Monetary Policy and the Equity Premium*

Christopher Gust and David López-Salido†
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Abstract

Recent research has emphasized that risk premia are important for understanding the monetary transmission mechanism. An important result in this literature is that unanticipated changes in monetary policy affect equity prices primarily through changes in risk rather than through changes in real interest rates. To account for this finding, we develop a DSGE model in which monetary policy generates endogenous movements in risk. The key feature of our model is that asset and goods markets are segmented, because it is costly for households to transfer funds between these markets, and they may only infrequently update their desired allocation of cash across these two markets. Our model emphasizes that time-varying risk is driven by costly portfolio rebalancing of financial accounts rather than limited participation in financial markets. We show that the model can account for the mean returns on equity and the risk-free rate, and generates variations in the equity premium that help explain the response of stock prices to monetary shocks.

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†Corresponding Author: David López-Salido. Email addresses: christopher.gust@frb.gov, david.lopez-salido@frb.gov

1 Introduction

Monetary policy primarily affects the macroeconomy through its effect on financial markets. In standard monetary models, this interaction between the financial and real sides of the economy occurs through short-term interest rates, as changes in monetary policy affect the conditional mean of the short-term interest rate which in turn macroeconomic variables such as output, employment, and inflation. These models, however, abstract from another channel through which monetary policy affects financial markets and the macroeconomy. In these models, there is little or no role for monetary policy to influence the conditional variances of variables or the perceived riskiness of the economy.[1] In contrast, Bernanke and Kuttner (2005) provide evidence that monetary policy does affect risk, suggesting that standard monetary models are potentially missing an important channel through which monetary shocks propagate from the financial to the real economy. Using high-frequency data, they show that, while an unanticipated easing of monetary policy lowers real short-term interest rates, it also has a large quantitative effect on equity returns occurring through a reduction in the equity premium.

In this paper, we develop a DSGE model in which monetary policy affects the economy through the standard interest rate channel and through its effect on economic risk. The key feature of our model is that asset and goods markets are segmented, because it is costly for households to transfer funds between these markets. Accordingly, they may only infrequently update their desired allocation of cash between an account devoted to purchasing goods and a brokerage account used for financial transactions. The optimal decision by an individual household to rebalance their cash holdings is a state-dependent one, reflecting that doing so involves paying a fixed cost in the presence of uncertainty. Households are heterogenous in this fixed cost, and only those households that rebalance their portfolios during the current period matter for determining asset prices. Because the fraction of these household changes over time

[1]See Alvarez, Atkeson, and Kehoe (2007a) for an extended discussion of this point. In a companion paper, Alvarez, Atkeson, and Kehoe (2007b) argue that the evidence on the relationship between interest rates and exchange rates is consistent with movements in these variables that are driven mainly by changes in conditional variances.

in response to both real and monetary shocks, risk in the economy is both time-varying and endogenous.

We show that our model, unlike standard monetary models, is able to account for the Bernanke and Kuttner (2005) evidence. In particular, a monetary easing in our model leads to a fall in real interest rates and a reduction in the equity premium. Furthermore, for reasonable calibrations of the model, the reduction in the equity premium is an important reason why stock prices rise in response to a monetary easing.

In addition to comparing our model to the evidence of Bernanke and Kuttner (2005), we also examine the model's ability to account for the mean returns on equity and the risk-free rate. As shown by Mehra and Prescott (1985), standard representative agent models with power utility have difficulty quantitatively accounting for these moments. For reasonable calibrations of monetary and technology shocks, our model is able to match the observed means on equity and risk-free rates with a power utility function that implies constant relative risk aversion equal to two. [2] We show that to match these moments, the average fraction of households that reallocates funds across markets can not be too large. For our benchmark calibration, about 15 percent of households, on average, rebalance their portfolios in a quarter. Underlying this average fraction of rebalancing, there is a considerable degree of heterogeneity across households in our model, with some rebalancing every period and another fraction never rebalancing away from their initial allocation.

Recent microdata on household finance provides strong support for infrequent portfolio rebalancing. For instance, in two recent papers, Calvet, Campbell, and Sodini (2008) and Calvet, Campbell, and Sodini (2009) document that, while there is little rebalancing of the financial portfolios of stockholders by the average household, there is a great deal of heterogeneity at the micro level with some households rebalancing these portfolios very frequently. In addition, using information on asset holdings from the PSID, Bilias, Georgarakos, and Haliassos (2008) and Brunnermeier and Nagel (2008) provide evidence that household portfolio allocation dis-

[2] In a companion paper, Gust and López-Salido (2009), we focus on the asset pricing implications of the endogenous rebalancing model.

play substantial inertia. Surveys conducted by the Investment Company Institute (ICI) and the Securities Industry Association (SIA) also suggest that households rebalance their portfolios infrequently. For instance, in 2004, the median number of total equity transactions for an individual was four. In addition, sixty percent of equity investors did not conduct any equity transactions during 2004. Finally, in a 2005 survey, the ICI reports that more that two-thirds of the time the proceeds from the sales of stocks by households are fully reinvested.[3]

Our model is most closely related to and builds on the analysis of Alvarez, Atkeson, and Kehoe (2007b). They introduce endogenously segmented markets into an otherwise standard cash-in-advance economy and show how changes in monetary policy can induce fluctuations in risk. However, our model differs from theirs in two important respects. First, we incorporate production and equity returns. Second and more importantly, in their model, risk is endogenous, because the fraction of households that participates in financial markets is state-dependent. In our model, all households participate in financial markets, but it is costly to reallocate cash between the asset and goods markets from a household's initial allocation. In other words, endogenous asset segmentation occurs along an intensive margin in our model rather than an extensive margin. This distinction is important, because we show that for reasonable calibrations we can not account for the Bernanke and Kuttner (2005) evidence or match the average equity premium in their framework.[4]

Our paper is also related to portfolio choice models that emphasize infrequent adjustment. In this literature, the paper most closely related to ours is Abel, Eberly, and Panageas (2007).[5] They also model infrequent adjustment of cash between a transaction account used to purchase goods and another account used to purchase financial assets. Their framework differs from ours, since they do not consider the role of monetary policy and use a partial equilibrium framework

[3]See, Figure F.7, "Disposition of Proceeds from most recent sale of individual stocks", in ICI 2005.

[4]Polkovnichenko (2004), Vissing-Jorgensen (2002), Vissing-Jorgensen (2003), and Gomes and Michaelides (2006) also show that it is difficult to match the equity premium in a model with endogenous stock market participation. Guvenen (2005) considers a model in which stock market participation is fixed exogenously and shows that this feature in addition to heterogeneity in preferences can help account for the average equity premium.

[5]Also, see Lynch (1996), Marshall and Parekh (1999), and Gabaix and Laibson (2001).

in which returns are exogenous. However, they show that infrequent portfolio adjustment can arise due to rational inattention on the part of households.

The rest of this paper proceeds as follows. The next section describes the model and its calibration. Section 3 presents the results, emphasizing the model's ability to match unconditional moments such as the mean returns on equity and a risk-free asset as well as the conditional responses of these variables to a monetary policy shock. Section 4 concludes and discusses directions for future research.

2 The Model

Our model builds on the cash-in-advance economy of Alvarez, Atkeson, and Kehoe (2007b), which we extend to incorporate equity prices. Our model differs from theirs, since we emphasize that time-varying risk is driven by costly portfolio rebalancing of financial accounts rather than limited participation in financial markets.

Our economy is populated by a large number of households, firms, and a government sector. There are infinite number of periods, and in periods $t \geq 1$ trade occurs in financial and goods markets. In period $t = 0$ there is an initial round of trade in bonds in the asset market with no trade in goods markets. In the asset market, households trade a complete set of state-contingent claims, and in the goods market, households use money to buy goods subject to a cash-in-advance constraint.

We assume that trade in these two markets takes place in separate locations so that they are segmented from each other. A household can pay a fixed cost to transfer funds between the asset and goods markets. This fixed cost is constant over time but varies across households. We refer to a household that pays this fixed cost as actively rebalancing cash allocated for consumption and asset markets, and those that do not as inactive.

There are two sources of uncertainty in our economy — aggregate shocks to technology, Z_t, and money growth, μ_t. We let $s_t = (z_t, \mu_t)$ index the aggregate event in period t, and $s^t = (s_1, ..., s_t)$ denote the state, which consists of the aggregate shocks that have occurred through period t.

2.1 Firms

There is large number of perfectly competitive firms, which each have access to the following technology for converting capital, $K(s^{t-1})$, and labor, $L(s^t)$, into output, $Y(s_t)$ at dates $t \geq 1$:

$$Y(s^t) = K(s^{t-1})^\alpha \left[\exp(z_t) L(s^t)\right]^{1-\alpha}. \tag{1}$$

We assume that the technology shock follows a first-order autoregressive process:

$$z_t = \rho_z z_{t-1} + \epsilon_{zt}, \tag{2}$$

where $\epsilon_{zt} \sim N(0, \sigma_z^2)$ for all $t \geq 1$. Capital does not depreciate, and there exists no technology for increasing or decreasing its magnitude. We adopt the normalization that the aggregate stock of capital is equal to one. Labor is supplied inelastically by households, its supply is also normalized to one.

Following Boldrin, Christiano, and Fisher (1997), we assume that firms have a one-period planning horizon. To operate capital in period $t+1$, a firm must purchase it at the end of period t from those firms operating during period t. To do so, a firm issues equity $S(s^t)$ and purchases capital subject to its financing constraint,

$$P_k(s^t) K(s^t) \leq S(s^t), \tag{3}$$

where $P_k(s^t)$ denotes the price of capital in state, s^t.

A firm derives revenue from its sale of output, $P(s^{t+1})Y(s^{t+1})$, and the sale of its capital stock, $P_k(s^{t+1})K(s^t)$, at the end of period $t+1$. A firm's expenses include its obligations on equity, $(1 + R^e(s^{t+1}))S(s^t)$, and payments to to labor, $W(s^{t+1})L(s^{t+1})$. A firm's net revenues, $V(s^{t+1})$, including its expenses, must be greater than zero in each state so that:

$$V(s^{t+1}) = P(s^{t+1})Y(s^{t+1}) + P_k(s^{t+1})K(s^t) - (1 + R^e(s^{t+1}))S(s^t) - W(s^{t+1})L(s^{t+1}) \geq 0. \tag{4}$$

The firm's problem at date $t+1$ is to maximize $V(s^{t+1})$ across states of nature by choice of $K(s^t)$ and $L(s^{t+1})$ subject to (1) and (3). This problem implies that the financing constraint

(3) is satisfied as a strict equality in equilibrium. The equilibrium real wage, $w(s^{t+1})$ is given by:

$$w(s^{t+1}) = \frac{W(s^{t+1})}{P(s^{t+1})} = (1-\alpha)\frac{Y(s^{t+1})}{L(s^{t+1})}, \tag{5}$$

Linear homogeneity of the firm's objective, together with the weak inequality in equation (4) imply that $V(s^{t+1}) = 0$ for all s^{t+1} so that:

$$1 + r^e(s^{t+1}) = \frac{1 + R^e(s^{t+1})}{\pi(s^{t+1})} = \frac{\left[\alpha \frac{Y(s^{t+1})}{K(s^t)} + p_k(s^{t+1})\right]}{p_k(s^t)}. \tag{6}$$

In the above, $p_k(s^t) = \frac{P_k(s^t)}{P(s^t)}$ denotes the real price of capital and $\pi(s^{t+1}) = \frac{P(s^{t+1})}{P(s^t)}$ is the economy's inflation rate.

2.2 The Government

The government issues one-period state-contingent bonds and controls the economy's money stock, M_t. At date 0, the government also issues an annuity at price, P_A, which has a constant payoff A_0 in units of consumption. Its budget constraints at date 0 is given by:

$$\bar{B} = \int_{s_1} q(s_1)B(s_1)ds_1 + P_A A_0, \tag{7}$$

where $\bar{B}$ is given and $q(s_1)$ denotes the price of the state-contingent bond, $B(s_1)$. At dates $t \geq 1$, the government's budget constraint is given by:

$$B(s^t) + M_{t-1} + P(s^t)A_0 = M_t + \int_{s_{t+1}} q(s^t, s_{t+1})B(s^t, s_{t+1})ds_{t+1}, \tag{8}$$

with $M_0 > 0$ given. Finally, the government injects cash into the economy via a first-order autoregressive process for money growth, $\mu_t = \frac{M_t}{M_{t-1}}$:

$$\mu_t = (1-\rho_\mu)\bar{\mu} + \rho_\mu \mu_{t-1} + \epsilon_{\mu t}, \tag{9}$$

where $\bar{\mu} > 0$, and $\epsilon_{\mu t} \sim N(0, \sigma_\mu^2)$ for all $t \geq 1$.

