I proxy the European members of G6 by Western Europe from 1960-1965 and the EU-6 (Belgium, France, Germany, Italy, Luxembourg and the Netherlands) plus United Kingdom for the period 1966-1985. Data on bilateral ”valuation effects” are taken from Bertaut and Tryon (2007). ”Valuation effects” are defined as the adjusted valuation changes of stocks and bonds that the U.S. holds in G6 (U.S.’s claims) minus the adjusted valuation changes of stocks and bonds G6 countries hold in the U.S. (U.S.’s liabilities).

B Appendix B

Data are from Bertaut and Tryon (2007). ”Valuation effects” are defined as the adjusted valuation changes of stocks and bonds that the U.S. holds (U.S.’s claims) minus the adjusted valuation changes of stocks and bonds foreign countries hold in the U.S. (U.S.’s liabilities).

Figure 6: U.S.-Rest Of The World Non-FDI Valuation Effects
C Appendix C

Exogenous processes:

\[ \log(z_t) = \rho \log(z_{t-1}) + \varepsilon_t^z \quad (C1) \]
\[ \log(z_t^*) = \rho \log(z_{t-1}^*) + \varepsilon_t^{z*} \quad (C2) \]
\[ \log(g_t) = (1 - \rho_g) \log(\bar{g}) + \rho_g \log(g_{t-1}) + \varepsilon_t^g \quad (C3) \]
\[ \log(g_t^*) = (1 - \rho_g) \log(\bar{g}) + \rho_g \log(g_{t-1}^*) + \varepsilon_t^{g*} \quad (C4) \]

Output processes:

\[ y_t = z_t g_t (\pi_t)^\lambda \quad (C5) \]
\[ y_t^* = z_t^* g_t^* (\pi_t)^{1-\lambda} \quad (C6) \]
\[ \pi_t = \frac{g_t^*}{g_t} \pi_{t-1}^{1-2\lambda} \quad (C7) \]

where \( \pi_t = \frac{\pi_t^*}{\pi_{t-1}} \) is the ratio of the two trend processes. \( \pi_t \) is a state variable at time \( t \) and converges to one in the steady state.

Euler equations in detrended form become:

\[ c_t^{-\sigma} = \beta \left( \frac{c_t}{y_t} \right)^{-\phi} E_t[c_{t+1}^{-\sigma} (g_t \pi_t^\lambda)^{-\sigma} R_{t+1}] \quad (C8) \]
\[ E_t[c_{t+1}^{-\sigma} R_{t+1}] = E_t[c_{t+1}^{-\sigma} R_{t+1}^*] e^{-\tau} \quad (C9) \]
\[ c_t^{*-\sigma} = \beta \left( \frac{c_t^*}{y_t^*} \right)^{-\phi} E_t[c_{t+1}^{*-\sigma} (g_t \pi_t^\lambda)^{-\sigma} R_{t+1}^*] \quad (C10) \]
\[ E_t[c_{t+1}^{*-\sigma} R_{t+1}] e^{-\tau} = E_t[c_{t+1}^{*-\sigma} R_{t+1}^*] \quad (C11) \]

while the domestic agent’s budget constraint and market clearing conditions are now:

\[ g_t \pi_t^\lambda w_{t+1} + c_t = \theta_t w_t R_t + (1 - \theta_t) w_t R_t^* e^{-\tau} + (1 - \theta^*) w_t^* R_t (1 - e^{-\tau}) + (1 - \alpha) q_{t+1} R_t \quad (C12) \]
\[ R_t = g_t \pi_t^\lambda q_{t+1} \quad (C13) \]
\[ R_t^* = g_t \pi_t^\lambda q_{t+1}^* \quad (C14) \]
\[ c_t^* = c_t + c_t^* \quad (C15) \]
\[ q_{t+1} = \theta_{t+1} w_{t+1} + (1 - \theta_{t+1}^*) w_{t+1}^* \quad (C16) \]
\[ q_{t+1} + q_{t+1}^* = w_{t+1} + w_{t+1}^* \quad (C17) \]
D Appendix D

In this appendix, I show the derivation of equation (51).

