# Beyond Cobb-Douglas: Estimation of a CES Production Function with Factor Augmenting Technology*

Devesh Raval†

November 15, 2010

**Abstract**

Both the recent literature on production function identification and a considerable body of other empirical work assume a Cobb-Douglas production function. Under this assumption, all technical differences are Hicks neutral. I provide evidence from US manufacturing plants against Cobb-Douglas and present an alternative production function that better fits the data. A Cobb Douglas production function has two empirical implications that I show do not hold in the data: a constant cost share of capital and strong comovement in labor productivity and capital productivity (revenue per unit of capital). Within four digit industries, differences in cost shares of capital are persistent over time. Both the capital share and labor productivity increase with revenue, but capital productivity does not. A CES production function with labor augmenting differences and an elasticity of substitution between labor and capital less than one can account for these facts. To identify the labor capital elasticity, I use variation in wages across local labor markets. Since the capital cost to labor cost ratio falls with local area wages, I strongly reject Cobb-Douglas: capital and labor are complements. Many results in economic growth and macroeconomics depend both upon the bias of technical change and the value of the elasticity of substitution. Specifying the correct form of the production function is more generally important for empirical work, as I demonstrate by applying my methodology to address questions of misallocation of capital.

*I would especially like to thank my committee members Ali Hortacsu, Sam Kortum, and Chad Syverson for all of their support and guidance on this paper. I have also benefited from conversations with Fernando Alvarez, Costas Arkolakis, Alejo Costa, Chang-Tai Hsieh, Erik Hurst, Matthias Kehrig, Steven Levitt, Asier Mariscal, Benni Moll, Emi Nakamura, Ezra Oberfield, Adi Rom, Andy Zuppann, as well as participants at the UChicago IO Lunch, Micro Lunch, and Trade Working Group. I would also like to thank Frank Limehouse for all of his help at the Chicago Census Research Data Center and Randy Becker for assistance with deflators for the micro data. Any opinions and conclusions expressed herein are those of the author and do not necessarily represent the views of the U.S. Census Bureau. All results have been reviewed to ensure that no confidential information is disclosed.

†PhD Candidate, University of Chicago

# 1 Introduction

Plant level capital labor ratios are extremely dispersed, even within narrowly defined four digit SIC industries. I document this dispersion in Table 1. In the 1987 Census of Manufacturers for the US, the 75th percentile plant has two and a half times the capital per worker of the 25th percentile plant for the median industry. Looking further into the tails of the distribution, the 90th percentile plant has more than six times the capital per worker of the 10th percentile plant. This dispersion is not solely due to differences in wages or worker quality across plants. Capital shares of total cost are also disperse, with the 75th percentile plant having a capital share 1.8 times larger than the 25th percentile plant and the 90th percentile plant almost four times larger than the 10th percentile plant.

A Cobb-Douglas production function, the most common choice of the empirical literature, implies that the capital share is constant. As I showed above, the capital share within industry is far from constant. This dispersion in capital shares is not just temporary, due perhaps to measurement error or adjustment costs. I find that the capital share and capital-labor ratio are both strongly autocorrelated over a span of 10 years, about as autocorrelated as conventionally measured TFP. I also show that the capital share is strongly correlated with firm output, with the largest firms within the same industry having a 30% higher capital share than the smallest plants in 1987 and 80% higher capital share in 2002. Labor productivity and capital productivity (output per unit of capital) should also move together if the production function is Cobb Douglas. But in the US Census data, the average revenue product of labor rises with revenue while the average revenue product of capital is flat beyond the smallest plants.

A production function with non neutral productivity can better explain these facts. Productivity is always Hicks neutral when the production function is Cobb Douglas; improvements in productivity do not affect the relative marginal products of capital and labor and so do not alter the relative allocations of the factors. Under a CES production function with an elasticity of substitution not equal to one, productivity that augments labor affects the ratio of the marginal products of the factors. Labor augmenting productivity is akin to having more effective labor for the same number of workers. If labor and capital are complements (so the elasticity of substitution is less than one), firms with more effective labor have a higher marginal product of capital relative to labor. Cost minimizing firms set marginal products equal to the factor prices they face, so firms with more labor augmenting productivity expand their capital labor ratios. Labor augmenting productivity is thus labor saving as well.

Profit maximizing firms facing a downward sloping demand curve from differentiated products demand expand output when labor augmenting productivity rises. These firms also increase their capital per worker and capital share. Bigger firms then have larger capital shares and a much higher average revenue product of labor but not average revenue product of capital. These patterns are exactly what I see in the US Census data.

The response of firms to labor augmenting productivity depends upon the value of the long run elasticity of substitution. The elasticity of substitution measures how much firms change their capital intensity when factor prices change. When labor and capital are complements, firms increase their capital labor ratio less than an increase in wages, so the factor cost ratio falls. Under a Cobb-Douglas production function, the capital-labor ratio rises exactly in proportion to the rise in wages, so the factor cost ratio is constant. The slope of the relation between wages and the factor cost ratio then identifies the elasticity of substitution.

I find sharp rejections of the Cobb-Douglas specification when I use differences in wages across local areas for identification. I construct wages from both worker data and establishment data and estimate the elasticity using state, MSA, and county differences in wages, as well as county differences in wages within a state. As local area wages rise, the factor cost ratio falls. For overall manufacturing, this decreasing relationship implies that the elasticity of substitution is between .45 and .65.

I then estimate the elasticity of substitution separately for major industries at the 2 digit level within manufacturing. I can reject Cobb-Douglas for 17 out of 19 two digit industries using state level wages, 17 out of 19 using county level wages, and 15 out of 19 using within state county level wages. I also construct a sample of ten large four digit industries with considerable geographic variation. I reject Cobb-Douglas for 8 out of 10 industries with state wages and 10 out of 10 with county wages.

I also examine the elasticities of capital with skilled labor and unskilled labor separately, where I use production workers as a proxy for unskilled labor, nonproduction workers for skilled labor, the high school wage for the unskilled labor wage and the college wage for the skilled labor wage. The elasticity of capital with unskilled labor is still lower than one for most of my specifications. Consistent with skill capital complementarity, the elasticity of capital with skilled labor is below .5 in all specifications and much lower than the elasticity with unskilled labor.

The production function is one of the basic building blocks of economic theory. Thus, my results on the production function have implications for a whole host of economic questions. In Industrial Organization, economists are investigating how productivity differences across firms are related to market structure, and more generally what causes differences in productivity across firms (Bartlesman and Doms (2000),Syverson (forthcoming)). Many macroeconomic models assume that productivity shocks cause business cycle fluctuations. Productivity plays a central role in the recent trade literature as well. In these trade models, productivity is heterogeneous across firms and high productivity firms decide to enter into trade (Bernard et al. (2003),Melitz (2003)).

I can now look at the implications of both Hicks neutral productivity and labor augmenting productivity. Assuming cost minimization, I can identify a labor augmenting productivity from the plant's first order conditions for labor and capital. With data on revenue, I can identify Hicks neutral productivity together with price differences across plants. I find that labor augmenting productivity is correlated with both size and size growth, which holds up using employment or value added as a measure of size. My Hicks neutral measure is negatively correlated with both, although this result may be due to large and growing firms having low prices.

The long run elasticity of substitution is central to many questions of growth theory, including changes in income shares and relative convergence over time. The qualitative implications of many growth models depend on whether the elasticity of substitution is below or above 1, but the value of the elasticity is important for many questions as well. Since innovation and improvements in productivity drive economic growth, the bias of productivity affects how and why innovation occurs. Acemoglu (forthcoming) characterizes when technology improvements are labor saving, for example.

The type of technical differences we see has important implications for questions of misallocation as well. A recent literature studies whether poor countries are poor because resources are not allocated efficiently (Banerjee and Duflo (2005),Restuccia and Rogerson (2008)). Productive firms do not get enough capital while unproductive firms get too much capital, and firms may operate under output constraints or benefit from subsidies. These allocation frictions lower aggregate productivity. Hsieh and Klenow (2009) take this theory to the micro data and find that eliminating misallocation frictions would increase aggregate TFP by 40% in the US and more than 100% in China and India. In their model, firms with Cobb-Douglas production functions face capital and output wedges in a static environment.

Identifying allocation frictions in the data requires assumptions on the form of the production function and productivity. In a Cobb-Douglas world with misallocation frictions, a high capital share of cost implies a low capital wedge and a low labor share of revenue implies a high output wedge. Labor augmenting productivity would both increase the capital share of cost and decrease the labor share of revenue, as well as increase revenue. Thus, labor augmenting productivity would imply a set of testable implications for the misallocation wedges and revenue. Using data from Chile, I find that firms with low capital wedges have high output wedges and high revenue, as differences in labor augmenting productivity would predict.

Table 1: Dispersion in K/L and Capital Share within 4 digit Industries for the 1987 Census of Manufacturers

| | | Median | 25% | 75% |
|---|---|---|---|---|
| Capital-Labor Ratio | 75/25 Ratio | 2.5 | 2.3 | 2.8 |
| | 90/10 Ratio | 6.4 | 5.5 | 8.2 |
| Capital Share | 75/25 Ratio | 1.8 | 1.6 | 2.05 |
| | 90/10 Ratio | 3.85 | 3.2 | 4.6 |

For each industry, I calculated the 75/25 ratio and 90/10 ratio for each variable. I have then reported the median, 25%, and 75% of these ratios across industries.

## 1.1 Literature Review

The Cobb-Douglas production function has been the most widely used production function in empirical work. However, the Cobb-Douglas is extremely restrictive as it sets the elasticity of substitution between factors to one. In the early 1960s and 70s economists began to relax the restrictions imposed by Cobb-Douglas. Arrow et al. (1961) introduce the CES class of production functions which encompass any production function with a constant elasticity of substitution. The translog production function due to Christensen et al. (1973) is even more general. It does not impose that the elasticity of substitution is constant across plants or across multiple inputs. The translog is a second order approximation to any production function, as it contains all first and second order terms of the inputs.

Economists have been concerned since Marshak and Andrews (1944) with endogeneity problems in production function estimation. Since a firm takes into account its productivity when making input decisions, inputs are generally positively correlated with productivity. This correlation biases production function parameters estimated using OLS. Because of these endogeneity problems, economists gradually returned to assuming the simple Cobb-Douglas.

Olley and Pakes (1996) provide an approach to the endogeneity problem in the Cobb-Douglas case.[1] They start from a structural model that implies that investment is a monotonic function of productivity and capital. They then invert this function and replace productivity with the inverted function in the production function. The endogeneity problem is then gone. Output is a nonparametric function of investment, capital, and labor, and additive measurement error. This estimation procedure allows them to get rid of the measurement error. They then assume that productivity is first order Markov, in which case one can form the innovation in productivity as a function of model parameters. A set of timing assumptions lead to a GMM estimator of moments where observed variables or their lags are uncorrelated with the innovation in productivity. For example, capital is uncorrelated with the innovation since investment is made before the innovation is known.

Gandhi et al. (2009) apply the Olley Pakes type methodology to a wider array of production functions. They use revenue share equations from the first order conditions of the production function rather than a nonparametric input demand equation to separate measurement error from Hicks neutral productivity. They then estimate the production function parameters with similar GMM moments to Olley Pakes. The revenue share equations allow them to estimate more complicated production functions than Cobb Douglas, such as the CES or translog. By adding an input demand equation, they can handle imperfect competition as well. In their CES estimation case, they find an elasticity considerably above 1, very different from both my results and the rest of the literature. However, they assume only Hicks neutral productivity, so labor augmenting productivity would become measurement error in their approaches. Ignoring labor augmenting productivity could lead to severe biases in their estimation procedure.

