Dealing with the Trilemma: Optimal Capital Controls with Fixed Exchange Rates

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Mundell’s trilemma states the incompatibility of free capital flows, independent monetary policy and a fixed exchange rate. We lay down a standard macroeconomic model of a small open economy in a currency union and study optimal capital controls. We provide sharp analytical and numerical characterizations. We find that capital controls are employed to respond to some shocks but not others. We discuss how the solution depends on the degree of nominal rigidity and the openness of the economy. We find that capital controls are much more effective stabilization tools than government spending. We also find that capital controls may be optimal even if the exchange rate is not fixed if wages, in addition to prices, are sticky. Finally, we compare the single country’s optimum to a coordinated solution for the union as a whole. Our results show a very limited need for coordination. In particular, the noncooperative equilibrium features the same capital controls as the coordinated solution.

1 Introduction

Mundell’s trilemma states that a country cannot simultaneously have free capital flows, independent monetary policy, and a fixed exchange rate. Countries often maintain a fixed exchange rate, either by independent choice or as members of a currency union. How then should these countries cope with macroeconomic shocks? To what degree should they give up on free capital mobility to regain monetary policy? Although the International Monetary Fund has recently sided more sympathetically with the use of capital controls (Ostry et al., 2010), we still lack a theoretical framework to answer these questions. Our goal is to fill in this gap by studying optimal capital control policy in a standard open economy model with fixed exchange rates.

Our model, which builds on Galí and Monacelli (2005, 2008), introduces capital controls in a standard open economy model with nominal rigidities. Goods are differentiated
by country and variety. Countries experience different macroeconomic shocks. These shocks can be of several kinds, as we allow for productivity shocks, foreign demand for exports shocks, foreign consumption shocks and shocks to wealth—the latter captures fluctuations in the price of commodities. We analyze different price setting assumptions: flexible prices, rigid prices, one-period in advance sticky prices, and Calvo price setting. We characterize the way a country should optimally use capital controls to maximize welfare in response to shocks. We show that this depends crucially on the nature of the shock, on the stickiness of prices, and on the openness of the economy. We also show that there are gains from coordination. We characterize how countries would optimally use capital controls if they could perfectly coordinate.

We first start with the case of flexible prices. Even with flexible prices, optimal capital controls are generally nonzero, a point explored in detail in Costinot et al. (2011). In present context, with a small open economy there is no ability to affect the world interest rate. However, each country still has some monopoly power over their terms of trade. Without trade barriers capital controls emerge as an imperfect tool to manipulate terms of trade. By reallocating spending intertemporally a country can raise their export prices in some periods and lower them in others. This effect is not the focus of this paper. Indeed, we isolate a few cases where this effect is not at play: when shocks are permanent, or in the Cole-Obstfeld parametrization (unitary inter- and intra-temporal elasticities). But more generally, the case with flexible prices acts as a benchmark to compare our results with nominal rigidities.

We first contrast the case of flexible prices with its polar opposite: perfectly rigid prices. Thus, we assume that the exchange rate and prices are fixed forever. As before, capital controls are not employed in response to permanent shocks. In response to transitory shocks, however, capital controls are typically the reverse of what would be optimal with flexible prices. A useful example is the Cole-Obstfeld case with a trend in productivity. In this case, optimal capital controls are zero when prices are flexible, but they take the form of a tax on inflows (or a subsidy on outflows) when prices are completely rigid. With flexible prices, the country’s price index decreases over time. This expected deflation raises the real interest rate and increases the growth rate of consumption. With rigid prices, the real interest rate is fixed and hence the growth rate of consumption is too low, inducing a boom relative to the flexible price allocation. By taxing inflows (or subsidizing outflows) the country can increase its nominal interest rate, cooling off the economy. The growth rate of output is also increased, but by less, moving the trade balance into surplus. In contrast, with flexible prices trade is always balanced in the Cole-Obstfeld case. This underscores that capital controls are a second best tool, allowing the country to regain
some monetary autonomy and therefore some control over the intertemporal allocation of spending. However, this reallocation is costly since it introduces a wedge between the intertemporal prices for home and foreign households. Moreover, capital controls cannot affect the composition of spending between home and foreign goods, which is determined by relative prices that are unchanged since the exchange rate and prices are fixed.

We then study the same model under the intermediate assumption that prices are set one period in advance, just as in Obstfeld and Rogoff (1995). This case is more complex for the following reason. As explained above, capital controls lead to intertemporal wealth transfers. Wealth transfers, in turn, lead to terms of trade and real exchange rate changes—familiar at least since Keynes (1929) and Ohlin (1929). Taking these effects into account complicates the analysis, yet we are able to provide tight characterizations for the use of capital controls in this context, both for transitory and for permanent shocks of different kinds.

We then consider the case of Calvo price setting. In this setting, capital controls affect inflation dynamics in more complex way. This allows us to capture a prudential role of capital controls, where the government is concerned about the effect of temporary shocks on absolute and relative prices. A temporary capital inflow, for example, may heat up the economy and affect prices in a way that is harmful once the flows are reverted.

With Calvo pricing, the planning problem is more involved because there is a now a cost to inflation due to the induced price dispersion. Nevertheless, we are able to provide analytical characterization in the limit of a closed economy and explore the solution numerically away from this limit case. In general, capital controls are employed countercyclically. One important insight is that the more closed the economy, the more effective are capital controls.

There is a growing theoretical literature in international macro demonstrating, among other things, how restrictions on international capital flows may be welfare-enhancing in the presence of various credit market imperfections; see e.g. Calvo and Mendoza (2000), Caballero and Lorenzoni (2007), Aoki et al. (2010), Jeanne and Korinek (2010), and Martin and Taddei (2010). An older literature emphasized the trilemma, see for example McKinnon and Oates (1966). To the best of our knowledge ours is the first paper to study optimal capital controls in a micro-founded model with nominal rigidities.
2 A Small Open Economy

We build on the framework by Galí and Monacelli (2005, 2008), who develop a model composed of a continuum of open economies. Our main focus is on policy in a single country, which we call Home, taking as given the rest of the world, which we call Foreign. However, we also explore the joint policy problem for the entire world when coordination is possible. We make a few departures from Gali and Monacelli’s model to adapt it to the questions we address. First and foremost, they studied monetary and fiscal policy, so we must extend the model to include capital controls. Second, in contrast to their simplifying assumption of complete markets, we prefer to assume international financial markets are incomplete. No risk sharing between countries is allowed, only risk free borrowing and lending. Given this assumption, to keep the analysis tractable, we limit our attention to one-time unanticipated shocks to the economy.\(^1\) Third, Gali and Monacelli confine their normative analysis to the Cole-Obstfeld parameter specification, where various elasticities of substitution are unity. This case is more tractable and we will make extensive use of this fact here too. However, we also include some results outside of this parameter configuration. Finally, we also consider a larger variety of shocks to the economy and consider extensions where we include wage rigidity alongside price stickiness.

### 2.1 Households

There is a continuum measure one of countries \(i \in [0, 1]\). We focus attention on a single country, which we call Home, and can be thought of as a particular value \(H \in [0, 1]\). In every country, there is a representative household with preferences represented by the utility function

\[
\sum_{t=0}^{\infty} \beta^t \left[ \frac{C_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\phi}}{1+\phi} \right]
\]

where \(N_t\) is labor, and \(C_t\) is a consumption index defined by

\[
C_t = \left[ (1 - \alpha)^{\frac{1}{\eta}} C_{H,t}^{\frac{1}{\eta}} + \alpha^{\frac{1}{\eta}} C_{F,t}^{\frac{1}{\eta}} \right]^{\eta - 1}.
\]

\(^1\)Relative to the literature, this is not a limitation since most studies, including Gali-Monacelli, work with linearized equilibrium conditions, so that the response to shocks is unaffected by the presence of future shocks.
where $C_{H,t}$ is an index of consumption of domestic goods given by

$$C_{H,t} = \left( \int_0^1 C_{H,t}(j)^{\frac{\epsilon - 1}{\epsilon}} \, dj \right)^{\frac{\epsilon}{\epsilon - 1}},$$

where $j \in [0, 1]$ denotes an individual good variety. Similarly, $C_{F,t}$ is a consumption index of imported goods given by

$$C_{F,t} = \left( \int_0^1 \Lambda_{i,t}^{\frac{\gamma - 1}{\gamma}} C_{i,t}^{\frac{\gamma - 1}{\gamma}} \, di \right)^{\frac{\gamma}{\gamma - 1}}$$

where $C_{i,t}$ is, in turn, an index of the consumption of varieties of goods imported from country $i$, given by

$$C_{i,t} = \left( \int_0^1 C_{i,t}(j)^{\frac{\epsilon - 1}{\epsilon}} \, dj \right)^{\frac{\epsilon}{\epsilon - 1}}.$$

Thus, $\epsilon$ is the elasticity between varieties produced within a given country, $\eta$ the elasticity between domestic and foreign goods, and $\gamma$ the elasticity between goods produced in different foreign countries. An important special case obtains when $\sigma = \eta = \gamma = 1$. We call this the Cole-Obstfeld case, in reference to Cole and Obstfeld (1990). This case is more tractable and has some special implications that are worth highlighting. Thus, we devote special attention to it, although we will also derive results away from it.

The parameter $\alpha$ indexes the degree of home bias, and can be interpreted as a measure of openness. Consider both extremes: as $\alpha \to 0$ the share of foreign goods vanishes; as $\alpha \to 1$ the share of home goods vanishes. Since the country is infinitesimal, the latter captures a very open economy without home bias; the former a closed economy barely trading with the outside world.

We have included a taste shifter $\Lambda_{i,t}$, which is always normalized so that $\int \Lambda_{i,t} \, di = 1$, that affects the utility of imports from country $i$. All countries experience the same taste configuration $\{\Lambda_{i,t}\}$. Thus, variations in $\Lambda_{H,t}$ allow us to consider variations in the demand for Home’s exports.

Households seek to maximize their utility subject to the sequence of budget con-
While absence of arbitrage for residents of country \( i \) requires \( D \) holding: domestic currency. The portfolio of home agents is composed of home and foreign bond holding: \( D_t \) is home bond holdings of home agents, \( \hat{\tau}_i \) is a tax on capital inflows and subsidy on capital outflows in the home country, and similarly \( \tau_i^j \) is a tax on capital inflows and subsidy on capital outflows in country \( i \). The proceeds of these taxes are rebated lump sum to the households at Home and country \( i \), respectively. The Home country taxes inflows to make the after tax (net of any subsidy paid by their own country of origin) return to foreign investors \( (1 + \tau_{t-1})/ (1 + \tau_{t-1}) \) in domestic currency.\(^2\)

We can re-express the household budget constraint as

\[
P_t C_t + D_{t+1} + \int_{0}^{1} E_i D_{t+1}^i di \\
\leq W_t N_t + \Pi_t + T_t + (1 + i_{t-1}) D_t + \int_{0}^{1} \frac{1 + \tau_{t-1}^i}{1 + \hat{\tau}_i^j} E_i^j (1 + \hat{i}_{t-1}^j) D_j^i di,\]

for \( t = 0, 1, 2, \ldots \). In this inequality, \( P_{H,t}(j) \) is the price of domestic variety \( j \), \( P_{i,t} \) is the price of variety \( j \) imported from country \( i \), \( W_t \) is the nominal wage, \( \Pi_t \) represents nominal profits and \( T_t \) is a nominal lump sum transfer. All these variables are expressed in domestic currency. The portfolio of home agents is composed of home and foreign bond holding: \( D_t \) is home bond holdings of home agents, \( D_j^i \) is bond holdings of country \( i \) of home agents. The returns on these bonds are determined by the nominal interest rate in the home country \( i_t \), the nominal interest rate \( i_t^j \) in country \( i \), and the evolution of the nominal exchange rate \( E_{i,j} \) between home and country \( i \). Capital controls are modeled as follows: \( \tau_i \) is a tax on capital inflows and subsidy on capital outflows in the home country, and similarly \( \tau_i^j \) is a tax on capital inflows and subsidy on capital outflows in country \( i \). The proceeds of these taxes are rebated lump sum to the households at Home and country \( i \), respectively. The Home country taxes inflows to make the after tax (net of any subsidy paid by their own country of origin) return to foreign investors \( (1 + \tau_{t-1})/ (1 + \tau_{t-1}) \) in domestic currency.\(^2\)

In other words, if we take after tax return to be \( (1 - \hat{\tau}_{t-1}^j)(1 + \hat{i}_{t-1}^j) \) then this represents a tax equal to \( \hat{\tau}_{t-1}^j = \tau_{t-1} / (1 + \tau_{t-1}) \). To see why this is required assume \( \tau_{t-1}^j = 0 \). Absence of arbitrage by domestic residents requires

\[
1 + i_{t-1} = (1 + \tau_{t-1}^j) \frac{E_i^j}{E_{i-1}^j} (1 + \hat{i}_{t-1}^j),
\]

while absence of arbitrage for residents of country \( i \) requires

\[
(1 - \hat{\tau}_{t-1}^j) \frac{E_{i-1}^j}{E_i^j} (1 + i_{t-1}) = 1 + \hat{i}_{t-1}^j.
\]

The two are compatible if and only if \( \hat{\tau}_{t-1}^j = \tau_{t-1} / (1 + \tau_{t-1}) \).
using price indices

\[ P_t = [(1 - \alpha)P_{H,t}^{1-\eta} + \alpha P_{F,t}^{1-\eta}]^{\frac{1}{1-\eta}}, \]

\[ P_{H,t} = \left( \int_0^1 \frac{P_{H,t}(j)^{1-\epsilon}}{dj} dj \right)^{\frac{1}{1-\epsilon}}, \]

\[ P_{F,t} = \left( \int_0^1 \frac{\Lambda_{i,t} P_{i,t}^{1-\gamma}}{di} di \right)^{\frac{1}{1-\gamma}}, \]

\[ P_{i,t} = \left( \int_0^1 \frac{P_{i,t}(j)^{1-\epsilon}}{dj} dj \right)^{\frac{1}{1-\epsilon}}. \]

Here \( P_t \) is the Home consumer price index (CPI), \( P_{H,t} \) is the Home producer price index (PPI), \( P_{F,t} \) is a price index of imported goods at Home, while \( P_{i,t} \) is country \( i \)'s PPI.

### 2.2 Firms

**Technology.** A typical firm in the home economy produces a differentiated good with a linear technology given by

\[ Y_t(j) = A_t N_t(j) \]  

where \( A_t \) is productivity in the home country. We denote productivity in country \( i \) by \( A_t(i) \).

We allow for a constant employment tax \( 1 + \tau^L \), so that real marginal cost deflated by Home PPI is given by

\[ MC_t = \frac{1 + \tau^L}{A_t} \frac{W_t}{P_{H,t}}. \]

We take this employment tax to be constant in our model. We explain below how it is determined.

**Price-setting assumptions.** We will consider a variety of price setting assumptions: flexible prices, one-period in advance sticky prices, and sticky prices a la Calvo. Across all specifications we will maintain the assumption that the Law of One Price (LOP) holds so that at all times, the price of a given variety in different countries is identical once expressed in the same currency. This assumption is sometimes known as Producer Currency Pricing (PCP).

First, consider the case of flexible prices. Firm \( j \) optimally sets its price \( P_{H,t}(j) \) to ma-
imize

$$\max_{P_{H,t}(j)} (P_{H,t}(j) Y_{t|t} - P_{H,t}MC_{t} Y_{t|t})$$

where

$$Y_{t|t} = \left( \frac{P_{H,t}(j)}{P_{H,t}} \right)^{-\epsilon} C_{t+k}$$

Second, consider the case where prices are set one period in advance. Since we consider only one time-unanticipated shocks around the symmetric deterministic steady state, this simply means that prices are fixed at \( t = 0 \) and flexible for \( t \geq 1 \).

Third, consider Calvo price setting, where in every period, a randomly selected fraction \( 1 - \delta \) of firms can reset their prices. Those firms that get to reset their price choose a reset price \( P_{r,t} \) to solve

$$\max_{P_{r,t}} \sum_{k=0}^{\infty} \delta^k \Pi_{h=1}^k \frac{1}{1 + \delta_{t+h}} (P_{r,t} Y_{t+k|t} - P_{H,t}MC_{t} Y_{t+k|t})$$

where

$$Y_{t+k|t} = \left( \frac{P_{r,t}}{P_{H,t+k}} \right)^{-\epsilon} C_{t+k}$$

### 2.3 Market Clearing

Defining aggregate output to be

$$Y_{t} = \left( \int_{0}^{1} Y_{t}(j)^{\epsilon-1} dj \right)^{\frac{1}{\epsilon-1}}$$

we obtain the goods market clearing condition

$$Y_{t} = (1 - \alpha) \left( \frac{P_{H,t}}{P_{t}} \right)^{-\eta} C_{t} + \alpha \Lambda_{H,t} \int_{0}^{1} \left( \frac{P_{H,t}}{E_{i,t} P_{F,t}} \right)^{-\gamma} \left( \frac{P_{i,t}}{P_{F,t}} \right)^{-\eta} C_{i} \alpha_{i} dj.$$  \hspace{1cm} (3)

Labor market clearing in the home economy is then

$$N_{t} = \frac{Y_{t}}{A_{t}} \int_{0}^{1} \left( \frac{P_{t}(j)}{P_{t}} \right)^{-\epsilon} \alpha_{i} dj$$  \hspace{1cm} (4)

where \( N_{t} = \int_{0}^{1} N_{t}(j) dj \).

