A NEW POVERTY DECOMPOSITION

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Abstract: This note proposes a new poverty decomposition that can be used to explain changes in poverty over time. The change in poverty is derived as the exact sum of four elements: (i) the overall growth effect, assuming inequality in the distribution does not change; (ii) the impact of differences in growth rates between the groups; (iii) the effect of the change in inequality within the different groups; (iv) the impact of changes in the population shares of the various groups.

Key words: Economic Growth, Poverty, and Inequality

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1: Introduction

The relationship between growth and inequality is of great interest and has still remained much debated in the community of development economists. Kuznets initiated one of the first empirical studies on this subject as early as 1955, yet the issue of how growth interacts with inequality during the course of economic development is largely inconclusive. The cross-country studies – including Forbes (2000), Deininger and Squire (1998), Anand and Kanbur (1993) - that attempt to capture the overall relation between growth and inequality are mixed. What is more, using time series analysis, Gottschalk and Smeeding (1997) and Williamson (1999, 1991) argue that a country experience on the two shows a wide difference.[1]

Similarly, the growth-poverty relation has been studied extensively. A large number of cross-country evidence – including a recent study by Dollar and Kraay (2000) – suggests that there is a positive correlation between economic growth and poverty reduction. Nevertheless, results emerging from cross-country analysis tend to be inconclusive on the growth-poverty relation (Brock and Durlauf 2000, Bourguignon 2000, Deininger and Okidi 2001). In addition, Ravallion (2001) finds that of those countries experiencing an increase in living standards in his sample of 50 developing countries, the annual reduction in poverty is much larger for countries where inequality is falling. In this context, the growth-poverty relation is complex and is also related to the level and changes in inequality.

There can be many ways of explaining the interrelation between growth, inequality and poverty. This note attempts to explain this interrelationship through an idea of poverty decomposition. The main objective of this note is to derive a new poverty decomposition that can be utilized to explain changes in the poverty incidence over time in terms of growth, inequality and migration.

[1] Aghion, P., Caroli, E., and Garcia-Penalosa, C. (1999) provide an excellent discussion of the relation between inequality and growth.

Suppose the population is divided into various segments on the basis of different criteria. The proposed decomposition then shows that for each classification the change in the incidence of poverty can be broken down into four elements: (i) one that reflects shifts in population between segments that have different degrees of poverty; (ii**)** a second one that measures the impact of the overall growth in income in the economy; (iii) a third one that takes into account the fact that different segments experienced different growth rates; (iv) a fourth one reflecting changes in the distribution of income within each segment.

2: A General Class of Decomposable Poverty Measures

Suppose income *x* of an individual is a random variable with density function *f (x)*. Let *z* be the poverty line - the threshold income below which one is considered to be poor. Then a general class of additively separable poverty measures can be written as:

$$P = \int_0^z \theta(z,x) f(x) dx \qquad (1)$$

where $\theta(z, x)$ is a homogenous function of degree zero in *z* and *x*, satisfying the restrictions

$$\frac{\partial \theta(z,x)}{\partial x} < 0$$

$$\frac{\partial^2 \theta(z,x)}{\partial x^2} > 0$$

Foster, Greer, and Thorbecke (1984) for example proposed a class of poverty measures that is obtained by substituting

$$\theta(\mathrm{z}, \mathrm{x}) = \left(\frac{z-x}{z}\right)^{\alpha} \qquad (2)$$

in (1), where α is the parameter of inequality aversion. When the headcount ratio is used as the poverty measure, $\alpha = 0$. For α =1 and 2, θ measures the poverty gap ratio and the severity of poverty, respectively.

Let us now divide the total population into *k* mutually exclusive socioeconomic and demographic groups. Then a poverty measure is said to be additively decomposable if we can write the total poverty as the weighted average of poverty within each group.

$$P = \sum_i f_i P_i \tag{3}$$

where f_i and P_i are the population share and poverty index of the ith group, respectively.

Kakwani (1993) has demonstrated that the entire class of additively separable measures in (1) is additively decomposable. Our proposed poverty decomposition derived in the next section is based on the assumption that the poverty measures are additively decomposable. This means that our decomposition is valid for the entire class of additively separable poverty measures given in (1).[2]

3: A New Poverty Decomposition Derived

Let us define the change in poverty between two periods as

$$\Delta P = P_2 - P_1 \tag{4}$$

where $P_1 = \sum_i f_{1i} P_{1i}$, $P_2 = \sum_i f_{2i} P_{2i}$, P_{1i} and P_{2i} being the poverty incidence in the ith group in years 1 and 2, respectively, and f_{1i} and f_{2i} are the population shares of the ith group in years 1 and 2, respectively. Equation (4) can be written as

$$\Delta P = \frac{1}{2}\left[\sum_i f_{1i}(P_{2i} - P_{1i}) + \sum_i f_{2i}(P_{2i} - P_{1i})\right] + \frac{1}{2}\left[\sum_i P_{1i}(f_{2i} - f_{1i}) + \sum_i P_{2i}(f_{2i} - f_{1i})\right]$$

This expression shows that the change in total poverty can be written as the sum of two components. The first component measures the effect on total change in poverty due to changes in within-group poverty and the second component estimates the change in total poverty due to changes in population shares between groups.