2.3 Households

Households inelastically supply their labor to firms and purchase the output of the firms. A household can also purchase and sell bonds and stocks in asset markets. However, we assume that the asset and goods markets are segmented so that a household must pay a real fixed cost, γ, to transfer cash between them. This cost is constant for a household but differs across households according to the distribution $F(\gamma)$ with density $f(\gamma)$. Specifically, we assume some positive mass has a zero fixed cost so that $F(0) > 0$, while the distribution for the remaining households is given by $\tilde{\gamma} \sim N(\tilde{\gamma}_m, \sigma_\gamma)$, where $\tilde{\gamma} = log(\gamma)$.

At the beginning of period t, a household of type γ starts the period with money balances, $M(s^{t-1}, \gamma)$, in the goods market. They can then decide whether to transfer additional funds, $P(s^t)x(s^t, \gamma)$, between the goods and asset markets. We define the indicator variable $z(s^t, \gamma)$ to equal zero if these transfers are zero and one if a household makes a transfer. We also allow the stream of income from an annuity purchased by the household at date 0 to be available for consumption at dates $t \geq 1$. Unlike the fixed cost of transferring $x(s^t, \gamma)$ across markets, we assume that there are no fixed costs associated with the annuity, so that a household's initial allocation of cash across markets is a relatively costless one.

With these sources of cash available for consumption, a household faces the cash-in-advance constraint:

$$P(s^t)c(s^t, \gamma) = M(s^{t-1}, \gamma) + P(s^t)x(s^t, \gamma)z(s^t, \gamma) + P(s^t)A(\gamma), \tag{10}$$

where $A(\gamma)$ is the constant stream of income in units of consumption derived from the annuity purchased by household γ at date 0. Equation (10) holds as a strict equality, implying that a household can not store cash in the goods market from one period to the next.[6] This assumption greatly simplifies our analysis, and in the appendix, we provide sufficient conditions for this constraint to hold strictly as an equality. In the asset market, a household of type γ begins the period with holdings of bonds, $B(s^t, \gamma)$, and receives a return on any equity purchased in the

[6] For models in which households choose to store cash in the goods market, see Abel, Eberly, and Panageas (2007), Khan and Thomas (2007), and Alvarez, Atkeson, and Edmond (2003).

previous period. Both of these sources of funds are available as cash in the asset market and can either be reinvested, or if the household pays its fixed cost, the funds can be transferred to the goods market. Accordingly, the household's cash constraint in the asset market at dates $t \geq 1$ is given by:

$$B(s^t,\gamma) + (1+R^e(s^t))S(s^{t-1},\gamma) = \int_{s_{t+1}} q(s^t,s_{t+1})B(s^t,s_{t+1},\gamma)ds_{t+1} + \\ S(s^t,\gamma) + P(s^t)[x(s^t,\gamma)+\gamma]z(s^t,\gamma). \tag{11}$$

As shown in the appendix, this constraint is satsified as a strict equality as long as nominal interest rates are positive. In period 0, a household's cash constraint in the asset market reflects its purchases of the annuity and its initial level of assets, $\overline{B}(\gamma)$:

$$\overline{B}(\gamma) = \int_{s_1} q(s^1)B(s^1)ds_1 + P_A A(\gamma). \tag{12}$$

The constant payment stream $A(\gamma)$ is independent of the state of nature but does depend on a household's type. A household with a relatively high γ will in general demand a different quantity of the annuity than a low cost household. In addition, a household that purchases a positive amount of the annuity, may still choose to pay its fixed cost γ, and make a state-dependent transfer, $x(s^t,\gamma)$, in periods in which a large enough shock occurs. Since doing so essentially involves reallocating cash in the two markets away from a household's initial allocation, $A(\gamma)$, we call this version of the model, the "endogenous rebalancing model."

Given that both the cash-in-advance constraint, equation (10), and asset market constraint, equation (11), hold with strict equality, we can write a household's budget constraint for $t \geq 1$ simply as:

$$M(s^t,\gamma) = P(s^t)w(s^t). \tag{13}$$

This condition greatly simplifies our analysis, because it implies that at the beginning of each period, all households regardless of type hold the same amount of cash. Consequently, we do not need to keep track of each households' money holdings over time. It is also convenient to substitute this expression for $M(s^t,\gamma)$ into the cash-in-advance constraint, and rewrite it as:

$$c(s^t,\gamma) = \frac{w(s^{t-1})}{\pi(s^t)} + x(s^t,\gamma)z(s^t,\gamma) + A(\gamma). \tag{14}$$

When $A(\gamma) = 0$ for all γ, then our model is similar to the one studied by Alvarez, Atkeson, and Kehoe (2007b). In this case, asset markets are completely segmented from goods markets for agents who do not transfer $x(s^t, \gamma)$ between them. As a result, consumption of these inactive households is simply based on their previous period's wage income, as their consumption is completely isolated from either stock or bond returns. We call this version of our model, the "endogenous participation model", because the decision to transfer funds from asset to the goods market determines whether a household participates in financial markets. In contrast, in the "endogenous rebalancing" model, all households participate in financial markets. Instead, their decision to transfer funds $x(s^t, \gamma)$ at date t amounts to a rebalancing of cash from their initial allocation determined at date 0.

In the endogenous rebalancing model, a household's problem is to choose $A(\gamma)$ and $\{c(s^t, \gamma), x(s^t, \gamma), z(s^t, \gamma), M(s^t, \gamma), B(s^t, \gamma), S(s^t, \gamma)\}_{t=1}^{\infty}$ to maximize:

$$\sum_{t=1}^{\infty} \beta^t \int_{s^t} U(c(s^t, \gamma)) g(s^t) ds^t \tag{15}$$

subject to equations (10)-(13), taking prices and initial holdings of money, bonds, and stocks as given. In equation (15), the function $g(s^t)$ denotes the probability distribution over history s^t. We also assume that preferences are given by:

$$U(c) = \frac{c^{1-\sigma}}{1-\sigma}, \tag{16}$$

where σ is the coefficient of constant relative risk aversion.

2.4 Market Clearing

The resource constraint in our economy is given by:

$$Y(s^t) = \int_0^{\infty} \left[c(s^t, \gamma) + \gamma z(s^t, \gamma)\right] f(\gamma) d\gamma, \tag{17}$$

while market clearing in factor markets requires $K(s^t) = 1$ and $L(s^t) = 1$. In asset markets, for stock and bond markets to clear at dates $t \geq 0$ we have:

$$S(s^t) = \int_0^{\infty} S(s^t, \gamma) f(\gamma) d\gamma \qquad\qquad B(s^{t+1}) = \int_0^{\infty} B(s^{t+1}, \gamma) d\gamma.$$

At date 0, we also have $\bar{B} = \int_0^\infty \bar{B}(\gamma)f(\gamma)d\gamma$ and $A_0 = \int_0^\infty A(\gamma)f(\gamma)d\gamma$. Market clearing in the money market for $t \geq 1$ is given by:

$$\int_0^\infty \left\{M(s^{t-1},\gamma) + P(s^t)[x(s^t,\gamma) + \gamma]z(s^t,\gamma) + P(s^t)A(\gamma)\right\} f(\gamma)d\gamma = M_t, \tag{18}$$

where the presence of the fixed cost reflects it is paid using cash from the asset market.

An equilibrium is a collection of asset prices, $\{P_A, q(s^t), R^e(s^t)\}$, wages, and goods prices $\{P_k(s^t), P(s^t)\}$ which together with money, asset holdings, and allocations, $\{c(s^t,\gamma), x(s^t,\gamma), z(s^t,\gamma))\}$, for households that for each transfer cost γ, the asset holdings and allocations solve the the households' optimization problem. These prices together with the allocations $\{S(s^t), K(s^t), Y(s^{t+1}), L(s^{t+1})\}$ satisfies the firms' optimization problems. Finally, the government budget constraint holds and the resource constraint along with the market clearing conditions for capital, labor, bonds, stocks, and money are all satisfied.

2.5 Equilibrium Characterization

We now solve for the equilibrium consumption and transfers of households in the endogenous rebalancing model. We show that there will be a variable fraction of active households that choose to make the state-dependent transfer and a remaining fraction that does not. We then characterized the link between the consumption of active households and equity returns.

2.5.1 Consumption and Transfers

We begin by characterizing a household's consumption conditional on their choice of paying the fixed cost of making a state dependent transfer (i.e., their choice of $z(s^t,\gamma)$. To do so, we use the fact that market clearing of factor markets implies that $Y(s^t) = \exp((1-\alpha)z_t)$. In addition, we can determine the economy's price level and inflation rate by substituting equations (10) and (17) into the money market clearing condition to write:

$$P(s^t) = \frac{M_t}{\exp[(1-\alpha)z_t]}, \qquad \pi(s^t) = \frac{\mu_t}{\exp[(1-\alpha)(z_t - z_{t-1})]}. \tag{19}$$

With these expressions, we can show that the consumption of an inactive household (i.e., one that sets $z(s^t,\gamma)=0$) is given by:

$$c_I(s^t,\gamma) = \frac{w(s^t)}{\mu_t} + A(\gamma) = \frac{(1-\alpha)\exp[(1-\alpha)z_t]}{\mu_t} + A(\gamma). \tag{20}$$

From this expression, we can see that inflation is distortionary, since it reduces the consumption of inactive households. This effect induces some households to pay the fixed cost and become active. An increase in technology will also induce households to become active. While the consumption of inactive households rises due to an increase in wages following a technology shock, the benefits of being active are even greater, reflecting that active consumption is also boosted by higher capital income.

There is perfect risk-sharing amongst active households, and these agents have identical consumption for aggregate state s^t:

$$c_A(s^t,\gamma) = c_A(s^t). \tag{21}$$

Thus, the consumption of active households is independent of γ. To further characterize, the consumption of active and inactive households, we need to determine $A(\gamma)$, the initial allocation of cash from the asset market that is devoted to consumption. In the appendix, we show that the price of the annuity is given by

$$P_A = \sum_{t=1}^{\infty} \int_{s^t} Q(s^t)P(s^t)ds^t, \tag{22}$$

where $Q(s^t) = \Pi_{j=1}^t q(s^j)$ and a household's choice of $A(\gamma)$ must satisfy:

$$\sum_{t=1}^{\infty} \beta^t \int_{s^t} \left[U'(c_A(s^t)) - U'(c_I(s^t,\gamma))\right](1-z(s^t,\gamma))g(s^t)ds^t = 0. \tag{23}$$

This latter condition states that in the states of the world in which a household is inactive (i.e., $z(s^t,\gamma)=0$), the household chooses $A(\gamma)$ to equate the expected discounted value of marginal utility of its consumption to the expected discounted value of the marginal utility of the consumption of the active households in those states of the world. Thus, the annuity provides a minimal level of consumption insurance to a household whose fixed cost γ is so large that they would never actively rebalance their cash holdings away from $A(\gamma)$. Moreover, an agent who never actively rebalances their cash accounts will choose $A(\gamma) > 0$. This reflects that consumption of the active households reflects capital income from equity markets, and the consumption of a household that was always inactive and did not purchase the annuity would not. Accordingly, the only way for a completely inactive household to satisfy equation (23) is to purchase the annuity at date 0 (i.e., set $A(\gamma) > 0$) and in this way get some of the proceeds from capital income.

While equation (23) places restrictions on the choice of $A(\gamma)$ for agents that are inactive in at least one state of the world, this condition is irrelevant for agents that actively rebalance their portfolios in each state of the world. In this case, a household's choice of $A(\gamma)$ is redundant, since they can use $x(s^t, \gamma)$ to achieve their desired level of consumption. Consequently, it is convenient to define the function $A(\gamma)$ only for γ such that $\gamma > \gamma_{Min} \geq 0$, where γ_{Min} is defined as the household which is active in every state of the world.

With the $A(\gamma)$ defined only for $\gamma > \gamma_{Min} \geq 0$, we now turn to characterizing a household's decision to actively rebalance or not. In the appendix, we provide sufficient conditions for the existence of an equilibrium. In addition, we show that the optimal choice of $z(s^t, \gamma)$ has a cutoff rule in which households with $\gamma \leq \bar{\gamma}(s^t)$ pay the fixed cost and consume $c_A(s^t)$. In addition, the equilibrium values of the fixed cost of the marginal household, $\bar{\gamma}(s^t)$, and $c_A(s^t)$ satisfy:

$$F(\bar{\gamma}(s^t))c_A(s^t) + \int_{\bar{\gamma}(s^t)}^{\infty} [\frac{w(s^t)}{\mu_t} + A(\gamma)]f(\gamma)d\gamma = \exp[(1-\alpha)z_t] - \int_0^{\bar{\gamma}(s^t)} \gamma f(\gamma)d\gamma, \tag{24}$$

$$U[c_A(s^t)] - U[\frac{w(s^t)}{\mu_t} + A(\bar{\gamma}(s^t)] = U'[c_A(s^t)] \left[c_A(s^t) - \frac{w(s^t)}{\mu_t} - A(\bar{\gamma}(s^t)) + \bar{\gamma}(s^t) \right]. \tag{25}$$

Equation (24) is the resource constraint expressed using the equilibrium consumptions of inactive and active types as well as $\bar{\gamma}(s^t)$. Equation (25) states that the net gain for the marginal household of actively rebalancing is equal to the cost of transferring funds across the two markets. The net gain, $U(c_A(s^t)) - U(\frac{w(s^t)}{\mu_t} + A(\bar{\gamma}(s^t))$, is simply the difference in the level of utility from being active as opposed to inactive, while the cost reflects the fixed cost of transferring funds plus the amount transferred by the marginal household (i.e., $x(s^t, \bar{\gamma}(s^t)) = c_A(s^t) - \frac{w(s^t)}{\mu_t} - A(\bar{\gamma}(s^t))$.