Other literatures also estimate the elasticity of substitution between labor and capital. A macro time series literature investigates the aggregate elasticity of substitution between labor and capital. These papers use aggregate time series data on labor, capital, output,

[1] Their approach can be generalized (to say the translog) assuming Hicks neutral productivity.

and factor payments and estimate regressions based upon the CES first order conditions. Berndt (1976) could not reject that the aggregate production function was Cobb-Douglas in the US time series. Recently, however, a number of papers have challenged this result, as the Berndt paper assumed that all technical change was Hicks neutral. Antras (2004) and Klump et al. (2007) both allow labor augmenting technical change through time trends or other parametric functional forms and estimate the elasticity to be .8 and .6 for the US, respectively. Depending upon the type of technical change and country studied, researchers have found the elasticity to be below, equal, or greater than one. León-Ledesma et al. (2010) show in Monte Carlo simulations that estimation approaches that use both the production function and its first order conditions jointly can identify the elasticity of substitution.

Another literature estimates the firm level elasticity of substitution using shocks to the rental rate of capital, such as changes in capital taxes or investment tax credits. Chirinko (2008) provides a recent survey of this literature. Since adjustment costs mean that capital can not adjust instantaneously, the short run elasticity of substitution can be much lower than the long run elasticity of substitution. One approach here models the response of investment to changes in the rental rate of capital while accounting for adjustment costs. Another approach estimates the elasticity assuming that the basic frictionless first order conditions hold. While estimates of the elasticity from both of these approaches have a wide range, the most recent papers using firm level panel data find estimates in the range of .4 to .6, which are in line with my results. In these approaches all the variation in the user cost of capital comes from changes over time, so panel data is required. I use differences in wages across local areas and so only need a cross section of plants. I am also able to estimate the elasticity of substitution for individual industries.

## 2 Basic Theory

The production function posits a relationship between a set of inputs, which in this study are labor and capital, and output. Mathematically, we represent this relationship by a function $F$, as follows:

$$Y = F(K, L) \tag{1}$$

where $Y$ is output, $K$ is capital, and $L$ is labor. We can then predict how much the firm will produce if we know how much capital and labor it is using. However, this specification will not fit the data well; over time firms produce more with the same amounts of inputs, and across firms some firms produce much more than others with the same amount of inputs. To rescue the production function, we have to add terms for productivity. The productivity term measures how much more one firm cab produce than another, given the same inputs and same base production function $F$.

We can introduce productivity into the production function in a number of different ways. If productivity is Hicks neutral, improvements in productivity affect labor and capital symmetrically:

$$Y = AF(K, L) \tag{2}$$

where $A$ is Hicks neutral productivity.

Hicks neutral productivity increases the marginal productivity of capital and labor by the same percentage:

$$MPK = AF_K(K, L) \tag{3}$$

$$MPL = AF_L(K, L) \tag{4}$$

Improvements in productivity could also be specific to a factor in the production function. For example, if productivity is labor augmenting (Harrod neutral), the production function is:

$$Y = F(K, BL) \tag{5}$$

where $B$ is labor augmenting productivity.

Thus, an increase in labor augmenting productivity is equivalent to having more labor. Labor augmenting productivity changes have different effects on the marginal products of capital and labor, changing (in general) the ratio of marginal products. The ratio of marginal products of capital and labor is:

$$\frac{MPK}{MPL} = \frac{F_K(K, BL)}{BF_L(K, BL)} \tag{6}$$

I now assume the CES production function to show how the ratio of marginal products changes with labor augmenting productivity. As its name suggests, the CES production function assumes a constant elasticity of substitution $\sigma$ between labor and capital. The CES Production Function with labor and capital and both Hicks neutral and labor augmenting technical differences is:

$$Y = A(\alpha K^{\frac{\sigma-1}{\sigma}} + (1-\alpha)(BL)^{\frac{\sigma-1}{\sigma}})^{\frac{\sigma}{\sigma-1}} \tag{7}$$

Here, $A$ is Hicks neutral productivity and $B$ is labor augmenting productivity. The distribution parameter $\alpha$ governs how much capital contributes to output relative to labor. If the elasticity is not equal to 1, $\alpha$ can not be identified separately from $A$ and $B$. This production function is also constant returns to scale. This formulation is slightly different than that of Arrow et al. (1961) who only allow Hicks neutral productivity. One useful property of the CES production function is that it nests a number of famous simple cases. When the elasticity of substitution $\sigma$ converges to 0, we have the Leontief production function, where labor and capital are extreme complements:

$$Y = A\min(\alpha K, (1-\alpha)BL) \tag{8}$$

When the elasticity of substitution $\sigma$ is 1, we have the Cobb-Douglas production function:

$$Y = AK^{\alpha}(BL)^{1-\alpha} \tag{9}$$

When the elasticity of substitution $\sigma$ converges to infinity, we have the linear production function, where labor and capital are extreme substitutes:

$$Y = A(\alpha K + (1-\alpha)BL) \tag{10}$$

## 2.1 Elasticity of Substitution:

Clearly, the CES production function depends on the elasticity of substitution, as the previous three production functions have very different properties. Robinson (1933) offers the following definition of the elasticity of substitution:

$$\sigma = -\frac{d\log(K/L)}{d\log(r/w)} \tag{11}$$

Here, we are assuming that the firm is cost minimizing facing constant factor prices for both labor and capital, where $r$ is the rental rate of capital and $w$ is the wage. Thus, the elasticity of substitution tells us how much the capital-labor ratio adjusts to changes in factor prices. In a Leontief world, factor input ratios are constant so changes in factor prices do not affect the capital labor ratio at all. With perfect substitution, the firm employs either only capital or only labor depending on factor prices, so a rise in the wage would cause some firms using capital to use labor.

For the Cobb-Douglas production function, the change in the capital-labor ratio exactly offsets the change in the factor price ratio, so the capital cost to labor cost ratio ($rK/wL$ ) is kept constant.

Hicks (1932) provides another definition of the elasticity of substitution, which is formally equivalent to Robinson's definition under constant returns to scale. Hicks shows that:

$$\sigma = \frac{F_k F_l}{F_{kl} F} \tag{12}$$

Thus, the elasticity of substitution depends upon the curvature of the production function through the second derivative $F_{kl}$. We can see this curvature in the isoquants of the production function. For the linear production function, the isoquants are linear. As the elasticity of substitution falls, the isoquants become more and more curved, until they take an L shape with the Leontief production function.

## 2.2 Factor bias under CES:

In the CES case, the elasticity of substitution determines what happens to the ratio of marginal products. The CES production function implies that:

$$\frac{MPK}{MPL} = (B)^{\frac{1-\sigma}{\sigma}} (\frac{K}{L})^{-\frac{1}{\sigma}} \frac{\alpha}{1-\alpha} \tag{13}$$

Keep the capital labor ratio constant. If the elasticity of substitution $\sigma$ is less than 1, the marginal product of capital rises relative to that of labor when labor augmenting productivity increases. If the elasticity of substitution is greater than 1, so labor and capital are substitutes, increases in labor augmenting productivity decrease the marginal product of capital relative to labor. In the Cobb-Douglas case, where $\sigma$ equals 1, increases in productivity do not affect the ratio of marginal products.

So far we have kept factor proportions constant. If the firm is cost minimizing, then marginal products will be set equal to factor prices and firms will adjust the amounts of the factors they have. Cost minimization then implies:

$$\frac{r}{w} = \frac{MPK}{MPL} = (B)^{\frac{1-\sigma}{\sigma}} (\frac{K}{L})^{-\frac{1}{\sigma}} \frac{\alpha}{1-\alpha} \tag{14}$$

If $\sigma < 1$, firms respond to the increase in labor augmenting productivity by raising their capital-labor ratio until marginal products are equal to factor prices. Thus, increases in labor augmenting productivity are also *labor saving* if $\sigma < 1$. If $\sigma > 1$, then firms decrease their capital-labor ratio when labor augmenting productivity rises.

The intuition here is that increases in labor augmenting productivity $B$ are equivalent to increases in labor for firms. If labor and capital are complements, firms then want to increase the amount of capital they hold, so the marginal product of capital rises until firms increase capital relative to labor. If labor and capital are substitutes, the opposite intuition holds.

The elasticity of the capital-labor ratio to changes in labor augmenting productivity $B$ is:

$$\frac{K}{L} = B^{1-\sigma}(\frac{r}{w})^{-\sigma} \tag{15}$$

$$\frac{d\log(K/L)}{d\log B} = (1-\sigma) \tag{16}$$

Thus, while the capital-labor ratio increases when $B$ increases, it does so less than proportionately unless the production function is Leontief.

The factor cost ratio is:

$$\frac{rK}{wL} = (\frac{r}{w})^{1-\sigma}(B)^{1-\sigma}\frac{\alpha}{1-\alpha} \tag{17}$$

If the elasticity of substitution $\sigma$ is less than 1, the factor cost ratio rises with labor augmenting productivity. Since the factor cost ratio also rises with the ratio of factor prices, increases in wages will reduce the ratio.

## 2.3 Factor bias under Cobb-Douglas:

In the Cobb-Douglas case we can not separate Hicks neutral from factor augmenting productivity. The labor augmenting productivity $B$ can be merged with the Hicks neutral productivity $A$ to form a new Hicks neutral shifter $\tilde{A}$:

$$Y = AK^{\alpha}(BL)^{1-\alpha}$$

$$Y = AB^{1-\alpha}K^{\alpha}L^{1-\alpha}$$

$$Y = \tilde{A}K^{\alpha}L^{1-\alpha}$$

In effect, all productivity differences are Hicks neutral when the production function is Cobb-Douglas. In the Cobb-Douglas case, the coefficient on capital also determines the capital share of cost under cost minimization:

$$\frac{rK}{rK + wL} = \alpha \tag{18}$$

$$\frac{rK}{wL} = \frac{\alpha}{1-\alpha} \tag{19}$$

Thus, a simple test of the Cobb-Douglas production function is whether the capital share is constant over time or across firms in a given industry.

## 2.4 Cost Minimization:

Cost minimizing firms set the ratio of the marginal product of capital to marginal product of labor equal to the ratio of factor prices. Thus, the ratio of marginal products of both factors should be constant across firms. The average products of the factors do respond to changes in productivity. The average product of capital is:

$$\frac{Y}{K} = A(\alpha + (1-\alpha)B^{\frac{\sigma-1}{\sigma}}(\frac{L}{K})^{\frac{\sigma-1}{\sigma}})^{\frac{\sigma}{\sigma-1}} \tag{20}$$

Increases in Hicks neutral productivity, $A$, will raise the average product of capital.

Increases in labor augmenting productivity have two effects. First, the overall increase in productivity will increase the average product of capital. But, cost minimizing firms will respond by increasing the capital-labor ratio and so lowering the average product of capital. Since the capital-labor ratio rises less than proportionately (with elasticity $1-\sigma$) to increases in $B$, the average product of capital will still increase.

The average product of labor is:

$$\frac{Y}{l} = A((1-\alpha)B^{\frac{\sigma-1}{\sigma}} + (\alpha)(\frac{k}{l})^{\frac{\sigma-1}{\sigma}})^{\frac{\sigma}{\sigma-1}} \tag{21}$$

Increases in Hicks neutral productivity, $A$, will raise the average product of labor as well. Increases in labor augmenting productivity $B$ directly boost the average product of labor through the production function. Since cost minimizing firms increase their capital-labor ratio, the average product of labor should increase through this channel as well. Thus, the average product of labor rises faster with $B$ than the average product of capital.

## 2.5 Profit Maximization:

Cost minimization does not place any assumptions on the level of output. To complete the model, I introduce a demand side through a isoelastic demand function. Each firm produces a differentiated product and has a downward sloping demand curve for their product. The demand curve is proportional to price as follows:

$$q \propto p^{-\epsilon} \tag{22}$$

where here $\epsilon$ is the elasticity of demand and so greater than 1.