### 2.4 Terms of Trade, Nominal and Real Exchange Rates

Let \( E_{i,t} \) be nominal exchange rate between home and \( i \) (an increase in \( E_{i,t} \) is a depreciation of the home currency). Because the Law of One Price holds, we can write \( P_{i,t}(j) = E_{i,t} P_{i,t}(j) \) and \( P_{i,t} = E_{i,t} P_{i,t} \) where \( P_{i,t}(j) \) is country \( i \)’s price of variety \( j \) expressed in its own
currency, and $P_{i,t}^i = \left( \int_0^1 P_{i,t}^i (j)^{1-e} \right)^{\frac{1}{1-e}}$ is country $i$’s domestic PPI (in terms of country $i$’s currency, as opposed to $P_{i,t}$ which is expressed in Home currency). We therefore have

$$P_{F,t} = E_t P_t^*$$

where

$$P_t^* = \left( \int_0^1 \Lambda_{i,t} P_{i,t}^{1-\gamma} di \right)^{\frac{1}{1-\gamma}}$$

is the world price index and

$$E_t = \frac{\left( \int_0^1 \Lambda_{i,t} E_{i,t}^{1-\gamma} P_{i,t}^{1-\gamma} di \right)^{\frac{1}{1-\gamma}}}{\left( \int_0^1 P_{i,t}^{1-\gamma} di \right)^{\frac{1}{1-\gamma}}}$$

is the effective nominal exchange rate.

The effective terms of trade are defined by

$$S_t \equiv \frac{P_{F,t}}{P_{H,t}} = \left( \int_0^1 \Lambda_{i,t} S_{i,t}^{1-\gamma} di \right)^{\frac{1}{1-\gamma}} \quad (5)$$

where $S_{i,t} = P_{i,t}/P_{H,t}$ is the terms of trade of home versus $i$. The terms of trade can then be expressed as

$$S_t = \frac{E_t P_t^*}{P_{H,t}}. \quad (6)$$

The terms of trade can be used to rewrite the home CPI as

$$P_t = P_{H,t}\left[ 1 - \alpha + \alpha S_t^\eta \right]^{\frac{1}{1-\eta}}. \quad (7)$$

Finally we can define the real exchange rate between home and $i$ as $Q_{i,t} = E_{i,t} P_t^i / P_t$. Similarly let the effective real exchange rate be

$$Q_t = \frac{E_t P_t^*}{P_t}.$$
2.5 First-Order Conditions

First-order conditions for the household problem. The first-order conditions for the household problem include the labor-leisure condition

\[ C_i^\sigma N_i^\phi = \frac{W_i}{P_t} \]  

(8)

the Euler equation

\[ \beta \frac{C_{i+1}^{-\sigma}}{C_i^{-\sigma}} \frac{P_i}{P_{i+1}} = \frac{1}{1 + i_t} \]  

(9)

and the arbitrage conditions for all \( i \in [0, 1] \)

\[ 1 + i_t = \frac{1 + \tau_i}{1 + \tau'_i} \frac{E_{i,t+1}}{E_{i,t}}. \]  

(10)

This last equation indicates that capital control introduce a wedge in the Uncovered Interest Parity (UIP) equation—an observation that will play an important role in our analysis. Combining (9) and (10), we can derive a modified version of the Backus-Smith condition in the presence capital controls:

\[ C_t = \Theta_{i,t} \theta_i^j Q_{i,t}^{\frac{1}{\phi}}, \]  

(11)

where

\[ \frac{\theta_{i+1}}{\theta_i} = \left( \frac{1 + \tau_i}{1 + \tau'_i} \right)^{\frac{1}{\phi}}. \]

First-order conditions for the firm problem. The first-order conditions for the firm problem depend on the price setting assumption.

If prices are flexible, then firm \( j \) sets its price \( P_{H,t}(j) \) equal to a constant markup \( M = \frac{\epsilon}{\epsilon - 1} \) over nominal marginal cost \( \frac{1 + \tau'_i}{A_i} W_t \) so that

\[ MC_t = \frac{1}{M}. \]

If prices are set one period in advance, then prices are simply fixed at \( t = 0 \), and set flexibly as above for \( t \geq 1 \).

If prices are set a la Calvo, then a firm that gets to reset its price at \( t \) chooses

\[ P_t^r = M \sum_{k=0}^{\infty} \delta^k \prod_{h=1}^{1 + \tau'_i + h} \frac{1}{Y_{t+k}} (P_{H,t+k} MC_{t+k}) \frac{1}{\prod_{h=1}^{1 + \tau'_i + h}}. \]  

(12)
2.6 Equilibrium Conditions with Symmetric Rest of the World

We now summarize the equilibrium conditions. For simplicity of exposition, we focus on the case where all foreign countries are identical. Moreover, we assume that foreign countries do not impose capital controls. We denote foreign variables with a star. Taking foreign variables as given, equilibrium in the home country can be described by the following equations. We find it convenient to group these equations into two blocks, which we refer to as the demand block and the supply block.

The demand block is independent of the nature of price setting. It is composed of the Backus-Smith condition

\[ C_t = \Theta_t C^*_t Q_{t,1}^{\frac{1}{\sigma_t}}, \]

the equation relating the real exchange rate to the terms of trade

\[ Q_t = \left[ (1 - \alpha) (S_t)^{\eta-1} + \alpha \right]^{\frac{1}{\eta-1}}, \]

the goods market clearing condition

\[ Y_t = C^*_t \left[ (1 - \alpha) \Theta_t Q_{t,1}^{\frac{1}{\sigma_t}} \left( \frac{Q_t}{S_t} \right)^{-\eta} + \alpha \Lambda_{H,t} S_t^\gamma \right], \]

the labor market clearing condition

\[ N_t = \frac{Y_t}{A_t} \Delta_t \]

where \( \Delta_t \) is an index of price dispersion \( \Delta_t = \int_0^1 \left( \frac{P_{H,t}(j)}{P_{H,t}} \right)^{-\epsilon} \), the Euler equation

\[ 1 + i_t = \beta^{-1} \frac{C_{t+1}}{C_t} \Pi_{t+1} \]

where \( \Pi_t = \frac{P_{t+1}}{P_t} \) is CPI inflation, the arbitrage condition between home and foreign bonds

\[ \frac{\Theta_t^{\sigma_t}}{\Theta_{t+1}^{\sigma_{t+1}}} = \frac{1 + i_t}{1 + i^*_t \frac{E_t}{E_{t+1}}}, \]

and the country budget constraint

\[ NFA_t = -C^*_t \left( S_t^{-1} Y_t - Q_t^{-1} C_t \right) + \beta (1 + \tau^*_t) NFA_{t+1} \]

where \( NFA_t \) is the country’s net foreign assets at \( t \), which for convenience, we measure
in the foreign price at home \(P_{F,t}\) as the numeraire, and which we adjust by the foreign marginal utility of consumption \(C_i^e\). The country budget constraint is derived from the consumer’s budget constraint after substituting out the lump-sum transfer. Under government budget balance the transfer equals the sum of the revenue from the labor tax and the tax on foreign investors, net of the revenue lost to subsidize domestic residents’ investments abroad.\(^3\)

The supply block varies with the nature of price setting. With flexible prices, it boils down to the following condition which combines the household labor leisure condition (8) and the optimal markup condition

\[
C_i^e S_i^{-1} Q_i = M \frac{1 + \tau^L}{A_t} N_i^{\phi}
\]

together with the no price dispersion assumption \(\Delta_t = 1\). With one period in advance price stickiness, the only difference is that at \(t = 0\), all price are fixed. This means that \(S_0 = E_0 \frac{P_0^e}{P_{H,0}}\) where \(P_0^e\) and \(P_{H,0}\) are fixed. Finally with Calvo price setting, the supply block is more complex. It is composed of the equations summarizing the first-order condition for optimal price setting

\[
\frac{1 - \delta \Pi_{H,t}^{-1}}{1 - \delta} = \left( \frac{F_t}{K_t} \right)^{e-1},
\]

\[
K_t = \frac{\epsilon}{\epsilon - 1} \frac{1 + \tau^L}{A_t} Y_t N_i^{\phi} \Pi_{H,t}^e + \delta \beta K_{t+1},
\]

\[
F_t = Y_t C_i^e S_i^{-1} Q_i \Pi_{H,t}^e + \delta \beta F_{t+1},
\]

together with an equation determining the evolution of price dispersion

\[
\Delta_t = h(\Delta_{t-1}, \Pi_{H,t})
\]

where \(h(\Delta, \Pi) = \delta \Pi^e + (1 - \delta) \left( \frac{1 - \delta \Pi^{-1}}{1 - \delta} \right)^{e-1}\), and the equation relating CPI and PPI inflation

\[
\Pi_t = \Pi_{H,t} \left[ \frac{(1 - \alpha) + \alpha S_i^{1 - \eta}}{(1 - \alpha) + \alpha S_{i-1}^{1 - \eta}} \right]^{\frac{1}{1 - \eta}}.
\]

For most of the paper, we will be concerned with fixed exchange rate regimes (either pegs or currency unions) in which case we have the additional restriction that \(E_t = E_0\) for

\(^3\)Of course, we do not not require budget balance but since Ricardian equivalence holds here, all other government financing schemes have the same implications.
all $t \geq 0$ where $E_0$ is predetermined.

### 2.7 Labor Tax

We assume that each country chooses their labor tax optimally assuming a symmetric deterministic steady state with flexible prices.

To solve for this optimal labor subsidy it is useful to consider the following relaxed problem, which allows taxes to vary over time. As it turns out, at a steady state, the optimal tax is constant. Assume that the world is at a symmetric deterministic steady state. Each country takes the rest of the world as given and uses a time-varying tax on capital inflow and subsidy on capital outflow controls $\tau_t$ and labor tax $\tau_{L,t}$ to maximize the welfare of its households. For example, the home country solves

$$
\max_{\{C_t, Y_t, N_t, \Theta_t, Q_t, S_t\}} \sum_{t=0}^{\infty} \left[ \frac{C_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\phi}}{1+\phi} \right]
$$

subject to

$$
C_t = \Theta_t C^* Q_t^{\frac{1}{\sigma}}, \\
Y_t = C^* \left[ (1 - \alpha) Q_t^{\frac{1}{\sigma} - \eta} S_t^{\eta} \Theta_t + \alpha S_t^{\gamma} \right], \\
Q_t = \left[ (1 - \alpha) S_t^{\eta - 1 + \alpha} \right]^{\frac{1}{\eta - 1}}, \\
N_t = \frac{Y_t}{A}, \\
0 = \sum_{t=0}^{\infty} \beta^t C^*^{1-\sigma} \left( S_t^{-1} Y_t - Q_t^{-1} C_t \right).
$$

We can then back out the optimal labor tax using and the optimal capital controls using

$$
S_t^{-1} \Theta_t^{-\sigma} C^*^{1-\sigma} = M \frac{1 + \tau_t^L}{A} N_t^{\phi}, \\
\frac{\Theta_t+1}{\Theta_t} = (1 + \tau_t)^{\frac{1}{\sigma}}.
$$

Using this program we can prove the following.

**Proposition 1 (Steady State Tax).** At a symmetric deterministic steady state with flexible prices where each country can set time-varying labor subsidies and capital controls, the optimal labor tax
is given by $\tau_L = \frac{1}{M} \frac{\eta+\gamma-1}{1-\alpha+\eta+\gamma-1} - 1$ and optimal capital controls are equal to zero.

Importantly, since the optimal labor tax is constant, this also solves the problem we are interested in: where the labor tax is required to be constant.

From each country’s perspective, the labor tax is the result of a balancing act between offsetting the monopoly distortion of individual producers and exerting some monopoly power as a country. The two terms in the optimal tax formula reflect the two legs of this tradeoff. Reflecting the former leg of the tradeoff, the optimal labor tax is decreasing in the degree of monopoly power of individual firms ($M$). Reflecting the second leg of the tradeoff, the optimal labor tax is increasing in the degree of openness ($\alpha$), and decreasing in the elasticity of substitutions between home and foreign goods ($\eta$) as well as in the elasticity of substitution between goods of different countries ($\gamma$).

3 Perfectly Flexible and Perfectly Rigid Prices

In this section, we focus on two extreme cases: perfectly flexible or perfectly rigid prices. To simplify the exposition, in this section and in the next we assume that the economy is initially at the deterministic symmetric steady state. This assumption plays no material role in our results.

We consider shocks of four kinds: productivity shocks (shocks to $\{A_t\}$), export demand shocks (shocks to $\{\Lambda_{H,t}\}$), foreign consumption shocks (shocks to $\{C^*_t\}$) and wealth shocks (shocks to $NFA_0$). The only shocks that require some elaboration are the wealth shocks. In the small open economy context, there are many interpretations to the shock to initial net foreign asset position $NFA_0$. It may capture the return on investments in risky assets, the default on debt held abroad, etc. Another interesting interpretation is the following. Suppose in addition to the goods described previously, the Home country owns an endowment of a commodity good $X_t$ (e.g. oil, copper, soybeans, etc.) that it does not consume and exports to international markets taking the price $P_{X,t}$ as given in foreign currency. Thus, the budget constraint becomes

$$P_tC_t + D_{t+1} + \int_0^1 E_{t,i} D_{i+1}^i di \leq W_t N_t + \Pi_t + E^*_t P^*_t X_t + T_t + (1 + i_{t-1}) D_t + \int_0^1 \frac{1 + \tau_{i-1}^t E_i^t (1 + i_{t-1}) D_{t-1}^i}{1 + \tau_{i-1}^t}.$$

where $E_t^*$ is the exchange rate against the reference country for which the price $P^*_t$ is quoted. The only new element is the presence of $E^*_t P^*_t X_t$ on the right hand side of this
constraint. The model is then summarized by the same equilibrium conditions as before if we reinterpret $NFA_t$ as also capturing the present value (discounting using the foreign interest rate) of the revenue from exports of this commodity. Under this interpretation, a shock to the price path $\{P^*_X, t\}$ or the endowment path $\{X_t\}$ can be captured by it’s impact on the present value $\sum_{t=0}^\infty \beta C^*_t - \sigma \sum_{t=0}^\infty \phi M(1+\tau) \Theta_t$.

We characterize the optimal use of capital controls from an individual country’s perspective in response to these different shocks, assuming that other countries do not use capital controls.

3.1 Flexible Prices

We start with the case of flexible prices. We can write the corresponding planning problem as

$$\max_{\Theta_t} \sum_{t=0}^\infty \beta^t C^*_t = \frac{\Theta^1 - \sigma}{1 - \sigma} - \frac{1}{1+\phi M(1+\tau)} \Theta^\sigma \left((1-\alpha)Q^\sigma t - \eta S^H t - \Theta t + \Lambda t S^\gamma t - 1\right)$$

subject to

$$0 = \sum_{t=0}^\infty \beta^t C^*_t = \frac{1}{1+\phi M(1+\tau)} S^H t - \Theta t,$$

$$1 = M(1+\tau) \frac{1}{A^1 + \phi} C^*_t = \frac{1}{\Theta^\sigma} \left((1-\alpha)Q^\sigma t - \eta S^H t - \Theta t + \alpha \Lambda t S^\gamma t - 1\right)^\phi,$$

$$Q_t = \left[(1-\alpha)S^H t - 1 + \alpha \right]^{\frac{1}{\eta - 1}}.$$

In this problem if the sequence of productivity and export demand shocks $\{A_t, \Lambda_{H,t}\}$ are constant, then the solution is constant, as long as the program is sufficiently convex. We have verified convexity in the Cole-Obstfeld case. To simplify, we assume it holds away from this case.

Interestingly, even with flexible prices, when the paths for $\{A_t\}$, $\{\Lambda_{H,t}\}$ are not constant, it is generally optimal to use capital controls. Optimal capital controls can be inferred by taking the first-order conditions of the planning problem above and using the fact that $\frac{\Theta_{t+1}}{\Theta_t} = (1+\tau)^{\frac{1}{\beta}}$.

**Proposition 2** (Flexible Prices). Suppose prices are flexible. Then in general, optimal capital controls are non zero. Optimal capital controls are equal to zero for permanent shocks to productivity $A_t = A'$ for all $t \geq 0$ where $A' \neq A$, for permanent export demand shocks $\Lambda_{H,t} = \Lambda_H$.
for all \( t \geq 0 \) where \( \Lambda_{H} \neq 1 \), for permanent foreign consumption shocks \( C^{*}_{t} = C^{*}' \) for all \( t \geq 0 \) where \( C^{*}' \neq C^{*} \), and for wealth shocks \( W_{0} \neq 0 \). In the Cole-Obstfeld case \( \sigma = \eta = \gamma = 1 \), optimal capital controls are equal to zero in response to arbitrary productivity shock \( \{ A_{t} \} \) or foreign consumption shocks \( \{ C^{*}_{t} \} \), and optimal capital controls \( \tau_{t} \) have the same sign as \( \Lambda_{t+1} - \Lambda_{t} \) in response to export demand shocks.

**Proof.** The first part of the proposition follows immediately from the planning problem, provided the program is sufficiently convex, as we assume. We therefore focus on the Cole-Obstfeld case. Then the first order condition for the planning problem is

\[
\phi + \alpha - \frac{(1-\alpha)^{2} \phi}{1 + \phi} \frac{\Theta_{t}}{\alpha \Lambda_{t} + (1-\alpha) \Theta_{t}} + \frac{(1-\alpha) \alpha \Lambda_{t}}{1 + \phi} \frac{1}{\Theta_{t}} - \Gamma \Theta_{t} = 0
\]

where \( \Gamma \) is chosen such that

\[
\alpha \sum_{t=0}^{\infty} \beta^{t} (\Theta_{t} - \Lambda_{t}) = NFA_{0}.
\]

For a given value of \( \Gamma \), then \( \Theta_{t} \) is independent of \( A_{t} \) and increasing in \( \Lambda_{t} \). Since \( \frac{\Theta_{t+1}}{\Theta_{t}} = (1 + \tau_{t})^{\frac{1}{\sigma}} \) the result follows.