The percentage change in total poverty, thus, can be written as follows:

[2] Kakwani's (1980) class of poverty measures, of which Sen's (1976) poverty measure is a particular case, is not additively decomposable and thus our decomposition is not valid for this class of measures.

$$\frac{\Delta P}{P} = \sum_i \frac{\overline{f}_i P_i}{P}\left(\frac{\Delta P_i}{P_i}\right) + \sum_i \frac{\overline{P}_i f_i}{P}\left(\frac{\Delta f_i}{f_i}\right) \quad (5)$$

where $\overline{f}_i = \frac{f_{1i} + f_{2i}}{2}$ and $\overline{P}_i = \frac{P_{1i} + P_{2i}}{2}$. Note that the first term in Equation (5) estimates the percentage change in total poverty explained by changes in poverty within groups. The second term estimates the percentage change in total poverty due to a shift in population between groups. The shift in population is deemed pro-poor if the second term is negative because it leads to a reduction in poverty. Such a situation could occur if the migration flow is from a poorer to a richer region, assuming rural areas are poorer than urban areas. If migration takes place from urban to rural areas, then the second component is positive because it increases poverty. In this case, the population shift is not pro-poor.

Kakwani (2000) has proposed a poverty decomposition that explains the percentage change in poverty at the aggregate level as the sum of two components. One is the growth effect, measuring the change in poverty when mean income changes but inequality remains fixed. The other component is the inequality effect, which measures changes in poverty when inequality changes but the mean income remains constant. This methodology can now be applied within each group.

A general poverty measure is characterized as

$$P = P\big(z, \mu, L(p)\big)$$

where *z* is the poverty line, μ is the mean income of society, and *L (p)* is the Lorenz curve. The Lorenz curve measures the effect of inequality on poverty. Following from Kakwani (2000), the percentage change in poverty can be written as

$$\Delta P = (\Delta P)_m + (\Delta P)_I$$

$$= \textit{Growth effect} + \textit{Inequality effect} \quad (6)$$

where $(\Delta P)_m$ is the change in poverty if mean income changes from μ_1 in period 1 to μ_2 in period 2 but the Lorenz curve remains fixed. Thus, $(\Delta P)_m$ can be written as

$$(\Delta P)_m = \frac{1}{2}\left[P(z,\mu_2, L_1(p)) - P(z,\mu_1, L_1(p)) + P(z,\mu_2, L_2(p) - P(z,\mu_1, L_2(p))\right]$$

where $L_1(p)$ and $L_2(p)$ are the Lorenz curves in periods 1 and 2, respectively. Note that in deriving the growth effect, we can fix the Lorenz curve either for the initial period or for the terminal period. Neither of the two periods of Lorenz curves can be justified and thus we have taken the average of the two periods.[3]

Similarly, the inequality component can be derived as

$$(\Delta P\ )_I = \frac{1}{2}\left[P(z,\mu_1, L_2(p)) - P(z,\mu_1, L_1(p)) + P(z,\mu_2, L_2(p) - P(z,\mu_2, L_1(p))\right]$$

This expression estimates the change in poverty if inequality measured by the Lorenz curve changes from $L_1(p)$ in the initial period to $L_2(p)$ in the terminal period but mean income is fixed between the two period. The sum of the mean and inequality effects gives rise to the total changes in poverty. Our new poverty decomposition is based on Kakwani's exact decomposition methodology. It differs from Kakwani's in a way that ours takes into account differential growth and inequality effects between groups and population shift between sectors as derived below.

We apply the decomposition in (6) within each group, which results in

$$\frac{\Delta P_i}{P_i} = \frac{(\Delta P_i)_m}{P_i} + \frac{(\Delta P_i)_I}{P_i} \qquad (7)$$

for $i = 1, 2, 3, \ldots\ldots, k$.