According to equations (24) and (25), the fixed cost of the marginal investor as well as the consumption of active rebalancers depend on the current levels of technology and money growth and not future or past values of the shocks. These variables also depend on the initial allocation of funds devoted to the goods market, $A(\gamma)$. More generally, equations (23)-(25) jointly determine $c_A(s^t)$, $\bar{\gamma}(s^t)$), and the function $A(\gamma)$ for $\gamma > \gamma_{Min}$.

To solve for these variables, we need to approximate the function $A(\gamma)$ for $\gamma > \gamma_{Min}$. Given $A(\gamma)$, we can then use equations equations (24) and (25) to determine $c_A(s^t)$ and $\bar{\gamma}(s^t)$ exactly. To approximate $A(\gamma)$, we re-express equation (23) as a stochastic difference equation and following Judd (1999) use the linear Fredholm integral equations and quadrature to solve $A(\gamma)$ at a finite number of points. We then determine $A(\gamma)$ using piecewise linear interpolation. For details regarding this solution procedure, see the appendix.

2.5.2 Consumption of Rebalancers and the Equity Premium

In our economy, the asset pricing kernel depends on the consumption of the rebalancers and is given by:

$$m(s^t, s_{t+1}) = \beta \frac{U'[c_A(s^{t+1})]}{U'[c_A(s^t)]}. \tag{26}$$

This pricing kernel is the state-contingent price of a security expressed in consumption units and normalized by the probabilities of the state. This pricing kernel can be used to determine the real risk-free rate (r^f) as well as the real return on equity (r^e). These returns are given by:

$$[1 + r^f(s^t)]^{-1} = \int_{s_{t+1}} m(s^t, s_{t+1}) g(s_{t+1}|s^t) ds_{t+1}, \tag{27}$$

$$1 = \int_{s_{t+1}} m(s^t, s_{t+1})[1 + r^e(s^t, s_{t+1})] g(s_{t+1}|s^t) ds_{t+1}, \tag{28}$$

where $g(s_{t+1}|s^t) = \frac{g(s^{t+1})}{g(s^t)}$ denotes the probability of state s_{t+1} conditional on state s^t. From equation (29) in the firm's problem, the equilibrium real return on equity is given by:

$$1 + r^e(s^{t+1}) = \frac{[\alpha \exp[(1-\alpha) z_{t+1}] + p_k(s^{t+1})]}{p_k(s^t)}. \tag{29}$$

Using these two equations, we can then define the equity premium in our economy as:

$$\frac{\mathrm{E}_t[1 + r^e_{t+1}]}{1 + r^f_t} = 1 - \mathrm{cov}_t \left(m_{t+1}, 1 + r^e_{t+1} \right), \tag{30}$$

where for convenience we have switched notation to express both the expected return on equity and the covariance between the pricing kernel and the return on equity, which are both conditional on the state of the world at date t.

To solve for asset prices in our economy, we need to determine the price of capital, which is equivalent to stock prices in our economy. To do so, we use equations (28) and (29) to express the price of capital as a stochastic difference equation. As discussed in the appendix, we then use the linear Fredholm integral equations and the Nystrom extension to determine the decision rule for the price of capital.

2.6 Parameter Values

For our benchmark calibration, we set the discount factor, $\beta = 0.99$, to be consistent with a quarterly model. The economy's capital share, α, is equal to 0.36. For the coefficient of relative risk aversion, we used a range of values between 2 and 4, broadly consistent with the survey of the literature in Hall (2008). For the distribution over the fixed cost, we choose benchmark values of $\gamma_m = \exp(\tilde{\gamma}_m) = 0.02$, $\sigma_\gamma = 0.35$, and $F(0) = 0$. Since this distribution is crucial to our analysis, we also consider a range of values for γ_m. In our model, absent the small average transaction cost, the aggregate consumption process corresponds to the technology shock. Therefore, we calibrated the parameters governing the process for technology based on the time series properties of aggregate consumption. We set $\rho_z = 0.97$ and based on annual consumption data from 1889-2004, we set $\sigma_z = 0.013$. These values imply that the annualized standard deviation for consumption growth in our model is slightly higher than 3 percent. For the money growth process, we used quarterly data on M2 over the sample period 1959:Q1-2007:Q4 and estimated $\rho_\mu = 0.68$ and $\sigma_\mu = 0.007$. We set $\bar{\mu}$ to be consistent with an average, annualized money growth rate of 4%.

3 Results

Before discussing the model's implications for monetary policy and the equity premium, it is helpful to characterize the non-stochastic steady state of our model.

3.1 Deterministic Steady State

In a deterministic environment, the endogenous rebalancing model reduces to a representative agent economy. This model only becomes interesting in the presence of uncertainty; nevertheless, it is useful to compare its deterministic steady state with the version of the model with endogenous participation.

In the non-stochastic steady state of the endogenous rebalancing model, all households will have the same level of consumption. According to equation (23), a household that chooses to be inactive obtains the same level of consumption as an active household. An inactive can obtain such a level of consumption by choosing their annuity such that $c_A = c_I = \frac{1-\alpha}{\mu} + A$, where A is the constant value of the annuity for all inactive households. With consumption the same across households, all households with $\gamma > 0$ will never rebalance their portfolios, and the households with $\gamma = 0$ will be indifferent between rebalancing or using the non-state contingent transfer, A.

In the non-stochastic steady state of the endogenous participation model (i.e., $A(\gamma) = 0$ for all γ), the consumption of active households exceeds the consumption of inactive households, who only receive $c_I = \frac{1-\alpha}{\mu}$. By choosing to be inactive, these agents do not receive the capital income associated with participating in the stock market. Given this difference in consumption levels, all households will participate in financial markets for our benchmark value of γ_m and σ_γ. Later, we consider alternative calibrations in which the participation rate in financial markets is less than one.

3.2 Endogenous Rebalancing and the Equity Premium

Figure 1 shows the sample averages for the risk-free rate and the equity premium (see the red dot labeled "U.S. Data") taken from Cecchetti, Lam, and Mark (1993). We also report 5% confidence ellipse, based on their estimates. The figure also shows the average equity premium and risk-free rate in the endogenous rebalancing model using our benchmark calibration with $\sigma = 3$.[7] This calibration leads to an average equity premium of 6.6% and a risk-free rate of 1.5%.

Moving from southeast to northwest along the blue line with circles, each point reports the equity premium and risk-free rate for different values of γ_m, which leads to a different average fraction of rebalancers. There are two basic results that can be evinced from this line. First, decreasing the average fraction of rebalancers (by increasing γ_m) produces a rise in the equity premium and fall in the risk-free rate. Second, if the fraction of household rebalancers lies between 10% and 20%, then our model lies within the 95% confidence region. The green line with squares in Figure 1 shows the results if we vary σ in the endogenous rebalancing model. Raising the coefficient of relative risk aversion has a similar effect on the equity premium and risk-free rate as lowering the average fraction of rebalancers via γ_m. As the coefficient of relative risk aversion increases, the mean equity premium rises and the risk-free rate falls. For values of σ between 2 and 4, the combination of mean returns on equity and the risk-free asset lies within the 95% confidence region. For comparison purposes, the magenta line with triangles in the southeast corner of the figure shows the results for the standard cash-in-advance economy with a representative agent. In this model, as in Mehra and Prescott (1985), the only way to match the observed equity premium is to increase the coefficient of relative risk aversion to an implausibly high level.

Figure 2 shows the mean of the equity premium and risk-free rate for the endogenous participation model of Alvarez, Atkeson, and Kehoe (2007b) with $\sigma = 3$. The square along the green line in southeast corner of the figure shows the results for our calibration of this model for different values of the capital share, α. For $\alpha = 0.36$, the model fails to match the

[7] These results are based on simulating the model economy 500 times using a sample size of 200 observations.

observed value of the equity premium, as the average participation rate is 1. Moving towards the northwest along the squared-green line, the value of α is lowered so that the capital income share becomes smaller. Only when the income obtained from equity is implausibly small can the limited participation model account for the observed equity premium.[8]

3.3 Understanding the Mechanism

Figure 2 demonstrates the critical role that the annuity plays in our analysis in accounting for the equity premium puzzle. When $A(\gamma) = 0$ for all γ, as in the endogenous participation model, a fraction of agents are excluded from financial markets. Given a reasonable capital share, there is a large incentive to participate in equity markets and receive a share of the capital income. This incentive makes it difficult for this model to match the average equity premium. In contrast, in the endogenous rebalancing model, through their choice of $A(\gamma)$ at date 0, all households participate in financial markets. However, some households, because of their fixed cost, rely entirely on a non-state contingent plan for transferring funds between asset and goods markets.

The top panel of Figure 3 shows the demand schedule for the annuity for households with a fixed cost greater than zero (i.e., households that rebalance at least once in their lifetime). Demand for the annuity is increasing in a household's fixed cost up to a threshold value of γ. This function is increasing, because a household with a higher fixed cost anticipates that they will be rebalance their portfolio less frequently and demands a larger value of the annuity to help insure against consumption losses. Households at or beyond the threshold value of γ never actively use state-contingent transfers between asset and goods markets. Accordingly, these households all choose the same level of the annuity. Figure 3 also makes clear that in equilibrium, the demand for the annuity is relatively constant across households, varying between 0.361 and 0.364, or a level slightly higher than the economy's capital share.

[8]We also raised the value of γ_m, which has the effect of lowering the average participation rate in financial markets. Values of γ_m that imply participation rates as low as 10% are still outside the 95% confidence region for the mean risk-free rate and equity premium.

To understand the role of $A(\gamma)$ in helping the endogenous rebalance model account for the data, we begin by noting that the equilibrium in our model can be described by equations (24) and (25) for a given $A(\gamma)$. These two equations determine the combination of equilibrium values of the consumption of rebalancers and the fixed cost of the marginal rebalancer. Given that $A(\gamma)$ is nearly constant, it is helpful for illustrative purposes to assume momentarily that $A(\gamma)$ is constant for all inactive households. Furthermore, To get closed form solution for these equations and without loss of generality, we also assume that $\sigma = 2$ and $F(\gamma)$ is uniformly distributed between $[0, \gamma_u]$.[9] Under these assumptions, it is convenient to rewrite these two equation as follows:[10]

$$\frac{\bar{\gamma}}{\gamma_u} c_A + (1 - \frac{\bar{\gamma}}{\gamma_u}) c_I = \exp[(1-\alpha)z] - \frac{\bar{\gamma}^2}{2\gamma_u}, \tag{31}$$

$$(c_A - c_I)^2 = c_I \bar{\gamma}. \tag{32}$$

The first equation represents the combination of values of $\bar{\gamma}$ and c_A that satisfy goods market clearing, and can be used to express the fraction of rebalancers (i.e., $\frac{\bar{\gamma}}{\gamma_u}$) as a function of their consumption. We call this schedule the GM curve for goods market clearing. For $c_A > c_I$ where $c_I = \frac{1-\alpha}{\mu} + A$, the middle panel of Figure 3 shows that this curve is downward sloping. This reflects that an increase in γ raises the average level of transaction costs in the economy, reducing real resources, and lowering the consumption of rebalancers.

The second equation is defined over the same variables and characterizes the marginal household's decision to rebalance its portfolio (i.e., the type $\bar{\gamma}$ household). We call this schedule the MR curve in reference to this marginal rebalancer. For $\sigma = 2$, the middle panel of Figure 3 shows that this curve is a parabola with a minimum occurring at c_I. For $c_A > c_I$, the state-contingent transfer of an active household (i.e. $x = c_A - c_I$) is positive, as is the cost of transferring funds. However, a rise in the consumption of active households, all else equal, makes it more attractive to make the state contingent transfer, and thus for $c_A > c_I$, the MR schedule is increasing.

[9]Using alternative values for σ, and a normal distribution for γ, does not alter the subsequent analysis but it eliminates an analytical expression to characterize the equilibrium of the model.

[10]Given that these equations are static, we simplify our notation, ignoring that these variables depend on s^t.