In tensor notation, (49) after substitutions is:

\[
[D_2]_i[\tilde{R}_2]_j[\sum]^{i,j} + (E[\tilde{R}_t - \tilde{R}_t]^* - [\tilde{R}_4]_i,j[\sum]^{i,j}][D_3]_k[v_k]^k + [\tilde{R}_4]_i,j[D_3]_k[\sum]^{i,j}[v_k]^k
\]

\[
+[\tilde{R}_2]_i([\tilde{D}_3]_{k,j} + [D_1][\tilde{R}_2]_j[\gamma_k])[[\sum]^{i,j}[v_k]^k + [D_2]_i([\tilde{R}_5]_{k,j} + [\tilde{R}_1][\tilde{R}_2]_j[\gamma_k])[[\sum]^{i,j}[v_k]^k]
\]

\[
-\sigma[\tilde{R}_2]_i((\tilde{C}_2^H)_{j,k} - [\tilde{C}_3^F]_{j,k})[[\sum]^{i,j}[v_k]^k
\]

\[
+ \frac{1}{2}([\tilde{R}_4^1_i][\tilde{R}_2^2]_j - [\tilde{R}_2^2_i][\tilde{R}_4^1]_j)[\tilde{D}_3]_k[[\sum]^{i,j}[v_k]^k + \frac{1}{2}[\tilde{D}_2]_i[\tilde{R}_2]_j([\tilde{R}_3^1]_k + [\tilde{R}_3^2]_k)\sum[i,j][v_k]^k
\]

\[
= \frac{\tau}{\sigma} + \frac{\tau}{6\sigma}(([[\tilde{R}_4^1]_j + [\tilde{R}_3^2]_j] - \sigma([\tilde{C}_3^H]_j + [\tilde{C}_3^F]_j))[v_k]^k (D1)
\]

Note that from (35)

\[
[D_2]_i[\tilde{R}_2]_j[\sum]^{i,j} = \frac{\tau}{\sigma} (D2)
\]

and

\[
E[\tilde{R}_t - \tilde{R}_t]^* = \frac{1}{2} \left( ([\tilde{R}_2]_i[\tilde{R}_2]_j - [\tilde{R}_4]_i,j[\tilde{R}_2]_j) + \sigma((\tilde{C}_2^H)_i + [\tilde{C}_2^F]_i)[\tilde{R}_2]_j \right) \sum^{i,j} (D3)
\]

Substitute (D3) into (D1)

\[
[D_2]_i[\tilde{R}_2]_j[\sum]^{i,j} - \frac{\sigma}{2} [D_2]_i[\tilde{R}_2]_j(\tilde{C}_3^H)_k + [\tilde{C}_3^F]_k)[\sum]^{i,j}[v_k]^k
\]

\[
+ [\tilde{R}_2]_i([\tilde{D}_3]_{k,j} + [D_1][\tilde{R}_2]_j[\gamma_k])[[\sum]^{i,j}[v_k]^k + [D_2]_i([\tilde{R}_5]_{k,j} + [\tilde{R}_1][\tilde{R}_2]_j[\gamma_k])[[\sum]^{i,j}[v_k]^k
\]

\[
+ \frac{1}{2}[\tilde{D}_2]_i[\tilde{R}_2]_j([\tilde{R}_3^1]_k + [\tilde{R}_3^2]_k)[\sum]^{i,j}[v_k]^k
\]

\[
= \frac{\tau}{\sigma} + \frac{\tau}{6\sigma}(([[\tilde{R}_4^1]_k + [\tilde{R}_3^2]_k] - \sigma([\tilde{C}_3^H]_k + [\tilde{C}_3^F]_k))[v_k]^k (D4)
\]

Substitute (D2) into (D4), note that (D4) holds for \( \forall k \) implies:

\[
[\tilde{R}_2]_i([\tilde{D}_3]_{k,j} + [D_1][\tilde{R}_2]_j[\gamma_k])[[\sum]^{i,j} + [D_2]_i([\tilde{R}_5]_{k,j} + [\tilde{R}_1][\tilde{R}_2]_j[\gamma_k])[[\sum]^{i,j}
\]

\[
= \frac{1}{3} \frac{\sigma}{\sigma} \left( ([\tilde{C}_3^H]_k + [\tilde{C}_3^F]_k) - ([\tilde{R}_3^1]_k + [\tilde{R}_3^2]_k) \right) (D5)
\]

Since \( R_1 = 0 \), the solution for \( \gamma \) is:

\[
\gamma_k = -\frac{[\tilde{R}_2]_i[D_5]_{k,j}[[\sum]^{i,j} + [D_2]_i[\tilde{R}_5]_{k,j}[[\sum]^{i,j}]}{[\tilde{R}_2]_i[D_1][\tilde{R}_2]_j} + \frac{1}{3} \frac{\sigma}{\sigma} \left( ([\tilde{C}_3^H]_k + [\tilde{C}_3^F]_k) - ([\tilde{R}_3^1]_k + [\tilde{R}_3^2]_k) \right) (D6)
\]