The production function has constant returns to scale, so the marginal cost of production does not depend upon the amount produced. Since the firm faces a downward sloping demand curve, the equilibrium price that it charges falls as it produces more quantity. The firm then has an optimal size as the cost of output is constant and the revenue from output is decreasing. [2]

Profit maximizing firms will then choose revenue such that:

$$PY \propto (AB)^{\gamma}(1+(\frac{w}{Br})^{\sigma-1})^{(\frac{1}{1-\sigma})(\epsilon-1)} \quad (23)$$

Firms with high Hicks neutral productivity $A$ and high labor augmenting productivity $B$ will generate more revenue. The average revenue products of labor and capital are:

$$\frac{PY}{L} \propto 1+(\frac{Br}{w})^{1-\sigma} \quad (24)$$

$$\frac{PY}{K} \propto 1+(\frac{w}{Br})^{1-\sigma} \quad (25)$$

Hicks neutral productivity does not affect the average revenue products. Improvements in Hicks neutral productivity induce the firm to produce more until the marginal return of factors meets factor prices. This increase in production pushes the firm down its demand curve until the price falls and average revenue products remain constant.

Labor augmenting productivity, by contrast, shifts the average revenue products of labor and capital in opposite directions. A firm with high labor augmenting productivity $B$ increases its labor ratio, depressing its average revenue product of capital and pushing up its average revenue product of labor.

[2] In effect, the isoelastic demand function is also isomorphic to decreasing returns to scale in the revenue production function. The isoelastic demand function implies that the optimal price will be a simple markup over marginal cost.

When labor and capital are complements, large persistent differences in labor augmenting productivity across plants implies:

1. The capital share of cost varies within plants in an industry and is persistent.
2. Large plants have a higher capital share of cost than small plants.
3. Large plants have a much higher average revenue product of labor than small plants.
4. Large plants do not have a much higher average revenue product of capital than small plants.

# 3 Results

## 3.1 Data

In this study, I primarily use US data on manufacturing plants from the Census of Manufacturers and Annual Survey of Manufacturers (ASM). The Census of Manufacturers is a census of all manufacturing plants taken every five years. For some small plants with less than 5 workers, called Administrative Record plants, the Census only records payroll and employment gathered from IRS data. Since capital is an important variable in my study, I drop these plants as is common in the literature.

The primary data constraint for this study is data on capital. Before 1987 the Census did not ask questions on capital stocks for non ASM plants. Thus, I use the 1987, 1997, and 2002 Census of Manufacturers. The Annual Survey of Manufacturers tracks about 50,000 plants over five year periods and is more heavily weighted towards big plants. I use the manufacturing Censuses for my main results, but I do look at the ASM only plants for some robustness checks.

For the manufacturing Census samples, I drop all plants that enter in the given Census year. Entering plants could have high levels of inputs (for example capital) but no output if the plant entered late in the year. In this case, the plant would look extremely unproductive only because the true levels of inputs are measured incorrectly. Similarly, if the entering plant has bought its capital but not yet employed workers, the capital-labor ratio may look high relative to other plants in the industry.

I also clean the data for outliers. First, I drop all observations where data on a number of variables is either missing, zero, or negative: these include the average product of capital, average product of labor, capital share of cost, capital-labor ratio, and wage (measured as payroll over employment). I also drop outlier observations in the bottom .5% and top 99.5% tails of these variables relative to their industry, which amounts to about 4% of the dataset for each Census. This data cleaning prevents huge outliers due to mismeasurement from affecting the results, but the main results are similar when these outliers are included.

For the 2002 Census of Manufacturers, data on the monetary benefits given to employees are available for most plants. I use this benefits data to better measure payments for labor. I also then drop data from plants whose benefits data is imputed.

I measure capital by the end year book value of capital, deflated using a current cost to historic cost deflator. The 1987 Census has book values for equipment capital and structures capital separately, so I construct capital stocks for each and then combine them. For the general Census of Manufacturers I can not use perpetual inventory methods, because investment is not recorded in non-Census years.

To measure the capital share of cost, I also need measures of factor prices. I use unpublished 2 digit BLS rental rates calculated by standard formulas converting capital prices to rental rates. For wage payments I use the total wage bill of the plant, except in 2002 when I add benefits payments as well.

I measure age for plants using the Longitudinal Business Database, which records the first year and last year of each plant. However, the LBD began in 1975 so any plants existing before 1975 are given a first year of 1975. To better measure age for older plants, I record the first year of the plant as 1972 if it existed in the 1972 Census of Manufacturers.

### 3.2 Estimates of the Elasticity of Substitution

The first order conditions for capital and labor of the CES production function under cost minimization imply that:

$$\log(rk/wl) = -(1-\sigma)\log(w/r) + (1-\sigma)\log B + \log\frac{\alpha}{1-\alpha} \tag{26}$$

The response of the factor cost ratio to factor price changes comes straight from the Robinson definition of the elasticity of substitution. If the production function is Cobb-Douglas, plants adjust their capital-labor ratio proportionately to increases in wages, so the factor cost ratio remains constant. If the elasticity of substitution is less than one, plants do not increase their capital intensity enough to compensate for the rise in wages and so the factor cost ratio falls. Thus, the slope of the relationship between the wage and the factor cost ratio depends upon the sign of the elasticity of substitution.

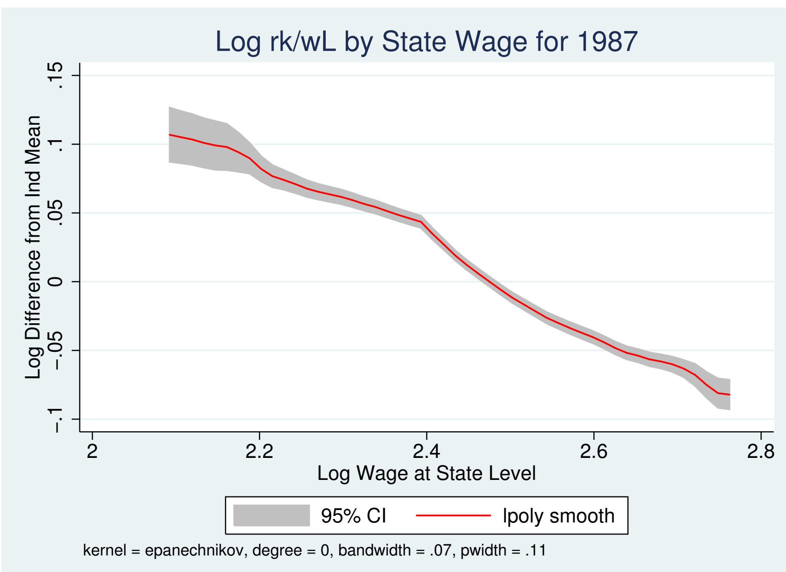
I compute the factor cost ratio as before, from data on capital and the wage bill at the plant level. I then use local area wage variation to estimate the elasticity of substitution.

I calculate measures of local area wages from a worker based dataset and an employer based dataset. The first source of wage variation is from the Census 5% sample of Americans, where I match the 1990 Census to the 1987 Census of Manufacturers and the 2000 Census to the 1997 Census of Manufacturers. I calculate the individual's wage as wage and salary income divided by the number of hours worked times weeks worked, for men working in the private sector with ages between 25 and 55. This wage is thus an individual hourly wage. I then construct the average log wage for each state and each MSA. I have experimented with using median log wages or average log wages for manufacturing workers only in unpublished results and get similar estimates to those below.

The second source of wage variation is on the firm side, using the Longitudinal Business Database (LBD). The LBD contains employment and payroll data for every establishment in the US (so around 7 million establishments). I define the wage as payroll divided by employment, so this wage is the average yearly wage for the establishment. I then construct average log wages for each state, MSA, and county in the United States. Since the LBD is yearly, I match the Manufacturing Censuses to wages from the appropriate year of the LBD.

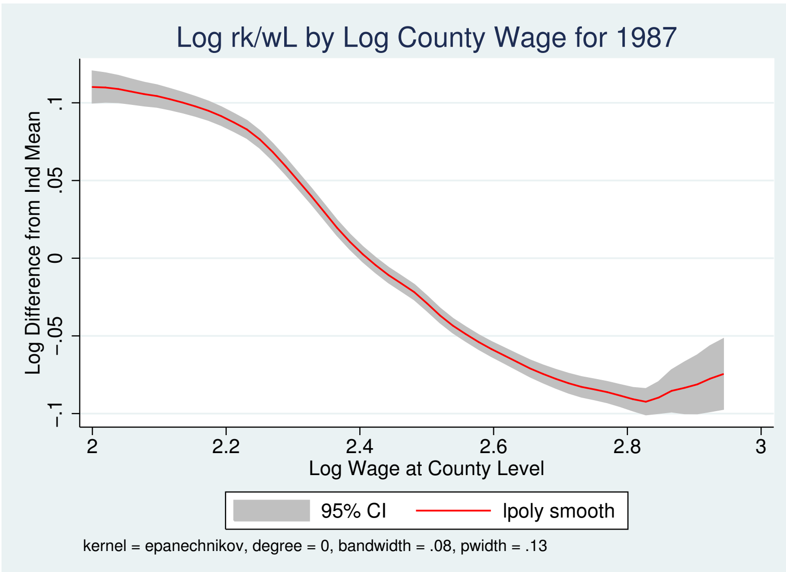
Figures 1 and 2 nonparametrically plot the industry demeaned factor cost ratio against the state level wage and the county level wage. In both plots, the factor cost ratio is strongly decreasing in the local area wage. The relationship does flatten out for very low wages and very high wages, especially in the county level wage graph. However, there are not many plants in areas with these extreme wages, so the confidence bands are quite wide.

Figure 1: Factor Cost Ratio by State Level Wage for 1987




The X axis is the average log wage at the state level, where the wage is calculated as the wage and salary income over total number of hours worked for a worker in the Census 5% sample data. The Y axis is the log factor cost share after taking out industry averages.

Figure 2: Factor Cost Ratio by County Level Wage for 1987




The X axis is the average log wage at the county level, where the wage is computed as payroll/number of employees at the establishment level for establishments in the Longitudinal Business Database. The Y axis is the log factor cost share after taking out industry averages.

Table 2 displays estimates of the elasticity of substitution for all of manufacturing. Each column of the table provides estimates of the elasticity of substitution from a different source of wage variation. In these regressions, I assume that the elasticity of substitution is the same for every 4 digit industry, but the distribution parameter $\alpha$ and average level of labor augmenting productivity $B$ can vary across industries through industry fixed effects. I cluster standard errors at the 2 digit SIC- local area level where the local area is based on the source of the wage variation, so the state-level regressions have standard errors clustered at the 2 digit SIC- state level, the MSA level regressions have standard errors clustered at the 2 digit SIC- MSA level, etc. This level of clustering adjusts the standard errors for correlated shocks to the factor cost ratio in local areas for plants in the same broad industry.

Table 2: Elasticities of Substitution between Labor and Capital for All Manufacturing

| | State Level | MSA Level | State Level | County Level | County Level, Within State |
|---|---|---|---|---|---|
| CMF 1987 | .53<br>*(.03)* | .66<br>*(.03)* | .53<br>*(.02)* | .60<br>*(.02)* | .65<br>*(.02)* |
| CMF 1997 | .45<br>*(.03)* | .69<br>*(.03)* | .47<br>*(.03)* | .60<br>*(.01)* | .67<br>*(.01)* |
| Source of Wage Data | Census 5% individual samples | | Longitudinal Business Database | | |
| State Dummies | No | No | No | No | Yes |
| N | ∼180,000 | ∼125,000 | ∼180,000 | ∼180,000 | ∼180,000 |

Note: All regressions include industry dummies and have standard errors clustered at the 2 digit industry-area level (so for state-level regressions, 2 digit sic-state, etc.) Wages used are the average log wage for the geographic area, where the wage is computed as payroll/number of employees at the establishment level for the LBD wages and the wage is wage and salary income over total number of hours worked for the Census 5% sample data.

