Prototype Model for a Single Country
Real Computable General Equilibrium Model

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Introduction

The World Bank has been asked by several governments to assist in the development of economic models for policy analysis with a structural and/or long-term focus. These tools are intended to assist policy makers in assessing the impacts of various policy options—fiscal, structural, development and trade. The tools need to elucidate the impacts on the composition of output and demand, income distribution, poverty and macroeconomic indices such as GDP growth and trade. The prototype model is a real computable general equilibrium model that is adapted to the needs and specifications of each country. The model is multi-sectoral, multi-factor (differentiating different labor skills, capital, land and other factors of production), multi-household to assist in distribution and poverty analysis, and multi-trading partner enabling the analysis of various changes to trade policies.

This document presents a prototype model for the CGE analytical tool. The prototype has some key features for assessing structural and poverty impacts:

- Labor markets disaggregated by skill level
- Land and capital markets disaggregated by type of capital/land
- A production structure which differentiates the substitutability of unskilled labor on the one hand, and skilled labor and capital on the other hand
- Differentiation of production of like-goods (e.g. small- and large-scale farms, or public versus private production)
- Detailed income distribution
- Intra-household transfers (e.g. urban to rural), transfers from government, and remittances
- Multiple households
- A tiered structure of trade (differentiating across various trading partners)
- Possibility of influencing export prices
- Internal domestic trade and transport margins
- Various potential factor mobility assumptions

This document also reflects recent changes to the model specification, including:

- Agent-specific import demand replacing the economy-wide import demand specification
- Agent-specific import tariffs allowing for implementation of the duty-drawback system, a key feature of Chinese fiscal policies
- Endogenously determined unemployment
- Scale economies and imperfect competition
- A simple debt module

Some versions of the model have been adapted to include alternative specifications for determining wage markups, such as efficiency wages or negotiated wage packages.

The rest of the document proceeds to describe all of the model details using the standard circular flow description of the economy. It starts with production \( (P) \), income distribution \( (Y) \), demand \( (D) \), trade \( (T) \), domestic trade and transport margins \( (M) \), goods market equilibrium \( (E) \), macro closure \( (C) \), factor market equilibrium \( (F) \), macroeconomic identities \( (I) \), and growth \( (G) \).

Table 1 describes the indices used in the equations. Note that the model differentiates between production activities, denoted by the index \( i \), and commodities, denoted by the index \( k \). In many models, the two will
overlap exactly. However, this differentiation allows for the same commodity to be produced by one or more sectors, and to differentiate these commodities by source of production. For example, it could be used in a model of economies in transition where commodities produced by the public sector have a different cost structure than commodities produced by the private sector, and the commodities themselves could be differentiated by consumers.\(^1\) Another example, could be small- versus large-scale agricultural producers.

Table 1: Indices used in the model

<table>
<thead>
<tr>
<th>i</th>
<th>Production activities</th>
</tr>
</thead>
<tbody>
<tr>
<td>k</td>
<td>Commodities</td>
</tr>
<tr>
<td>l</td>
<td>Labor skills</td>
</tr>
<tr>
<td>ul</td>
<td>Unskilled labor</td>
</tr>
<tr>
<td>sl</td>
<td>Skilled labor</td>
</tr>
<tr>
<td>kt</td>
<td>Capital types</td>
</tr>
<tr>
<td>lt</td>
<td>Land types</td>
</tr>
<tr>
<td>e</td>
<td>Corporations</td>
</tr>
<tr>
<td>gz</td>
<td>Geographic zones (rural, urban, national)</td>
</tr>
<tr>
<td>h</td>
<td>Households</td>
</tr>
<tr>
<td>f</td>
<td>Final demand accounts(^b)</td>
</tr>
<tr>
<td>a</td>
<td>Armington agents including the trade and transport margin accounts(^c)</td>
</tr>
<tr>
<td>tr</td>
<td>Tariff regimes</td>
</tr>
<tr>
<td>m</td>
<td>Trade and transport margin accounts(^d)</td>
</tr>
<tr>
<td>r</td>
<td>Trading partners</td>
</tr>
</tbody>
</table>

Notes:  
\(^a\) The unskilled and skilled labor indices, \(ul\) and \(sl\), are subsets of \(l\), and their union composes the set indexed by \(l\).
\(^b\) The standard final demand accounts are ‘Gov’ for government current expenditures, ‘ZIp’ for private investment, ‘ZIg’ for public investment, ‘TMG’ for international export of trade and transport services, and ‘DST’ for changes in stocks.
\(^c\) The index \(a\) is the union of production activities, \(i\), households, \(h\), and other final demand accounts, \(f\), and a separate domestic margins account denoted by ‘\(margn\)’.
\(^d\) The standard trade and transport margin accounts are ‘\(D\)’ for domestic goods, ‘\(M\)’ for imported goods, and ‘\(X\)’ for exported goods.