Figure 3 displays the equilibrium in the endogenous rebalancing model as the intersection of these two curves. In the endogenous rebalancing model, this intersection occurs at a point in which $c_A > c_I$, though consumption of a rebalancer is not much higher than the consumption of a non-rebalancer, reflecting their purchase of the annuity. In contrast, the bottom panel of Figure 3 shows that the equilibrium in the endogenous participation model occurs at a point in which active consumption is much higher than inactive consumption (given by the minimum of the parabola). The equilibrium financial market participation rate shown in the bottom panel is also much higher than the the rate of rebalancing that occurs in equilibrium in the middle panel.

This difference in the equilibrium positions of the two economies has important implications for the volatility of the consumption of active households and therefore the equity premium. To demonstrate this using our illustrative example, Figure 4 displays the effects of a deterministic increase in technology on the equilibrium allocations implied by equations (31) and (32). An increase in technology shifts the GM curve upward and to the right, as the economy's resources expand. The MR curve shifts to the right, as the wage income of non-rebalancers rises. This boosts the consumption of non-rebalancers from c_{I0} to c_{I1}.

With the initial equilibrium occurring near the minimum of the parabola, there is a large increase in the consumption of non-rebalancers that exceeds both the increase in non-rebalancer consumption and technology. This large increase reflects that the technology shock, by raising the return on equity, also involves a redistribution away from non-rebalancers to rebalancers. While this redistribution occurs in the endogenous participation model, its effects on active consumption are modest, given that the initial equilibrium is at a point at which the MR curve is relatively steep.

An important implication of Figure 4 is that the consumption of rebalancers will be considerably more volatile than the consumption of non-rebalancers and aggregate consumption, if the fraction of rebalancers is relatively small. To demonstrate this, Figure 5 returns to the benchmark calibration of the endogenous rebalancing model in which the fixed cost is normally distributed. The upper panel displays that the probability of rebalancing is monotonically decreasing in a household's fixed cost. Roughly ten percent of households will rebalance more than 80 percent of the time, while more than 50 percent of the households do not reallocate cash from asset to goods markets after making their initial non-state contingent allocation. Thus, in line with the microdata, there is considerable heterogeneity in portfolio rebalancing, with a large group of households exhibiting substantial inertia in their portfolio allocation.

The middle panel of Figure 5 displays the mean level of consumption for different types of households. As suggested by our illustrative example involving the GM and MR curves, households that rebalance more frequently have a slightly higher average level of consumption than non-rebalancers. In addition, the consumption of a household that frequently rebalances its portfolio is about four times more volatile than a household that keeps their portfolio unchanged at its initial allocation. In effect, households that rebalance frequently are trading off higher consumption volatility against a higher mean level of consumption. Interestingly, this does not contradict recent evidence provided by Parker and Vissing-Jorgensen (2009). Using data from the Consumer Expenditure (CEX) Survey, they found that the consumption of 'high-consumption' households is more exposed to fluctuations in aggregate consumption (and income) than that of low-consumption households. In addition, they did quantify that exposure to aggregate consumption growth of households in the top 10 percent of the consumption distribution is about five times that of households in the bottom 80 percent.

This higher volatility of the consumption of rebalancers helps explain why the endogenous rebalancing model can account for the equity premium puzzle. In the standard CIA model, the pricing kernel depends on aggregate consumption. While aggregate consumption growth and the return on equity are positively correlated, the covariance between these variables is small, reflecting the low volatility of aggregate consumption. In the endogenous rebalancing model, the volatility of aggregate consumption growth is also low, but the pricing kernel depends on the consumption of active rebalancers.

3.4 Monetary Policy and Equity Prices

In the endogenous rebalancing model, technology shocks account for roughly 80 percent of the mean excess return on equity. Still, monetary policy shocks can induce important fluctuations in equity prices and the equity premium.[11] In this section, we investigate this relationship and compare our model's implications to the estimates of Bernanke and Kuttner (2005).

Using high-frequency data on the federal funds rate, Bernanke and Kuttner (2005) construct a measure of unanticipated changes in monetary policy. They find that a broad index of stock prices registers a one-day gain of 1 percent in reaction to a 25 basis point easing of the federal funds rate. Building on the analysis of Campbell (1991) and Campbell and Ammer (1993), Bernanke and Kuttner (2005) then use a structural VAR to decompose the response of stock prices into three components: changes in current and expected future dividends, changes in current and expected future real interest rates, and changes in expected future excess equity returns or equity premia. While an unanticipated easing lowers interest rates, they conclude that an important channel in which stock prices increase occurs through changes in the equity premia or the perceived riskiness of stocks.

To compare our model's implications to these stylized facts, we compute impulse response functions from the endogenous rebalancing model. Since our model is nonlinear, it is important to define how we construct these impulse responses. Follow the discussion in Hamilton (1994), we define the impulse response of variable, $y(s^t)$, at date t to a monetary innovation that occurs at date 1 as:

$$E[log\left(y(s^t)\right) \mid \mu_1, z_0] - E[log\left(y(s^t)\right) \mid \mu_0, z_0], \forall t \geq 1, \quad (33)$$

where $\mu_0 = \bar{\mu}$ and $z_0 = \bar{z}$. Thus, an impulse response to a monetary shock occurring at date 1 is defined as the revision in expectations from a variable's unconditional mean. For log-linear models, equation (33) simplifies to the usual analytical representation in which (up to a scaling factor) the model's linear coefficients characterize the impulse response function. Since in our

[11] In Gust and López-Salido (2009) we provide further analysis on the role of technology shocks in accounting for asset pricing puzzles.

context evaluating the expectations in equation (33) involves multidimensional integrals, we use Monte Carlo integration to compute the impulse response functions. The details of this procedure are discussed in the appendix.

Figure 6 shows the impulse response to a monetary innovation that raises money growth a quarter of a standard deviation at date 1. The solid blue lines in the figure show the results in which money growth, as in Alvarez, Atkeson, and Kehoe (2007b), has an autocorrelation of 0.9, and the dashed red line shows the results for our estimated autocorrelation of 0.68. Starting with the blue line, the nominal interest rate falls about 100 basis points on impact, and the real rate falls about 150 basis points. Thus, as the limited participation models of Lucas (1990) and Fuerst (1992), our economy displays a significant liquidity effect. Moreover, as in AAK (2001), this liquidity effect is persistent, as interest rates gradually rise back to their unconditional means.

The unanticipated monetary expansion induces a significant increase in the price of equity. Equity prices rise about 3 percent on impact, implying a multiplier of 3 from the 100 basis point easing in monetary policy. Such a multiplier is in line with Bernanke and Kuttner (2005), who estimate multipliers between 3 and 6, when they take into account sampling uncertainty. On impact, the equity premium moves down 1.5 percentage points, and its response mirrors that of equity prices. Dividends in our model are simply a function of technology and do not change in response to the monetary innovation. Thus, the increase in stock prices reflects both the fall in real rates and the decline in the risk premium, with these variables playing a roughly equal role in accounting for the higher return on equity. Thus, our model is broadly consistent with the evidence presented in Bernanke and Kuttner (2005).

Figure 6 also shows the response of consumption of rebalancers and non-rebalancers and the fraction of rebalancers. A monetary injection has a redistributive effect, as it reduces the consumption of non-rebalancers, whose real money balances available for consumption fall, and raises the consumption of those that choose to rebalance. This redistributive effect induces more households to rebalance and the fraction of rebalancers rises about 0.6 percentage points on impact. In this regard, our model has a similar mechanism to Alvarez, Atkeson, and Kehoe

(2007b), but our focus is on the intensive margin (i.e., the frequency in which a household uses a state contingent transfers rather than the non-state contingent transfer) rather than the extensive margin (i.e., the participation rate in financial markets).

To understand why a monetary easing induces a decline in the equity premium, it is helpful to revisit our illustrative example and consider the effect of a deterministic change in money growth on the GM and MR schedules. The top panel of Figure 7 shows the effects of a small positive increase in the money growth. This increase shifts the GM curve to the right, since the consumption of rebalancers rises for a fixed $\bar{\gamma}$. In addition, the lower consumption of the non-rebalancers (which occurs at the minimum of the parabola) shifts the MR curve upward and to the left, implying that the benefit to making the state-contingent transfer has gone up. Hence, the deterministic increase in money growth leads to an equilibrium with both higher consumption of active rebalancers and a higher fraction of rebalancers.

With the increase in money growth occurring from an initial equilibrium close to the minimum of the parabola, a small monetary expansion may induce a relatively large increase in the consumption of rebalancers. However, this effect diminishes as the monetary shock becomes larger, reflecting the concavity of the MR schedule. Near the initial equilibrium, small increases in monetary policy induce relatively large increases consumption, while larger shocks lead to smaller effects on consumption. Intuitively, as the shock becomes larger, a greater fraction of households rebalance their portfolios, so that the nominal shock begins to have smaller and smaller real effects.

Building on this intuition, the top two panels of Figure 8 show the first and second derivatives of the logarithm of active consumption with respect to the logarithm of money growth in the neighborhood of the unconditional mean for money growth. The top panel shows that the first derivative (i.e., $\frac{\partial \log c_A(\mu_t)}{\partial \log \mu_t}$) is positive and decreasing, reflecting that higher money growth boosts active consumption but by progressively less. The middle panel shows that the second derivative is increasing, reflecting the high degree of concavity of active consumption in the neighborhood of the unconditional mean rate of money growth.

This nonlinearity drives the endogenous fluctuations in risk in our model. An increase in money growth reduces the sensitivity of active consumption to expected future changes in money growth, as the fraction of active rebalancers increases. Thus, for higher rates of money growth, active consumption growth becomes less volatile and its covariance with the return on equity diminishes, leading to a decline in the equity premium. The middle panel of Figure 7 also suggests that this logic holds in reverse for small monetary contractions. In particular, a monetary contraction induces a relatively large decline in active consumption, with active consumption becoming more volatile. With the return on equity also falling in response to a monetary contraction, active rebalancers demand a higher risk premium on equity.

Another important nonlinearity in our model is that the fraction of rebalancers may actually rise for a large enough contraction in money growth. This possibility is illustrated in the bottom panel of Figure 7 which shows that the GM curve becomes an upward sloping function that intersects the MR curve to the left of its minimum value. For large monetary contractions, households real money balances are high and the cost of making the state-contingent transfer becomes negative, since $x = c_A - c_I < 0$. Thus, more households choose to become active and transfer funds into their brokerage accounts.

To illustrate that larger shocks induce a higher fraction of rebalancers, the bottom panel of Figure 8 displays the regions of inactive and active rebalancing for the distribution of households as a function of the deterministic rate of money growth. The blue line in the figure indicates the marginal household, who is indifferent between keeping its cash portfolio unchanged and using the state-contingent transfer. For larger shocks, whether positive or negative, more households opt to become active and make use of their state contingent transfer.

4 Conclusions

We have developed a dynamic stochastic general equilibrium model in which monetary policy affects the economy through its effect on risk premium. Our model can be viewed as an extension of the neoclassical framework by incorporating segmentation between asset and goods markets. In this sense, we hope to the next link in the chain beginning with Lucas (1990) and recently extended by Alvarez, Atkeson, and Kehoe (2002). However, we depart from the former in two important respects. First, we explicitly model a production economy with equity returns. Second and more importantly, all households participate in financial markets in our environment, but it is costly to make state contingent transfers between asset and goods markets. In other words, our model emphasizes endogenous asset segmentation along an intensive margin (i.e., portfolio rebalancing decision) rather than along the extensive margin (i.e., participation decision).

This modification allows to introduce inertia in risk-taking as a key source of heterogeneity in household's portfolio allocation. Through this mechanism, we are able to account for the average excess returns on equity and endogenous movements in risk following a monetary shock. In addition, in line with the evidence of Bernanke and Kuttner (2005), a monetary easing can lead to a decline in equity premium, because more households choose to transfer funds between asset and goods markets.

A natural extension of this paper will be to incorporate endogenous capital and labor supply considerations to examine whether the model can account for key features of both asset prices and business cycles.

References

Abel, A. B., J. C. Eberly, and S. Panageas (2007). Optimal Inattention to the Stock Market and Information Costs and Transaction Costs. mimeo, Wharton School of the University of Pennsylvania.

Alvarez, F., A. Atkeson, and C. Edmond (2003). On the Sluggish Response of Prices to Money in an Inventory-Theoretic Model of Money Demand. National Bureau of Economic Research Working Paper 10016.

Alvarez, F., A. Atkeson, and P. Kehoe (2007a). If Exchange Rates are Ramdom Walks, Then Almost Everything we Say About Monetary Policy is Wrong. Federal Reserve Bank of Minneapolis, Research Dept. Staff Report 388,.

Alvarez, F., A. Atkeson, and P. Kehoe (2007b). Time-Varying Risk, Interest Rates, and Exchange Rates in General Equilibrium. Federal Reserve Bank of Minneapolis, Research Dept. Staff Report 371,.

Alvarez, F., A. Atkeson, and P. J. Kehoe (2002). Money, Interest Rates, and Exchange Rates with Endogenously Segmented Markets. *Journal of Political Economy 110*, 73–112.