The elasticity of substitution is .53 in the state-level wage regressions for 1987 and .45 or .47 in the state-level wage regressions for 1997. My estimates are not sensitive to the source of the state level wages, as estimates from the 5% Census sample wage regressions and LBD wage regressions are very similar. When I use more disaggregated wages at the MSA and county level, estimates are slightly higher. At the MSA level, I estimate that the elasticity is .66 for 1987 and .69 for 1997. Since the MSA level regressions drop all plants not located in an MSA, these estimates are based only on plants in major metropolitan areas. Using wages at the county level, I can incorporate plants outside major metro areas and still have a more disaggregated measure of the wage. County level wage regressions imply that the elasticity of substitution is .6. In all of these cases, I can comfortably reject that the production function is Cobb-Douglas.

I also run county-level wage regressions with state level fixed effects, so that all of the wage variation is within state. The previous regressions could have problems if state-level regulations affect both wages and the factor cost ratio.[3]

Using within state county level wage variation, I find that the elasticity of substitution is .65 for 1987 and .67 for 1997. These estimates are slightly higher than those without state fixed effects, but I can still comfortably reject an elasticity of 1.

So far, I have assumed that the elasticity of substitution is constant across all industries. In Table 3, I show estimates of the elasticity of substitution using state, county, and within state county level wage variation for each two digit SIC industry for 1987. I exclude the tobacco industry because it is much smaller than the other two digit industries. Two digit SIC industries are major broad industry groupings within manufacturing. For example, Textiles or Primary Metals are two digit SIC industries, while Carpets and Rugs (SIC 2273) and Steel Blast Furnaces (SIC 3312) are four digit SIC industries within these broader two digit SIC industries.

[3] For example, right to work laws make it more difficult for firms to unionize, which could both lower wages and make it easier for firms to automate and change their factor cost ratio. Holmes (1998) shows that plants do indeed respond to right to work laws, as industrial activity is higher than average in right to work states adjacent to non right to work states.

Table 3: Elasticities of Substitution between Labor and Capital for 2 digit SIC Industries

| SIC Two Digit Industry: | Level of Wage Variation | | | N |
|---|---|---|---|---|
| | State Level, 1987 | County Level, 1987 | County Level, Within State, 1987 | |
| 20: Food Products | .68<br>*(.09)* | .66<br>*(.04)* | .64<br>*(.05)* | ∼10,000 |
| 22: Textiles | .20<br>*(.11)* | .55<br>*(.09)* | .87<br>*(.09)* | ∼3,500 |
| 23: Apparel | .6<br>*(.1)* | .92<br>*(.06)* | 1.17<br>*(.06)* | ∼12,000 |
| 24: Lumber and Wood | .52<br>*(.12)* | .39<br>*(.05)* | .41<br>*(.05)* | ∼15,000 |
| 25: Furniture | .43<br>*(.14)* | .38<br>*(.05)* | .43<br>*(.07)* | ∼6,000 |
| 26: Paper | .05<br>*(.12)* | .35<br>*(.06)* | .39<br>*(.08)* | ∼4,000 |
| 27: Printing and Publishing | .68<br>*(.04)* | .67<br>*(.03)* | .68<br>*(.03)* | ∼26,000 |
| 28: Chemicals | .37<br>*(.13)* | .4<br>*(.09)* | .38<br>*(.09)* | ∼6,500 |
| 29: Petroleum Refining | .43<br>*(.22)* | .7<br>*(.12)* | .86<br>*(.14)* | ∼1,500 |
| Source of Wage Data | Census 5% individual samples | Longitudinal Business Database | | |
| State Dummies | No | No | Yes | |

| SIC Two Digit Industry: | Level of Wage Variation | | | N |
|---|---|---|---|---|
| | State Level, 1987 | County Level, 1987 | County Level, Within State, 1987 | |
| 30: Rubber | .37<br>*(.14)* | .46<br>*(.05)* | .54<br>*(.05)* | ∼8,500 |
| 31: Leather | .73<br>*(.24)* | .77<br>*(.11)* | .86<br>*(.18)* | ∼1,000 |
| 32: Stone, Clay, Glass, Concrete | .39<br>*(.11)* | .62<br>*(.04)* | .75<br>*(.05)* | ∼9,000 |
| 33: Primary Metal | .39<br>*(.11)* | .6<br>*(.06)* | .69<br>*(.08)* | ∼4,000 |
| 34: Fabricated Metal | .44<br>*(.08)* | .47<br>*(.04)* | .52<br>*(.04)* | ∼20,000 |
| 35: Machinery | .6<br>*(.07)* | .65<br>*(.02)* | .68<br>*(.03)* | ∼25,000 |
| 36: Electrical Machinery | .45<br>*(.11)* | .53<br>*(.07)* | .64<br>*(.06)* | ∼8,000 |
| 37: Transportation Equip | .63<br>*(.14)* | .64<br>*(.06)* | .77<br>*(.07)* | ∼5,000 |
| 38: Instruments | .76<br>*(.1)* | .61<br>*(.06)* | .55<br>*(.09)* | ∼4,500 |
| 39: Misc | .73<br>*(.13)* | .51<br>*(.04)* | .51<br>*(.06)* | ∼6,500 |
| Source of Wage Data | Census 5% individual samples | Longitudinal Business Database | | |
| State Dummies | No | No | Yes | |

Note: All regressions include industry dummies and have standard errors clustered at the 2 digit industry-area level (so for state-level regressions, 2 digit sic-state, etc.) Wages used are the average log wage for the geographic area, where the wage is computed as payroll/number of employees at the establishment level for the LBD wages and the wage is wage and salary income over total number of hours worked for the Census 5% sample data.

The elasticity of substitution does vary considerably between 2 digit industries, but most of these differences are not statistically significant. For an example of statistically significant differences, the lumber and wood industry has an elasticity of about .4 which is significantly different from the leather industry with an elasticity of .77. While there are some differences in estimates between different types of wage variation within the same industry, only for Textiles and Apparel are these estimates significantly different from each other.

I can reject that the elasticity of substitution is one for 17 of 19 industries using state level wages, 17 of 19 industries using county level wages, and 15 of 19 industries using within state county level wages. In only one case, for the Apparel industry with within state county level wage variation, do I find a point estimate of the elasticity that is above one. Thus, disaggregating to the SIC 2 digit level does not alter my main conclusion that the Cobb-Douglas specification can be rejected.

## 3.3 Potential Caveats to Estimation

### 3.3.1 Local Area Agglomerations

Many industries exhibit agglomeration effects, which lead firms in those industries to concentrate in particular geographic areas. For example, much of the US furniture industry is located in North Carolina, the computer industry in Silicon Valley, and the auto industry in Detroit. If the plants inside the agglomeration area were no different than the plants outside, such local area industry concentrations would mean less wage variation to identify the elasticity of substitution and so higher standard errors on estimates.

To bias the regression estimates, plants in the agglomeration area must have a different production function than those outside and wages would have to be different in agglomeration areas than non agglomeration areas. For the furniture industry, North Carolina plants would need to pay higher wages than the rest of the US and have a production function with a lower capital share (or vice versa). Holmes and Stevens (2010) argue that some industries are characterized by a large mass (bottom 80%) of small producers producing for specialized local demand, and a few (top 20%) of producers producing on a mass scale for the entire market. If the production functions for local demand and global demand were different, agglomeration areas where the big producers operate could have systematic differences in their factor cost ratio from the rest of the US.

To check whether these forces are driving my results, I look at 10 four digit SIC industries with substantial geographic variation. I select all the industries that are located in at least 300 MSAs or at least 250 MSAs and 48 states, dropping industries that have less than 1,000 plants or are miscellaneous industries (plants that the SIC code did not classify anywhere else are sometimes put into miscellaneous industries). Newspaper Publishing, Commercial Lithographic Printing, and Ready Mixed Concrete are among the biggest of these industries. Ready Mixed Concrete is perhaps the best test case; since ready mixed concrete can not be shipped very far, every location that has construction activity must have concrete plants. For concrete, there are no agglomerations and no differences between local and non-local producers. Table 4 displays the estimates of the elasticity of substitution using state and county level wages for these industries. I can reject Cobb-Douglas for eight out of ten industries with state level wages and ten out of ten industries with county level wages. For ready mixed concrete, I find an elasticity of substitution of .36 in the state-level regressions and .8 in the county level regressions.

Table 4: Elasticities of Substitution between Labor and Capital for Geographically Varying Industries

| SIC Four Digit Industry | Level of Wage Variation | | N |
|---|---|---|---|
| | State Level, 1987 | County Level, 1987 | |
| 2711: Newspaper Publishing | .52<br>*(.11)* | .75<br>*(.07)* | ~4,000 |
| 2752: Commercial Printing, Lithographic | .8<br>*(.05)* | .71<br>*(.03)* | ~12,000 |
| 3272: Concrete Products, Except Block and Brick | .54<br>*(.16)* | .8<br>*(.07)* | ~2,000 |
| 3273: Ready Mixed Concrete | .36<br>*(.16)* | .8<br>*(.07)* | ~4,000 |
| 3441: Fabricated Structural Metal | .47<br>*(.17)* | .48<br>*(.11)* | ~1,500 |
| 3444: Sheet Metal Work | .54<br>*(.12)* | .45<br>*(.09)* | ~3,000 |
| 2051: Bread and other Bakery Products, except Crackers | .77<br>*(.21)* | .63<br>*(.12)* | ~1,000 |
| 2421: Sawmills and Planing Mills | .94<br>*(.21)* | .75<br>*(.08)* | ~3,000 |
| 2431: Millwork | .01<br>*(.14)* | 0<br>*(.09)* | ~1,500 |
| 2434: Wood Kitchen Cabinets | .35<br>*(.19)* | .23<br>*(.11)* | ~2,000 |
| Source of Wage Data | Census 5% individual samples | Longitudinal Business Database | |
| State Dummies | No | No | |

Note: All regressions include industry dummies and have standard errors clustered at the 2 digit industry-area level (so for state-level regressions, 2 digit sic-state, etc.) Wages used are the average log wage for the geographic area, where the wage is computed as payroll/number of employees at the establishment level for the LBD wages and the wage is wage and salary income over total number of hours worked for the Census 5% sample data.

To improve on this analysis, I plan to estimate the elasticity of substitution separately for the industries that Holmes and Stevens identify as not having differences between local demand plants and mass market plants. I will also estimate the elasticity of substitution separately for industries with low agglomerations using an agglomeration index incorporating differences in output across areas, as in Ellison and Glaeser (1997), and not just dispersion in location of plants.

### 3.3.2 Different Types of Workers

So far in this paper, I have combined all labor into one plant level aggregate. This aggregation implies that different types of workers are perfect substitutes for each other and complementary with capital with the same elasticity of substitution. Capital could be more complementary with some kinds of workers than others.

The simplest disaggregation of labor is into skilled workers and unskilled workers. In fact, one major agenda of the recent labor literature has been investigating skill biased technical change, where productivity increases are biased towards skilled workers. Also, many papers assume that capital and skilled labor are complementary. In the Manufacturing Censuses, I do have information on the number of production workers and non production workers and their wages. I use production workers are a proxy for unskilled workers and nonproduction workers for skilled workers, as is done by a number of papers in the literature including Kahn and Lim (1998) and Blum (2010).

I then estimate the elasticity of substitution between capital and skilled labor, capital and unskilled labor, and skilled and unskilled labor. The regression equations are similar to that for capital and labor, except that I use the high school average log wage from the Census 5% files for a measure of the unskilled wage and the college average log wage for a measure of the skilled wage. Table 5 shows the results using state and MSA level wage variation. I find that the elasticity between capital and skilled workers is always significantly lower than that between capital and unskilled workers, consistent with capital-skill complementarity.