Model Equations

Production

Production, like in most CGE models, relies on the substitution relations across factors of production and intermediate goods. The simplest production structure has a single constant-elasticity-of-substitution (CES\(^2\)) relation between capital and labor, with intermediate goods being used in fixed proportion to output. In the production structure described below, there are multiple types of capital, land and labor, and they are combined in a nested-CES structure which is intended to represent the various substitution possibilities across these different factors of production. Typically, intermediate goods will enter in fixed proportion to output, though at the aggregate level, the model allows for a degree of substitutability between aggregate intermediate demand and value added.\(^3\) The decomposition of value added has several components (see figure 1 for a representation of the multiple nests). First, land is assumed to be a

---

1 The model allows for perfect substitution, in which case consumers are indifferent regarding who produces the good. An example might be electricity.
2 See the annex for a description of the generic CES function and its properties, as well as its counterpart function, the constant elasticity of transformation function (CET).
3 Deviations from this structure might include isolating some key inputs, for example energy, or agricultural chemicals in the case of crops, and feed in the case of livestock.
substitute for an aggregate capital labor bundle.\(^4\) The latter is then decomposed into unskilled labor on the one hand, and skilled labor cum capital on the other hand. This conforms to recent observations which suggest that capital and skilled labor are complements which can substitute for unskilled labor. The four aggregate factors—unskilled and skilled labor, land and capital, are decomposed by type in a final CES nest.

**Top-level nest and producer price**

The top-level nest has output, \(XP\), produced as a combination of value added, \(VA\), and an aggregate demand for goods and non-factor services, \(ND\). In most cases, the substitution elasticity will be assumed to be zero, in which case the top-level CES nest is a fixed-coefficient Leontief production function. Equations (P-1) and (P-2) represent the optimal demand conditions for the generic CES production function, where \(PND\) is the price of the \(ND\) bundle, \(PVA\) is the aggregate price of value added, \(VC\) is the unit variable cost of production, and \(\sigma^p\) is the substitution elasticity. If the latter is zero, both \(ND\) and \(VA\) are used in fixed proportions to output, irrespective of relative prices. Equation (P-3) represents the unit variable cost function, \(VC\). It is derived from the CES dual price formula.\(^5\)

\[
\begin{align*}
ND_i &= \alpha_i^{nd} \left( \frac{VC_i}{PND_i} \right)^{\sigma^p} XP_i \quad \text{(P-1)} \\
VA_i &= \alpha_i^{va} \left( \frac{VC_i}{PVA_i} \right)^{\sigma^p} XP_i \quad \text{(P-2)} \\
VC_i &= \left[ \alpha_i^{nd} PND_i^{1-\sigma^p} + \alpha_i^{va} PVA_i^{1-\sigma^p} \right]^{1/(1-\sigma^p)} \quad \text{(P-3)}
\end{align*}
\]

The model allows for increasing-returns-to-scale. Total cost is divided into a unit variable cost and a fixed per unit cost that is composed of fixed units of labor and capital. Firms are assumed to be symmetric, i.e. they have identical cost structures and the fixed costs are specified on a per firm basis. Let \(N\) be the number of firms, then sectoral unit fixed cost (i.e. total across all firms), \(FC\), is given by equation (P-4), where \(KF\) and \(LF\) represent respectively the fixed units of capital and labor needed to produce the initial unit of output. As output expands, the fixed cost declines per unit of output. Total unit cost, \(TC\), is simply the sum of the fixed unit cost, \(FC\), and the unit variable cost, \(VC\), and is given by equation (P-5). In the case of constant-returns-to-scale, fixed costs are zero and total unit cost is equated to variable unit cost. Equation (P-6) defines the cost disadvantage ratio, \(CDR\). It is the ratio of fixed cost to total cost. Under constant-returns-to-scale the \(CDR\) is 0.