The fact that capital controls are in general useful even though prices are flexible might seem surprising given the fact that we are considering a small open economy, with no ability to affect the world interest rate. This can be understood by noting that capital controls, by allowing a country to reallocate demand intertemporally, allow this country to manipulate its terms of trade, raising them in some periods and lowering them in others. This margin is in general useful, unless of course the shocks are permanent, in which case no gain can be reached by engaging in this kind of intertemporal terms of trade manipulation. Moreover, in the Cole-Obstfeld case, it is optimal not to use capital controls in response to any path of productivity or foreign consumption shocks or to wealth shocks.

**3.2 Rigid Prices**

After having considered the case of perfectly flexible prices, we now consider the opposite extreme case where prices are entirely rigid (fixed for all \( t \geq 0 \)). This implies that \( S_{t} = Q_{t} = 1 \) for all \( t \geq 0 \). The planning problem is now
$$\max \sum_{t=0}^{\infty} \beta^t \left[ \Theta_t^{1-\sigma} C_t^{1-\sigma} - (a \Lambda_t + \Theta_t(1 - \alpha))^1_{1^{1+\phi}} A_t^{-1(1+\phi)} C_t^{1+\phi} \right]$$

subject to,

$$0 = \sum_{t=0}^{\infty} \beta^t C_t^{1-\sigma} \alpha [\Lambda_t - \Theta_t].$$

**Proposition 3 (Rigid Prices).** Suppose that prices are entirely rigid. Optimal capital controls \( \tau_t \) have the same sign as \( A_{t+1} - A_t \) in response to productivity shocks. Optimal capital controls \( \tau_t \) have the opposite sign as \( \Lambda_{t+1} - \Lambda_t \) in response to export demand shocks. Optimal capital controls \( \tau_t \) have the opposite sign as \( C^*_t - C^*_t \) in response to foreign consumption shocks. Optimal capital controls are zero in response to initial wealth shocks.

**Proof.** The problem is convex: it features a concave objective and linear constraint in \( \Theta_t \). Putting a multiplier \( \Gamma > 0 \) on the left-hand side of the budget constraint, the necessary and sufficient first-order condition is

$$\Theta_t^{1-\sigma} - (1 - \alpha) (a \Lambda_t + \Theta_t(1 - \alpha))^\phi A_t^{-1(1+\phi)} C_t^{1+\phi} - \Gamma \alpha = 0.$$

The results in the proposition follow. \( \square \)

It is interesting to contrast the use of capital controls when prices are flexible with their use when prices are entirely rigid. There are both similarities and differences. In both cases, capital controls are not used in response to permanent productivity, export demand, or initial wealth shocks. But the use of capital controls in response to transitory shocks is quite different. For example, at least in the Cole-Obstfeld case, in response to productivity shocks or foreign consumption shocks, capital controls are zero when prices are flexible, but have the opposite sign as \( C^*_{t+1} - C^*_t \) or the same sign as \( A_{t+1} - A_t \) when prices are entirely rigid. In response to export demand shocks, optimal capital controls \( \tau_t \) have the same sign as \( \Lambda_{t+1} - \Lambda_t \) when prices are flexible, but the opposite sign when prices are entirely rigid.

A useful example is the Cole-Obstfeld case when productivity is increasing over time. In this case, optimal capital controls are zero when prices are flexible while they take the form of a tax on inflows / subsidy on outflows when prices are rigid. With flexible prices, the country’s price index decreases over time. This expected deflation raises the real interest rate and increases the growth rate of consumption. With rigid prices, the real interest rate is fixed and hence the growth rate of consumption is too low compared to the flexible price allocation. By taxing inflows / subsidizing outflows the country can increase its nominal interest rate and reduce the growth rate of consumption. In the process,
the growth rate of output is also decreased, but less so, so that the county’s trade balance moves towards surplus. This would not happen with flexible prices where trade would remain balanced. This underscores that capital controls are a second best tool. They allow the country to regain some monetary autonomy and therefore some control over the intertemporal allocation of spending. However, this reallocation is costly since it introduces a wedge between the intertemporal prices for home and foreign households. Moreover, capital controls cannot affect the division of spending between home and foreign goods when prices are completely rigid.

4 One Period in Advance Price Stickiness

We use dynamic programming to break down the planning problem into two, at \( t = 0 \) and after, with net foreign assets as the only endogenous state variable.

For \( t \geq 1 \), the planning problem is

\[
V(NFA_1) = \max_{\{c_t, \theta_t, n_t, y_t, s_t, q_t\}_{t=1}^\infty} \sum_{t=1}^\infty \beta^{t-1} \left[ \frac{c_t^{1-\sigma}}{1-\sigma} - \frac{n_t^{1+\phi}}{1+\phi} \right]
\]

subject to

\[
c_t = \theta_t c_t^* Q_t^\gamma
\]

\[
y_t = c_t^* \left[ (1-\alpha) Q_t^{1-\eta} s_t \theta_t + \alpha \Lambda H_t s_t^\gamma \right]
\]

\[
q_t = \left[ (1-\alpha) (s_t)^{\eta-1} + \alpha \right]^{1/\eta-1}
\]

\[
n_t = \frac{y_t}{A_t^{\gamma}}
\]

\[
c_t^{-\sigma} s_t^{-1} q_t = M \frac{1+\tau^L}{A_t} n_t^\phi
\]

for all \( t \geq 1 \) and

\[
NFA_1 = -\sum_{t=0}^\infty \beta^t c_t^{1-\sigma} \left( s_t^{-1} y_t - q_t^{-1} c_t \right)
\]

The \( t = 0 \) planning problem is

\[
\max_{c_0, \theta_0, n_0, y_0, NFA_1} \left( \frac{c_0^{1-\sigma}}{1-\sigma} - \frac{n_0^{1+\phi}}{1+\phi} + \beta V(NFA_1) \right)
\]
subject to

\[ C_0 = \Theta_0 C^*_0, \]
\[ Y_0 = C_0 \left[ (1 - \alpha) + \alpha \Lambda_0 \Theta_0^{-1} \right], \]
\[ N_0 = \frac{Y_0}{A_0}, \]
\[ NFA_0 = -C^*_0 \sigma (Y_0 - C_0) + \beta NFA_1, \]

where we have used the fact that \( S_0 \) and \( Q_0 \) are given with \( S_0 = Q_0 = 1 \).

With one-period in advance sticky prices, we are able to provide tight results for temporary productivity and export demand shocks, as well as for permanent productivity, export demand, and wealth shocks in the Cole-Obstfeld case.

**Proposition 4 (Transitory Shocks with Sticky Prices).** Suppose that prices are sticky one period in advance. Then in response to a transitory productivity shock \( A_0 \neq A \) and \( A_1 = A \) for all \( t \geq 1 \), optimal capital controls \( \tau_t \) are zero for \( t \geq 1 \), and optimal capital controls \( \tau_0 \) are a decreasing function of \( A_0 \) with \( \tau_0 = 0 \) when \( A = A_0 \). In response to a transitory export demand shock \( \Lambda_{H,0} \neq 1 \) and \( \Lambda_{H,t} = 1 \) for all \( t \geq 1 \), in the limit of small time intervals \( (\beta \to 1) \), optimal capital controls \( \tau_t \) are zero for \( t \geq 1 \), and optimal capital controls \( \tau_0 \) are an increasing function of \( \Lambda_{H,0} \) with \( \tau_0 = 0 \) when \( \Lambda_{H,0} = 1 \). In response to a transitory foreign consumption shock \( C^*_0 \neq C^* \) and \( C^*_t = C^* \) for all \( t \geq 1 \), when \( \sigma = 1 \), optimal capital controls \( \tau_t \) are zero for \( t \geq 1 \), and optimal capital controls \( \tau_0 \) have the same sign as \( C^*_0 - 1 \).

**Proof.** We start with transitory productivity shocks. The optimal allocation is constant for \( t \geq 1 \). Let \( \Theta \) be the corresponding value of \( \Theta \), which is related to \( \Theta_0 \) by \( (\Theta - 1) = \frac{1 - \beta}{\beta} (1 - \Theta_0) \). This immediately implies that \( \tau_0 = 0 \) for \( t \geq 1 \). Then we need to solve the \( t = 0 \) problem \( \max_{\Theta_0, A_0} U(\Theta_0, A_0) \) where

\[
U(\Theta_0, A_0) = C^* 1 - \sigma \Theta_0^{1-\sigma} \frac{1-\sigma}{A_0^{1+\phi}} - \frac{C^* 1+\phi [(1-\alpha)\Theta_0 + \alpha]^{1+\phi}}{1+\phi} + \beta V \left( \frac{1}{\beta} C^* 1-\sigma \alpha (1 - \Theta_0) \right).
\]

It is easily verified that \( U_{A_0,\Theta_0} > 0 \). This implies that \( \Theta_0 \) is an increasing function of \( A_0 \). This in turn implies that \( \Theta \) is a decreasing function of \( A_0 \). The first result in the proposition follows since capital controls \( \tau_0 \) are given by \( \frac{\Theta}{\Theta_0} = (1 + \tau_0)^{\frac{1}{\beta}} \).

Now consider temporary export demand shocks. The optimal allocation is constant allocation for \( t \geq 1 \), so we drop the \( t \) subscripts. This immediately implies that \( \tau_t = 0 \) for \( t \geq 1 \). We are interested in the limit of small time intervals \( (\beta \to 1) \). We have \( \frac{1 - \beta}{\beta} \alpha (\Lambda_0 - \Theta_0) = \alpha \left( Q^{1-\eta} \Theta - S^{\gamma-1} \Lambda \right) \). Using the other constraints, we can write the
right hand side as a function $J(\Theta)$ with $J(1) = 0$. We find that to a first order in $1 - \beta$ we can write

$$\Theta = 1 + \frac{1}{J'(1)}(1 - \beta)\alpha (\Lambda_0 - \Theta_0) + O(1 - \beta)^2.$$  

We can then write welfare for $t \geq 1$ as a function $H(\Theta)$. We find that up to second order terms in $1 - \beta$, we have to solve $\max_{\Theta_0} U(\Theta_0, \Lambda_0)$ where

$$U(\Theta_0, \Lambda_0) = C^{1-\sigma} \frac{\Theta_0^{1-\sigma}}{1 - \sigma} - \frac{C^{1+\phi}}{A^{1+\phi}} \frac{[(1 - \alpha)\Theta_0 + \alpha\Lambda_0]^{1+\phi}}{1 + \phi} + \frac{H'(1)}{J'(1)} \alpha (\Lambda_0 - \Theta_0).$$  

It is easily verified that $U_{\Lambda_0, \Theta_0} < 0$. This implies that $\Theta_0$ is a decreasing function of $\Lambda_{H,0}$. Moreover, up to first order terms in $1 - \beta$, $\Theta$ is constant. The second result in the proposition follows since capital controls $\tau_0$ are given by $\frac{\Theta_0}{\Theta_0} = (1 + \tau_0)^{\frac{\phi}{1}}$.

Finally consider foreign consumption shocks. The optimal allocation is constant for $t \geq 1$. Let $\Theta$ be the corresponding value of $\Theta$, which is related to $\Theta_0$ by $C^{1-\sigma} (\Theta - 1) = \frac{1 - \beta}{\beta} C_0^{1-\sigma} (1 - \Theta_0)$. This immediately implies that $\tau_i = 0$ for $t \geq 1$. We have to solve $\max_{\Theta_0} U(\Theta_0, \Lambda_0)$ where

$$U(\Theta_0, C_0^*) = C^{1-\sigma} \frac{\Theta_0^{1-\sigma}}{1 - \sigma} - \frac{C^{1+\phi}}{A^{1+\phi}} \frac{[(1 - \alpha)\Theta_0 + \alpha\Lambda_0]^{1+\phi}}{1 + \phi} + \beta V \left( \frac{1}{\beta} C_0^{1-\sigma} \alpha (1 - \Theta_0) \right).$$  

We find

$$U_{C_0^*, \Theta_0} (\Theta_0, C_0^*) = (1 - \sigma) C_0^{1-\sigma} \Theta_0^{1-\sigma} - (1 - \alpha)(1 + \phi) \frac{C^{1+\phi}}{A^{1+\phi}} [(1 - \alpha)\Theta_0 + \alpha]^{1+\phi}$$

$$- (1 - \sigma) C_0^{1+\phi} \alpha V' \left( \frac{1}{\beta} C_0^{1-\sigma} \alpha (1 - \Theta_0) \right)$$

$$- (1 - \sigma) C_0^{1+\phi} C_0^{1-\sigma} (1 - \Theta_0) \frac{1}{\beta} \alpha^2 V'' \left( \frac{1}{\beta} C_0^{1-\sigma} \alpha (1 - \Theta_0) \right).$$  

Clearly, for $\sigma = 1$, we have $U_{C_0^*, \Theta_0} (\Theta_0, C_0^*) < 0$. The third result follows. \qed

To gain some intuition for these results, consider the case of a temporary negative productivity shock in the Cole-Obstfeld case. Similar intuitions can be derived for general preferences, for export demand shocks, foreign consumption shocks and wealth shocks but we omit them in the interest of space. With flexible prices, the allocation would feature constant labor, a temporary decrease in output, consumption, and exports (but with constant export revenues). At $t = 0$, there would be a temporary improvement in the terms of trade brought about by an increase in the prices of home goods, and a constant wage. The nominal interest rate would be unchanged, but there would be a temporary
high real interest rate brought about by expected deflation as the prices of home goods revert to their original values.

Now consider what happens with one period in advance sticky prices. If the exchange rate were flexible, we would achieve the same real allocation by temporarily raising the nominal interest rate, letting the exchange rate appreciate at \( t = 0 \) and depreciate back at \( t = 1 \). This can be seen as a version of the classical argument in favor of flexible exchange rate famously put forth by Milton Friedman.

If the exchange rate is not flexible, then this solution cannot be attained because the terms of trade are fixed at \( t = 0 \). Without capital controls, at \( t = 0 \), labor temporarily increases, consumption and output are constant. Compared with the flexible price allocation, output, labor and consumption are higher. Similarly, at \( t = 0 \), inflation and the real interest rate are unchanged and are respectively higher and lower than at the flexible price allocation. It is therefore intuitively desirable to impose positive capital controls and increase the nominal interest rate to decrease consumption at \( t = 0 \). At \( t = 0 \), output then also decreases, but by less than consumption. Therefore the country runs positive net exports at \( t = 0 \), which implies higher consumption and lower output for \( t \geq 1 \).

This analysis underscores that capital controls are a second best instrument. They allow the country to regain some monetary autonomy and therefore some control over the intertemporal allocation of spending. However, this reallocation is costly since it introduces a wedge between the intertemporal prices for home and foreign households. Moreover, capital controls cannot affect the division of spending between home and foreign goods in the short run when prices are fixed.

Our next result deals with permanent shocks. The proof is contained in the appendix.

**Proposition 5 (Permanent Shocks with Sticky Prices).** Suppose that prices are sticky one period in advance. Consider the Cole-Obstfeld case \( \sigma = \eta = \gamma = 1 \). Then in response to any shock to the path \( \{A_t\} \) of productivity, optimal capital controls \( \tau_t \) are zero for \( t \geq 1 \), and optimal capital controls \( \tau_0 \) are a decreasing function of \( A_0 \) with \( \tau_0 = 0 \) when \( A_0 = A \). In response to a small enough permanent export demand shocks \( \Lambda_{H,t} = \Lambda_H \) for all \( t \geq 1 \) where \( \Lambda_H \neq 1 \), optimal capital controls \( \tau_t \) are zero for \( t \geq 1 \), and optimal capital controls \( \tau_0 \) are an increasing function of \( \Lambda_H \) with \( \tau_0 = 0 \) when \( \Lambda_H = 1 \). In response to any shock to the path \( \{C^{*}_{t}\} \) of foreign consumption, optimal capital controls \( \tau_t \) are zero for \( t \geq 1 \), and optimal capital controls \( \tau_0 \) are an increasing function of \( C^{*}_0 \) with \( \tau_0 = 0 \) when \( C^{*}_0 = C^* \). In response to a small enough wealth shock \( W_0 \neq 0 \), optimal capital controls \( \tau_t \) are zero for \( t \geq 1 \), and optimal capital controls \( \tau_0 \) are an increasing function of \( NFA_0 \) with \( \tau_0 = 0 \) when \( NFA_0 = 0 \). The last result is true more generally for any positive shock \( NFA_0 > 0 \).

It is interesting to contrast the use of capital controls when prices are flexible with
their use when prices are sticky. First, we know that in response to permanent shocks to productivity, export demand, foreign consumption, or wealth, capital controls are not used when prices are flexible. By contrast, capital controls are used to deal with these shocks when prices are sticky. Second, at least in the Cole-Obstfeld case, in response to transitory shocks to productivity and export demand, capital controls are used in the opposite direction when prices are sticky than when they are flexible. Indeed, following a negative productivity shock ($A_0 < A$) or a positive foreign consumption shock ($C^*_0 > C^*$), capital controls $\tau_0$ are zero with flexible prices but positive with sticky prices, and following a transitory positive export demand shock ($\Lambda_{H,0} > 1$), capital controls $\tau_0$ are negative with flexible prices but positive with sticky prices.