Bernanke, B. and K. Kuttner (2005). What Explains the Stock Markets Reaction to Federal Reserve Policy? *Journal of Finance LX*, 1221–1257.

Bilias, Y., D. Georgarakos, and M. Haliassos (2008). Portfolio Inertia and Stock Market Fluctuations. CFS Working Paper 2006/4. Revised July 2008.

Boldrin, M., L. J. Christiano, and J. D. Fisher (1997). Habit Persistence and Asset Returns in an Exchange Economy. *Macroeconomic Dynamics 1*, 312–332.

Brunnermeier, M. and S. Nagel (2008). Do Wealth Fluctuations Generate Time-Varying Risk Aversion? Micro-Evidence from Individuals Asset Allocation. *American Economic Review 98*, 713–736.

Calvet, L. E., J. Y. Campbell, and P. Sodini (2008). Fight or Flight? Portfolio Rebalancing by Individual Investors. *Quarterly Journal of Economics*. forthcoming.

Calvet, L. E., J. Y. Campbell, and P. Sodini (2009). Measuring the Financial Sophistication of Households. *American Economic Review, Papes and Proceedings*. forthcoming.

Campbell, J. (1991). A Variance Decomposition for Stock Returns. *Economic Journal 101*, 157–179.

Campbell, J. and J. Ammer (1993). What Moves the Stock and Bond Markets? A Variance Decomposition for Long-Term Asset Returns. *Journal of Finance 48*, 3–37.

Cecchetti, S., P.-S. Lam, and N. Mark (1993). The Equity Premium and the Risk-Free Rate: Matching the Moments. *Journal of Monetary Economics 31*, 21–45.

Fuerst, T. S. (1992). Liquidity, Loanable Funds, and Real Activity. *Journal of Monetary Economics 29*, 3–24.

Gabaix, X. and D. Laibson (2001). The 6D Bias and the Equity-Premium Puzzle. In B. Bernanke and J. Rotemberg (Eds.), *NBER Macroeconomics Annual*. MIT Press.

Gomes, F. and A. Michaelides (2006). Asset Pricing with Limited Risk Sharing and Heterogeneous Agents. mimeo, London School of Economics.

Gust, C. and D. López-Salido (2009). Portfolio Inertia and Asset Pricing. mimeo Federal Reserve Board.

Guvenen, F. (2005). A Parsimonious Macroeconomic Model for Asset Pricing: Habit Formation or Cross-sectional Heterogeneity? mimeo, University of Rochester.

Hall, R. (2008). Sources and Mechanisms of Cyclical Fluctuations in the Labor Market. mimeo, Stanford University.

Hamilton, J. D. (1994). *Time Series Analysis*. Princeton, NJ: Princeton University Press.

Judd, K. L. (1999). *Numerical Methods in Economics*. Cambridge, MA: The MIT Press.

Khan, A. and J. K. Thomas (2007). Inflation and Interest Rates with Endogenous Market Segmentation. Federal Reserve Bank of Philadelphia Working Paper 07-1,.

Lucas, R. E. (1990). Liquidity and Interest Rates. *Journal of Economic Theory 50*, 237–264.

Lynch, A. (1996). Decision Frequency and Synchronization Across Agents: Implications for Aggregate Consumption and Equity Returns. *Journal of Finance LI*, 1479–1497.

Marshall, D. and N. Parekh (1999). Can Costs of Consumption Adjustment Explain Asset Pricing Puzzles? *Journal of Finance LIV*, 623–654.

Mehra, R. and E. Prescott (1985). The Equity Premium Puzzle. *Journal of Monetary Economics 15*, 145–166.

Parker, J. and A. Vissing-Jorgensen (2009). Who Bears Aggrgate Fluctuations and How? *American Economic Review, Papes and Proceedings*. forthcoming.

Polkovnichenko, V. (2004). Limited Stock Market Participation and the Equity Premium. *Finance Research Letters 1*, 24–34.

Vissing-Jorgensen, A. (2002). Towards an Explanation of Household Portfolio Choice Heterogeneity: Non-Financial Income Participation Costs Structures. National Bureau of Economic Research Working Paper 8884.

Vissing-Jorgensen, A. (2003). Perspectives on Behavioral Finance: Does Irrationality Disappear with Wealth? Evidence from Expectations and Actions. In M. Gertler and K. Rogoff (Eds.), *NBER Macroeconomics Annual*. MIT Press.

Figure 1: Endogenous Rebalancing, Standard CIA Model, and the Equity Premium

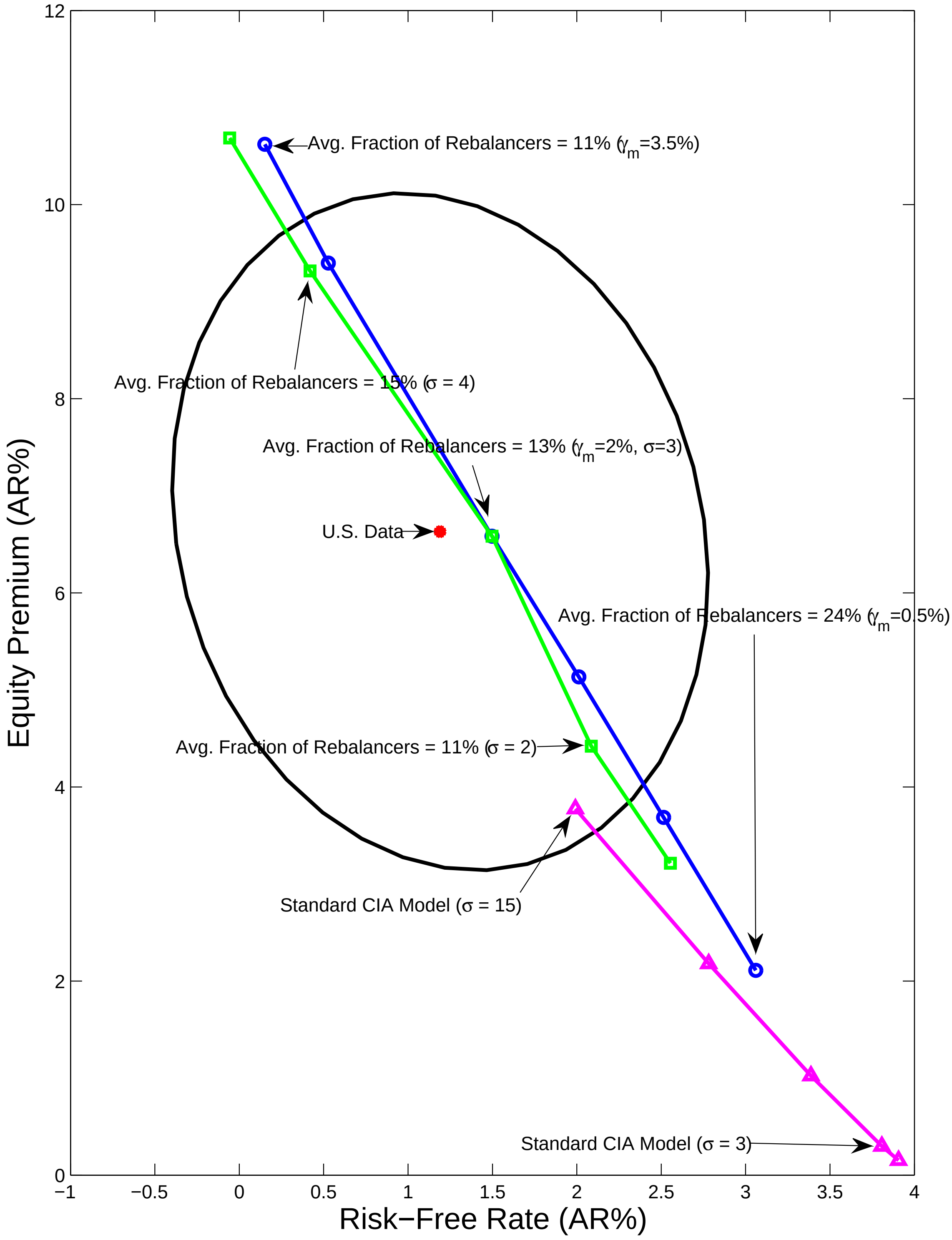


Figure 2: Endogenous Rebalancing, Endogenous Participation, and the Equity Premium