The model allows for either perfect or imperfect competition. In the case of the latter, the producer sets the markup over variable cost (including the tax on value added) and the producer price, \(PX\), is equal to variable cost adjusted by the markup \(\pi\), equation (P-7). Finally, the producer price, \(PP\), inclusive of the output tax, \(\tau^p\), is given by equation (P-8). To reiterate, the model easily conforms to the standard perfect competition and constant-returns-to-scale specification by setting fixed costs and the price markup to zero. In this case the producer price is equal to the unit cost of production, all taxes included.

\(^4\) In some sectors the model also allows for a sector-specific factor of production, for example, coal mining and oil production require reserves which cannot be used for any other activity. In this case, the nesting follows the same general structure as depicted in Figure 1.

\(^5\) See Annex B for a description of the CES function.
\[
FC_i = N_i \left[ \sum_{kt} R_{i,kt} KF_{i,kt}^d + \sum_l W_{i,j} LF_{i,j}^d \right] / XP_i \tag{P-4}
\]

\[
TC_i = VC_i + FC_i \tag{P-5}
\]

\[
CDR_i = \frac{TC_i - VC_i}{TC_i} = \frac{FC_i}{TC_i} \tag{P-6}
\]

\[
PX_i = (VC_i + VatdY_i / XP_i) (1 + \pi_i) \tag{P-7}
\]

\[
PP_i = (1 + \tau_i^p) PX_i \tag{P-8}
\]

**Second-level production nests**

The second-level nest has two branches. The first decomposes aggregate intermediate demand, \(ND\), into sectoral demand for goods and services, \(XA\), at the Armington level of demand. The model uses a generic CES functional form, though typically a Leontief structure will be imposed by setting the substitution elasticity to 0. Thus equation (P-9) describes the demand for good \(k\) by sector \(j\), where the coefficient \(a\) represents the proportion between \(XA\) and \(ND\), possibly influenced by the relative price of the input. The price of the \(ND\) bundle, \(PND\), is the weighted average of the price of goods and services, \(PA\), using the technology coefficients as weights, equation (P-10). The so-called Armington price is the price of the composite \(XA\) good.

\[
XA_{k,j} = a_{k,j} \left( \frac{PND_j}{PA_{k,j}} \right)^{\sigma_j} ND_j \tag{P-9}
\]

\[
PND_j = \left[ \sum_k a_{k,j} (PA_{k,j})^{1-\sigma_j} \right]^{1/(1-\sigma_j)} \tag{P-10}
\]

The second branch decomposes the aggregate value added bundle, \(VA\), into three components: aggregate demand for capital and labor, \(KL\), aggregate land demand, \(TT^d\), and a sector-specific resource, \(NR\), see equations (P-11) through (P-13). The relevant component prices are \(PKL\), \(PTT\) and \(PR\), respectively, and the substitution elasticity is given by \(\sigma\). Equation (P-13) allows for the possibility of factor productivity changes as represented by the \(\lambda\) parameter. The price of value added, \(PVA\), is the CES aggregation of the three component prices, as defined by equation (P-14).

---

6 The latter will typically be zero in most sectors.
\[ KL_i = \alpha_i^{kl} \left( \frac{PVA_i}{PKL_i} \right)^{\sigma_i^{kl}} VA_i \] (P-11)

\[ TT_i^d = \alpha_i^{\alpha_i^{pr}} \left( \frac{PVA_i}{PTT_i} \right)^{\sigma_i^{pr}} VA_i \] (P-12)

\[ NR_i^d = \alpha_i^{\alpha_i^{pr}} \left( \frac{PVA_i}{PR_i} \right)^{\sigma_i^{pr}} VA_i \] (P-13)

\[ PVA_i = \left[ \alpha_i^{kl} PKL_i^{1-\sigma_i^{kl}} + \alpha_i^{\alpha_i^{pr}} PTT_i^{1-\sigma_i^{pr}} + \alpha_i^{\alpha_i^{pr}} \left( \frac{PR_i}{\lambda_i^{pr}} \right)^{1-\sigma_i^{pr}} \right]^{\frac{1}{1/(1-\sigma_i^{pr})}} \] (P-14)

**Third-level production nest**

The third-level nest decomposes the aggregate capital-labor bundle, \( KL \), into two components. The first is the aggregate demand for unskilled labor, \( UL \), with an associated price of \( PUL \). The second is a bundle composed of skilled labor and capital, \( KSK \), with a price of \( PKSK \). Equations (P-15) and (P-16) reflect the standard CES optimality conditions for the demand for these two components, with a substitution elasticity given by \( \sigma_i^{kl} \). The price of capital-labor bundle, \( PKL \), is defined in equation (P-17).