5 Calvo Pricing

In this section, we assume that the exchange rate is fixed and focus on the case of Calvo price setting. As most of the literature on Calvo pricing, we find it convenient to work with a linearized model. More precisely, we log-linearize the model around the symmetric deterministic steady state. At $t = 0$, the economy is hit with a one time unanticipated shock. We describe the flexible price allocation with no intervention (the allocation that would prevail if prices were flexible and capital controls were not used). We call this allocation the natural allocation. We then summarize the behavior of the sticky price economy with capital controls in log-deviations (gaps) from the natural allocation. For both the natural and the sticky price allocation with capital controls, the behavior of the rest of the world is taken as given.

We find it convenient to work in continuous time. In continuous time, with sticky prices, no price index can jump. This greatly facilitates the derivation of the initial conditions for the economy. We denote the instantaneous discount rate by $\rho$, and the instantaneous arrival rate for price changes by $\rho_\delta$.

We use lower cases variables to denote gaps from the symmetric deterministic steady state. We denote the natural allocation with bars, and the gaps from the natural allocation with hats.

5.1 Summarizing the Economy and the Experiment

The natural allocation. Let $\nu = -\log(1 + \tau^L)$, $\mu = \log M$, $\omega = \sigma \gamma + (1 - \alpha)(\sigma \eta - 1)$, and $\hat{\sigma} = \frac{\sigma}{1 - \alpha + \alpha \omega}$.

The natural level of output is given by
\[
\tilde{y}_t = \frac{\sigma^{-1}(1 + \phi)}{1 + \phi\sigma^{-1}} a_t - \frac{\alpha(\omega - 1)}{1 + \phi\sigma^{-1}} c_t^* - \frac{\alpha \omega}{1 + \phi\sigma^{-1}} \tilde{\theta},
\]
the natural interest rate is given by
\[
\bar{r}_t - \rho = \frac{1 + \phi}{1 + \phi\sigma^{-1}} \hat{a}_t + \frac{\alpha(\omega - 1)\phi^*}{1 + \phi\sigma^{-1}} c_t^*,
\]
and the natural terms of trade are given by
\[
\bar{s}_t = \frac{1 + \phi}{1 + \phi\sigma^{-1}} a_t - \frac{\sigma + \phi}{1 + \phi\sigma^{-1}} c_t^* - \frac{\sigma + \phi(1 - \alpha)}{1 + \phi\sigma^{-1}} \tilde{\theta},
\]
where \(\tilde{\theta}\) is such that the country budget constraint holds:
\[
\tilde{\theta} = \frac{\left(\frac{\omega}{\sigma} - 1\right) \sigma}{1 + \left(\frac{\omega}{\sigma} - 1\right) \sigma(1 - \alpha)} \int_0^\infty e^{-\rho t} (\hat{y}_t - y_t^*) dt.
\]
We can also compute the natural levels of employment and consumption from the equations \(y_t = a_t + n_t\) and \(c_t = \theta_t + c_t^* + \frac{1 - \alpha}{\sigma} s_t\).

Note that the natural interest rate \(\bar{r}_t\), the natural terms of trade \(\bar{s}_t\), the foreign nominal interest rate \(i_t^*\) and foreign inflation \(\pi_t^*\) must be related by
\[
i_t^* - \bar{r}_t = -\hat{s}_t + \pi_t^*.
\]

**Summarizing the system in gaps.** The economy with sticky prices and capital controls can be conveniently summarized in gaps from the natural allocation.

The demand block is summarized by three equations, the Euler equation
\[
\dot{\tilde{y}}_t = \hat{\sigma}^{-1} [i_t - \pi_{H,t} - \bar{r}_t] - \alpha \omega \hat{\theta}_t,
\]
the UIP equation
\[
i_t = i_t^* + \sigma \hat{\theta}_t,
\]
and the country budget constraint
\[
\int_0^\infty e^{-\rho t} \hat{\theta}_t dt = \frac{\left(\frac{\omega}{\sigma} - 1\right) \sigma}{1 + \left(\frac{\omega}{\sigma} - 1\right) \sigma(1 - \alpha)} \int_0^\infty e^{-\rho t} \tilde{y}_t dt.
\]
The supply block consists of one equation, the New-Keynesian Philips Curve

$$\pi_{H,t} = \rho \pi_{H,t+1} - \kappa \hat{y}_t - \lambda \sigma \omega \theta_t$$

where $\lambda = \rho_\delta (\rho + \rho_\delta)$ and $\kappa = \lambda (\phi + \sigma)$.

Finally, we have the initial condition

$$\hat{y}_0 = (1 - \alpha) \hat{\theta}_0 - \sigma^{-1} \bar{s}_0$$

which formalizes the requirement that the terms of trade are predetermined at $t = 0$ i.e. that $\bar{s}_0 = -\bar{s}_0$.

Note that we can back out the terms of trade gap $\bar{s}_t$ from the equation

$$\hat{y}_t = (1 - \alpha) \hat{\theta}_t - \sigma^{-1} \bar{s}_t.$$ 

Similarly, we can back out the employment gap $\hat{n}_t$ and the consumption gap $\hat{c}_t$ from the equations

$$\hat{y}_t = \hat{n}_t,$$

and

$$\hat{y}_t = -\alpha \hat{\theta}_t + \hat{c}_t + \omega \sigma \bar{s}_t.$$

Finally, capital controls can be inferred from the UIP equation

$$\tau_t = i_t - i_t^*.$$ 

**An experiment.** Suppose we start in steady state where $i_t = i_t^* = \bar{r}_t = \bar{r}$, $\pi_{H,t} = \pi_t^*$ = 0, $\hat{y}_t = 0$. Now suppose there is an unexpected shock that upsets the whole sequences \{\bar{r}_t, \bar{s}_t, i_t^*, \pi_t^*\} but still satisfying the restriction $i_t^* - \bar{r}_t = -\bar{s}_t + \pi_t^*$. For simplicity, we assume that $\bar{r}_t - i_t^* = r_{shock} e^{-\rho r t}$.

This parametrization is flexible enough to accommodate pure terms of trade shocks (permanent shocks to $\bar{s}_t$ with no shock to $\bar{r}_t$), pure natural interest rate shock (shocks to $\bar{r}_t$ with no shocks to $\bar{s}_0$), and any combination of the two such as for example mean-reverting productivity shocks.

Note also that $i_t, \bar{r}_t$ and $i_t^*$ enter in the equations describing the system purely through $i_t - i_t^*$ and $\bar{r}_t - i_t^*$. As a result, positive world interest rate shocks to $i_t^*$ have the same effects as positive domestic natural interest rate shocks to $\bar{r}_t$. This is true both for the
allocation without capital controls, and for the optimal allocation with capital controls (the only difference is that the path of domestic interest rates is shifted by the shock to \(i^*_t\)).

We now explain how to set up the planning problem for the home economy. We focus on a special case, the Cole-Obstfeld case, where a second order approximation to the welfare function is available.

### 5.2 Optimal Capital Controls in the Cole-Obstfeld Case

The Cole-Obstfeld case is attractive for two reasons. First, with flexible prices, it is optimal not to use capital controls. Second, it is relatively easy to derive a second order approximation of the welfare function around the symmetric deterministic steady state.

**Loss function.** When \(\sigma = \gamma = \eta = 1\), we can derive a simple second order approximation of the welfare function. The corresponding loss function (in consumption equivalent units) can be written as

\[
(1 - \alpha)(1 + \phi) \int_0^\infty e^{-\rho t} \left[ \frac{1}{2} \alpha_\pi \pi^2_{H,t} + \frac{1}{2} y^2_t + \alpha_\theta \frac{1}{2} \theta^2_t \right] dt
\]

where \(\alpha_\pi = \frac{\epsilon}{\lambda(1+\phi)}\) and \(\alpha_\theta = \frac{\alpha}{1+\phi} \left( \frac{2-\alpha}{1-\alpha} + 1 - \alpha \right) \theta^2_t\).

The first two terms in the loss function are familiar in New-Keynesian models. The inflation term \(\frac{1}{2} \alpha_\pi \pi^2_{H,t}\) captures the cost of inflation: with staggered price setting a la Calvo, inflation comes with spurious dispersion in markups across firms—a form of misallocation. The output gap term \(\frac{1}{2} y^2_t\) represents the average markup distortion—because of sticky prices, the average markup can differ from the markup desired by firms.

The third term in the loss function \(\alpha_\theta \frac{1}{2} \theta^2_t\) is new and captures the cost of capital controls. In the Cole-Obstfeld case, the country budget constraint is simply \(\int e^{-\rho t} \theta_t dt = 0\). Capital controls can be used to reallocate demand across time by running nonzero trade balances \(-\alpha \theta_t\). However, this reallocation is costly since it introduces a wedge between the intertemporal prices for home and foreign households.

Note that \(\alpha_\pi\) is independent of \(\alpha\) but that \(\alpha_\theta\) goes to zero when \(\alpha\) goes to zero. Hence in the closed economy limit (\(\alpha \to 0\)), the cost of capital controls vanishes. The reason is that for a given path of \(\theta_t\), the associated trade balances \(-\alpha \theta_t\) vanish as \(\alpha\) goes to zero, and so do the distortions associated with the wedge between home and foreign intertemporal prices.
**Planning problem.** The planning problem is

\[
\min_{\{\pi_H, t, ˆy_t, ˆθ_t\}} \int_0^\infty e^{-\rho t} \left[ \frac{1}{2} \alpha \pi_H^2 + \frac{1}{2} \dot{y}_t^2 + \alpha \theta \frac{1}{2} \dot{θ}_t^2 \right] dt
\]

s.t.

\[
\dot{\pi}_H = \rho \pi_H - \dot{k} - \lambda \dot{θ}_t,
\]

\[
\dot{y}_t = (1 - \alpha)(i_t - i_t^*) - \pi_H + i_t^* - \bar{r}_t,
\]

\[
\dot{θ}_t = i_t - i_t^*,
\]

\[
\int_0^\infty e^{-\rho t} \dot{θ}_t dt = 0,
\]

\[
\dot{y}_0 = (1 - \alpha)\dot{θ}_0 - \bar{s}_0.
\]

This planning problem makes clear that in the Cole-Obstfeld case, welfare is necessarily lower with sticky prices and optimal capital controls than with flexible prices and no capital controls. Therefore the natural allocation represents an upper bound for welfare. Moreover, by taking the limit of flexible prices, i.e. by increasing \(\lambda\) and simultaneously varying \(\pi_H = \frac{\epsilon}{\lambda(1 + \phi)}\) and \(\dot{k} = \lambda(1 + \phi)\), it is easily confirmed that when prices are flexible, it is optimal not to use capital controls at all.

To get a better feel for the planning problem, it is useful to imagine how it would change if the exchange rate were flexible. The only differences are in the UIP equation and the initial condition, which become \(\dot{θ}_t = i_t - i_t^* - \dot{r}_t\) and \(\dot{y}_0 = (1 - \alpha)\dot{θ}_0 - \bar{s}_0 + \bar{e}_0\) where the initial level of the exchange rate \(\bar{e}_0\) and the rate of exchange rate depreciation \(\dot{e}_t\) are additional control variables. It is clear that the solution features \(\pi_H = \dot{y}_t = \dot{θ}_t = 0\) and \(i_t = i_t^* + \frac{\bar{r}_t - i_t^*}{1 - \alpha}\). The associated path for the exchange rate is determined by \(e_t = \bar{s}_t\) for all \(t\), which is equivalent to \(e_0 = \bar{s}_0\) and \(\dot{e}_t = \frac{\bar{r}_t - i_t^*}{1 - \alpha}\).

With flexible exchange rate, and for the shocks that we consider, the natural allocation can be attained. This observation about the stabilizing role of flexible exchange rates goes back to Milton Friedman. But with a fixed exchange rate, perfect macroeconomic stabilization cannot be achieved. A way to understand this is to go back to Mundell’s trilemma, which states that it is impossible to have at the same time free capital flows, independent monetary policy, and a fixed exchange rate.

By introducing capital controls, the home country is able to regain some monetary policy autonomy, even with a fixed exchange rate. But this comes at a cost since capital controls distort the intertemporal allocation of consumption, for a given path of output, and also impacts inflation dynamics. Optimal policy requires optimally trading off the
gain in monetary autonomy against the the resulting distortions. In this section, we make this intuition formal and characterize how to optimally use capital controls to accommodate macroeconomic shocks.

**Formulation as an optimal control problem.** We can incorporate a multiplier $\Gamma$ on the intertemporal budget constraint. We are left with the following optimal control problem

$$\min_{\{\pi_{H,t}, \hat{y}_t, \hat{\theta}_t\}} \int_0^\infty e^{-\rho t} \left[ \frac{1}{2} \alpha \pi_{H,t}^2 + \frac{1}{2} \hat{y}_t^2 + \alpha \theta \frac{1}{2} \hat{\theta}_t^2 + \Gamma \hat{\theta}_t \right] dt$$

subject to

$$\dot{\pi}_{H,t} = \rho \pi_{H,t} - \hat{\kappa} \hat{y}_t - \lambda \alpha \hat{\theta}_t,$$

$$\dot{\hat{y}}_t = (1 - \alpha) (i_t - i_t^*) - \pi_{H,t} + i_t^* - \bar{r}_t,$$

$$\dot{\hat{\theta}}_t = i_t - i_t^*,$$

$$\hat{y}_0 = (1 - \alpha) \hat{\theta}_0 - \bar{s}_0.$$

Let $\mu_{\pi,t}, \mu_{y,t}$ and $\mu_{\theta,t}$ be the costates. We find the following first-order conditions

$$\dot{\mu}_{\pi,t} = -\alpha \pi_{H,t} + \mu_{y,t},$$

$$\dot{\mu}_{y,t} = -\hat{y}_t + \mu_{\pi,t} \hat{\kappa} + \rho \mu_{y,t},$$

$$\dot{\mu}_{\theta,t} = -\alpha \hat{\theta}_t - \Gamma + \mu_{\pi,t} \lambda \alpha + \rho \mu_{\theta,t},$$

$$(1 - \alpha) \mu_{y,t} + \mu_{\theta,t} = 0,$$

$$\mu_{\pi,0} = 0,$$

$$\mu_{\theta,0} = -(1 - \alpha) \mu_{y,0},$$

together with the transversality conditions $\lim_{t \to \infty} e^{-\rho t} \mu_{y,t} \hat{y}_t = 0$, $\lim_{t \to \infty} e^{-\rho t} \mu_{\theta,t} \hat{\theta}_t = 0$, and $\lim_{t \to \infty} e^{-\rho t} \mu_{\pi,t} \hat{\pi}_t = 0$.

We can use the fourth and third of these first-order conditions to find

$$\mu_{\pi,t} \left( \hat{\kappa} + \lambda \frac{\alpha}{1 - \alpha} \right) = \hat{y}_t + \frac{\alpha \theta}{1 - \alpha} \hat{\theta}_t + \frac{\Gamma}{1 - \alpha}.$$

We can then summarize the optimal allocation as a system of differential equations in $\hat{y}_t, \pi_{H,t}, \hat{\theta}_t$ and $\mu_{y,t}$:
\[
\hat{y}_t = -\frac{\alpha_0}{(1-\alpha)^2} + \alpha_\pi \left( \hat{\kappa} + \lambda \frac{\alpha}{1-\alpha} \right) \pi_{H,t} + \frac{\kappa + \lambda \frac{\alpha}{1-\alpha}}{1 + \frac{\alpha_0}{(1-\alpha)^2}} \mu_{y,t} - \frac{\alpha_0}{(1-\alpha)^2} \left( \hat{r}_t - i^*_t \right) \\
\hat{\pi}_{H,t} = \rho \pi_{H,t} - \lambda \hat{\theta}_t, \\
\hat{\theta}_t = \frac{1}{1-\alpha} \left( 1 - \alpha_\pi \left( \hat{\kappa} + \lambda \frac{\alpha}{1-\alpha} \right) \right) \pi_{H,t} + \frac{1}{1-\alpha} \left( \kappa + \lambda \frac{\alpha}{1-\alpha} \right) \mu_{y,t} + \frac{r_t - i^*_t}{1-\alpha}, \\
\hat{\mu}_{y,t} = -\frac{\lambda \frac{\alpha}{1-\alpha}}{\kappa + \lambda \frac{\alpha}{1-\alpha}} \hat{y}_t + \rho \mu_{y,t} + \frac{\kappa}{\hat{\kappa} + \lambda \frac{\alpha}{1-\alpha}} + \frac{\hat{\kappa} \frac{\alpha}{1-\alpha}}{\hat{\kappa} + \lambda \frac{\alpha}{1-\alpha}} \hat{\theta}_t,
\]

with the initial condition
\[
\hat{y}_0 + \frac{\alpha_0}{1-\alpha} \hat{\theta}_0 = -\frac{\Gamma}{1-\alpha},
\]

where \(\hat{y}_0, \hat{\theta}_0, \pi_{H,0}, \mu_{y,0}\) and \(\Gamma\) must be chosen such that \(\int_0^\infty e^{-\rho t} \hat{\theta}_t dt = 0\) and the transversality conditions are verified.