The elasticity of substitution between capital and skilled labor is very low, at .26 for 1987 and .14 for 1997 using state wages. Estimates at the MSA level are higher, at .36 for 1987 and .45 for 1997, but are still fairly low.

Table 5: Partial Elasticities of Substitution between Capital and Skilled Labor, Capital and Unskilled Labor, and Skilled and Unskilled Labor, for all Manufacturing

| Elasticity | Level of Wage Variation | | | |
|---|---|---|---|---|
| | State Level, 1987 | MSA Level, 1987 | State Level, 1997 | MSA Level, 1997 |
| Capital and Unskilled Labor | .57<br>*(.03)* | .75<br>*(.03)* | .72<br>*(.05)* | .93<br>*(.05)* |
| Capital and Skilled Labor | .26<br>*(.04)* | .36<br>*(.03)* | .14<br>*(.04)* | .45<br>*(.03)* |
| Skilled and Unskilled Labor | 1.14<br>*(.055)* | 1.16<br>*(.04)* | 1.31<br>*(.04)* | 1.13<br>*(.03)* |
| State Dummies | No | No | No | No |
| N | ∼180,000 | ∼110,000 | ∼180,000 | ∼130,000 |

Note: I run regressions with the high school wage (wage of high school completers from the Census 5% samples) to compute the unskilled labor-capital elasticity, regressions with the college wage (wage of college completers from the Census 5% samples) to calculate the skilled labor-capital elasticity, and regressions with the relative wage of high school completers to college completers to calculate the skilled labor-unskilled labor elasticity.

The elasticity of substitution between capital and unskilled labor is higher, at .57 for 1987 and .72 for 1997 using state level wages. I can still reject that the elasticity of substitution between capital and unskilled labor is 1 for three of the four estimates. Only for MSA level wages in 1997 is the estimate of the elasticity .93, high enough to be insignificantly different than one. The estimates of the elasticity of substitution between skilled and unskilled labor indicate that skilled and unskilled labor are substitutes, with elasticities slightly, but significantly, above one.

The basic CES production function can not rationalize these estimates, as the CES form implies that the partial elasticities between all factors would be equal. The nested CES production function of Sato (1967) and the translog production function, as well as a whole range of other possibilities considered in Fuss and McFadden, eds (1978) are all candidate production functions to explain these estimates. However, exploring these is beyond the scope of this paper.

#### 3.3.3 Different Production Functions than CES

So far, I have assumed that the elasticity of substitution between labor and capital is constant. If a more broad production function characterizes an industry, the elasticity of substitution will not necessarily be constant. The translog production function, for example, does allow the elasticity of substitution to vary across plants. One way to test this is to run a regression of the factor cost ratio against the local area wage including polynomial terms in the local area wage as well. If the elasticity of substitution is constant, the relationship between the factor cost ratio and local area wage should be linear. In this case, one should not be able to reject that the coefficient on all of the non linear terms are jointly zero.

I include quadratic, cubic, and quartic local area wage terms in the regressions using all of manufacturing and conduct joint tests that the coefficients on all of these terms are equal to zero. In all cases, I reject the null hypothesis, raising the possibility that a more general production function than the CES characterizes the data. I plan to examine how large these deviations from linearity are, especially outside the tails of the local area wage distribution.

Another way to explain the above facts is that plants within the same industry have different Cobb-Douglas production functions. Differences in Cobb-Douglas production functions would lead plants to set different factor cost ratios. Since high wage areas have lower factor cost ratios, the Cobb-Douglas capital coefficient would have to be lower in high wage areas.

In models where plants can choose either their production technology or their location, plants have higher Cobb-Douglas capital coefficients in high wage areas. If a firm can choose between two different Cobb-Douglas production technologies, the relative cost of these technologies depends on factor prices. As wages increase, the more capital intensive Cobb-Douglas technology is favored because its relative cost falls. Firms could also choose where to locate. High capital intensive technology firms should be more likely to locate in high wage areas, as their costs rise less with high wages. Both of these scenarios predict that high capital share technologies would be observed in high wage areas, the opposite of what I find.

### 3.3.4 Endogeneity Concerns

My identification strategy relies upon differences in wages across local areas in the US. One potential concern is that the local area wage is endogenous to factors that affect firm factor cost ratios. To explore this, I first examine what causes differences in wages across local areas. If there are no frictions preventing people from moving around the country, people move seeking higher wages until the real wage that the worker faces is the same across locations. The wage important for my study is the wage that the plant pays its workers. In areas with a high cost of living (because of housing prices, for example), wages faced by the plant will be high even if real wages for workers are the same across areas. Thus, cost of living differences can lead to local area wage differences.

Migration costs are another reason for local area wage dispersion. Increases in labor demand will increase wages when labor supply is relatively fixed. Labor demand is affected by many factors, including the number of firms in the area and demand shocks to local industries. Local demand shocks will not affect the plant's optimal ratio of factor costs, however, as changes in demand do not affect the cost minimization conditions. Hicks neutral productivity improvements will also increase labor demand but not affect the plant's factor cost ratio. Only improvements in labor augmenting productivity $B$ could both change the wage, by increasing labor demand, and change the plant's factor cost ratios. Even in this narrow case, manufacturing may not affect local area wages very much. Manufacturing is a small percentage of total employment, at 17.4% of total employment in 1987 and 14.5% of total employment in 1997.

Small local industries such as ready mixed concrete should have little affect on local area labor demand. To recall previous results in Table 4, I find an elasticity of substitution less than one for these industries as well. I plan to examine local areas where manufacturing is small relative to the local economy as another robustness check.

Another potential concern is that the plant's choice of its level of labor augmenting productivity $B$ depends on the local wage. If plants adjust their level of labor augmenting technology because of the local wage, $B$ will be related to the wage. When wages are high, labor augmenting technology that saves on labor is more valuable. Such wage based technology adoption would cause high wage areas to have high levels of labor augmenting technology and high capital shares. Thus, the relationship between the local area wage and labor augmenting technology would bias the estimate of the elasticity of substitution towards one.

## 3.4 Stylized Facts about the Capital Share

Since I have found that the elasticity of substitution is less than one, I now look at stylized facts on the capital share of cost that could be generated by labor augmenting productivity.

### 3.4.1 Persistence over time:

The capital-labor ratio and capital share of cost are both persistent over time. If labor augmenting productivity is persistent, the capital share of cost should be as well. On the other hand, measurement error or adjustment costs should not lead to persistent differences in the capital share of cost and capital labor ratio. Measurement error should only cause temporary dispersion in capital shares unless measurement error in capital is serially correlated over a ten year span. Firms that face adjustment costs will not always match the static first order conditions of the simple theory above. Instead, they will only adjust capital infrequently when their capital stock gets too far away from the optimal capital stock. However,

Table 6: Persistence in Capital Share and Capital-Labor Ratio between CMF87 and CMF97

| | Ten Year | Implied One Year | Ten Year | Implied One Year |
|---|---|---|---|---|
| Log(Capital-Labor Ratio) | .42<br>*(.004)* | .92<br>*(.001)* | .48<br>*(.003)* | .93<br>*(.001)* |
| Log(Capital Share) | .28<br>*(.004)* | .88<br>*(.001)* | .27<br>*(.002)* | .88<br>*(.001)* |
| TFP | .27<br>*(.003)* | .88<br>*(.001)* | .39<br>*(.003)* | .91<br>*(.001)* |
| Weights | No | No | Value Added | Value Added |

All regressions contain 4 digit SIC industry dummies. TFP is measured by log(Value Added) minus log capital and log labor (no of employees) weighted by 4 digit industry level cost shares. The implied one year coefficient is the ten year coefficient to the 1/10 power.

over a period of ten years the firm should have readjusted its capital stock, so adjustment costs would not predict long run persistence in the capital share or capital-labor ratio.

First, I regress each variable in 1997 against its value for the same plant in 1987, controlling for industry dummies. I compare these values with the autocorrelation values for conventionally measured TFP, measured by log value added subtracting log capital and log labor (total number of employees), both weighted using the industry level cost shares of the input. I use TFP as a comparison since TFP is well known to be autocorrelated over time. Table 6 contains the estimates.

The capital labor ratio is extremely autocorrelated over time, with a 10 year coefficient of .42 implying a one year auto-correlation of .92. The capital share is somewhat less autocorrelated, with a 10 year coefficient of .28 implying a one year autocorrelation of .88. Small differences in the one year auto correlation rates can lead to big differences in the 10 year rates. TFP is about as autocorrelated as the capital share.

I also run weighted regressions with value added weights, which measure the autocorre-

Table 7: Transition Table between Quartiles of Capital-Labor Ratio from CMF87 to CMF97

| Quartile, CMF 87 | Quartile, CMF 97 | | | |
|---|---|---|---|---|
| | Q1 | Q2 | Q3 | Q4 |
| Q1 | **36.3%** | 29.1% | 21.2% | 13.4% |
| Q2 | 21.26% | **29.1%** | 29% | 20.7% |
| Q3 | 14.7% | 22.4% | **31.4%** | 31.5% |
| Q4 | 9.5% | 13.7% | 24.6% | **52.2%** |

Here Quartile 1 have the smallest 25% of plants by within industry capital-labor ratio and Quartile 4 the largest 25% of plants by within industry capital-labor ratio.

Table 8: Transition Table between Quartiles of Capital Share from CMF87 to CMF97

| Quartile, CMF 87 | Quartile, CMF 97 | | | |
|---|---|---|---|---|
| | Q1 | Q2 | Q3 | Q4 |
| Q1 | **39.3%** | 27% | 19.4% | 14.3% |
| Q2 | 22.5% | **30.3%** | 23.9% | 23.4% |
| Q3 | 18.9% | 26.1% | **26.8%** | 28.2% |
| Q4 | 12% | 19.3% | 21.6% | **47.2%** |

Here Quartile 1 have the smallest 25% of plants by within 4 digit industry capital share and Quartile 4 the largest 25% of plants by within 4 digit industry capital share.

lation of the biggest plants in the industry. In the weighted regressions, both TFP and the capital-labor ratio become even more autocorrelated, with ten year rates of .93 and .91. The capital share coefficients remain the same.

Another way to look at persistence is to compute transition tables. Within industry year, I assign plants to quartiles based on their capital share or capital-labor ratio. I then examine how much movement there is between quartiles over ten years. If the variables are not persistent, a plant in 1987 should be equally likely to be in either of the four quartiles in 1997, regardless of what its initial quartile was. Tables 7 and 8 contain these transition tables.

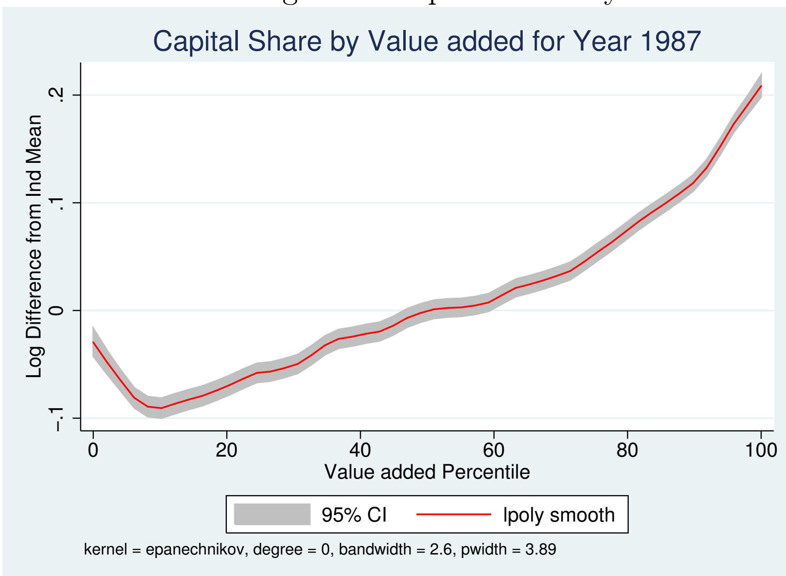
For the capital-labor ratio, more than 50% of the largest quartile plants in 1987 are in the largest quartile of plants in 1997. For the capital share, 47.2% of the largest quartile plants in 1987 are in the largest quartile of plants in 1997. Results are similar but smaller for the smallest quartile plants, with 37% and 39% for the capital-labor ratio and capital share. Thus, the capital-labor ratio and capital share have a high degree of persistence over time.