[3] This issue has an analogy to construction of price indices. The Laspeyres price index fixes the base year consumer basket whereas Paasche index fixes the terminal year consumer basket. We have taken the average of the two following the idea behind Fisher's ideal index number.

where

$$(\Delta P_i)_m = \frac{1}{2}\left[P(z,\mu_{2i},L_{1i}(p)) - P(z,\mu_{1i},L_{1i}(p)) + P(z,\mu_{2i},L_{2i}(p)) - P(z,\mu_{1i},L_{2i}(p))\right]$$

and

$$(\Delta P_i)_I = \frac{1}{2}\left[P(z,\mu_{1i},L_{2i}(p)) - P(z,\mu_{1i},L_{1i}(p)) + P(z,\mu_{2i},L_{2i}(p)) - P(z,\mu_{2i},L_{1i}(p))\right]$$

where μ_{ti} is the mean income of the ith group in year t and L_{ti} (p) is the Lorenz curve of the i^{th} group in year t (t = 1,2).

From (5) and (7), the percentage change in total poverty can be expressed as

$$\frac{\Delta P}{P} = \sum_i \frac{\overline{f}_i P_i}{P}\frac{(\Delta P_i)_m}{P_i} + \sum_i \frac{\overline{f}_i P_i}{P}\frac{(\Delta P_i)_I}{P_i} + \sum_i \frac{\overline{P}_i f_i}{P}\left(\frac{\Delta f_i}{f_i}\right)$$

$$= \textit{Within group growth Effect + Within group inequality Effect + Population Shift} \qquad (8)$$

The first term in (8) measures the effect of growth within each group on the overall change in the poverty incidence, when the distribution within each group remains the same over time. This term can be further decomposed into two terms:

$$\sum_i \frac{\overline{f}_i P_i}{P}\frac{(\Delta P_i)_m}{P_i} = \sum_i \frac{\overline{f}_i P_i}{P}\frac{(\Delta P_i)_g}{P_i} + \sum_i \frac{\overline{f}_i P_i}{P}\frac{(\Delta P_i)_{bg}}{P_i} \qquad (9)$$

where

$$(\Delta P_i)_g = \frac{1}{2}\left[P(z,\mu^*{}_{2i},L_{1i}(p)) - P(z,\mu_{1i},L_{1i}(p)) + P(z,\mu^*{}_{2i},L_{2i}(p)) - P(z,\mu_{1i},L_{2i}(p))\right]$$

$$(\Delta P_i)_{bg} = \frac{1}{2}\left[P(z,\mu_{2i},L_{1i}(p)) - P(z,\mu^*{}_{2i},L_{1i}(p)) + P(z,\mu_{2i},L_{2i}(p)) - P(z,\mu^*{}_{2i},L_{2i}(p))\right]$$

and $\mu^*{}_{2i} = \mu_{1i}(1+g)$

which, g being the average growth rate of the whole population, is the mean income of the i^{th} group in year 2 if the income of the i^{th} group were growing at the same rate as the average growth rate of the whole population.

The first term on the right hand side of (9) measures the effect of growth on the percentage change in poverty, assuming all groups enjoy the same uniform growth rates. The second term on the right hand side of (9) takes into account the fact that the actual growth rates vary from one group to the other. Substituting (9) into (8), we arrive at our final poverty decomposition that may be summarized as follows.

The percentage change in the incidence of poverty is expressed as the sum of four components. The first one is the overall growth effect, assuming inequality does not change. The second one takes into account the fact that growth rates vary from one group to the other. The third element reflects the impact of the change in inequality within the different groups. The last component is the consequence of changes in the population shares of the various groups. This is an exact decomposition and there will not be any residual term. This decomposition does not refer to a specific inequality measure. It uses the idea of shift in that part of the Lorenz curve, which affects the poor. This decomposition is general and does not require any restriction except that poverty measures should be additively decomposable. Thus, in view of Kakwani's (1993) result, the decomposition will be valid for the entire class of additively separable poverty measures defined in (1), of which the Foster, Greer and Thorbecke's class of measure is a particular case.

4: Concluding Remarks

Poverty is never static. It keeps changing over time. In order to formulate policies to reduce poverty, it is important that we understand the dynamics of poverty. How is poverty changing over time? The poverty decomposition proposed in the paper provides a methodology to understand the changes in poverty over time. The decomposition explains the percentage change in the poverty incidence in terms of four components.

The first component will always be negative if there is a positive growth in the economy. The second component can be either negative or positive. If it is positive (negative), the difference in growth rates of different groups has contributed to an increase (decrease) in total poverty. The third component can again be either positive or negative. If it is positive (negative), it indicates that a change in inequality within group has contributed to an increase (decrease) in the total poverty incidence. Finally, the fourth component measures the effect of migration of population between groups on the total poverty incidence, which attempts to capture the impact of migration on changes in poverty.

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