\[ UL_i = \alpha_i^{\alpha_i^{pr}} \left( \frac{PKL_i}{PUL_i} \right)^{\sigma_i^{pr}} KL_i \] (P-15)

\[ KSK_i = \alpha_i^{sk} \left( \frac{PKL_i}{PKSK_i} \right)^{\sigma_i^{sk}} KL_i \] (P-16)

\[ PKL_i = \left[ \alpha_i^{\alpha_i^{pr}} PUL_i^{1-\sigma_i^{pr}} + \alpha_i^{sk} PKSK_i^{1-\sigma_i^{sk}} \right]^{1/(1-\sigma_i^{pr})} \] (P-17)

**Fourth-level production nest**

The fourth-level nest decomposes the capital-skilled labor bundle into a capital component, \( KT^d \), and a skilled labor component, \( SKL \). Equations (P-18) and (P-19) represent the optimality conditions where the relevant component prices are \( PKT \) and \( PSKL \), and the substitution elasticity is given by \( \sigma_i^{sk} \). Equation (P-20) determines the price of the \( KSK \) bundle, \( PKSK \).
\[ SKL_i = \alpha_i^s \left( \frac{PKSK_i}{PSKL_i} \right)^{\sigma_{ii}} KSK_i \] (P-18)

\[ KT_i^d = \alpha_i^s \left( \frac{PKSK_i}{PKT_i} \right)^{\sigma_{ii}} KSK_i \] (P-19)

\[ PKSK_i = \left[ \alpha_i^s PKSL_i^{-\sigma_{ii}} + \alpha_i^s PKT_i^{-\sigma_{ii}} \right]^{1/(1-\sigma_{ii})} \] (P-20)

**Demand for labor by sector and skill**

Equations (P-21) and (P-22) decompose the demands for aggregate unskilled and skilled labor, respectively, across their different components. The variable \( LV^d \) represents variable labor demand in sector \( i \) for labor of skill level \( l \). The relevant wage is given by \( W \) which is allowed to be both sector and skill-specific. The respective cross-skill substitution elasticities are \( \sigma^u \) and \( \sigma^s \). Both equations (P-21) and (P-22) incorporate sector and skill specific labor productivity, represented by the variable \( \lambda^l \). The aggregate unskilled and skilled price indices are determined in equations (P-23) and (P-24), respectively \( PUL \) and \( PSKL \). Total labor demand by sector, \( L^d \), is the sum of fixed labor demand (aggregated over all firms) and variable labor demand, equation (P-25).

\[ LV_{i,ul}^d = \alpha_{i,ul}^l \left( \frac{PUL}{W_{i,ul}} \right)^{\sigma_{ii}} UL_i \quad \text{for } ul \in \{ \text{Unskilled labor} \} \] (P-21)

\[ LV_{i,sl}^d = \alpha_{i,sl}^l \left( \frac{PSKL_i}{W_{i,sl}} \right)^{\sigma_{ii}} SKL_i \quad \text{for } sl \in \{ \text{Skilled labor} \} \] (P-22)

\[ PUL_i = \left[ \sum_{ul \in \{ \text{Unskilled labor} \}} \alpha_{i,ul}^l \left( \frac{W_{i,ul}}{\lambda_{i,ul}^l} \right)^{1-\sigma_{ii}} \right]^{1/(1-\sigma_{ii})} \] (P-23)

\[ PSKL_i = \left[ \sum_{sl \in \{ \text{Skilled labor} \}} \alpha_{i,sl}^l \left( \frac{W_{i,sl}}{\lambda_{i,sl}^l} \right)^{1-\sigma_{ii}} \right]^{1/(1-\sigma_{ii})} \] (P-24)

\[ L_{i,j}^d = N_i LF_{i,j}^d + LV_{i,j}^d \] (P-25)

**Demand for capital and land across types**

The aggregate land and capital bundles, \( KT^d \) and \( TT^d \) respectively, are disaggregated across types, leading to type- and sector-specific capital and land demand, \( KV^d \) and \( T^d \). The decomposition is represented in
equations (P-26) and (P-29), where the respective prices are \( R \) and \( PT \) which are both type- and sector-specific. The equations also incorporate productivity factors. Equations (P-27) and (P-30) represent the price indices for aggregate capital and land, respectively \( PKT \) and \( PTT \). Finally, equation (P-28) represents the total sectoral demand for capital by type, \( K^d \), as the sum of fixed and variable capital demand.