### 3.4.2 Correlation with Value added:

I next look at how the capital share moves with output. For output, I use real double-deflated value added. [4]For each plant, I calculate its output percentile relative to the rest of the industry I then calculate each firm's log capital share taking out SIC 4 digit industry dummies (or 6 digit NAICS dummies, for the 2002 Census). Using local polynomial regression, I regress the demeaned plant log capital share on the plant output percentile. I use local polynomial regressions to avoid placing functional form assumptions upon the relationship between the variables. Figure 3 and Figure 4 show the local polynomial graphs for 1987 and 2002. In both cases, the largest plants of the industry have a much higher capital share of cost than the smallest plants- for 1987 about 30% higher and for 2002 about 80% higher. This increase is not just the biggest plants having a larger capital share than the smallest plants, however. Even among the largest 20% of plants in a given industry we see a substantial increase in the capital share for larger plants.

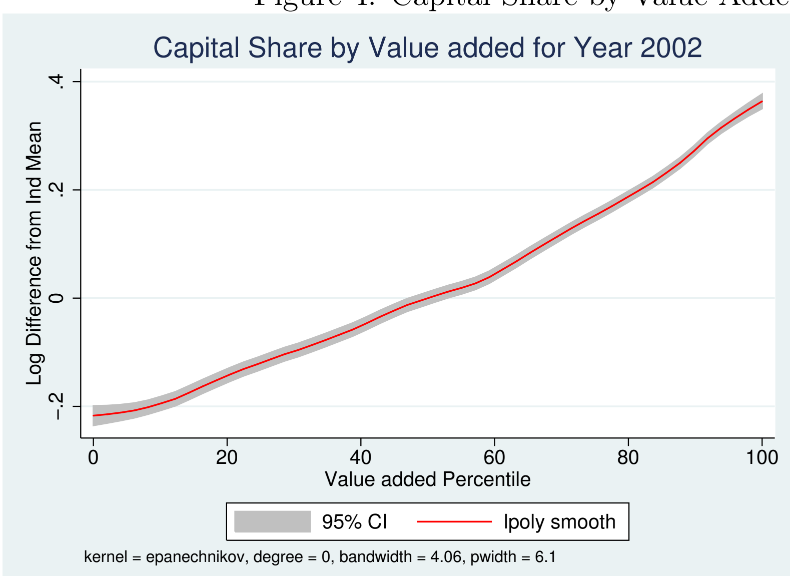
[4] As Basu and Fernald (1997) point out, value added is the correct measure of output if firms are perfectly competitive or materials are Leontief with labor and capital. I am assuming the elasticity of substitution between materials and labor and capital together is zero, so value added works as a measure of output. However, I do find the same patterns using total sales as the size measure instead of value added.

Figure 3: Capital Share by Value Added for Year 1987




The X axis is a plant's percentile of value added relative to its industry. The Y axis is the log cost share of capital after taking out industry averages. The graph was generated using local polynomial regression.

Figure 4: Capital Share by Value Added for Year 2002

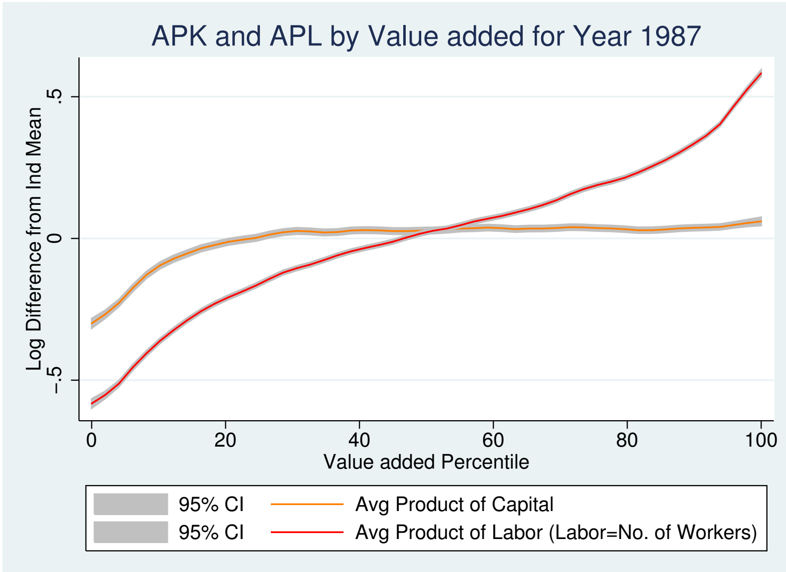



The X axis is a plant's percentile of value added relative to its industry. The Y axis is the log cost share of capital after taking out industry averages. The graph was generated using local polynomial regression.

For 1987 the capital share dips slightly for the smallest plants, which is not the case in 2002. In 1997 this dip is substantially bigger. The rise in the capital share after the smallest 20% of plants is always present, however. I can only measure the amount of capital owned by firms, not the amount of capital used by the firms. If a few firms shut down or produce much less than usual, they are not using much of their capital stock. These firms will have very low output but a high capital share relative to the industry. This example illustrates a general problem: capital utilization rates are not observed. Differences in utilization lower the true capital share for low output firms and raise the true capital share for high output firms, and so should only bolster my findings of a rising capital share with output.

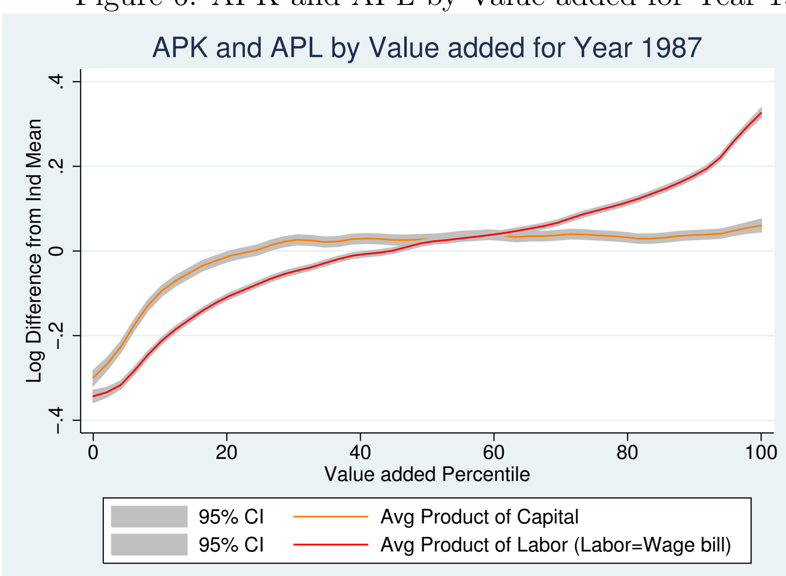
I also look at the correlation of value added with the average revenue product of capital and average revenue product of labor. I run similar local polynomial regressions to those for the capital share using the 1987 Census. I measure labor both as the total number of employees, in Figure 5, and as the wage bill in Figure 6. The average revenue product of capital is increasing, by about 40%, but only for the smallest 20% of plants. For the rest of the plants the average revenue product of capital is constant. The average revenue product of labor is increasing enormously, by more than 170% when using total number of employees as the measure of labor and by about 100% using the wage bill as the measure of labor. Thus the average product of labor is increasing when the average product of capital is constant for the upper 80% of plants. These relationships look similar in 1997. In 2002, the average product of capital is actually falling for the largest plants.

Figure 5: APK and APL by Value added for Year 1987, where Labor=Number of Workers




The X axis is a plant's percentile of value added relative to its industry. The Y axis is the log average product of capital or average product of labor after taking out industry averages. Here labor is defined as the number of workers at the plant. The graph was generated using local polynomial regression.

Figure 6: APK and APL by Value added for Year 1987, where Labor=Wage Bill




The X axis is a plant's percentile of value added relative to its industry. The Y axis is the log average product of capital or average product of labor after taking out industry averages. Here labor is defined as the plant's wage bill. The graph was generated using local polynomial regression.

Table 9: Correlations with Size for K/L Ratio, Capital Share, APK, APL

| | 1987 | | 1997 | |
|---|---|---|---|---|
| Log(Capital-Labor Ratio) | .15<br>*(.0015)* | .18<br>*(.005)* | .124<br>*(.002)* | .17<br>*(.015)* |
| Log(Capital Share) | .04<br>*(.001)* | .07<br>*(.003)* | .01<br>*(.001)* | .07<br>*(.007)* |
| Log(APK) | .07<br>*(.002)* | .03<br>*(.006)* | .099<br>*(.001)* | .074<br>*(.02)* |
| Log(APL) | .22<br>*(.001)* | .21<br>*(.005)* | .22<br>*(.001)* | .24<br>*(.008)* |
| Weights | No | Value Added | No | Value Added |

All of these coefficients are from regressions with the LHS variable as the dependent variable and log of value added as the independent variable. Controls include dummy variables for age and state, single establishment status and 4 digit SIC industry. I use robust standard errors.

Table 9 contains regressions where each cell represents a different regression with log capital share, the log capital-labor ratio, the log average product of capital and the log average product of labor as the dependent variables and log value added as the independent variable, as well as controls for state, age, industry, and single establishment status, using the 1987 and 1997 Censuses. The basic results of the graphs remain, though a linear relation is not the best functional form for the relationship. A linear regression finds a positive linear relationship between the average product of capital and value added, even though all of the increase was only for the smallest 20% of plants. Consistent with this, the regressions weighted with value added have a lower coefficient on the average product of capital than the unweighted coefficients. Still, the average product of labor is increasing much faster than the average product of capital in these regressions.

I also examine whether the increasing relationship between capital share and value added holds for single establishment plants and multiestablishment plants separately. If multiestablishment plants are larger and have employees that contribute to production at the plant but are in other establishments, such as senior managers, the capital share of big firms could be overstated. To check for this, I regress the log capital share and the log capital labor ratio against log value added and controls, plus interaction terms between single establishment status and log value added. Table 10 contains these results. I find the capital-labor ratio and capital share to be higher for large plants for both single-establishment and multiestablishment plants, with coefficients of similar order of magnitude for both sets of plants. For example, in 1987 the capital share increases by 4% with value added for single establishment plants and 5% for multiestablishment plants in the unweighted regressions, or 6% and 7% in the weighted regressions. The only case where the single establishment and multi establishment plants move differently is for the 1997 unweighted regressions. In the 1997 unweighted regressions, the capital share increases by 5% with value added for multiestablishment plants but falls by 1% for single establishment plants. In the weighted regressions, though, I find that the capital share increases by 8% for single establishment plants and 6.5% for multiestablishment plants. I believe that the differences between the weighted and unweighted regressions are due to low capital utilization for the smallest single establishment plants in 1997.