**Rigid prices.** In the case where prices are completely rigid, the optimal allocation can be found in closed form.

**Proposition 6** (Rigid Prices, Calvo). *In the case where prices are completely rigid, the optimal allocation is given by*

\[
\hat{y}_t = -\frac{\mu^{\text{shock}}}{1-\alpha + \frac{\alpha_0}{1-\alpha}} \left[ (1-\alpha) \frac{1}{\rho + \rho_r} + \frac{\alpha_0}{1-\alpha} \left( 1 - e^{-\rho_r t} \right) \right] - \bar{s}_0, \\
i_t = i^*_t + \frac{1}{1-\alpha + \frac{\alpha_0}{1-\alpha}} (\bar{r}_t - i^*_t),
\]

*and capital controls are given by \(\tau_t = i_t - i^*_t\).*

When prices are rigid, capital controls are not used to accommodate a pure terms of trade shock (a permanent shock in \(\bar{s}_t\) with no shock to \(\bar{r}_t\)). By contrast, capital controls are used to smooth variations in output in response to a natural interest rate shock (\(\bar{r}_t\)), which requires \(i_t = i^*_t + \frac{1}{1-\alpha + \frac{\alpha_0}{1-\alpha}} (\bar{r}_t - i^*_t)\). Note that in this latter case, the interest must not be set equal to the natural interest rate \(\bar{r}_t\), nor must it be set to the world nominal interest rate \(i^*_t\). Note also that the smaller \(\alpha\), the more capital controls are used to accommodate natural interest rate shocks: \(i_t = i^*_t + \frac{1}{1-\alpha + \frac{\alpha_0}{1-\alpha}} (\bar{r}_t - i^*_t)\). When \(\alpha\) goes to zero (closed economy limit), the output gap is constant \(\hat{y}_t = -\rho^{\text{shock}} \frac{\rho + \rho_r}{\bar{s}_0} - \bar{s}_0\) with \(i_t = \bar{r}_t\).
Closed economy limit ($\alpha \to 0$). We can derive the optimal allocation in closed form in the closed-economy limit ($\alpha \to 0$). Note that $\alpha_\pi$ is independent of $\alpha$ and that $\alpha_\theta$ becomes zero. Let

$$\lambda_- = \frac{\rho - \sqrt{\rho^2 + 4\alpha_\pi^2\hat{\kappa}^2}}{2}$$

and

$$\Gamma = \left[ s_0 + \int_0^\infty e^{-\rho t}(\bar{r}_t - i_t^*) \right] \left[ 1 + \hat{k} + (1 - \alpha_\pi \hat{k}) \left( \frac{1}{\rho^2 \alpha_\pi} - \frac{1 - \alpha_\pi \hat{k}}{\alpha_\pi (\rho - \lambda_-)^2} \right) \right].$$

Proposition 7 (Closed economy limit). In the closed economy limit ($\alpha \to 0$), the optimal allocation is given by

$$\hat{y}_t = \Gamma \frac{1 - \alpha_\pi \hat{k}}{\lambda_- (\rho - \lambda_-)} (e^{\lambda_- t} - 1) - \Gamma,$$

$$\pi_{H,t} = \Gamma \frac{1 - \alpha_\pi \hat{k}}{\alpha_\pi (\rho - \lambda_-)} (e^{\lambda_- t} - 1) - \Gamma \frac{1}{\alpha_\pi} \left[ \frac{-\rho (1 - \alpha_\pi \hat{k}) + \rho - \lambda_-}{\rho (\rho - \lambda_-)} \right],$$

$$i_t = \bar{r}_t + (1 - \alpha_\pi \hat{k}) \pi_{H,t} - \frac{\hat{k}}{\rho} \Gamma = \bar{r}_t + \Gamma \frac{(1 - \alpha_\pi \hat{k})^2}{\alpha_\pi (\rho - \lambda_-)} e^{\lambda_- t} - \frac{1}{\rho \alpha_\pi} \Gamma,$$

and capital controls are given by $\tau_t = i_t - i_t^*$.  

Note that if $\Gamma \neq 0$, inflation does not return to zero in the long run (and hence $i_t - \bar{r}_t$ does not return to zero). By contrast, for $\alpha > 0$, inflation always returns to zero in the long run (and hence $i_t - \bar{r}_t$ also returns to zero). In other words, the double limits $\lim_{\alpha \to 0} \lim_{t \to \infty} \pi_{H,t} = 0$ and $\lim_{t \to \infty} \lim_{\alpha \to 0} \pi_{H,t} \neq 0$ are different.

Proposition 8. In the closed economy limit ($\alpha \to 0$), in the special case where $1 - \alpha_\pi \hat{k} = 0$, the solution features constant output gaps and inflation $\hat{y}_t = -\Gamma$, $\pi_{H,t} = -\frac{1}{\rho \alpha_\pi} \Gamma$ where $\Gamma = [s_0 + \int_0^\infty e^{-\rho t}(\bar{r}_t - i_t^*)] (1 + \hat{k})$. In addition the nominal interest rate is given by $i_t = \bar{r}_t - \frac{\hat{k} \Gamma}{\rho}$.

Note that by contrast with the case of rigid prices, capital controls are used to accommodate pure terms of trade shocks.

5.3 Numerical exploration

In this section we explore how capital controls can be used to accommodate pure terms of trade shocks, pure natural interest rate shocks, and mean-reverting productivity shocks. We focus on the Cole-Obstfeld case. We set $\phi = 3$, $\rho = 0.04$, $\delta = 1 - 0.754$, $\epsilon = 6$. We run two experiments, the first one with $\alpha = 0.4$ and the second one with $\alpha = 0.1$. 

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A **terms of trade shock.** Our first experiment is such that $\bar{s}_t = -0.05$ for all $t$ and $\bar{r}_t = 0$ for all $t$. This represents an improvement in the terms of trade that causes an appreciation in the real exchange rate. This can be traced back to a permanent shock to productivity $A_t$.

Figures 1 and 2 show the results two values of openness, $\alpha = 0.4$ and $\alpha = 0.1$, respectively. The green line is the outcome without interventions (zero capital controls). The blue line is the outcome with optimal capital controls.

To gain intuition, consider first the outcome without intervention. On the one hand, if prices were fully flexible there would be an immediate and permanent upward jump in the price of home goods $P_H$. On the other hand, with totally rigid prices the equilibrium features a permanent rise in output and consumption. In the intermediate case, with Calvo pricing, the equilibrium features a smoother transition towards a higher price for home goods. As shown in the figure, positive home inflation and positive output gaps are positive and asymptote to zero. Due to the Cole-Obstfeld parameterization, the transition does not affect trade balance. Output and consumption are higher, pushing for higher imports (foreign revenue is constant), but Home goods are also cheaper, encouraging the relative consumption of home goods over imports. These two effects cancel each other out.

Turning to the optimal intervention, of course, with flexible prices no intervention is required. With fully rigid prices, our result implies that the optimum does nothing and accepts the permanent change in output. Interestingly, in the intermediate case, with Calvo pricing, some intervention is optimal. In the short run, the optimal policy intervention lowers home consumption and output relative to the no-intervention equilibrium. The drop in consumption lowers imports, which creates a trade surplus. The rest of the dynamics are simply convergence. Note an tradeoff: the optimal policy lowers output and consumption. This makes inflation lower, but the adjustment in home prices is what we need. Note that when $\alpha$ is lower, the nominal interest rate $i_t$ is higher, so that capital controls are used more. The paths for the output gap and inflation become much smoother (in the closed economy limit, they would be completely smooth, although at a strictly positive level).

A **(mean-reverting) natural interest rate shock (or world interest rate shock).** Next we look at a shock to $\bar{r}_t$ which becomes negative (with $\bar{r}_0 = -0.05$) and then mean-reverts to zero with $\bar{s}_0 = 0$. The coefficient of mean reversion is set to a relatively high value of 1, so that the half-life of the shock is 0.7 years. This shock can be traced back to a temporary productivity shock where at $t = 0$, it becomes known that productivity will be decreasing.
in the future. This shock is isomorphic to a positive world interest rate shocks to \( i^*_t \), up to a shift (of \( i^*_t \)) in the path of the domestic interest rate \( i_t \).

Figures 3 and 4 show the results two values of openness, \( \alpha = 0.4 \) and \( \alpha = 0.1 \), respectively. The green line is the outcome without interventions (zero capital controls). The blue line is the outcome with optimal capital controls.

Consider first the outcome without intervention. If prices were flexible, there would be inflation in \( P_H \) with no change at impact. The inflation rate would be decreasing over time towards zero. This would reduce the real interest rate, more so in the short run. With sticky prices and a fixed nominal interest rate, this inflationary process is smoothed, so that inflation is at first lower, then higher and also converges to zero in the long run. The long-run increase in \( P_H \) is the same as with flexible prices. To understand the behavior of the output gap, it is useful to keep in mind the long-run Euler equation

\[
\hat{y}_t = \hat{y}_\infty - \int_t^\infty (i^*_s - \bar{r}_s - \pi_{H,s}) ds
\]

together with the fact that the long-run output gap \( \hat{y}_\infty \) is zero. Because the long-run increase \( \int_0^\infty \pi_{H,s} ds \) in \( P_H \) is the same as under flexible prices, the output gap is initially zero. Because inflation is initially lower and eventually higher than under flexible prices, the output gap is positive throughout. Once again, due to the Cole-Obstfeld parameterization, the transition does not affect trade balance.

We now turn to the optimal intervention. With fully rigid prices, capital controls would be such that the nominal interest rate \( i_t \) would equal

\[
i_t^* + \frac{1}{1-\alpha + \frac{\alpha \bar{r}}{1-\alpha}} (\bar{r}_t - i^*_t).
\]

The corresponding level of capital controls is displayed in red in the figures. This involves setting a subsidy on inflows / tax on outflows so as to reduce the nominal interest rate. The output gap would be positive throughout but initially zero. The country would initially run a trade surplus (such capital controls initially reduce consumption more than output) and eventually a trade deficit. With Calvo pricing, the solution has similar features. Additionally, there is inflation. But note that both the paths for the output gap and inflation are smoother with the optimal intervention than with no intervention. Note that when \( \alpha \) is lower, the nominal interest rate is much closer to

\[
i_t^* + \frac{1}{1-\alpha + \frac{\alpha \bar{r}}{1-\alpha}} (\bar{r}_t - i^*_t)
\]

which is turn closer to \( \bar{r}_t \), so that capital controls are used more. The paths for the output gap and inflation become much smoother (in the closed economy limit, they would be completely smooth, although at a strictly positive level).

**A mean-reverting productivity shock.** Finally we look at a temporary negative productivity shock such that initially \( \bar{s}_0 = -0.05 \). The coefficient of mean reversion is set to 0.2 so that the half-life of the shock is 3.5 years.

Figures 3 and 4 show the results two values of openness, \( \alpha = 0.4 \) and \( \alpha = 0.1 \), respectively. The green line is the outcome without interventions (zero capital controls). The
blue line is the outcome with optimal capital controls. This shocks is a combination of the first two shocks (with a positive sign on the first one and a negative sign on the second one). The figures can be understood accordingly.

**Welfare.** We can also compute the reduction in the loss function (in consumption equivalent units) brought about by the optimal use of capital controls. Since the numbers represent a single shock, it should be kept in mind that these gains should be scaled by the size and frequency of the shocks.

It is apparent from these numbers that the more closed the economy, the more effective capital controls.

### 5.4 Sticky wages

In this section we introduce sticky wages. We then look at the role of capital controls in a currency union. We can also look at the role of capital controls with flexible exchange rates.

**Introducing sticky wages in addition to sticky prices.** In this section, we incorporate sticky wages. We now assume that labor is an aggregate of different varieties of labor

\[
N_i = \left( \int_0^1 N_i(j) \frac{\bar{c}_{wp}}{\epsilon_w - 1} dj \right)^{\frac{\epsilon_w}{\epsilon_w - 1}}.
\]

We denote the corresponding wage index by

\[
W_i = \left( \int_0^1 W_i(j) \frac{1}{1 - \epsilon_w} dj \right)^{\frac{1}{1 - \epsilon_w}},
\]

and we let \(M_w \equiv \frac{\epsilon_w}{\epsilon_w - 1}\).

Wages are set a la Calvo, with parameter \(\delta_w\). The corresponding first-order condition
The planning problem. In order to write down the corresponding planning problem, we need to introduce a few notations. First we define
\[
\lambda_w = \frac{\rho_I}{1+\epsilon_w\\phi} (\rho_I + \rho_\delta w) + \epsilon_w\\phi
\]
where \(\rho_\delta w\) is the arrival rate of new wages.

The planning problem now features additional state variables: wage inflation \(\pi_t^{w}\) and the real wage gap \(\hat{\omega}_t\). It can be written as

\[
\min_{\{\pi_{H,t}, \pi_t^{w}, \hat{y}_t, \hat{\theta}_t, \hat{\omega}_t\}} \frac{1}{2} \int_0^\infty e^{-\rho t} \left[ \alpha_\pi \pi_t^{2} + \alpha \pi_w (\pi_t^{w})^2 + \hat{y}_t^2 + \alpha \theta_t^2 \right] dt
\]

\[
\pi_t^{w} = \rho \pi_t^{w} - \lambda w [(1 - \alpha) + \phi] \hat{y}_t - \lambda_w \alpha (2 - \alpha) \hat{\theta}_t + \lambda_w \hat{\omega}_t,
\]

\[
\pi_{H,t} = \rho \pi_{H,t} - \lambda w \hat{\omega}_t - \lambda \alpha \hat{y}_t + \lambda \alpha (1 - \alpha) \hat{\theta}_t,
\]

\[
\hat{y}_t = (1 - \alpha) i_t - (\pi_{H,t} + \bar{r}_t) + \alpha i_t^*,
\]

\[
\hat{\theta}_t = i_t - i_t^*,
\]

\[
\hat{\omega}_t = \pi_t^{w} - (1 - \alpha) (\pi_{H,t} - \alpha \pi_t^*) - \hat{\omega}_t,
\]

together with the initial conditions

\[
\hat{\omega}_0 = -\bar{\omega}_0,
\]

\[
-\bar{s}_0 = \hat{y}_0 - (1 - \alpha) \hat{\theta}_0,
\]

and the country budget constraint

\[
\int_0^\infty e^{-\rho t} \hat{\theta}_t = 0.
\]

By putting a multiplier \(\Gamma\) on the country budget constraint and incorporating it in the objective, we are left with an optimal control problem with state variables \(\pi_{H,t}, \pi_t^{w}, \hat{y}_t, \hat{\theta}_t, \hat{\omega}_t\), and control variable \(i_t\).
Flexible exchange rate. As Milton Friedman forcefully argued, flexible exchange rates can act as a substitute for flexible prices. When prices are sticky, but wages are not, a flexible exchange rate can be used to achieve the flexible price allocation. In this way, flexible exchange rates yield perfect macroeconomic stabilization. But this is no longer possible, in general, when wages are also sticky. Perfect macroeconomic stabilization is not possible, even with a flexible exchange rate. This raises the question we address here: are capital controls optimal when prices and wages are sticky but a flexible exchange rate is managed optimally?

In terms of the equilibrium conditions, two things change when the exchange rate is allowed to vary: first, the initial conditions and second, the relationship between $\dot{\hat{\theta}}_t$ and $i_t - i_t^*$ since we now have $\dot{\hat{\theta}}_t = i_t - i_t^* - \dot{\epsilon}_t$. The planning problem is

$$\min_{\{\pi_{H,t}, \tau_t^w, \hat{\gamma}_t, i_t, \hat{\omega}_t\}} \frac{1}{2} \int_0^\infty e^{-\rho t} [\alpha \pi_{H,t}^2 + \alpha \tau_t^w (\pi_t^w)^2 + \hat{y}_t^2 + \alpha \theta^2] dt$$

$$\pi_t^w = \rho \pi_t^w - \lambda_w [(1 - \alpha) + \phi] \hat{\gamma}_t - \lambda_w \alpha (2 - \alpha) \hat{\theta}_t + \lambda_w \hat{\omega}_t,$$

$$\pi_{H,t} = \rho \pi_{H,t} - \lambda \hat{\omega}_t - \lambda \alpha \hat{\gamma}_t + \lambda \alpha (1 - \alpha) \hat{\theta}_t,$$

$$\dot{\hat{\gamma}}_t = (1 - \alpha) i_t - (\pi_{H,t} + \bar{r}_t) + \alpha i_t^*,$$

$$\dot{\hat{\theta}}_t = i_t - i_t^* - \dot{\epsilon}_t,$$

$$\dot{\hat{\omega}}_t = \pi_t^w - (1 - \alpha) \pi_{H,t} - \alpha (\pi_t^* + \dot{\epsilon}_t) - \hat{\omega}_t,$$

together with the initial condition\(^4\)

$$\hat{\omega}_0 = -\bar{\omega}_0 - \alpha \bar{s}_0 - \alpha \hat{y}_0 + \alpha (1 - \alpha) \hat{\theta}_0,$$

and the country budget constraint

$$\int_0^\infty e^{-\rho t} \hat{\theta}_t = 0.$$

By putting a multiplier $\Gamma$ on the country budget constraint and incorporating it in the objective, we are left with an optimal control problem with state variables $\pi_{H,t}, \tau_t^w, \hat{\gamma}_t, \hat{\theta}_t$.

\(^4\)The initial condition comes from the two conditions

$$\dot{\hat{\omega}}_0 = -\bar{\omega}_0 - \alpha e_0,$$

$$e_0 - \bar{s}_0 = \hat{y}_0 - (1 - \alpha) \hat{\theta}_0.$$

where $e_0$ is the initial exchange rate with sticky prices and wages.
\( \omega_t \) and control variables \( i_t \) and \( \dot{e}_t \).

It seems that in general, there is a role for capital controls when prices and wages are sticky, even when the exchange rate is flexible. The role of capital controls is quite different than with a fixed exchange rate though. With a flexible exchange rate, each country already has the flexibility to set its own monetary policy. However, this per se is not enough to perfectly stabilize the economy. In this context, capital controls still emerge as a second best instrument.