In the form of matrix expression, it is:

\[
\gamma = \frac{-R_2 \Sigma D'_5 - D_2 \Sigma R'_5}{D_1 R_2 \Sigma R'_2} - \frac{1}{3} \frac{\sigma}{\sigma} \left( (-\sigma C_3^H + R_3^H) + (-\sigma C_3^F + R_3^F) \right) (D7)
\]

I have derived equation (51).
Appendix E

Impulse responses to a 4.5% transitory shock and a 0.2% trend shock

4.5 % positive transitory shock

0.2 % positive trend shock

Figure 7: Home-Foreign output and wealth ratios

Figure 8: Trade balance and current account as percentages of GDP
$4.5\%$ positive transitory shock  \hspace{1cm} $0.2\%$ positive trend shock

Figure 9: Asset price dynamics

Figure 10: Valuation effects as percentages of GDP
4.5% positive transitory shock

![Graph showing the fraction of an agent's wealth held in his country's equity after a 4.5% positive transitory shock.](image)

**Figure 11:** Fraction of an agent's wealth held in his country's equity

0.2% positive trend shock

![Graph showing the fraction of an agent's wealth held in his country's equity after a 0.2% positive trend shock.](image)

**Figure 11:** Fraction of an agent's wealth held in his country’s equity

Home's gross assets and liabilities as percentages of GDP

![Graph showing Home's gross assets and liabilities as percentages of GDP.](image)

**Figure 12:** Home’s gross assets and liabilities as percentages of GDP

NFAs

![Graph showing Net Foreign Assets.](image)

**Figure 13:** NFAs
This appendix shows simulation statistics when $\lambda = 0.017$. To keep the long run domestic equity holding $\theta$ is about 87 percent of an agent’s total portfolio, the foreign investment cost is calibrated $\tau = 0.00086$ , or .086%.

<table>
<thead>
<tr>
<th>Variables</th>
<th>Both shocks</th>
<th>Only Transitory</th>
<th>Only Trend</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\text{std}\left(\frac{Y_{t+1}}{Y_t}\right)$</td>
<td>Output growth</td>
<td>0.0506</td>
<td>0.0504</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.0041)</td>
<td>(0.0035)</td>
</tr>
<tr>
<td>$\text{std}\left(\frac{C_{t+1}}{C_t}\right)$</td>
<td>Consumption growth</td>
<td>0.0369</td>
<td>0.0356</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.0026)</td>
<td>(0.0027)</td>
</tr>
<tr>
<td>$\text{std}\left(\frac{TB}{Y_t}\right)$</td>
<td>Trade balance</td>
<td>0.0475</td>
<td>0.0395</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.0107)</td>
<td>(0.0051)</td>
</tr>
<tr>
<td>$\text{std}\left(\frac{CA}{Y_t}\right)$</td>
<td>Current account</td>
<td>0.0415</td>
<td><strong>0.0329</strong></td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.0075)</td>
<td>(0.0039)</td>
</tr>
<tr>
<td>$\text{std}\left(\frac{\Delta NFA_t}{Y_t}\right)$</td>
<td>Changes in NFA</td>
<td>0.0504</td>
<td><strong>0.0314</strong></td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.0093)</td>
<td>(0.0040)</td>
</tr>
<tr>
<td>$\text{corr}\left(\frac{VE_t}{Y_t}, \frac{CA_t}{Y_t}\right)$</td>
<td>Corr(Val. Eff,CA)</td>
<td>0.1951</td>
<td>-0.3271</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.1274)</td>
<td>(0.1100)</td>
</tr>
<tr>
<td>$\text{std}\left(\frac{UV E_t}{Y_t}\right)$</td>
<td>Unexpected Val. Eff.</td>
<td>0.0181</td>
<td>0.0047</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.0013)</td>
<td>(0.0003)</td>
</tr>
<tr>
<td>$\text{std}\left(\frac{EV E_t}{Y_t}\right)$</td>
<td>Expected Val. Eff.</td>
<td>0.0106</td>
<td>0.0057</td>
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<tr>
<td></td>
<td></td>
<td>(0.0050)</td>
<td>(0.0020)</td>
</tr>
<tr>
<td>$\text{std}\left(\frac{VE_t}{Y_t}\right)$</td>
<td>Valuation Effects</td>
<td>0.0213</td>
<td>0.0079</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.0039)</td>
<td>(0.0016)</td>
</tr>
</tbody>
</table>

Table 3: Volatility of current account, valuation effects and changes in NFAs, with $\lambda = 0.00086$