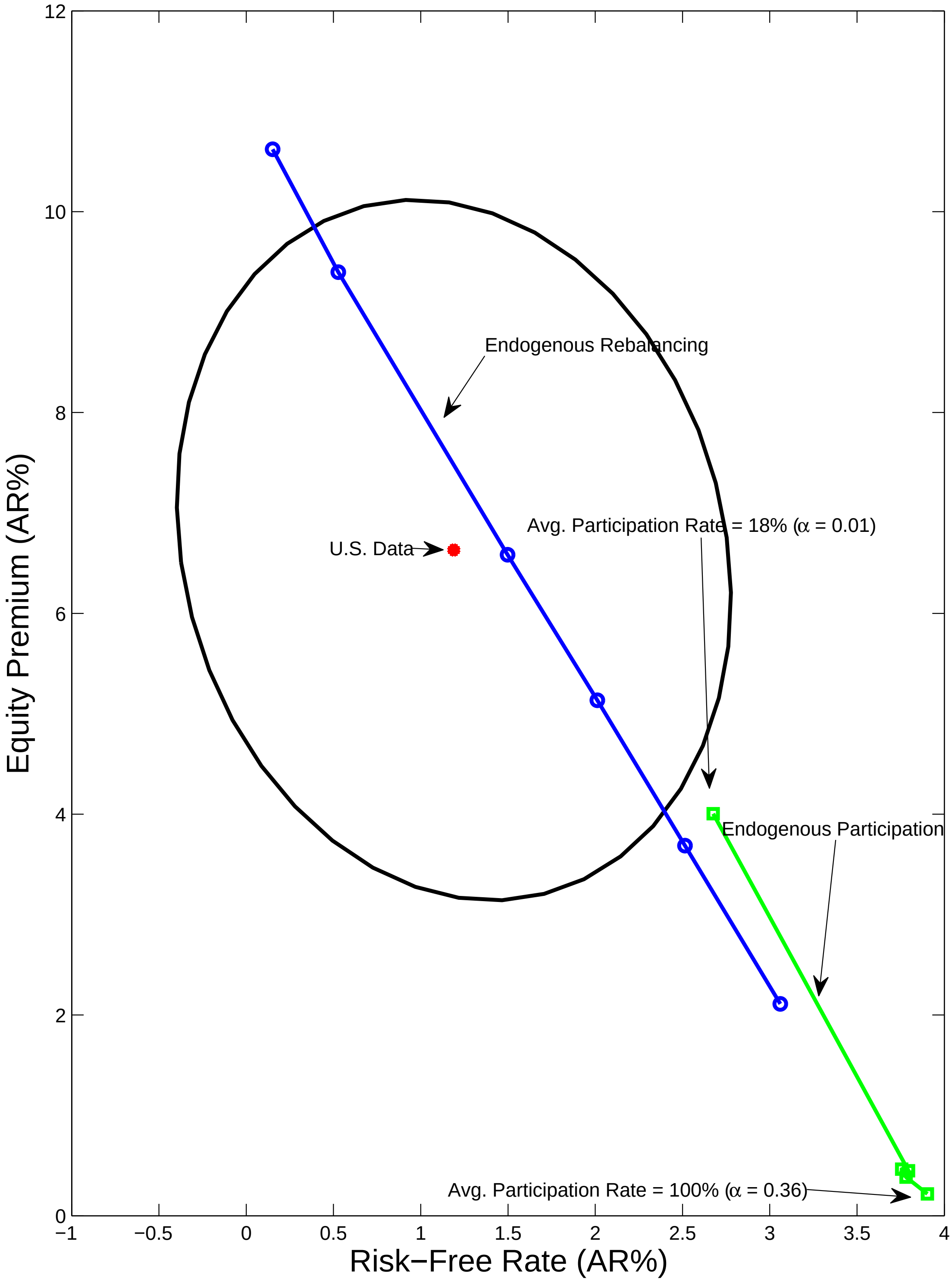


Figure 3: Demand for the Annuity and Equilibrium

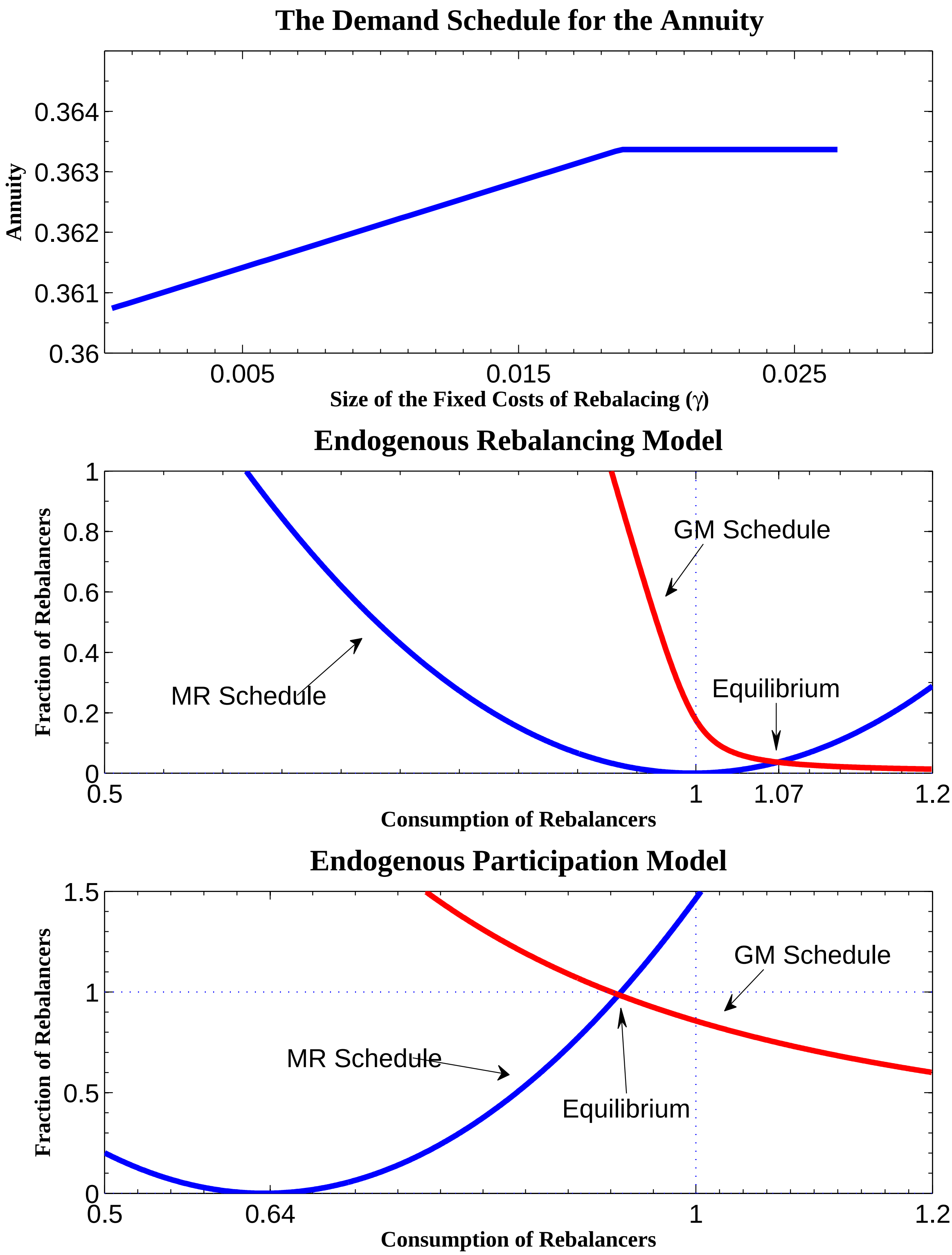


Figure 4: A Deterministic Increase in Technology in the Rebalancing Model

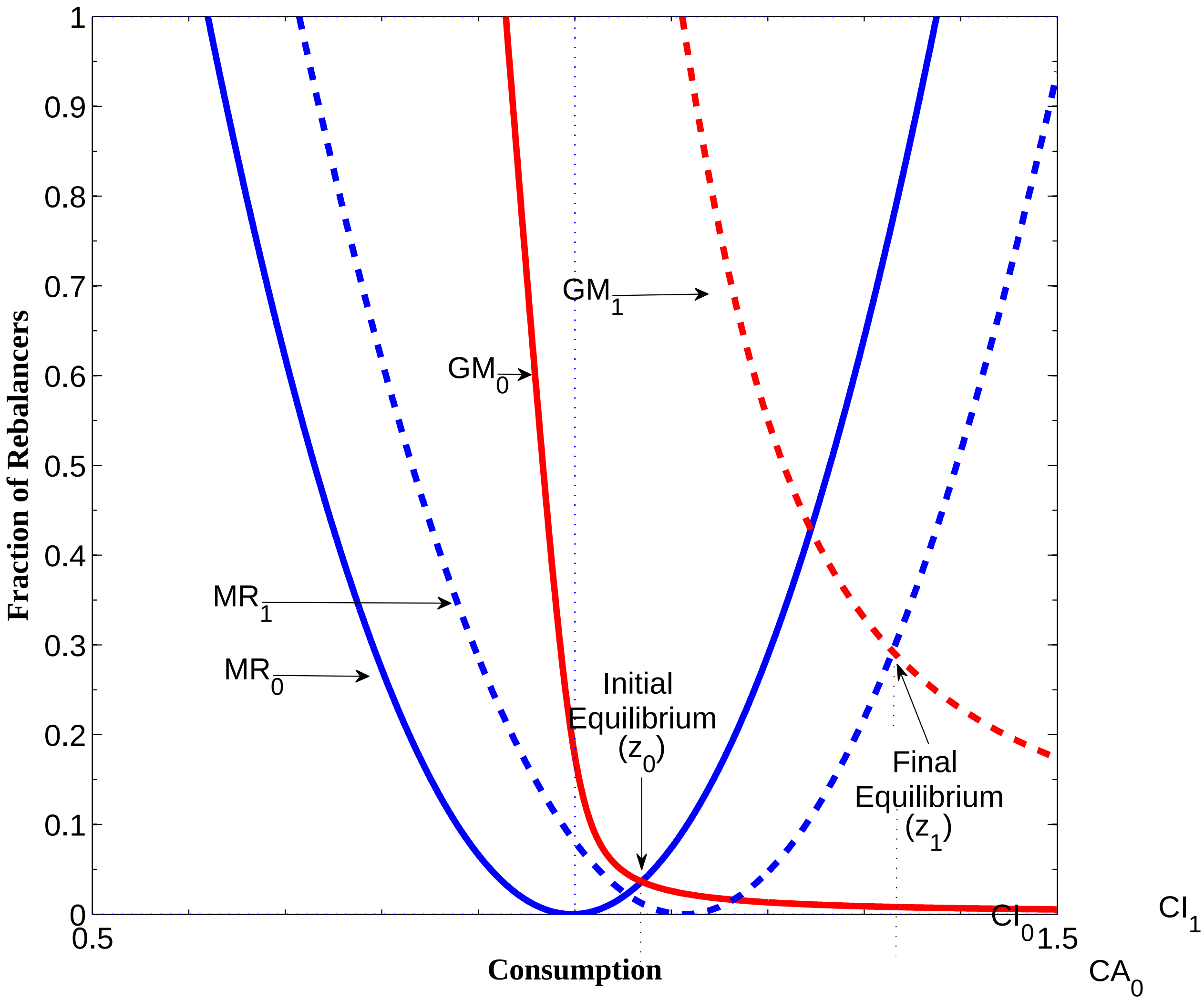


Figure 5: Consumption Distribution across Households

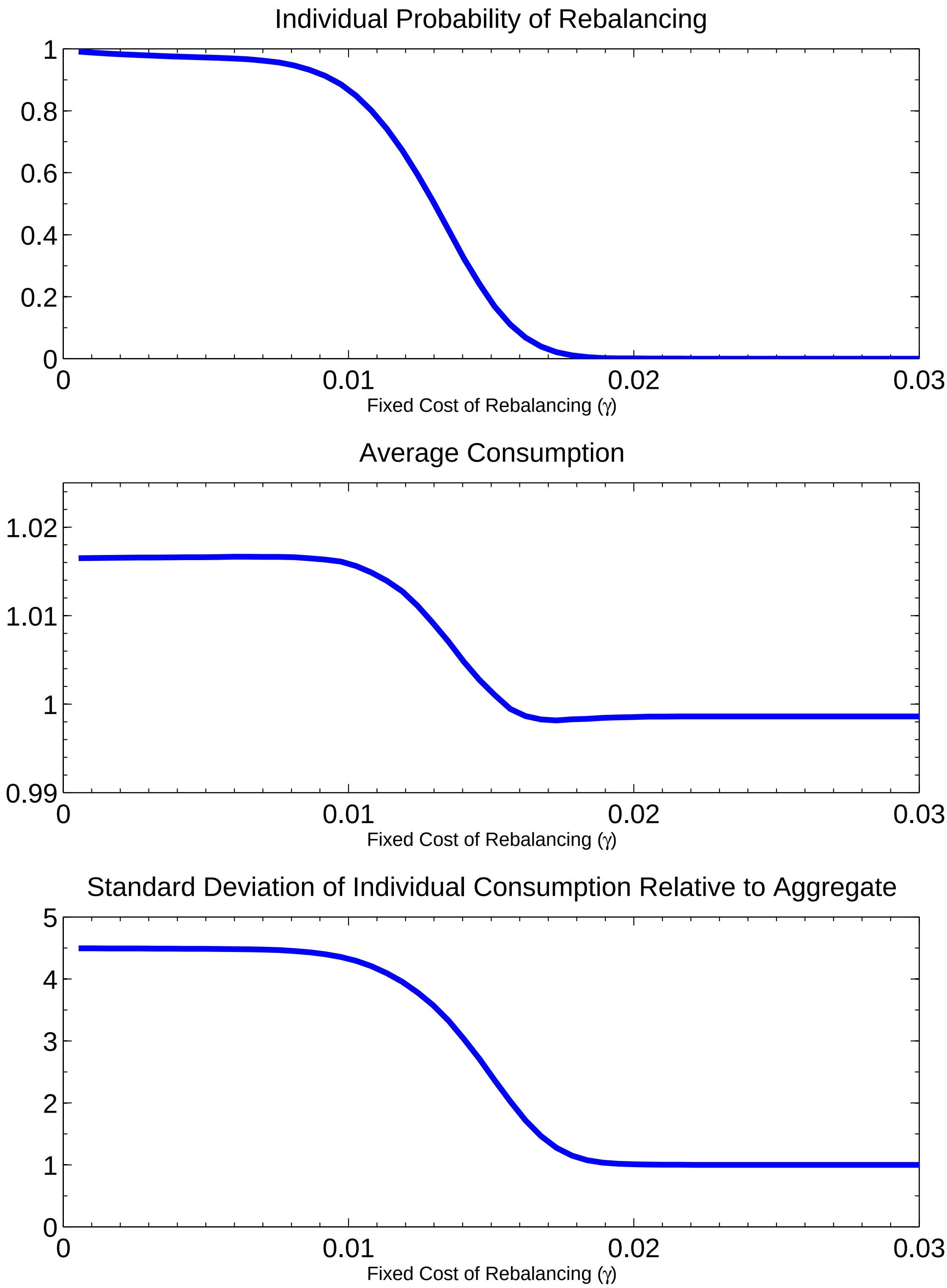


Figure 6: Impulse Response to an Expansionary Monetary Shock
(Deviation from Date 0 Expectation of a Variable)