However note that when prices are entirely rigid, this role for capital controls disappears entirely. The same is true in the closed economy limit.

**Proposition 9** (Sticky Prices, Wages, Flexible Exchange Rate). With sticky prices, sticky wages and a flexible exchange rate, optimal capital controls are generally nonzero. However, in the two limit cases of perfectly rigid prices and wages on the one hand, and closed economy limit \((\alpha \to 0)\) on the other hand, optimal capital controls are zero.

### 5.5 Government Spending

For comparison, we now introduce government expenditures, building more directly Galí and Monacelli (2008). Government expenditures \( G_t \) enter households’ utility as follows

\[
\sum_{t=0}^{\infty} \beta^t \left[ (1 - \chi) \frac{C_t^{1-\sigma}}{1-\sigma} + \chi \frac{G_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\phi}}{1+\phi} \right]
\]

where

\[
G_t = \left( \int_0^1 G_t(j) \frac{dj}{\tau} \right)
\]

where \( j \in [0, 1] \) denotes the good variety. Note that, following Galí and Monacelli (2008), this assumes that \( G_t \) is spent entirely on home goods.

To keep the exposition brief, we focus on the Cole-Obstfeld case, abstract from capital controls. We move directly to a continuous time formulation of the planning problem, set up as an optimal control:

\[
\min \int_0^\infty \left[ \frac{1}{2} \pi_{H,t}^2 + \frac{1}{2} \lambda_x (\dot{c}_t + (1 - \zeta) \dot{g}_t)^2 + \frac{1}{2} \lambda_y \dot{g}_t^2 \right] dt
\]

\[
\pi_{H,t} = \rho \pi_{H,t} - \bar{\kappa} [\dot{c}_t + (1 - \zeta) \dot{g}_t],
\]

\[
\dot{c}_t = (1 - \Gamma) [ -\pi_{H,t} - (\tilde{r}_t - i^*)],
\]

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and the initial condition
\[ \hat{c}_0 = -\bar{s}_0 (1 - \Gamma). \]

In this program, we have used the following notation. Parameter \( \Gamma = \frac{G}{Y} \) is the steady state share of government spending. Let \( \xi = \frac{1}{\Gamma + \phi} \). The parameter \( \hat{\kappa} \) is given by \( \hat{\kappa} = \frac{1}{(1-\Gamma)\xi} = \frac{\hat{\kappa}}{\hat{\ell}} \) and \( \lambda_S = \frac{\lambda}{\hat{\ell} - \xi} \). The variable \( \hat{c}_t \) denotes the log-deviation of consumption from the optimal allocation with flexible prices. The variable \( \hat{g}_t \) denotes the fiscal gap \( G_t - \bar{G}_t Y \) and the variable \( \hat{c}_t = \frac{C_t - \bar{C}_t Y}{\bar{C}_t Y} \) is the consumption gap, where bar variables denote the optimal allocation with flexible prices.

**A decomposition of government spending.** Following Werning (2012), we decompose government expenditures into two components.

\[ \hat{g}_t = \hat{g}^{opp}(\hat{c}_t) + \hat{g}^{stim} \]

where

\[ \hat{g}^{opp}(\hat{c}_t) = -\frac{\lambda_x (1 - \xi)}{\lambda_x (1 - \xi)^2 + \lambda_s} \hat{c}_t \]

and

\[ \hat{g}^{stim} = \frac{1 - \xi}{\lambda_x (1 - \xi)^2 + \lambda_s} \hat{\kappa} \hat{\mu}_{\pi,t} \]

The first component \( \hat{g}^{opp}(\hat{c}_t) \) is opportunistic spending. It is the level of expenditure that would be chosen by a government, taking the path for private consumption \( \hat{c}_t \) as given. Its determinants are purely microeconomic. It captures the idea that it makes sense to increase government spending when the opportunity cost of labor is low. The second component, stimulus spending, \( \hat{g}^{stim}_t \), is defined as the residual. Note that \( \hat{g}^{stim}_t = 0 \) whenever the costate \( \hat{\mu}_{\pi,t} \) is zero.

**Proposition 10 (Stimulus Spending).** Stimulus spending is initially zero so that \( \hat{g}^{stim}_0 = 0 \). In the special case where \( \hat{\kappa} \frac{1}{\Gamma + \phi} = \lambda_x \) then \( \mu_{\pi,t} = 0 \) for all \( t \) so that \( \hat{g}^{stim}_t = 0 \) for all \( t \). In this case, stimulus spending is zero throughout and government spending is purely opportunistic.

**Government spending and openness.** An important observation is that the planning problem for fiscal policy, expressed in gaps, is independent of the openness parameter \( \alpha \), and hence so is its solution. The openness of the economy only introduces a proportional
scaling factor in the associated loss function

\[ \frac{1}{2} (\Gamma + (1 - \Gamma)(1 - \rho)) \frac{\epsilon}{\lambda} \int_0^\infty \left[ \frac{1}{2} \pi_H t + \frac{1}{2} \lambda_x (\hat{c}_t + (1 - \xi)\hat{g}_t)^2 + \frac{1}{2} \lambda_g \hat{g}_t^2 \right] dt. \]

This is an important distinction with capital controls, which became more effective as the economy became more closed.

**Rigid prices.** With fixed prices, we can solve the system in closed form.

**Proposition 11 (Government Spending with Rigid Prices).** With rigid prices, the solution is given by

\[ \hat{c}_t = (1 - \Gamma) \left[-(\bar{r}_t - i^*)\right], \quad \hat{c}_0 = -\bar{s}_0(1 - \Gamma), \quad \hat{g}_t = \hat{g}_t^{opp}(\hat{c}_t) + \hat{g}_t^{stim}, \]

where \( \hat{g}_t^{opp}(\hat{c}_t) = -\frac{\lambda_x(1 - \xi)}{\lambda_x(1 - \xi)^2 + \lambda_g} \hat{c}_t \) and \( \hat{g}_t^{stim} = 0. \)

With rigid prices, the solution is simple: government spending is purely opportunistic, and stimulus spending is equal to zero.

### 6 Policy Coordination

Up to this point we have isolated the problem of a small open economy with a fixed exchange rate. The policy choices include a path for capital controls as well as a constant subsidy on labor. This country takes as given conditions in the rest of the world, including policy choices by other countries, the ensuing equilibrium interest rates and prices. We now consider a coordinated solution within a monetary union where countries cooperate on a constant tax and capital controls to maximize the sum of utilities. We will contrast this solution to the uncoordinated, noncooperative equilibrium, where each country acts in isolation.

An important consideration is the level of the labor tax. At a symmetric steady state with no coordination, the labor tax is given by

\[ M(1 + \tau_L) = \frac{1}{1 - \alpha}. \]

At a symmetric steady state with no coordination, the labor tax is instead given by

\[ M(1 + \tau_L) = 1. \]

The labor tax is lower in the latter case than in the former. Intuitively, coordination eliminates terms of trade manipulation. It is interesting to compare coordination vs. no coordination of the rest of policy instruments (capital controls in each country, and monetary policy at the aggregate level of the union), taking the labor tax as given.

#### 6.1 Uncoordinated labor tax

We start with the case where there is no coordination on the labor tax at the symmetric steady state so that

\[ M(1 + \tau_L) = \frac{1}{1 - \alpha}. \]

We denote with a double bar the corresponding
flexible price allocation with no capital controls. We denote with a double hat the gap of a variable from its flexible price counterpart. We denote with a tilde a variable minus its mean across countries. For example $\tilde{y}_i^t = \hat{y}_i^t - \hat{y}_i^*$ represents the deviation of country $i$’s output gap from the corresponding aggregate.

**Coordination.** For small $\alpha$, we can then write the coordinated planning problem as

$$\min \frac{1}{2} \int_0^\infty \int_0^1 e^{-\rho t} \left[ \alpha \pi (\hat{\pi}^i_{H,t})^2 + (\hat{y}_i^t)^2 + \alpha \theta (\hat{\theta}_i^t)^2 \right] dt$$

subject to

$$\hat{\pi}^{i}_{H,t} = \rho \hat{\pi}^{i}_{H,t} - \kappa \hat{y}^{i}_{t} - \lambda \alpha \hat{\theta}^{i}_{t},$$

$$\hat{y}^{i}_{t} = (1 - \alpha) \hat{\theta}^{i}_{t} - \hat{\pi}^{i}_{H,t} - \hat{\xi}^{i}_{t},$$

$$\int_0^\infty e^{-\rho t} \hat{\theta}^i_t dt = 0,$$

$$\tilde{y}_0^i = (1 - \alpha) \tilde{\theta}_0^i - \tilde{\xi}_0^i,$$

$$\int_0^1 \tilde{y}_i^t dt = 0,$$

$$\int_0^1 \hat{\pi}^i_{H,t} dt = 0,$$

$$\hat{\pi}^* = \rho \hat{\pi}^* - \kappa \hat{y}^*,$$

where the minimization is over the variables $\hat{\pi}^i_{H,t}, \hat{\pi}^*, \hat{y}^i, \hat{\theta}^i, \hat{\xi}$. Note that since $\int_0^1 \tilde{\xi}^i_t dt = 0$, the constraints imply that $\int_0^1 \hat{\theta}^i_t dt = 0$. This pins down controls through the equation $\hat{\theta}^i_t = \tau^i_t$. Intuitively, the extra linear term $\frac{\alpha}{1+\phi} \hat{y}^*$ can be traced back to the fact that the coordinated solution abstains from terms of trade manipulation. As a result the objective acquires a preference for higher output, leading to a classical inflationary bias.

It follows that we can break down the planning problem into two parts. First, there is an aggregate planning problem determining the average output gap and inflation $\hat{y}^*$ and

---

This representation is valid only for small $\alpha$, otherwise, a second order approximation of the constraints is required.
\[ \pi_t^* \]

\[ \min \frac{1}{2} \int_0^\infty e^{-\rho t} \left[ \alpha \pi_t^* (\pi_t^*)^2 + (\hat{\gamma}_t^*)^2 - \frac{\alpha}{(1-\alpha)(1+\phi)}\hat{\gamma}_t^* \right] dt \]  \hspace{1cm} (15) 

subject to

\[ \hat{\pi}_t^* = \rho \pi_t^* - \hat{\kappa} \hat{\gamma}_t^*. \]

Second, there is a disaggregated planning problem determining deviations from the aggregates for output gap, home inflation and consumption smoothing, \( \tilde{\gamma}_i^t, \tilde{\pi}_{H,t}^i \) and \( \tilde{\theta}_i^t \)

\[ \min \frac{1}{2} \int_0^\infty \int_0^1 e^{-\rho t} \left[ \alpha \pi_t (\tilde{\pi}_{H,t}^i)^2 + (\tilde{\gamma}_t^i)^2 + \alpha_\theta (\tilde{\theta}_t^i)^2 \right] dt \]  \hspace{1cm} (16) 

subject to

\[ \hat{\pi}_{H,t}^i = \rho \tilde{\pi}_{H,t}^i - \hat{\kappa} \tilde{\gamma}_t^i - \lambda \alpha \tilde{\theta}_t^i, \]
\[ \hat{\gamma}_t^i = (1-\alpha) \hat{\theta}_t^i - \tilde{\pi}_{H,t}^i - \tilde{\gamma}_t^i, \]
\[ \int_0^\infty e^{-\rho t} \tilde{\theta}_t^i dt = 0, \]
\[ \tilde{\gamma}_0^i = (1-\alpha) \hat{\theta}_0^i - \tilde{\gamma}_0^i, \]
\[ \int_0^1 \tilde{\gamma}_t^i dt = 0, \]
\[ \int_0^1 \tilde{\pi}_{H,t}^i dt = 0. \]

Consider dropping the last two aggregation constraints \( \int_0^1 \tilde{\gamma}_t^i dt = 0 \) and \( \int_0^1 \tilde{\pi}_{H,t}^i dt = 0 \). This relaxed planning problem can be broken down into separate component planning problems for each country \( i \in [0,1] \)

\[ \min \frac{1}{2} \int_0^\infty e^{-\rho t} \left[ \alpha \pi_t (\tilde{\pi}_{H,t}^i)^2 + (\tilde{\gamma}_t^i)^2 + \alpha_\theta (\tilde{\theta}_t^i)^2 \right] dt \]  \hspace{1cm} (16) 

subject to

\[ \hat{\pi}_{H,t}^i = \rho \tilde{\pi}_{H,t}^i - \hat{\kappa} \tilde{\gamma}_t^i - \lambda \alpha \tilde{\theta}_t^i, \]
\[ \hat{\gamma}_t^i = (1-\alpha) \hat{\theta}_t^i - \tilde{\pi}_{H,t}^i - \tilde{\gamma}_t^i, \]
\[ \int_0^\infty e^{-\rho t} \tilde{\theta}_t^i dt = 0, \]
\[ \tilde{y}_0^i = (1 - \alpha)\tilde{\theta}_0^i - \bar{s}_0^i. \]

**No coordination.** With no coordination, each country \( i \in [0, 1] \) takes the evolution of aggregates as given and solves the following uncoordinated component planning problem:

\[
\min \frac{1}{2} \int_0^\infty e^{-\rho t} \left[ \alpha \pi(\bar{\pi}_{H,t}^i)^2 + 2\alpha \pi^i \pi_{H,t}^i + (\bar{\pi}_t^i)^2 + 2\bar{\pi}_t^i \bar{\pi}_t^i + \alpha \theta(\bar{\theta}_t^i)^2 \right] dt
\]

subject to

\[
\begin{align*}
\dot{\pi}_{H,t}^i &= \rho \pi_{H,t}^i - \lambda \alpha \tilde{\theta}_t^i, \\
\dot{\tilde{y}}_t^i &= (1 - \alpha)\tilde{\theta}_t^i - \bar{\pi}_{H,t}^i - \bar{s}_t^i, \\
\int_0^\infty e^{-\rho t} \tilde{\theta}_t^i dt &= 0, \\
\bar{\pi}_t^i &= (1 - \alpha)\bar{\theta}_0 - \bar{s}_0^i
\end{align*}
\]

where the minimization is over the variables \( \pi_{H,t}^i, \tilde{y}_t^i, \tilde{\theta}_t^i \), taking \( \bar{\pi}_t^i \) and \( \pi_t^i \) as given. As usual, capital controls in country \( i \) can be computed by \( \tau_t^i = \dot{\theta}_t^i \). Note that the path for aggregates \( \{ \bar{\pi}_t^i, \pi_t^i \}_{t \geq 0} \) affects the solution to this problem solely through linear terms in the objective function.

A central monetary authority, by setting monetary policy, can choose aggregates \( \{ \bar{\pi}_t^i, \pi_t^i \} \) subject to the following constraints. First, it must ensure that the solutions to the uncoordinated component planning problems satisfy \( \int_0^1 \bar{\tilde{y}}_t^i dt = 0 \) and \( \int_0^1 \bar{\pi}_{H,t}^i dt = 0 \). This amounts to verifying a fixed point, that aggregates are actually equal to their proposed path. Second, it must ensure that the aggregate Phillips curve is verified, \( \pi_t^i = \rho \pi_t^i - \hat{\kappa} \bar{\pi}_t^i \). Both requirements define a set \( F \) of feasible aggregate outcomes \( \{ \bar{\pi}_t^i, \pi_t^i \}_{t \geq 0} \). The set is a linear space, which we characterize in closed form below, using the following definitions:

\[
A = \begin{bmatrix}
0 & \frac{\kappa(1-\alpha)+\lambda\alpha}{\rho(1-\alpha)+\lambda\alpha} \\
-\frac{\lambda\alpha}{\rho(1-\alpha)+\lambda\alpha} & 0
\end{bmatrix}, \quad E_1 = \begin{bmatrix}
1 \\
0
\end{bmatrix}, \quad E_2 = \begin{bmatrix}
0 \\
1
\end{bmatrix}.
\]

Which aggregate outcome in the feasibility occurs depends on the objective of the central monetary authority. For example, we can examine the case where the central monetary authority seeks to maximize aggregate welfare, taking into account that capital controls are set uncooperatively. For small \( \alpha \), this can be represented as the following planning problem

\[
\min \frac{1}{2} \int_0^\infty e^{-\rho t} \left[ \alpha \pi(\pi_t^i)^2 + (\bar{\pi}_t^i)^2 - \frac{\alpha}{(1-\alpha)(1+\phi)} \bar{\pi}_t^i \right] dt,
\]

where the minimization is over the feasible set \( F \).
Proposition 12 (Coordination vs. No Coordination with Uncoordinated Labor Tax). Suppose that countries do not coordinate on the labor tax. At the coordinated solution, the aggregates solve the aggregate planning problem (15): \( \hat{y}_t^0 = \frac{\alpha}{(1-\alpha)(1+\phi)} e^{\lambda t} \) and \( \pi_t^* = \frac{\rho}{(1-\alpha)(1+\phi)} \frac{k}{\rho-\lambda} e^{\lambda t} \) for all \( t \geq 0 \), where \( \lambda^- = \frac{\rho - \sqrt{\rho^2 + 4k\alpha\pi}}{2} \). At the uncoordinated solution, the aggregates \( \{\hat{y}_t^*, \pi_t^*\}_{t \geq 0} \) are in the feasible set \( F \) if and only if

\[
\hat{y}_t^* = \frac{k(1-\alpha) + \lambda\alpha}{1-\alpha} \int_t^\infty \pi_s^* E_1 e^{-A(s-t)} E_1 ds + \frac{kE_1^t A^{-1} E_2}{1-\alpha} \int_t^\infty \pi_s^* E_1 e^{-A(s-t)} E_1 ds
\]

\[
\pi_t^* = \rho \pi_t^* - \frac{k}{\rho-\lambda} \hat{y}_t^*.
\]

In particular, \( \hat{y}_t^* = \pi_t^* = 0 \) for all \( t \geq 0 \) is feasible but \( \hat{y}_t^* = \frac{\alpha}{(1-\alpha)(1+\phi)} e^{\lambda t} \) and \( \pi_t^* = \frac{\rho}{(1-\alpha)(1+\phi)} \frac{k}{\rho-\lambda} e^{\lambda t} \) for all \( t \geq 0 \) is not. Both at the coordinated and at the uncoordinated solutions, the disaggregated variables \( \tilde{\pi}_t^i, \tilde{y}_t^i, \tilde{\theta}_t^i \) solve the component planning problems (16).