So far I have used book values of capital for the capital stock. For the ASM plants it is possible to calculate capital using perpetual inventory methods. The advantage of perpetual inventory methods is that the vintage of each part of the capital stock is known, so I can depreciate each vintage by its age and deflate each vintage by its investment year's investment deflator. The disadvantage of the base perpetual inventory methods is that they do not take into account retirements of the capital stock. Plants retire their capital stock at a rate of about 4% a year, which is concentrated in a few plants retiring a lot of capital stock. Since firms retiring capital deduct the retirement values from their book value, the book value measures incorporate the depreciation from retirements. Data on retirements of capital stock are available for all years up to 1985, after which retirements are recorded only in Census years. I am constructing a perpetual inventory capital stock for the ASM plants

Table 10: Robustness Checks: Single vs. MultiUnit Establishments

| | 1987 | | 1997 | |
|---|---|---|---|---|
| Single Unit: | | | | |
| Log(Capital-Labor Ratio) | .16<br>*(.002)* | .16<br>*(.02)* | .11<br>*(.012)* | .19<br>*(.015)* |
| Log(Capital Share) | .04<br>*(.002)* | .06<br>*(.015)* | -.01<br>*(.001)* | .08<br>*(.008)* |
| Multi Unit: | | | | |
| Log(Capital-Labor Ratio) | .13<br>*(.002)* | .18<br>*(.006)* | .15<br>*(.002)* | .165<br>*(.016)* |
| Log(Capital Share) | .05<br>*(.001)* | .07<br>*(.004)* | .05<br>*(.002)* | .065<br>*(.007)* |
| Weights | No | Value Added | No | Value Added |

All of these coefficients are from regressions with the LHS variable as the dependent variable and log of value added as the independent variable. Controls include dummy variables for age and state, single establishment status and 4 digit SIC industry. The Single Unit and Multi Unit coefficients come from the same regression- I put an interaction of log(va) with multiunit status. I use robust standard errors.

that incorporates the retirement data to conduct robustness checks.

# 4 Productivity Estimation and Results

The stylized facts in the previous section imply that plants may differ on their levels of labor augmenting productivity. Since I have estimated the elasticity of substitution, I can now examine the productivity of each plant. I do not want to hard wire that productivity is completely Hicks neutral or completely labor augmenting. Thus, I estimate both a Hicks neutral productivity parameter $A$ and a labor augmenting productivity parameter $B$. Here, I assume that the quantity production function is a CES production function with constant returns to scale. Since the distribution parameter $\alpha$ can not be separated from labor augmenting productivity when the elasticity is less than one, I ignore it. Under these assumptions, cost minimization provides that:

$$\log B = \log(k/l) + \frac{\sigma}{1-\sigma}\log(\frac{rk}{wl}) \tag{27}$$

Thus, given the elasticity of substitution I can calculate the labor augmenting productivity of the plant straight from its factor allocations and payments, without using any data on output! I provide two estimates of $B$, the first where labor is measured as the number of employees and the second where labor is measured as the wage bill. The wage bill can account for higher quality labor but could also be influenced by higher local area wages or other factors. If labor is measured as the wage bill, my measure of labor augmenting productivity is effectively the factor cost ratio multiplied by $\frac{1}{1-\sigma}$, which increases the factor cost ratio since $\sigma$ is less than one. I set $\sigma$ to .6, which is the value of the elasticity of substitution I estimate in the county level wage regressions for all of manufacturing.

To estimate the Hicks neutral productivity $A$, I take the equation for the average product of capital and impose cost minimization conditions on the factors. The Hicks neutral productivity $A$ is then:

$$\log A = \log(Y/k) - \frac{\sigma}{1-\sigma}\log(\frac{rk}{rk+wl}) \tag{28}$$

Since I do not have data on quantity, only revenue, the measure of Hicks neutral productivity that I can calculate includes both Hicks neutral productivity from the quantity production function and differences in prices between plants in the industry:

$$\log A + \log P = \log(PY/k) - \frac{\sigma}{1-\sigma}\log(\frac{rk}{rk+wl}) \tag{29}$$

My approach to estimation of productivity is similar to the cost share approach for the Cobb-Douglas, but generalized to the CES production function and allowing for both Hicks neutral and labor augmenting productivity. So far, I have not used the information in the level of output to improve the estimates of $B$, but increases in $B$ should increase output as well as the capital share. Thus, it might be possible to provide better estimates of the labor augmenting productivity using this information.

I find that my estimates of $A$ and $B$ are highly negatively correlated. First, errors in capital will tend to move $A$ and $B$ in opposite directions as higher capital stocks will increase $B$ and decrease $A$ mechanically, leading to substantial negative correlation. Second, most models with decreasing demand where firms can decide their price imply that high productivity firms have low prices. Foster et al. (2008) find that physical TFP and the plant price are strongly negatively correlated. Since my measures of $A$ include differences in prices, plants that have low prices due to high $B$ will have a negative correlation between my measured $A$ and measured $B$.

I then examine some of the standard relationships between TFP and plant level variables found in the literature, checking how $A$ and $B$ vary with these variables. Table 11 examines

Table 11: Correlations between Productivity and Size

| | Log(VA) | | Log Employment | |
|---|---|---|---|---|
| Log(A) | -.05<br>*(.003)* | -.105<br>*(.002)* | -.08<br>*(.003)* | -.16<br>*(.002)* |
| Log(B) *(Labor=No of employees)* | .28<br>*(.003)* | .33<br>*(.003)* | .11<br>*(.004)* | .22<br>*(.003)* |
| Log(B) *(Labor= Wage Bill)* | .19<br>*(.003)* | .25<br>*(.003)* | .085<br>*(.004)* | .16<br>*(.003)* |
| TFP | .17<br>*(.001)* | .14<br>*(.001)* | .009<br>*(.001)* | .02<br>*(.0009)* |
| Weights | No | Value Added | No | Value Added |

All regressions contain 4 digit SIC industry dummies. TFP is measured by log(Value Added) minus log capital and log labor (no of employees) weighted by 4 digit industry level cost shares. Log(VA) or Log(Employment) are the independent variables.

how $A$, $B$, and TFP vary with the size of the plant. I construct TFP as before, as log value added minus an industry cost share weighted amounts of capital and labor. I use two measures of size, employment and value added. In the table, each cell is a separate regression with a log productivity measure as the dependent variable and a log size measure as the independent variable, along with industry dummies as controls. I find that TFP is positively correlated with revenue but not with employment. The labor augmenting productivity $B$ is positively correlated with both employment and value added, while my measure of $A$ is negatively correlated with both. These findings are consistent with simple models with decreasing demand, in which large productive plants have low prices and so low measured $A$.

I also look at 10 year size growth in Table 12, again measuring size both as employment and value added. Conventional TFP is positively correlated with employment growth but negatively correlated with value added growth. The labor augmenting productivity $B$ is

Table 12: Correlations between Productivity and Size Growth

| | Ten Year VA Growth | | Ten Year Employment Growth | |
|---|---|---|---|---|
| Log(A) | -.16<br>*(.006)* | -.16<br>*(.005)* | -.078<br>*(.008)* | -.12<br>*(.008)* |
| Log(B)<br>*(Labor=No of employees)* | -.002<br>*(.007)* | .055<br>*(.006)* | .2<br>*(.009)* | .285<br>*(.009)* |
| Log(B)<br>*(Labor= Wage Bill)* | .02<br>*(.007)* | .076<br>*(.006)* | .16<br>*(.009)* | .3<br>*(.009)* |
| TFP | -.164<br>*(.002)* | -.13<br>*(.002)* | .078<br>*(.002)* | .06<br>*(.003)* |
| Weights | No | Value Added | No | Value Added |

All regressions contain 4 digit SIC industry dummies. TFP is measured by log(Value Added) minus log capital and log labor (no of employees) weighted by 4 digit industry level cost shares.

positively correlated with both, while my measure of $A$ is negatively correlated with both.

These variables are also related to entry and survival. I define a plant as an entrant if it entered in the previous two years and a plant as surviving if it is still in business after the next two years. TFP is positively correlated with survival and negatively correlated with entry. I find that entering firms and surviving firms both have high $A$ and low $B$ in Table 13. This result would imply that exiting firms have high levels of labor augmenting productivity, which is strange as almost every model and previous data study find that firms that exit are less productive. However, my estimates of $A$ and $B$ rely upon firms cost minimizing. If exiting firms begin to reduce levels of inputs before exit, they would stop investment in capital, fire workers, and lower wages. If they can not sell their existing capital, however, they may look capital intensive with a higher capital-labor ratio and higher capital share, and so have a high $B$ for spurious reasons. In a sense, utilized capital is likely to be low but measured capital is high and so measured labor augmenting productivity $B$ is high.

I also examine the autocorrelation of productivity, as TFP is known to be highly corre-

Table 13: Correlations between Productivity and Entry and Survival

| | Entrant | | Survival | |
|---|---|---|---|---|
| Log(A) | .03<br>*(.012)* | .06<br>*(.02)* | .16<br>*(.013)* | .09<br>*(.02)* |
| Log(B)<br>*(Labor=No of employees)* | -.08<br>*(.014)* | -.2<br>*(.02)* | -.06<br>*(.015)* | .06<br>*(.02)* |
| Log(B)<br>*(Labor=Wage Bill)* | .001<br>*(.014)* | -.12<br>*(.02)* | -.17<br>*(.015)* | -.03<br>*(.02)* |
| TFP | -.03<br>*(.004)* | -.11<br>*(.006)* | .095<br>*(.004)* | .12<br>*(.006)* |
| Weights | No | Value Added | No | Value Added |

All regressions contain 4 digit SIC industry dummies. TFP is measured by log(Value Added) minus log capital and log labor (no of employees) weighted by 4 digit industry level cost shares.

lated. Table 14 contains estimates of the 10 year autocorrelation of $A$ and $B$, as well as TFP, between the 1997 and 1987 Manufacturing Censuses. All three measures are fairly highly autocorrelated over time.

# 5 Application to Misallocation

A proposed explanation for the vast differences in TFP between rich and poor countries is that resources are not allocated well in poor countries. In this view, some highly productive firms in a poor country do not have enough capital, while other less productive firms have too much capital. This low allocative efficiency can then cause countries to have low aggregate TFP. For the misallocation channel to be important, misallocation must generate large losses in aggregate TFP. Hsieh and Klenow (2009) solve for aggregate TFP in a setup where profit maximizing plants with Cobb-Douglas production functions face output and capital wedges. These wedges are meant to generalize many different reasons for misallocation.

Table 14: Autocorrelation of Productivity

| | Ten Year | Implied One Year | Ten Year | Implied One Year |
|---|---|---|---|---|
| Log(A) | .3<br>*(.004)* | .89 | .33<br>*(.004)* | .90 |
| Log(B) *(Labor=No of employees)* | .37<br>*(.004)* | .91 | .47<br>*(.004)* | .93 |
| Log(B) *(Labor= Wage Bill)* | .35<br>*(.004)* | .90 | .43<br>*(.004)* | .92 |
| TFP | .29<br>*(.004)* | .88 | .43<br>*(.004)* | .92 |
| Weights | No | No | Value Added | Value Added |

All regressions contain 4 digit SIC industry dummies. TFP is measured by log(Value Added) minus log capital and log labor (no of employees) weighted by 4 digit industry level cost shares. The implied one year coefficient is the ten year coefficient to the 1/10 power.

The dispersion in output and capital wedges lowers aggregate TFP, which in a frictionless world would only depend on the productivity of all of the plants. Applying their theory to plant level manufacturing data, they find that eliminating misallocation frictions can increase aggregate TFP by 40% in the US and more than 100% in China and India.

Midrigan and Xu (2009) explore a dynamic model with adjustment costs of capital and financial frictions and try to match the model to Korean plant data. They find that adjustment costs of capital and financial frictions lead to observed variation in the time series marginal product of capital but not the large cross-section differences in the marginal product of capital. Moll (2010) constructs a highly tractable dynamic model with financial frictions in which the autocorrelation of productivity determines whether there is misallocation. If firms are always highly productive they can easily self finance to obtain capital.