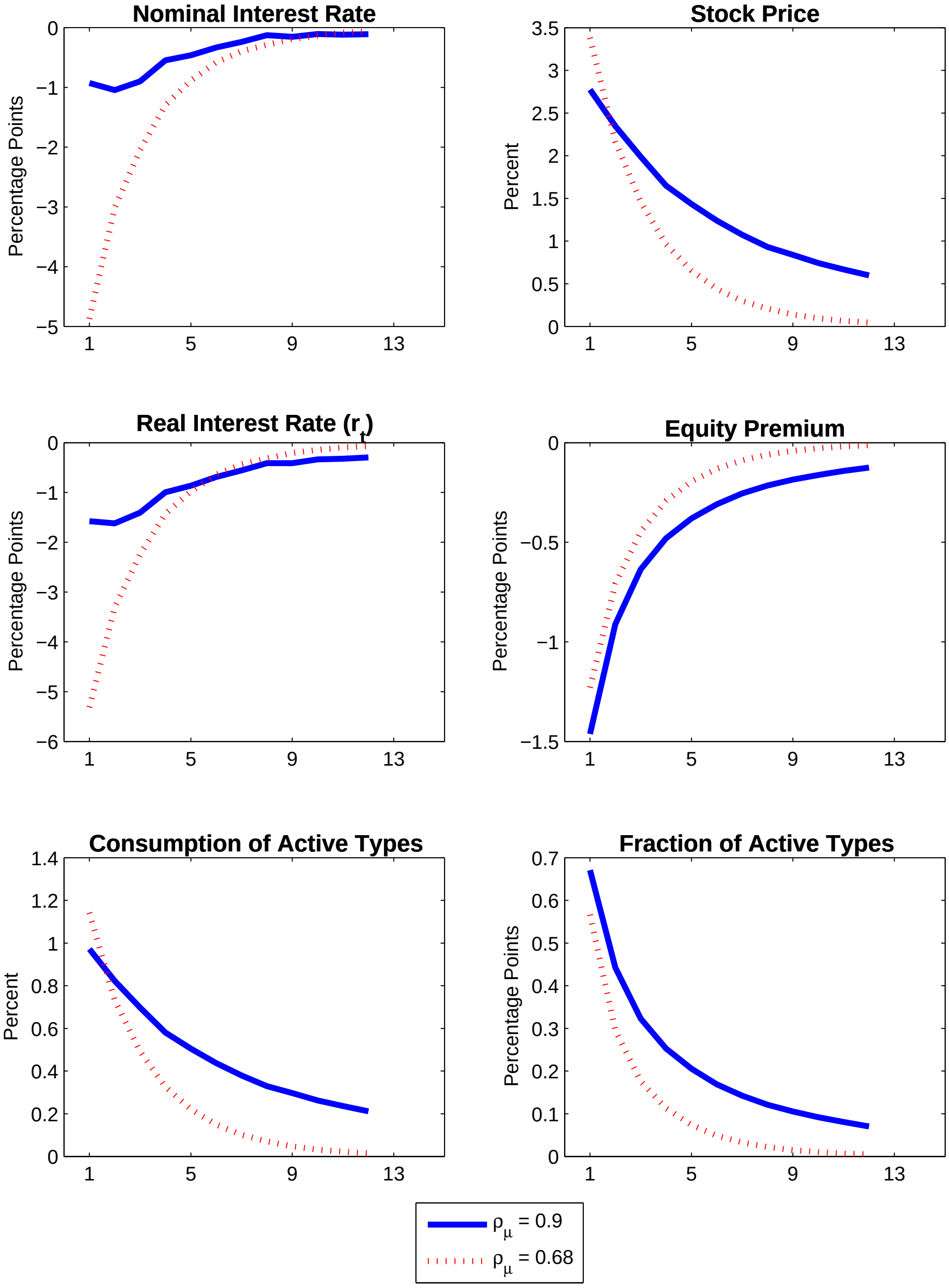


Figure 7: A Deterministic Change in Money Growth in the Rebalancing Model

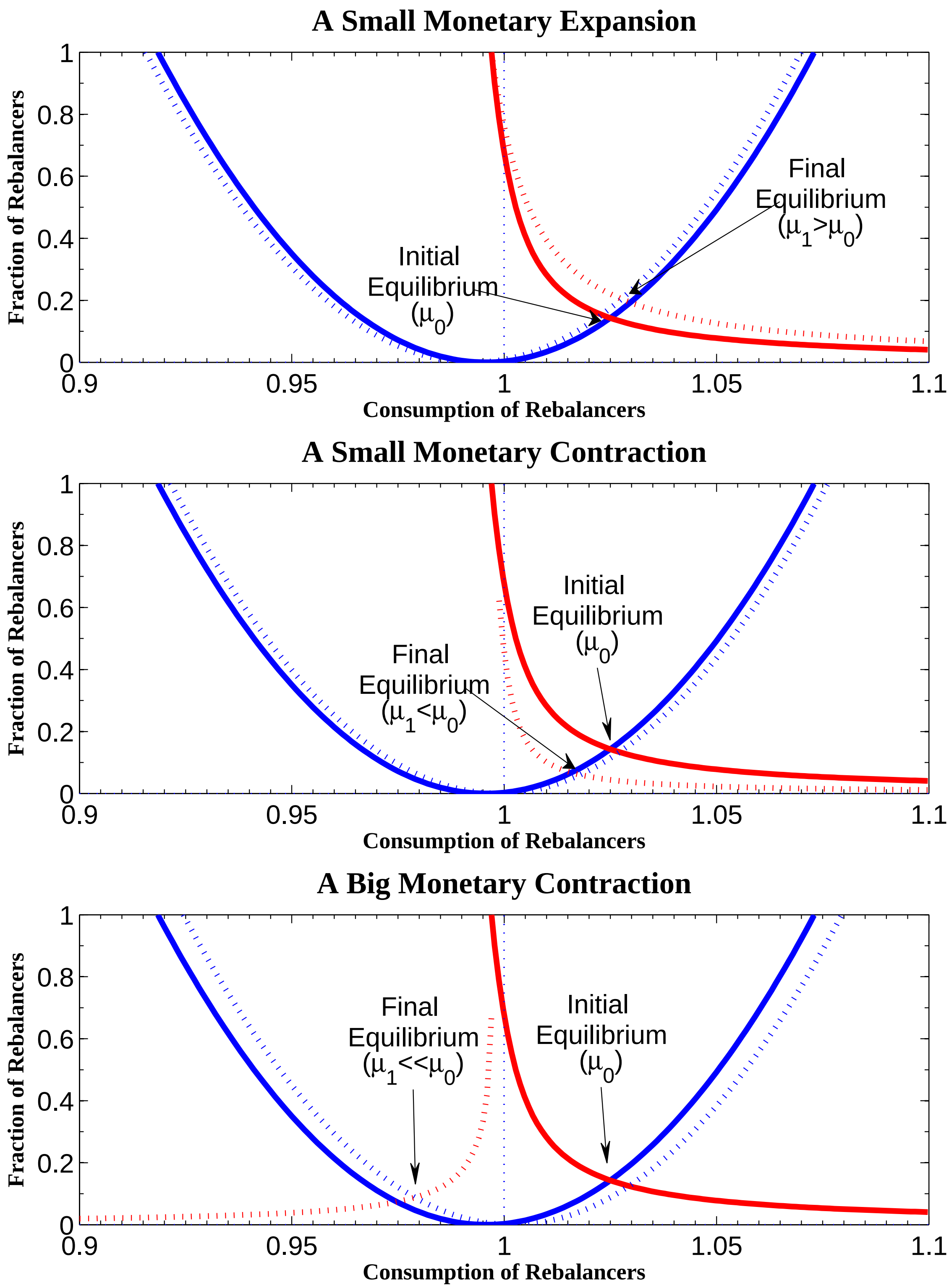
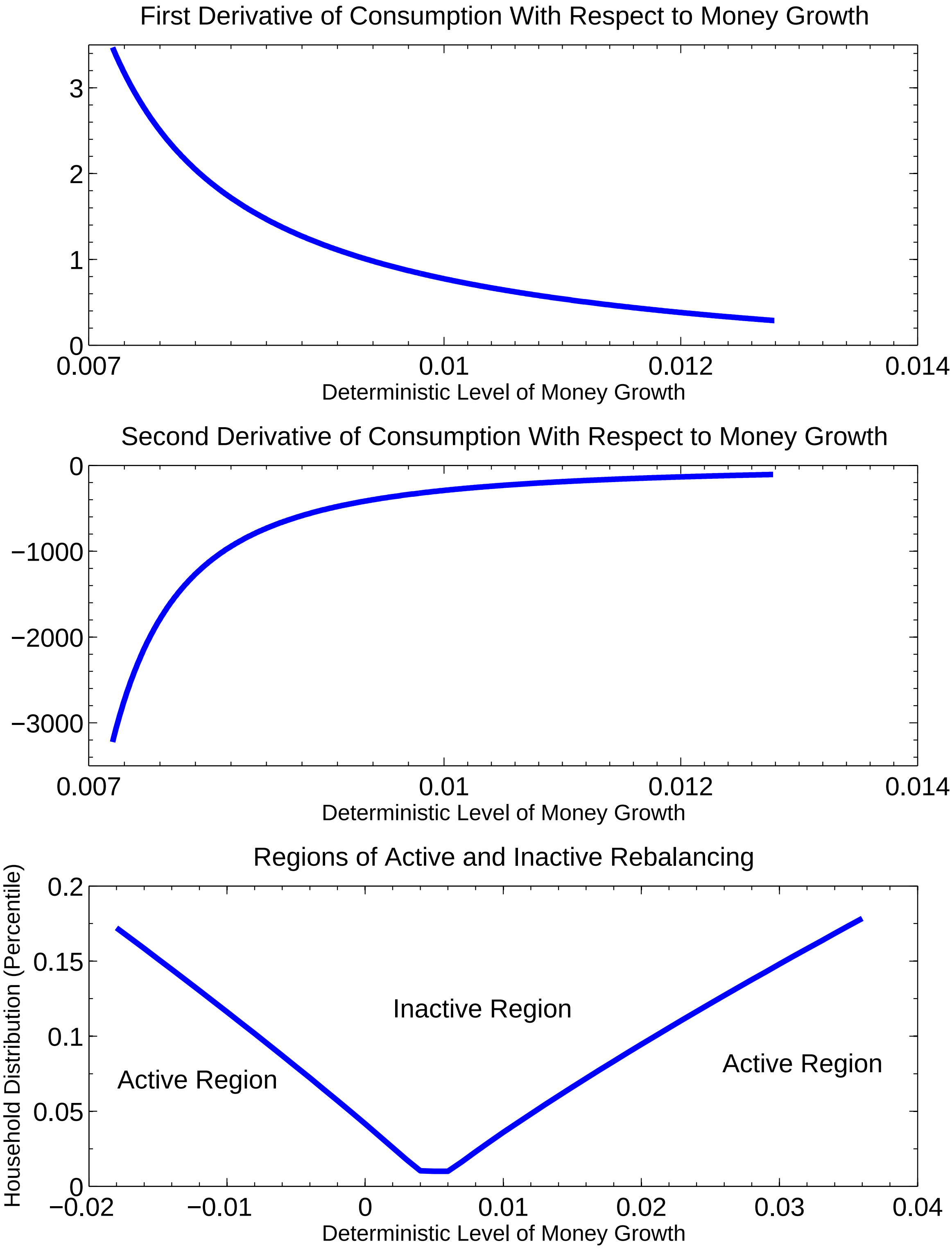


Figure 8: The Shape of Active Consumption and Regions of Active and Inactive Rebalancing



5 Technical Appendix

5.1 Sufficient Conditions for the Existence of Equilibrium

In this Appendix, we derive sufficient conditions that characterize an equilibrium where households do not carry over cash in either goods market or the asset market. Our strategy closely follow the Appendix in Alvarez, Atkeson, and Kehoe (2002). Hence, we first introduce in the cash-in-advance constraint the possibility that the household may hold cash, $a(s^t, \gamma) \geq 0$:

$$a(s^t, \gamma) = m(s^t, \gamma) + x(s^t, \gamma)z(s^t, \gamma) + (1+g)^t A(\gamma) - c(s^t, \gamma), \text{ for } \textit{all } t \geq 1$$

We also use the budget constraint to rewrite the money balances as as follows:

$$m(s^t, \gamma) = \frac{P(s^t)[w(s^t) + a(s^t, \gamma)]}{P(s^{t+1})} \tag{34}$$

In the asset market, we replace the period by period constraint with the sequence of budget constraints for $t \geq 1$:

$$B(s^t, \gamma) = \int_{s_{t+1}} q(s^t, s_{t+1})B(s^{t+1}, \gamma)ds_{t+1} + N(s^t, \gamma) - N(s^{t-1}, \gamma) + P(s^t)[x(s^t, \gamma) + (1+g)^t\gamma]z(s^t, \gamma) \tag{35}$$

where $N(s^{t-1}, \gamma)$ is the cash held over from the previous asset market and $N(s^t, \gamma)$ is cash held over into the next asset market. At time $t = 0$, the initial wealth is distributed as follows:

$$\overline{B}(\gamma) = \int_{s_{t+1}} q(s_1)B(s^1, \gamma)ds_1 + P_A A(\gamma) + N_0$$

where $N_0 = N(s^0)$ does not depend on γ.

We now proceed to develop the conditions that characterized an equilibrium sufficient conditions. In Lemma 1, we first characterize the household's optimal allocation of c and x, given prices and arbitrary rules for for m, a, $A(\gamma)$ and z. Second, in Lemma 2, we characterize the household trading rule z given arbitrary rules for m, a, and $A(\gamma)$–and the optimal rules specified in the Lemma 1. Next, we characterize the household's optimal choice for $A(\gamma)$, and finally, in Lemma 3 we provide sufficient conditions to ensure that a and N are always zero.