Proof. We start by showing that at the coordinated solution, the disaggregated variables \( \tilde{\pi}_t^i, \tilde{y}_t^i, \tilde{\theta}_t^i \) solve the component planning problems (16). The component planning problems for two different countries are identical linear quadratic problems. The only difference is the path for the forcing variable \( s_i \). We can use the fact that the solution is linear in the path of the forcing variable \( \{s_i\}_{t \geq 0} \) and that \( \int_0^1 \tilde{s}_i^t di = 0 \) to conclude that the solution of the relaxed planning problem satisfies the two aggregation constraints \( 0 = \int_0^1 \tilde{y}_i^t di \) and \( 0 = \int_0^1 \tilde{\pi}_i^t di \). This implies that the solution of the relaxed planning problem coincides with that of the original planning problem.

Next we show that at the uncoordinated solution, the disaggregated variables \( \tilde{\pi}_t^i, \tilde{y}_t^i, \tilde{\theta}_t^i \) also solve the component planning problems (16). The uncoordinated component planning problem (17) is linear quadratic. Its solution is therefore linear in \( \{\hat{y}_t^*, \pi_t^*, \tilde{s}_i\}_{t \geq 0} \). Imposing that \( 0 = \int_0^1 \tilde{y}_i^t di \) and \( 0 = \int_0^1 \tilde{\pi}_i^t di \) then immediately implies that if \( \{\hat{y}_t^*, \pi_t^*\}_{t \geq 0} \) is in \( F \), then the solution of the uncoordinated component planning problem coincides with the solution of the component planning problem (16).

The derivation of the feasible set can be found in the appendix. \( \square \)

6.2 Coordinated labor tax

We now consider the case where there is coordination on the labor tax at the symmetric steady state so that \( M(1 + \tau_L) = 1 \). With a slight abuse of notation, we keep denoting with a double bar the corresponding flexible price allocation with no capital controls. We denote with a double hat the gap of a variable from its flexible price counterpart. We
denote with a tilde a variable minus its mean across countries. For example \( \tilde{y}_i = \hat{y}_i - \hat{y}_i^* \) represents the deviation of country \( i \)'s output gap from the corresponding aggregate.

**Coordination.** We can then write the coordinated planning problem as

\[
\min \frac{1}{2} \int_0^\infty \int_0^1 e^{-\rho t} \left[ \alpha (\tilde{\pi}_{H,t})^2 + (\tilde{y}_i)^2 + (1 - \alpha)\alpha \theta (\tilde{\theta}_i)^2 + \alpha \pi (\pi^*_t)^2 + (\hat{y}_i^*)^2 \right] \, dt \, di
\]

subject to

\[
\begin{align*}
\hat{\pi}_{H,t} &= \rho \tilde{\pi}_{H,t} - \kappa \tilde{y}_i - \lambda \alpha \tilde{\theta}_i, \\
\hat{y}_i &= (1 - \alpha) \tilde{\theta}_i - \tilde{\pi}_{H,t} - \tilde{y}_i, \\
\int_0^\infty e^{-\rho t} \tilde{\theta}_i \, dt &= 0, \\
\tilde{y}_0 = (1 - \alpha) \tilde{\theta}_0 - \tilde{s}_0, \\
\int_0^1 \tilde{y}_i \, di &= 0, \\
\int_0^1 \tilde{\pi}_{H,t} \, di &= 0, \\
\hat{\pi}_t^* &= \rho \pi_t^* - \kappa \hat{y}_t^*,
\end{align*}
\]

where the minimization is over the variables \( \tilde{\pi}_{H,t}, \pi_t^*, \tilde{y}_i, \hat{y}_i^*, \tilde{\theta}_i \). Note that since \( \int_0^1 \tilde{s}_i \, di = 0 \), the constraints imply that \( \int_0^1 \tilde{\theta}_i \, di = 0 \). This pins down controls through the equation \( \hat{\theta}_i = \tau_i \). There are two differences between the coordinated planning problem with a coordinated labor tax (18) and and the one with an uncoordinated labor tax 14. First, the coefficient on the term \( \frac{1}{2} \hat{y}_i^2 \) is \( (1 - \alpha)\alpha \theta \) instead of \( \alpha \theta \). Second, there is no linear term in \( \hat{y}_i^* \).

As above we can break down the planning problem into two parts. First, there is an aggregate planning problem determining the average output gap and inflation \( \hat{y}_i^* \) and \( \pi_t^* \)

\[
\min \frac{1}{2} \int_0^\infty e^{-\rho t} \left[ \alpha \pi (\pi_t^*)^2 + (\hat{y}_i^*)^2 \right] \, dt
\]

subject to

\[
\hat{\pi}_t^* = \rho \pi_t^* - \kappa \hat{y}_t^*.
\]

Second, there is a disaggregated planning problem determining deviations from the ag-
gregates for output gap, home inflation and consumption smoothing, $\hat{y}_i, \hat{\pi}_{H,t}^i$ and $\hat{\theta}_i$

$$\min \frac{1}{2} \int_0^\infty \int_0^1 e^{-\rho t} \left[ \alpha \pi \left( \hat{\pi}_{H,t}^i \right)^2 + (\hat{y}_i)^2 + (1 - \alpha) \alpha \theta \left( \hat{\theta}_i \right)^2 \right] dt$$

subject to

$$\dot{\hat{\pi}}_{H,t}^i = \rho \hat{\pi}_{H,t}^i - \kappa \hat{y}_i^i - \lambda \alpha \hat{\theta}_i^i,$$
$$\dot{\hat{y}}_i^i = (1 - \alpha) \dot{\hat{\theta}}_i^i - \hat{\pi}_{H,t}^i - \bar{s}_i^i,$$

$$\int_0^\infty e^{-\rho t} \hat{\theta}_i^i dt = 0,$$
$$\hat{y}_i^0 = (1 - \alpha) \hat{\theta}_0^i - \bar{s}_i^0.$$

Consider dropping the last two aggregation constraints $\int_0^1 \hat{y}_i^i di = 0$ and $\int_0^1 \hat{\pi}_{H,t}^i di = 0$. This relaxed planning problem can be broken down into separate component planning problems for each country $i \in [0,1]$

$$\min \frac{1}{2} \int_0^\infty e^{-\rho t} \left[ \alpha \pi \left( \hat{\pi}_{H,t}^i \right)^2 + (\hat{y}_i)^2 + (1 - \alpha) \alpha \theta \left( \hat{\theta}_i \right)^2 \right] dt \quad (20)$$

subject to

$$\dot{\hat{\pi}}_{H,t}^i = \rho \hat{\pi}_{H,t}^i - \kappa \hat{y}_i^i - \lambda \alpha \hat{\theta}_i^i,$$
$$\dot{\hat{y}}_i^i = (1 - \alpha) \dot{\hat{\theta}}_i^i - \hat{\pi}_{H,t}^i - \bar{s}_i^i,$$

$$\int_0^\infty e^{-\rho t} \hat{\theta}_i^i dt = 0,$$
$$\hat{y}_i^0 = (1 - \alpha) \hat{\theta}_0^i - \bar{s}_i^0.$$

**No coordination.** With no coordination, each country $i \in [0,1]$ takes the evolution of aggregates as given and solves the following uncoordinated component planning prob-
\[
\min \frac{1}{2} \int_{0}^{\infty} e^{-\rho t} \left[ \alpha_\pi (\dot{\pi}_i^*)^2 + 2\alpha_\pi \pi_i^* \dot{\pi}_i^* + (\dot{\gamma}_i^*)^2 + 2\ddot{\gamma}_i^* \dot{\gamma}_i^* + \frac{\alpha}{1 + \phi} \ddot{\gamma}_i^* + (1 - \alpha)\alpha_\theta (\ddot{\theta}_i^*)^2 \right] dt
\]

subject to
\[
\begin{align*}
\dot{\pi}_i^* &= \rho \pi_i^* - \kappa \ddot{y}_i^* - \lambda \ddot{\theta}_i^*, \\
\dot{\gamma}_i^* &= (1 - \alpha) \dot{\theta}_i^* - \pi_i^* - \ddot{s}_i^*, \\
\int_{0}^{\infty} e^{-\rho t} \ddot{\theta}_i^* dt &= 0, \\
\ddot{y}_i^* &= (1 - \alpha) \ddot{\theta}_0^* - \dddot{s}_0^*,
\end{align*}
\]

where the minimization is over the variables \(\pi_i^*, \ddot{y}_i^*, \ddot{\theta}_i^*\), taking \(\dddot{y}_i^*, \pi_i^*\) as given. As usual, capital controls in country \(i\) can be computed by \(\tau_i^* = \ddot{\theta}_i^*\). Note that once again, the path for aggregates \(\{\dddot{y}_i^*, \pi_i^*\}_{t \geq 0}\) affects the solution to this problem solely through linear terms in the objective function. There are two differences between the uncoordinated planning problems with a coordinated labor tax (21) and the one with an uncoordinated labor tax (17). First, the coefficient on \((\dddot{\theta}_i^*)^2\) is \((1 - \alpha)\alpha_\theta\) instead of \(\alpha_\theta\). Second, there is a linear term in \(\dddot{y}_i^*\). Intuitively, the extra linear term \(\frac{\alpha}{1 + \phi} \dddot{y}_i^*\) can be traced back to the fact that the uncoordinated solution feature terms of trade manipulation. As a result the objective acquires a preference for lower output—a form of anti-inflationary bias.

Exactly as in the case where there is no coordination on the labor tax, a central monetary authority, by setting monetary policy, can choose aggregates \(\{\dddot{y}_i^*, \pi_i^*\}_{t \geq 0}\) subject to the following constraints. First, it must ensure that the solutions to the uncoordinated component planning problems satisfy \(\int_{0}^{1} \dddot{y}_i^* dt = 0\) and \(\int_{0}^{1} \pi_i^* dt = 0\). This amounts to verifying a fixed point, that aggregates are actually equal to their proposed path. Second, it must ensure that the aggregate Phillips curve is verified, \(\dddot{\pi}_i^* = \rho \pi_i^* - \dot{\kappa} \dddot{y}_i^*\). Both requirements define a set \(\mathcal{F}'\) of feasible aggregate outcomes \(\{\dddot{y}_i^*, \pi_i^*\}_{t \geq 0}\). Which aggregate outcome in the feasibility occurs depends on the objective of the central monetary authority. For example, we can examine the case where the central monetary authority seeks to maximize aggregate welfare, taking into account that capital controls are set uncooperatively. For small \(\alpha\), this can be represented as the following planning problem
\[
\min \frac{1}{2} \int_{0}^{\infty} e^{-\rho t} \left[ \alpha_\pi (\dddot{\pi}_i^*)^2 + (\dddot{y}_i^*)^2 \right] dt,
\]

where the minimization is over the feasible set \(\mathcal{F}'\).
Proposition 13 (Coordination vs. No Coordination with Coordinated Labor Tax). Suppose that countries coordinate on the labor tax. At the coordinated solution, the aggregates solve the aggregate planning problem (15):

\[ \hat{y}^*_t = 0 \] and \[ \pi^*_t = 0 \] for all \( t \geq 0 \). At the uncoordinated solution, the aggregates \( \{\hat{y}^*_t, \pi^*_t\}_{t \geq 0} \) are in the feasible set \( F' \) if and only if

\[ \hat{y}^*_t = -\frac{\alpha}{1+\phi} + \alpha \pi^*_t \left( 1 - \frac{\lambda \alpha}{1-\alpha} \right) \int_t^\infty \pi^*_s E_1 e^{-A(s-t)} E_1 ds + \frac{\hat{\kappa} E_1' A^{-1} E_2}{1 - \frac{\hat{\kappa}}{\hat{\kappa}(1-\alpha) + \lambda \alpha}} \int_0^\infty \pi^*_s E_1' A^{-A} E_1 ds, \]

\[ \pi^*_t = \rho \pi^*_t - \hat{\kappa} \hat{y}^*_t. \]

In particular, \( \hat{y}^*_t = \pi^*_t = 0 \) for all \( t \geq 0 \) is not feasible. Both at the coordinated and at the uncoordinated solutions, the disaggregated variables \( \tilde{\pi}^i_{H,t} \), \( \tilde{y}^i_t \), \( \tilde{\theta}^i_t \) solve the component planning problems (20).

**Proof.** The proof is almost identical to the one of Proposition 12. \( \square \)

Propositions 12 and 13 help understand the role of coordination. For a given labor tax, the aggregates \( \hat{y}^*_t \) and \( \pi^*_t \) associated with the coordinated solution and the uncoordinated solutions differ. Indeed, for a given labor tax, the aggregates corresponding to the coordinated solution are not feasible. By contrast, for a given labor tax, the disaggregated variables \( \tilde{\pi}^i_{H,t} \), \( \tilde{y}^i_t \), \( \tilde{\theta}^i_t \) (and hence capital controls) coincide at the coordinated and uncoordinated solutions. They depend only on the labor tax, and not on whether countries coordinate or not for a given labor tax. In other words, the lack of coordination impacts the solution by restricting the set of feasible aggregate outcomes, but does not impact the disaggregated variables and the associated capital controls for any feasible aggregate outcome.

**References**


A Appendix

A.1 Proof of Proposition 1

It is clear that it is optimal to pick constant values for \( \{C_t, Y_t, N_t, \Theta_t, Q_t, S_t\} \). This immediately implies that \( \tau_t = 0 \) and that \( \tau_L \) is constant.
Dropping the \( t \) subscripts and substituting some of the constraints, we can rewrite the planning problem as

\[
\max_S \left[ (1 - \alpha) (S)^{\eta-1} + \alpha \right] \frac{(1-\sigma)S^{\eta-1}}{1 - \sigma} S^{(1-\sigma)(\gamma-1)} C^{*1-\sigma} - \frac{1}{1 + \phi} \frac{C^{*1+\phi}}{A^{1+\phi}} \left[ (1 - \alpha) (S)^{\eta-1} + \alpha \right]^{1+\phi} S^{(1+\phi)\gamma}
\]

This yields a function \( S(C^*) \). We then need to solve for \( S(C^*) = 1 \). We can then back out the corresponding \( \tau^L \) from the labor-leisure condition.

We find

\[
0 = \left[ \frac{(1 - \alpha) \eta S^{\eta-1}}{S (1 - \alpha) (S)^{\eta-1} + \alpha} + \frac{(\gamma - 1)}{S} \right] \left[ (1 - \alpha) (S)^{\eta-1} + \alpha \right]^{(1-\sigma)\eta} S^{(1-\sigma)(\gamma-1)} C^{*1-\sigma} - \left[ \frac{(\eta - 1) S^{\eta-1}}{S (1 - \alpha) (S)^{\eta-1} + \alpha} + \frac{\gamma}{S} \right] \frac{C^{*1+\phi}}{A^{1+\phi}} \left[ (1 - \alpha) (S)^{\eta-1} + \alpha \right]^{1+\phi} S^{(1+\phi)\gamma}
\]

We now impose \( S = 1 \) and solve for \( C^* \)

\[
0 = [(1 - \alpha) \eta + (\gamma - 1)] C^{*1-\sigma} - [(\eta - 1) + \gamma] \frac{C^{*1+\phi}}{A^{1+\phi}}
\]

i.e.

\[
\frac{C^{*\phi+\sigma}}{A^{1+\phi}} = \frac{(1 - \alpha) \eta + \gamma - 1}{\eta + \gamma - 1}
\]

We can now plug this back in the labor-leisure condition

\[
1 = M(1 + \tau^L) \frac{C^{*\phi+\sigma}}{A^{1+\phi}}
\]

to obtain the proposition.

**A.2 Proof of Proposition 5**

In the Cole-Obstfeld case, we can write \( V(NFA_1) \) as the maximization over \( \{\Theta_t\}_{t \geq 1} \) of

\[
(1 - \alpha) \log(A_t) + \alpha \log C^*_t + \frac{1 - \alpha}{1 + \phi} \log(1 - \alpha) + \frac{\phi + \alpha}{1 + \phi} \log(\Theta_t)
\]

\[
- \frac{(1 - \alpha)\phi}{1 + \phi} \log [\alpha \Lambda_t + (1 - \alpha) \Theta_t] - \frac{(1 - \alpha)\Lambda_t + (1 - \alpha)\Theta_t}{1 + \phi}
\]
subject to

\[ NFA_1 = - \sum_{t=0}^{\infty} \beta^t C_t^{\phi} \alpha (\Lambda_t - \Theta_t) . \]

Consider first productivity shocks. For all \( t \geq 1 \) productivity \( A_t \), the solution of the planning problem for \( t \geq 1 \) features a constant allocation, for which we drop the \( t \) subscripts. This immediately implies that optimal capital controls are zero for \( t \geq 1 \). It is immediate that the optimal \( \Theta \) is independent of the path \( A_t \) for \( t \geq 1 \). The result for productivity shocks then simply follows from Proposition 4.

