

Technical Note Measuring poverty and analyzing changes in poverty over time

This note provides mathematical expressions for the most commonly used poverty measures and for their decomposition by sector or, more generally, by group. The note focuses on the first three poverty measures of the so-called FGT class (Foster, Greer, and Thorbecke 1984), namely the headcount, the poverty gap, and the squared poverty gap.

Poverty measures

Poverty Headcount: This is the share of the population which is poor, i.e. the proportion of the population for whom consumption or income y is less than the poverty line z . Suppose we have a population of size n in which q people are poor. Then the headcount index is defined as:

$$H = \frac{q}{n}$$

Poverty Gap: The poverty gap, which is often considered as representing the depth of poverty, is the mean distance separating the population from the poverty line, with the non-poor being given a distance of zero. The poverty gap is a measure of the poverty deficit of the entire population, where the notion of “poverty deficit” captures the resources that would be needed to lift all the poor out of poverty through perfectly targeted cash transfers. It is defined as follows:

$$PG = \frac{1}{n} \sum_{i=1}^q \left[\frac{z - y_i}{z} \right]$$

where y_i is the income of individual i , and the sum is taken only on those individuals who are poor (in practice, we often work with household rather than individual income, but individual income can still be defined as being equal, say, to the per capita income of the household). The poverty gap can be written as being equal to the product of the income gap ratio and the headcount index of poverty, where the income gap ratio is itself defined as:

$$PG = I * H, \text{ with}$$

$$I = \frac{z - y_q}{z} \quad \text{where} \quad y_q = \frac{1}{q} \sum_{i=1}^q y_i \quad \text{is the average income of the poor.}$$

It must be emphasized that the income gap ratio I in itself is not a good measure of poverty. Assume that some households or individuals who are poor but close to the poverty line are improving their standards of living over time, and thereby become non-poor. The Income gap ratio will increase because the mean distance separating the poor from the poverty line will increase (this happens because some of those who were less poor have emerged from poverty – so that those still in poverty are on average further away from the poverty line), suggesting a deterioration in welfare, while nobody is worst off and some people are actually better off. Although the income gap ratio I will increase, the poverty gap itself PG will decrease, because the headcount index of poverty will decrease, suggesting an improvement towards poverty reduction. The problem with the income gap ratio is that it is defined only on the population that is poor, while the poverty gap is defined over the population as a whole.

As mentioned above, the poverty gap is a useful statistics to assess how much resources would be needed to eradicate poverty through cash transfers **perfectly targeted** to the poor. Assume for example that the poverty gap is equal to 0.20. This means that the cash transfer needed to lift the poor out of poverty each poor person represents 20 percent of the poverty line. If the mean

income in the country is equal to twice the poverty line, the cash transfer would represent 10 percent of the country's mean income. Now, if it is the mean income of the non-poor which is equal to twice the poverty line, and if half the population is poor, it can be shown that the tax rate that would have to be imposed on the non-poor to lift the poor out of poverty with perfectly targeted transfers would be 20 percent again. If the mean income of the non-poor is equal to four times the poverty line, under the same assumption the necessary tax rate would be 10 percent. Such simple simulations can be used to communicate in an intuitive manner the meaning of the poverty gap. In practice however, given that perfectly targeted cash transfers to eradicate poverty are neither feasible nor necessarily a good thing (high tax rates could stifle economic growth and thereby future poverty reduction), one must be careful in their use.

Squared Poverty Gap: This is often described as a measure of the severity of poverty. While the poverty gap takes into account the distance separating the poor from the poverty line, the squared poverty gap takes the square of that distance into account. When using the squared poverty gap, the poverty gap is weighted by itself, so as to give more weight to the very poor. Said differently, the squared poverty gap takes into account the inequality among the poor. It is obtained as follows:

$$P2 = \frac{1}{n} \sum_{i=1}^q \left[\frac{z - y_i}{z} \right]^2$$

The headcount, the poverty gap, and the squared poverty gap are the first three measures of the Foster-Greer-Thorbecke class of poverty measures. The general formula for this class of poverty measures depends on a parameter α which takes a value of zero for the headcount, one for the poverty gap, and two for the squared poverty gap in the following expression:

$$P\alpha = \frac{1}{n} \sum_{i=1}^q \left[\frac{z - y_i}{z} \right]^\alpha$$

It is important to use the poverty gap or the squared poverty gap in addition to the headcount for evaluation purposes, since these measure different aspects of income poverty. Indeed, the basing evaluation on the headcount ratio would consider as more effective policies which lift the richest of the poor (those close to the line) out of poverty. On the basis of the poverty gap PG and the squared poverty gap $P2$, on the other hand, puts the emphasis on helping those who are further away from the line, the poorest of the poor.

Decompositions for changes in poverty over time

Two main decompositions have been used in the literature to analyze changes in poverty over time. The first decomposition deals with shifts in poverty between sectors or groups (Ravallion and Huppi 1991). The second decomposition deals with the contribution of income growth and changes in inequality to changes in poverty (Datt and Ravallion 1992; Kakwani 1997).

Sectoral decomposition

The poverty measures of the FGT class are additive. This means that the poverty measure for the population as a whole is equal to the weighted sum of the poverty measures for the population subgroups, with the weights defined by the population shares of the subgroups. This additive property makes it feasible to analyze the contribution of various population subgroups to changes in overall poverty over time. Assume that households or individuals can be classified according to various sectors in the economy. These may be industrial sectors, geographic sectors (urban versus rural), or any other sectors that the analyst may suggest. The overall change in poverty over time can be decomposed into: 1) changes in poverty within specific sectors, or intra-sectoral changes, 2) changes in poverty due to changes in the population shares

of sectors, or inter-sectoral changes, and 3) changes due to the possible correlation between intra-sectoral and inter-sectoral changes, or interaction effect. Denote by P_{it} the poverty measure in sector i at time t ; there are m sectors ($i=1, \dots, m$), with population share n_i in sector i , and two periods (1 and 2). Then, the overall change in poverty is equal to:

$$\Delta P = \sum_{i=1}^m n_{i1} (P_{i2} - P_{i1}) + \sum_{i=1}^m P_{i1} (n_{i2} - n_{i1}) + \sum_{i=1}^m (P_{i2} - P_{i1})(n_{i2} - n_{i1})$$

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Intra-sectoral

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Inter-sectoral

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Interaction effect

Growth and inequality decomposition

Changes in poverty rates can also be decomposed into changes due to economic growth (or mean income) in the absence of changes in inequality (or income distribution), and changes in inequality in the absence of growth. Denoting by $P(\mu_t, L_t)$ the poverty measure corresponding to a mean income in period t of μ_t and a Lorenz curve L_t , the decomposition is:

$$\Delta P = [P(\mu_2, L_r) - P(\mu_1, L_r)] + [P(\mu_r, L_2) - P(\mu_r, L_1)] + R_r$$

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Growth impact

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Inequality impact

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Residual

The first component is the change in poverty that would have been observed if the Lorenz curve had remained unchanged, while the second component is the change that would have been observed if mean income had not changed. The last component is a residual.