We rewrite the asset market constraint recursively by substituting out bond holdings to write a time $t = 0$ constraint as follows:

$$\overline{B}(\gamma) - N_0 \geq \sum_{t=1}^{\infty} \int_{s^t} Q(s^t) \left\{ P(s^t)[x(s^t, \gamma) + (1+g)^t\gamma]z(s^t, \gamma) + [N(s^t, \gamma) - N(s^{t-1}, \gamma)] \right\} ds_t + P_A A(\gamma) \tag{36}$$

where $Q(s^t) = \Pi_{j=1}^t q(s^j) = q(s^1)q(s^2)..q(s^t)$. Given these constraints, the households problem is to choose $\{m(s^t,\gamma),\ x(s^t,\gamma),\ c(s^t,\gamma),\ a(s^t,\gamma),\ N(s^t,\gamma)\}_{t=1}^{\infty}$ to maximize the following Lagrangian:

$$
\begin{aligned}
&Max\ \sum_{t=1}^{\infty}\int_{s^t}\beta^t U[c(s^t,\gamma)]g(s^t)ds^t \\
&+\sum_{t=1}^{\infty}\int_{s^t}\nu(s^t,\gamma)[m(s^t,\gamma)+x(s^t,\gamma)z(s^t,\gamma)+(1+g)^t A(\gamma)-c(s^t,\gamma)-a(s^t,\gamma)]ds^t \\
&+\sum_{t=1}^{\infty}\int_{s^t}\kappa(s^t,\gamma)[m(s^t,\gamma)-\frac{P(s^t)[w(s^t,\gamma)+a(s^t,\gamma)]}{P(s^{t+1})}]ds^t+ \\
&\lambda(\gamma)\left\{\overline{B}(\gamma)-\sum_{t=1}^{\infty}\int_{s^t}Q(s^t)\left\{P(s^t)[x(s^t,\gamma)+\gamma]z(s^t,\gamma)+[N(s^t,\gamma)-N(s^{t-1},\gamma)]\right\}ds_t-P_A A(\gamma)\right\}
\end{aligned}
$$

where $\nu(s^t,\gamma)$, $\kappa(s^t,\gamma)$, and $\lambda(\gamma)$ represent the Lagrange multipliers of the constraints (34)-(36), respectively.

The first order conditions for c and x are given by:

$$
\begin{aligned}
&\beta^t U'[c(s^t,\gamma)]g(s^t) = \nu(s^t,\gamma) \\
&\nu(s^t,\gamma)z(s^t,\gamma) - \lambda(\gamma)Q(s^t)P(s^t)z(s^t,\gamma) = 0
\end{aligned}
$$

Let suppose that $z(s^t,\gamma) = 1$ for some state of the world s^t, then we have:

$$\beta^t U'[c(s^t,\gamma)]g(s^t) = \lambda(\gamma)Q(s^t)P(s^t)$$

We assume an initial wealth $\overline{B}(\gamma)$, such that $\lambda(\gamma) = \lambda$, for all γ. This assumption implies that $B(\gamma_u) > B(0)$, i.e. that after $t = 0$, once the agents have decided the annuity, all the agents have the same wealth.

Lemma 1. All households that choose to pay the fixed cost (i.e. $z(s^t,\gamma) = 1$) for a given aggregate state s^t have identical consumption, so that $c(s^t,\gamma) = c_A(s^t)$. Thus,

$$x(s^t,\gamma) = c_A(s^t,\gamma) - a(s^t,\gamma) - m(s^t,\gamma) + A(\gamma) \tag{37}$$

Househods that choose not to pay the fixed cost (i.e. $z(s^t,\gamma) = 0$) have consumption equal to:

$$c(s^t,\gamma) = c_I(s^t,\gamma) = m(s^t,\gamma) - a(s^t,\gamma) + A(\gamma) \tag{38}$$

We next consider a household's optimal choice of whether to pay the fixed costs to trade, given prices $Q(s^t)$, $P(s^t)$, and P_A, for arbitrary choices for m and a. From Lemma 1 we have the form of the optimal allocation for c and x corresponding to the choices of $z(s^t,\gamma) = 1$ and $z(s^t,\gamma) = 0$. Substituting this rules into the utility function and the constraint (36) characterizes the problem of choosing $z(s^t,\gamma)$ and $c_A(s^t,\gamma)$:

$$\sum_{t=1}^{\infty}\int_{s^t}\beta^t U[c_A(s^t)]z(s^t,\gamma)g(s^t)ds^t+\sum_{t=1}^{\infty}\int_{s^t}\beta^t U[m(s^t,\gamma)+A(\gamma)-a(s^t,\gamma)](1-z(s^t,\gamma))g(s^t)ds^t$$
$$+\eta(\gamma)\left\{\overline{B}(\gamma)\text{-}\sum_{t=1}^{\infty}\int_{s^t}Q(s^t)\left\{\begin{array}{c}P(s^t)[c_A(s^t)+\gamma\text{-}(m(s^t,\gamma)+A(\gamma)\text{-}a(s^t,\gamma))]z(s^t,\gamma)\\+[N(s^t,\gamma)\text{-}N(s^{t-1},\gamma)]\end{array}\right\}ds_t\text{-}P_A A(\gamma)\right\}$$

We use a variational argument to characterize the optimal choice of $z(s^t,\gamma)$. The increment to the Lagrangian of setting $z(s^t,\gamma)=1$ in the state of the world s^t and for a given cost γ,

$$\beta^t U[c_A(s^t)]g(s^t)-\eta(\gamma)Q(s^t)P(s^t)[c_A(s^t)+\gamma\text{-}(m(s^t,\gamma)+A(\gamma)\text{-}a(s^t,\gamma))]ds^t \tag{39}$$

which corresponds to the difference between the utility gain minus the cost of transfering funds. The increment to the Lagrangian of setting $z(s^t,\gamma)=0$ in the state of the world s^t and for a given cost γ,

$$\beta^t U[m(s^t,\gamma)+A(\gamma)-a(s^t,\gamma)](1-z(s^t,\gamma))g(s^t) \tag{40}$$

which, in the abscense of trasnfers, it corresponds to the direct utility. The first order condition with respect to $c_A(s^t,\gamma)$ is given by:

$$\beta^t U'[c_A(s^t)]g(s^t)=\eta(\gamma)Q(s^t)P(s^t),$$

from which it follows $\eta(\gamma)=\eta$. Substracting (40) from (39) and using the first order conditions with respect to $c_A(s^t)$, we can define the following function $h(s^t,\gamma,A(\gamma))$:

$$h(s^t,\gamma,A(\gamma))=\beta^t\left\{\begin{array}{c}U[c_A(s^t)]-U[m(s^t,\gamma)+A(\gamma)-a(s^t,\gamma)]\\-U'[c_A(s^t)][c_A(s^t)+\gamma-(m(s^t,\gamma)+A(\gamma)-a(s^t,\gamma))]\end{array}\right\}g(s^t),$$

that can be used to determined the cutoff value for γ.

We also need to characterize the annuity $A(\gamma)$. The first order conditions for $A(\gamma)$:

$$\sum_{t=1}^{\infty}\int_{s^t}\beta^t U'[m(s^t,\gamma)+A(\gamma)\text{-}a(s^t,\gamma)](1\text{-}z(s^t,\gamma))g(s^t)ds^t=\eta P_A\text{-}\sum_{t=1}^{\infty}\int_{s^t}Q(s^t)P(s^t)z(s^t,\gamma)ds_t,$$

which can be rewritten as follows:

$$\eta P_A=\sum_{t=1}^{\infty}\int_{s^t}U'[c_A(s^t)]z(s^t,\gamma)g(s^t)ds_t+\sum_{t=1}^{\infty}\int_{s^t}\beta^t U'[m(s^t,\gamma)+A(\gamma)\text{-}a(s^t,\gamma)](1\text{-}z(s^t,\gamma))g(s^t)ds^t$$

This conditions is satisfied for *all* γ. We now assume that there will exist a $\gamma\in[0,\gamma_{Min}]$. For all these type of consumers, it follows:

$$\eta P_A=\sum_{t=1}^{\infty}\int_{s^t}U'[c_A(s^t)]z(s^t,\gamma)g(s^t)ds_t$$

which leaves $A(\gamma)$ indeterminate. Sustituting this expression into the previous first order condition for $\gamma > \gamma_{Min}$, we have

$$\sum_{t=1}^{\infty} \int_{s^t} \left\{ U'[c_A(s^t)] - U'[m(s^t,\gamma) + A(\gamma) - a(s^t,\gamma)] \right\} (1 - z(s^t,\gamma)) g(s^t) ds^t = 0,$$

which pins down the function $A(\gamma)$. We will now assume that $A(\gamma)$ is a non-negative non-decreasing function. Under this assumption, we can show that there is a unique cutoff value $\overline{\gamma}(c(s^t), s^t)$ such that:

$$h(s^t, \gamma, A(\gamma)) = 0$$

Formally, this means that a household chooses $z(s^t,\gamma) = 0$ if and only if $\gamma > \overline{\gamma}(c(s^t), s^t)$. The proof straightforwardly follows from noticing that $h(s^t, \gamma, A(\gamma))$ is a strightly decreasing function in γ. We will check that this condition is satisfied once we solve the model (see the main text).

Consider now the market clearing condition for the annuity. Agents with $\gamma > \gamma_{Min}$ choose $A(\gamma)$ and we will assume that households facing costs $\gamma \leq \gamma_{Min}$ will choose the same annuity A such that:

$$\frac{\gamma_{Min}}{\gamma_u} A_{Min} = -\frac{1}{\gamma_u} \int_{\gamma_{Min}}^{\gamma_u} A(\gamma) d\gamma$$

We now solve for the Lagrangian multiplier λ consitent with and equilibrium in which $N(s^t,\gamma) = 0$, for *all* s^t and γ. In this case, the multiplier λ satisfies expression (36) as an equality:

$$\overline{B}(\gamma) - P_A \widetilde{A}(\gamma) = \lambda^{-1} \sum_{t=1}^{\infty} \int_{s^t} U'[c_A(s^t) + \gamma - \frac{w(s^{t-1})}{\mu(s^{t-1})} - \widetilde{A}(\gamma)] ds_t$$

where

$$P_A = \frac{1}{\lambda} \sum_{t=1}^{\infty} \int_{s^t} U'[c_A(s^t)] g(s^t) ds^t, \qquad \widetilde{A}(\gamma) = \{ \begin{smallmatrix} A_{Min} & if & \gamma \leq \gamma_{Min} \\ A(\gamma) & if & \gamma_u \leq \gamma \leq \gamma_{Min} \end{smallmatrix}$$

Notice that the multilplier has the following expression:

$$\lambda = \frac{\sum_{t=1}^{\infty} \int_{s^t} U'[c_A(s^t) + \gamma - \frac{w(s^{t-1})}{\mu(s^{t-1})} - \widetilde{A}(\gamma)] ds_t}{\overline{B}(\gamma) - P_A \widetilde{A}(\gamma)}$$

To satisfy that λ does not depend upon γ, we need to allocate initial assets so that the condition is satisfied. We can also normalize $\overline{B}(\gamma)$ so that $\lambda = 1$. This non-zero value of the multiplier implies that the financial constraint is binding with $N(s^t,\gamma) = 0$ for all s^t. For this to be true, it must be the case that the nominal interest rate is always positive, $i(s^t) > 0$.

5.2 Global (Numerical) Solution

Given $A(\gamma)$, the resource constraint and equilibrium condition for the marginal rebalancer determine $c_A(s^t)$ and $\bar{\gamma}(s^t)$.

Following Judd (1999), we use the *linear Fredholm integral equations (Type 2)* to determine the price of capital from the stochastic difference equation:

$$p_k(s^t) = \int_{s_{t+1}} m(s^t, s_{t+1}) \left[\alpha \exp[(1-\alpha)z_{t+1}] + p_k(s^{t+1})\right] g(s_{t+1}|s^t) ds_{t+1},$$

where the pricing kernel depends on the consumption of rebalancers is defined as follows:

$$m(s^t, s_{t+1}) = \beta \left[\frac{c_A(s^t)}{c_A(s^{t+1})}\right]^{\sigma}.$$

We use Newton-Cotes quadrature to approximate the expectations associated with the normally-distributed technology and monetary shocks. As discussed in Judd (1999), this leads to a linear system of equations in the price of capital. These equations provide a solution for the price of capital at a finite number of points. To approximate the price of capital elsewhere, we use the Nystrom extension as discussed in Judd (1999).

We use a similar approach to determine $A(\gamma)$ for a fixed value of γ. To do so, we express equation (23) in recursive form:

$$v_A(s_0, \gamma) = \beta \int_s \left[U\left(c_A(s)\right) - U\left(c_I(s,\gamma)\right)\right] \left(1 - z(s,\gamma)\right) g(s|s_0) ds + \beta \int_s v_A(s,\gamma) g(s|s_0) ds.$$

With the expectations approximated using a quadrature formula, the resulting equations are linear in $v_A(s, \gamma)$. From these equations, we choose $A(\gamma)$ so that $v_A(\bar{s}, \gamma) = 0$ where $\bar{s}$ is a vector containing the unconditional mean levels of technology and monetary shocks, respectively.

5.3 Computation of Impulse-Response Function

An IRF of $y(s^t)$ to μ_1 is defined as the revision in expectations from a variable's conditional mean (Hamilton (1994)):

$$E[log\left(y(s^t)\right) \mid \mu_1, z_0] - E[log\left(y(s^t)\right) \mid \mu_0, z_0],$$

where $\mu_0 = \bar{\mu}$ and $z_0 = \bar{z}$ are the unconditional means of the shocks and $y(s^t)$ is the variable whose impulse response is of interest. Since evaluating the conditional expectations above involves multidimensional integrals, we use *Monte Carlo integration*.