Consider next foreign consumption shocks. For all \( t \geq 1 \) productivity \( C_t^* \), the solution of the planning problem for \( t \geq 1 \) features a constant allocation, for which we drop the \( t \) subscripts. This immediately implies that optimal capital controls are zero for \( t \geq 1 \). It is immediate that the optimal \( \Theta \) is independent of the path \( C_t^* \) for \( t \geq 1 \). The result for productivity shocks then simply follows from Proposition 4.

Consider next permanent export demand shocks \( \Lambda_{H,t} = \Lambda_H \) for all \( t \geq 0 \). For all \( t \geq 1 \) the solution of the planning problem for \( t \geq 1 \) features a constant allocation, for which we drop the \( t \) subscripts. This immediately implies that optimal capital controls are zero for \( t \geq 1 \). We also find it convenient to define \( \tilde{\Theta}_0 = \Theta_0 - \Lambda_H \) and \( \tilde{\Theta} = \Theta - \Lambda_H \). The country budget constraint establishes the following relationship between \( \tilde{\Theta} \) and \( \tilde{\Theta}_0 \): \( \tilde{\Theta} = -\frac{1-\beta}{\beta} \tilde{\Theta}_0 \). We are left with the following maximization problem

\[
\max \left[ \log [\Lambda_0 + \tilde{\Theta}_0] + \log C^* - \frac{1}{1 + \phi} \left( \frac{C^*}{A} \right)^{1+\phi} (\Lambda_H + (1-\alpha)\tilde{\Theta}_0)^{1+\phi} + \beta V (-\frac{\alpha}{\beta} \tilde{\Theta}_0) \right]
\]

where

\[
(1 - \beta) V(\tilde{\Theta}) = (1 - \alpha) \log(A) + \alpha \log C^* + \frac{1 - \alpha}{1 + \phi} \log(1 - \alpha) + \frac{\phi + \alpha}{1 + \phi} \log [\tilde{\Theta} + \Lambda_H] - \frac{(1 - \alpha)\phi}{1 + \phi} \log [\Lambda_H + (1 - \alpha)\tilde{\Theta}] - \frac{(1 - \alpha)\Lambda_H + (1 - \alpha)\Theta}{\Lambda + \Theta}
\]

Let \( U(\tilde{\Theta}_0, \Lambda_H) \) be the objective function. We can compute

\[
U_{\tilde{\Theta}_0, \Lambda_H} = \frac{\phi + \alpha}{1 + \phi} \frac{1}{(\tilde{\Theta} + \Lambda_H)^2} + \frac{\alpha(1 - \alpha)}{1 + \phi} \frac{\tilde{\Theta} - \Lambda_H}{(\Lambda_H + \tilde{\Theta})^3} - \frac{1}{(\Lambda_H + \tilde{\Theta}_0)^2}
\]

\[
- (1 - \alpha)\phi \left( \frac{C^*}{A} \right)^{1+\phi} (\Lambda_H + (1-\alpha)\tilde{\Theta}_0)^{\phi-1} - \frac{(1 - \alpha)^2\phi}{1 + \phi} \frac{1}{[\Lambda_H + (1 - \alpha)\tilde{\Theta}]^2}
\]

For \( \tilde{\Theta}_0 = 0 \) (and hence \( \tilde{\Theta} = 0 \)) we always have \( U_{\tilde{\Theta}_0, \Lambda_H}(0, \Lambda_H) < 0 \), so that at least
for small shocks, \( \hat{\Theta}_0 \) is decreasing in \( \Lambda_H \) and \( \hat{\Theta} \) is increasing in \( \Lambda_H \) with \( \hat{\Theta} = \hat{\Theta}_0 = 0 \) for \( \Lambda_H = 1 \). The result in the proposition follows since capital controls \( \tau_0 \) are given by

\[
\frac{\Lambda_H + \hat{\Theta}}{\Lambda_H + \hat{\Theta}_0} = 1 + \tau_0.
\]

Finally, let’s us consider wealth shocks \( W_0 \neq 0 \). For all \( t \geq 1 \) the solution of the planning problem for \( t \geq 1 \) features a constant allocation, for which we drop the \( t \) subscripts. This immediately implies that optimal capital controls are zero for \( t \geq 1 \). We also find it convenient to define \( \hat{\Theta}_0 = \Theta_0 - 1 - (1 - \beta)\frac{1}{\alpha}NFA_0 \) and \( \hat{\Theta} = \Theta - 1 - (1 - \beta)\frac{1}{2}NFA_0 \). The country budget constraint establishes the following relationship between \( \hat{\Theta} \) and \( \Theta_0 \):

\[
\hat{\Theta} = -\frac{1 - \beta}{\beta} \Theta_0. \]

We are left with the following maximization problem over \( \hat{\Theta}_0 \):

\[
\log \left( \hat{\Theta}_0 + 1 + (1 - \beta)\frac{1}{\alpha}NFA_0 \right) + \log C^* \\
- \frac{1}{1 + \phi} \left( \frac{C^*}{A} \right)^{1 + \phi} \left( (1 - \alpha)\hat{\Theta}_0 + (1 - \beta)\frac{1}{\alpha}NFA_0 + 1 \right)^{1 + \phi} + \beta V(\hat{\Theta})
\]

where

\[
(1 - \beta) V(\hat{\Theta}) = (1 - \alpha) \log(A) + \alpha \log C^* + \frac{1}{1 + \phi} \log(1 - \alpha)
\]

\[
+ \frac{\phi + \alpha}{1 + \phi} \log \left[ \hat{\Theta} + 1 + (1 - \beta)\frac{1}{\alpha}NFA_0 \right] - \frac{(1 - \alpha)\phi}{1 + \phi} \log \left[ (1 - \alpha)\hat{\Theta} + (1 - \beta)\frac{1}{\alpha}NFA_0 + 1 \right]
\]

\[
- \frac{(1 - \alpha)(1 - \alpha)\hat{\Theta} + (1 - \beta)\frac{1}{\alpha}NFA_0 + 1}{\hat{\Theta} + 1 + (1 - \beta)\frac{1}{2}NFA_0}
\]

Let \( U(\hat{\Theta}_0, NFA_0) \) be the objective function. We have

\[
\frac{1}{(1 - \beta)\frac{1}{\alpha}} U_{\Theta_0,NFA_0} = \frac{\phi + \alpha}{1 + \phi} \frac{1}{\Theta_0 + 1 + (1 - \beta)\frac{1}{\alpha}NFA_0}^2 - \frac{1}{\Theta_0 + 1 + (1 - \beta)\frac{1}{2}NFA_0}^2
\]

\[
- (1 - \alpha)^2 \phi \left( \frac{C^*}{A} \right)^{1 + \phi} \left( (1 - \alpha)\hat{\Theta}_0 + (1 - \beta)\frac{1}{\alpha}NFA_0 + 1 \right)^{\phi - 1}
\]

\[
- \frac{(1 - \alpha)^3 \phi}{1 + \phi} \frac{1}{\left[ (1 - \alpha)\hat{\Theta} + (1 - \beta)\frac{1}{\alpha}NFA_0 + 1 \right]^2} - \frac{2(1 - \alpha)}{1 + \phi} \frac{1}{\left[ \hat{\Theta} + 1 + (1 - \beta)\frac{1}{2}NFA_0 \right]^3}
\]

For \( \hat{\Theta}_0 = 0 \) (and hence \( \hat{\Theta} = 0 \)) we always have \( U_{\hat{\Theta}_0,NFA_0}(0, NFA_0) < 0 \), so that at least for small shocks, so that at least for small shocks, \( \hat{\Theta}_0 \) is decreasing in \( W_0 \) and \( \hat{\Theta} \) is increasing in \( W_0 \) with \( \hat{\Theta} = \hat{\Theta}_0 = 0 \) for \( \Lambda_H = 1 \). The result in the proposition follows since capital controls \( \tau_0 \) are given by

\[
\frac{1 + (1 - \beta)\frac{1}{2}NFA_0 + \hat{\Theta}}{1 + (1 - \beta)\frac{1}{2}NFA_0 + \hat{\Theta}_0} = 1 + \tau_0.
\]

More generally we have \( U_{\hat{\Theta}_0,NFA_0}(\hat{\Theta}_0, NFA_0) < 0 \) for all \( \hat{\Theta}_0 < 0 \) which implies, together with the concavity of \( U \).
in $\tilde{\Theta}_0$, that for positive shocks to $NFA_0$, $\tilde{\Theta}_0$ is a decreasing function of $NFA_0$ so that the result generalizes to any positive shock to initial wealth $NFA_0 > 0$.

### A.3 Derivation of the feasible sets $\mathcal{F}$ and $\mathcal{F}'$ in Propositions 12 and 13

We deal with the case where the labor tax is set at its uncoordinated level. The other case is similar. Because the uncoordinated component planning problem is linear quadratic, we know that $(\{\hat{y}_i^*\}_{t \geq 0}, \{\pi^*_i\}_{t \geq 0})$ is such that $0 = \int_0^1 \hat{y}_t^* dt$ and $0 = \int_0^1 \hat{\pi}^*_t dt$ if and only if the solution of the following minimization problem is $\hat{\pi}^*_t = \hat{y}_t = \hat{\theta}_t = 0$:

$$
\min \frac{1}{2} \int_0^\infty e^{-\rho t} \left[ \alpha \pi(\hat{\pi}^*_t)^2 + 2\alpha \pi \pi^*_t \hat{\pi}^*_t + (\hat{y}_t)^2 + 2\hat{y}^*_t \hat{y}_t + \alpha \theta (\hat{\theta}_t)^2 \right] dt
$$

subject to

$$
\dot{\hat{\pi}}^*_t = \rho \hat{\pi}^*_t - \hat{\kappa} \hat{y}_t - \lambda \alpha \hat{\theta}_t,
$$

$$
\dot{\hat{y}}_t = (1 - \alpha) \hat{\theta}_t - \hat{\pi}^*_t,
$$

$$
\int_0^\infty e^{-\rho t} \hat{\theta}_t dt = 0,
$$

$$
\hat{y}_0 = (1 - \alpha) \hat{\theta}_0,
$$

where the minimization is over the variables $\hat{\pi}^*_t, \hat{y}_t, \hat{\theta}_t$, taking $\hat{y}^*_t$ and $\pi^*_t$ as given. We put a multiplier $\Gamma$ on the constraint $\int e^{-\rho t} \hat{\theta}_t dt = 0$ and incorporate it in the objective to obtain an optimal control problem. The first-order conditions are

$$
-\mu_{\pi,t} = \alpha \pi \bar{\pi}_t + \alpha \pi \pi^*_t - \mu_{y,t},
$$

$$
\rho \mu_{y,t} - \mu_{y,t} = \hat{y}_t + \hat{y}^*_t - \hat{\kappa} \mu_{\pi,t},
$$

$$
\rho \mu_{\theta,t} - \mu_{\theta,t} = \alpha \theta \hat{\theta}_t + \Gamma - \lambda \alpha \mu_{\pi,t},
$$

$$
(1 - \alpha) \mu_{y,t} + \mu_{\theta,t} = 0,
$$

$$
\mu_{\pi,0} = 0.
$$

We can combine these equations to get

$$
\mu_{\pi,t} = \frac{1 - \alpha}{\hat{\kappa}(1 - \alpha) + \lambda \alpha} \hat{y}_t + \frac{\alpha \theta}{\hat{\kappa}(1 - \alpha) + \lambda \alpha} \hat{\theta}_t + \frac{\Gamma}{\hat{\kappa}(1 - \alpha) + \lambda \alpha} + \frac{1 - \alpha}{\hat{\kappa}(1 - \alpha) + \lambda \alpha} \hat{y}^*_t,
$$

50
leading to the following reduced system

\[
\begin{align*}
\frac{1 - \alpha}{\hat{k}(1 - \alpha) + \lambda \alpha} \dot{\hat{y}}_t + \frac{\alpha \theta}{\hat{k}(1 - \alpha) + \lambda \alpha} \dot{\hat{\theta}}_t + \frac{1 - \alpha}{\hat{k}(1 - \alpha) + \lambda \alpha} \dot{\hat{y}}_t^* &= -\alpha \pi \hat{\pi}_{H,t} + \mu_{y,t} - \alpha \pi \pi_t^*, \\
\rho \mu_{y,t} - \dot{\mu}_{y,t} &= \frac{\lambda \alpha}{\hat{k}(1 - \alpha) + \lambda \alpha} \dot{\hat{y}}_t - \frac{\hat{k} \alpha \theta}{\hat{k}(1 - \alpha) + \lambda \alpha} \dot{\hat{\theta}}_t - \frac{\hat{k} \Gamma}{\hat{k}(1 - \alpha) + \lambda \alpha} + \frac{\lambda \alpha}{\hat{k}(1 - \alpha) + \lambda \alpha} \dot{\hat{y}}_t^*, \\
\hat{\pi}_{H,t} &= \rho \hat{\pi}_{H,t} - \hat{k} \ddot{y}_t - \lambda \alpha \ddot{\theta}_t, \\
\ddot{y}_t &= (1 - \alpha) \ddot{\theta}_t - \hat{\pi}_{H,t}, \\
\ddot{y}_0 &= (1 - \alpha) \ddot{\theta}_0, \\
\frac{1 - \alpha}{\hat{k}(1 - \alpha) + \lambda \alpha} \ddot{y}_0 + \frac{\alpha \theta}{\hat{k}(1 - \alpha) + \lambda \alpha} \ddot{\theta}_0 + \frac{\Gamma}{\hat{k}(1 - \alpha) + \lambda \alpha} + \frac{1 - \alpha}{\hat{k}(1 - \alpha) + \lambda \alpha} \ddot{y}_0^* &= 0.
\end{align*}
\]

In order for \( \hat{\pi}_{H,t} = \ddot{y}_t = \ddot{\theta}_t = 0 \) to be the solution, we must have

\[
\frac{1 - \alpha}{\hat{k}(1 - \alpha) + \lambda \alpha} \ddot{y}_t^* = \mu_{y,t} - \alpha \pi \pi^*_t,
\]

\[
\rho \mu_{y,t} - \dot{\mu}_{y,t} = -\frac{\hat{k} \Gamma}{\hat{k}(1 - \alpha) + \lambda \alpha} + \frac{\lambda \alpha}{\hat{k}(1 - \alpha) + \lambda \alpha} \ddot{y}_t^*,
\]

\[
\frac{\Gamma}{\hat{k}(1 - \alpha) + \lambda \alpha} + \frac{1 - \alpha}{\hat{k}(1 - \alpha) + \lambda \alpha} \ddot{y}_0^* = 0.
\]

Let \( X_t = [\ddot{y}_t^*, \mu_{y,t}]' \) and \( B_t = \left[ -\frac{\hat{k}(1 - \alpha) + \lambda \alpha}{1 - \alpha} \alpha \pi \pi_t^*, \frac{\hat{k}}{\hat{k}(1 - \alpha) + \lambda \alpha} \Gamma \right]' \). We can write this system as

\[
\dot{X}_t = AX_t + B_t
\]

together with the initial condition

\[
E_1' X_0 = -\frac{1}{1 - \alpha} \Gamma.
\]

The characteristic polynomial of \( A \) is \( x^2 - \rho x + \frac{\lambda \alpha}{1 - \alpha} \). This implies that two eigenvalues of \( A \) are either positive, or complex with positive real parts. The solution is hence given by

\[
X_t = -\int_t^\infty e^{-A(s-t)} B_s ds.
\]
We can solve for $\Gamma$:

$$
\Gamma = - \frac{\hat{k}(1-a) + \lambda \alpha}{1-a - \frac{\hat{k}E_1 A^{-1} E_2}{\hat{k}(1-a) + \lambda \alpha}} \left( \alpha \pi \int_0^\infty \tau_s^* E_1' e^{-A \tau_s} E_1 ds \right).
$$

We conclude that $\{\hat{y}_t^*, \pi_t^*\}_{t \geq 0}$ is in the feasibility set $\mathcal{F}$ if and only if

$$
\hat{y}_t^* = \alpha \pi \frac{\hat{k}(1-a) + \lambda \alpha}{1-a} \int_t^\infty \tau_s^* E_1' e^{-A(s-t)} E_1 ds + \alpha \pi \frac{\hat{k}E_1' A^{-1} E_2 \int_0^\infty \tau_s^* E_1' e^{-A \tau_s} E_1 ds}{1-a - \frac{\hat{k}E_1' A^{-1} E_2}{\hat{k}(1-a) + \lambda \alpha}},
$$

$$
\hat{\pi}_t^* = \rho \pi_t^* - \hat{k}\hat{y}_t^*.
$$
Figure 1: Permanent terms of trade shock, $\alpha = 0.4$. 
Figure 2: Permanent terms of trade shock, $\alpha = 0.1$. 
Figure 3: Mean-reverting natural interest rate shock, $\alpha = 0.4$. 
Figure 4: Mean-reverting natural interest rate shock, $\alpha = 0.1$. 
Figure 5: Mean-reverting productivity shock, $\alpha = 0.4$. 

\[ \tau = i - i^* \]
Figure 6: Mean-reverting productivity shock, $\alpha = 0.1$. 
Figure 7: Government spending for a permanent terms of trade shock, $\alpha = 0.4$. 
Figure 8: Government spending for a mean-reverting natural interest rate shock, $\alpha = 0.4$. 
Figure 9: Government spending for a mean-reverting productivity shock, $\alpha = 0.4$. 