All of these models assume that the production function is Cobb-Douglas and so all differences in productivity are Hicks neutral. However, differences in labor augmenting productivity can cause dispersion in the capital cost share and so look like misallocation. Take the Hsieh and Klenow model. In their model, each plant faces downward sloping demand and has a constant returns to scale Cobb-Douglas quantity production function. All plants also face exogenous capital taxes, $\tau_k$, and output taxes, $\tau_y$. Thus each plant faces the following maximization problem:

$$\pi = (1 - \tau_y)PY - wL - (1 + \tau_k)rK \tag{30}$$

$$Y = AK^{\alpha}L^{1-\alpha} \tag{31}$$

Firms that face high capital taxes optimally choose low capital shares of cost, as the capital tax discourages them from purchasing capital. Firms with higher capital taxes also have lower revenue, as the capital tax implies a higher marginal cost and so higher prices.

Firms that face high output taxes also have less revenue as the output tax discourages them from producing more. These firms will also have a lower labor share of revenue, as their output restrictions mean higher prices and so higher revenue per unit produced. Hsieh and Klenow identify the frictions in the micro data as follows:

$$1 + \tau_k \propto \frac{wL}{rK} \tag{32}$$

$$1 - \tau_y \propto \frac{wL}{PY} \tag{33}$$

Capital taxes are proportional to the labor cost to capital cost ratio, while output taxes proportional to the inverse of the labor share of revenue.

They then solve for aggregate TFP and examine how aggregate TFP changes as the level of misallocation frictions change.

In a setup with labor augmenting productivity where capital and labor are complements, increases in labor augmenting productivity will increase the capital share of cost. If firms face a similar demand function as in Hsieh and Klenow, high labor augmenting productivity firms $B$ will also have high revenue and a low labor share of revenue, as I derived earlier. Thus, labor augmenting productivity would imply the following correlations for the measured misallocation taxes and revenue:

$$Corr(1 + \tau_k, 1 - \tau_y) > 0 \tag{34}$$

$$Corr(1 + \tau_k, PY) < 0 \tag{35}$$

$$Corr(1 - \tau_y, PY) < 0 \tag{36}$$

Under the misallocation setup, there is no reason for output taxes and capital taxes to have any correlation, as they are just exogenous frictions hitting firms. Firms facing capital taxes would have low revenue, as labor augmenting productivity would predict, but firms facing high output taxes would have lower revenue. I can then test in the micro data whether these restrictions hold. So far I have only run tests using the Chilean data in 1996, but I plan

Table 15: Correlations of Misallocation Frictions and Revenue for Chilean Plant Level Data in 1996

| | Unweighted | Value Added Weights |
|---|---|---|
| $Corr(1+\tau_k, 1-\tau_y)$ | .29 | .36 |
| $Corr(1+\tau_k, VA)$ | -.15 | -.32 |
| $Corr(1-\tau_y, VA)$ | -.55 | -.45 |

All of these correlations are within 4 digit ISIC Industry.

on repeating these for the US. Table 15 displays the estimates, where all correlations are within 4 digit ISIC industry. I do indeed find that firms with low capital taxes also have high output taxes, firms with low capital taxes have high output, and firms with high output taxes have high revenue. Thus, labor augmenting productivity $B$ may be able to explain patterns in the data for which misallocation theories would require two frictions. I am currently working on a more quantitative assessment on what kinds of apparent misallocation TFP losses labor augmenting productivity can generate.

So far, I have shown that labor augmenting productivity can generate micro data patterns that appear to be misallocation. Thus, labor augmenting productivity could explain why Hsieh and Klenow find increases in aggregate TFP of around 40% for the US just from removing allocation frictions. But a key result in Hsieh and Klenow is that China and India have much higher TFP gains from reallocating efficiently. Misallocation is certainly a reasonable explanation for the cross country differences that Hsieh and Klenow find. But another explanation is that labor augmenting productivity is much more disperse in China and India than in the US.

A simple model based on Acemoglu and Zilibotti (2001) can perhaps explain why India and China would have higher dispersion in labor augmenting productivity than the US. Acemoglu and Zilibotti develop a model where productivity innovations are generated in the North and respond to Northern factor prices. In the North, wages are high while in the South wages are low, so Northern innovation is primarily labor saving. High wages in the North force all firms in the North to adopt the new labor saving technologies. Since wages in the South are low, firms in the South do not always adopt the latest labor saving technologies, leading to big gaps in TFP between the North and the South. If the best practice firms in a country like India adopt the newest labor saving technologies, but most firms do not, labor augmenting productivity will be more disperse in the South than in the North. This model could thus explain why India and China would have a higher dispersion of labor augmenting productivity than the US.

## 6 Conclusion

If plants in an industry have a Cobb Douglas production function, the capital share of cost should be constant across plants. I find that neither of these implications hold in micro data from plants in the US Manufacturing Censuses. The capital share of cost varies considerably within four digit industries. This dispersion is not just measurement error or temporary deviations from the optimal capital share. Capital shares are highly correlated across time within the same plant, and are increase with the plant's revenue. The Cobb Douglas specification also implies that the average revenue product of capital and average revenue product of labor should move together. Instead, I show that the average revenue product of labor increases much more with revenue than the average revenue product of capital.

A CES production function with labor augmenting productivity can better explain these data facts. If the elasticity of substitution between labor and capital is less than one, labor augmenting productivity is labor saving as plants with higher labor augmenting productivity increase their capital labor ratios. Firms with higher labor augmenting productivity will also have higher capital shares. Given downward sloping demand, labor augmenting productivity improvements will increase a firm's revenue and average revenue product of labor, but not its average product of capital. This process induces a positive correlation between revenue and both the capital share and the average revenue product of labor, which I find in the data.

I then identify the labor capital elasticity of substitution using local labor market wage variation. Areas with higher wages have lower factor cost ratios, just as an elasticity less than one would predict. For manufacturing as a whole, I estimate the elasticity of substitution to be between .45 to .65, depending on the level of wage variation and year. Estimating SIC 2 industries separately, I can reject the Cobb-Douglas specification for 17 out of 19 industries with state level wages and 15 out 19 industries with within state county level wages. I also examine a set of 4 digit industries with wide geographic variation. I can reject Cobb Douglas for 8 out of 10 industries using state level wages and 10 out of 10 industries using county level wages. Separating workers into unskilled workers and skilled workers, I find workers for skill-capital complementarity as well, as the elasticity between capital and skilled labor is lower than that between capital and unskilled labor.

Using cost minimization conditions and my estimates of the elasticity of substitution, I can identify measures of both Hicks neutral productivity and labor augmenting productivity, though the Hicks neutral measure includes differences in prices among firms. My measure of labor augmenting productivity is positively correlated with both size and size growth measures. I also apply my methodology to questions of misallocation to show why measuring the type of production technology is important. The misallocation setup of Hsieh and Klenow (2009) assumes that plants face a set of capital and output wedges that lower aggregate TFP. Labor augmenting productivity can cause dispersion in the micro data that looks like misallocation of factors. A simple theory with labor augmenting productivity implies that firms with measured output constraints should also have low capital wedges, and that firms with high revenue have high output wedges and low capital wedges. I find that measured misallocation frictions do have the predicted correlations using Chilean plant level data.

## References


**Acemoglu, Daron**, "When Does Labor Scarcity Encourage Innovation?," *Journal of Political Economy*, forthcoming.

_ **and Fabrizio Zilibotti**, "Productivity Differences," *Quarterly Journal of Economics*, 2001, *116* (2).

**Antras, Pol**, "Is the US Aggregate Production Function Cobb-Douglas? New Estimates of the Elasticity of Substitution.," *Contributions to Macroeconomics*, 2004, *4* (1).

**Arrow, Kenneth, H.B. Chenery, B. Minhas, and Robert M. Solow**, "Capital-labor substitution and economic efficiency.," *Review of Economics and Statistics*, 1961, *43*, 225–50.

**Banerjee, Abhijit and Esther Duflo**, "Growth theory through the lens of development economics," *Handbook of Economic Growth*, 2005, *1.*

**Bartlesman, Eric and Mark Doms**, "Understanding Productivity: Lessons from Longitudinal Microdata," *Journal of Economic Literature*, 2000, *38* (3).

**Bernard, Andrew, Jonathan Eaton, Bradford Jensen, and Samuel Kortum**, "Plants and Productivity in International Trade," *American Economic Review*, 2003, *93* (4).

**Berndt, Ernst R.**, "Reconciling Alternative Estimates of the Elasticity of Substitution," *Review of Economics and Statistics*, 1976, *58* (1).

**Blum, Bernardo**, "Endowments, Output, and the Bias of Directed Innovation," *Review of Economic Studies*, 2010, *77* (2).

**Chirinko, Robert S.**, "Sigma: The long and short of it," *Journal of Macroeconomics*, 2008, *30*, 671–686.

**Christensen, Laurits R., Dale W. Jorgenson, and Lawrence J. Lau**, "Transcendental Logarithmic Production Frontiers Transcendental Logarithmic Production Frontiers," *Review of Economics and Statistics*, 1973, *55* (1).

**Ellison, Glenn and Edward Glaeser**, "Geographic Concentration in U.S. Manufacturing Industries: A Dartboard Approach," *Journal of Political Economy*, 1997, *105* (5).

**Foster, Lucia, John Haltiwanger, and Chad Syverson**, "Reallocation, Firm Turnover, and Efficiency: Selection on Productivity or Profitability?," *American Economic Review*, March 2008, *98* (1), 394–425.

**Fuss, Melvyn and Daniel McFadden, eds**, *Production Economics: A Dual Approach to Theory and Applications*, North Holland, 1978.

**Gandhi, Amit, Salvador Navarro, and David Rivers**, "Identifying Production Functions using Restrictions from Economic Theory," 2009. Mimeo.

**Hicks, J.R.**, *The Theory of Wages*, MacMillan and Co, 1932.

**Holmes, Thomas**, "The Effects of State Policies on the Location of Industry: Evidence from State Borders," *Journal of Political Economy*, August 1998, *106* (4), 667–705.

_ **and John Stevens**, "An Alternative Theory of the Plant Size Distribution with an Application to Trade," Technical Report, NBER Working Paper 2010.

**Hsieh, Chang-Tai and Peter Klenow**, "Misallocation and Manufacturing TFP in China and India," *Quarterly Journal of Economics*, 2009, *124* (4).

**Kahn, James and Jong-Soo Lim**, "Skilled Labor-Augmenting Technical Progress in U. S. Manufacturing," *Quarterly Journal of Economics*, 1998, *113* (4).

**Klump, R., P. McAdam, and A. Willman**, "Factor Substitution and Factor Augmenting Technical Progress in the US," *Review of Economics and Statistics*, 2007, *89* (1).

**León-Ledesma, Miguel A., Peter McAdam, and Alpo Willman**, "Identifying the Elasticity of Substitution with Biased Technical Change," *American Economic Review*, 2010, *100* (4).

**Marshak, J. and W. Andrews**, "Random simultaneous equations and the theory of production," *Econometricxs*, 1944, *12* (3-4), 143–205.

**Melitz, Marc**, "The Impact of Trade on Intra-Industry Reallocations and Aggregate Industry Productivity," *Econometrica*, 2003, *71.*

**Midrigan, Virgiliu and Daniel Xu**, "Accounting for Plant-Level Misallocation," Technical Report, New York University 2009.

**Moll, Benjamin**, "Productivity Losses from Financial Frictions: Can Self-financing Undo Capital Misallocation?," Technical Report, Princeton University 2010.

**Olley, G. Steven and Ariel Pakes**, "The Dynamics of Productivity in the Telecommunications Equipment Industry," *Econometrica*, 1996, *64* (6), 1263–1297.

**Restuccia, Diego and Richard Rogerson**, "Policy distortions and aggregate productivity with heterogeneous establishments," *Review of Economic Dynamics*, 2008, *11* (4).

**Robinson, Joan**, *The Economics of Imperfect Competition*, MacMillan and Co, 1933.

**Sato, Kazuo**, "A Two-Level Constant-Elasticity-of-Substitution Production Function," *The Review of Economic Studies*, 1967, *34* (2), 201–218.

**Syverson, Chad**, "What Determines Productivity?," *Journal of Economic Literature*, forthcoming.