\[
K^d_{i,kt} = \alpha_{i,kt} \left( \frac{PKT_i}{R_{i,kt}} \right)^{\sigma_i^d} \left( \frac{R_{i,kt}}{\lambda_{i,kt}^k} \right)^{1-\sigma_i^d} \]

\[
PKT_i = \left[ \sum_{kt} \alpha_{i,kt} \left( \frac{R_{i,kt}}{\lambda_{i,kt}^k} \right)^{1-\sigma_i^d} \right]^{1/(1-\sigma_i^d)} \]

\[
K^d_{i,kt} = N_i K^d_{i,kt} + K^v_{i,kt} \]

\[
T^d_{i,lt} = \alpha'_{i,lt} \left( \frac{PTT_i}{PT_{i,lt}} \right)^{\sigma_i^d} \left( \frac{PT_{i,lt}}{\lambda_{i,lt}^l} \right)^{1-\sigma_i^d} \]

\[
PTT_i = \left[ \sum_{lt} \alpha'_{i,lt} \left( \frac{PT_{i,lt}}{\lambda_{i,lt}^l} \right)^{1-\sigma_i^d} \right]^{1/(1-\sigma_i^d)} \]

Commodity supply and aggregation

Each activity produces an aggregate commodity, \( XP \), indexed by \( i \). The aggregate commodity may be broken out into multiple outputs (i.e. a multi-output production function), \( XS \), where a CET function is specified to optimally allocate the aggregate output across the \( XS \) markets with sales price \( PS \).\(^7\) Equation (P-31) determines the supply of commodity \( k \) produced by activity \( i \), where \( \omega \) is the elasticity of transformation. Both equations (P-31) and (P-32) allow for the possibility of perfect transformation (i.e. an infinite elasticity), in which case the supply equation is replaced by the law-of-one-price equation. Equation (P-32) defines the aggregate producer price, \( PP \), using the CET dual price formula. With perfect transformation, the CET dual price equation is replaced by the primal adding up equation.

---

\(^7\) A standard example may be a consolidated farming sector that has only one production function but produces multiple outputs, for example wheat, maize, barley, oats, etc. Nonetheless, in most cases a single activity produces a single commodity.
Consumption goods, indexed by $k$, are a combination of one or more produced goods. Aggregate domestic supply of good $k$, $X$, is a CES combination of one or more produced goods $XS$. In many cases, the CES aggregate is of a single commodity, i.e. there is a one-to-one mapping between a consumed good and its relevant production. There are cases, however, where it is useful to have consumed goods be an aggregation of produced goods, for example when combining similar goods with different production characteristics (e.g. public versus private, commercial versus small-scale, etc.) Equation (P-33) represents the optimality condition of the aggregation of produced goods into commodities. The producer price is $PS$, and the price of aggregate supply is $P$. The degree of substitutability across produced commodities is $\sigma$. Equation (P-34) determines the aggregate supply price, $P$. The model allows for perfect substitutability, in which case the law of one price holds and the produced commodities are simply aggregated to form aggregate output. Equation (P-35) is the equilibrium condition.

Income distribution

The prototype model has a rich menu of income distribution channels—factor income and intra-household, government and foreign transfers (i.e. remittances). The prototype also includes corporations used as a pass-through account for channeling operating surplus.

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8  Electricity is a good example of a homogeneous output but which could be produced by different production technologies, e.g. hydro-electric, nuclear, thermal, etc.
9  It is substituted out in the model.
Factor income

The first two equations define respectively revenue from the domestic value added tax, VatdY, and pure profits, Π. The domestic value added tax, equation (Y-1), is applied to producer revenues net of the value added tax itself and expenditures on goods and services. The residual is simply wage and capital expenditures and pure profits. Pure profit, equation (Y-2), is the difference between producer revenue and total costs (adjusted for the value added tax).

\[
\text{VatdY}_i = \tau_i^{v,d} \left( PX_i XP_i - \text{VatdY}_i - \sum_k PA_k XA_{k,i} \right) \quad \text{(Y-1)}
\]

\[
\Pi_i = XP_i \left( PX_i - TC_i \right) - \text{VatdY}_i \quad \text{(Y-2)}
\]

There are four broad factors—a sector specific resource, land, labor and capital—the latter three which can be sub-divided into various types. Equations (Y-3) through (Y-6) determine aggregate net-income from labor, LY, capital, KY, land, TY, each indexed by their respective sub-types, and the sector specific factor, RY. These are net incomes because the model incorporates factor taxes designated by \(\tau_l^t\), \(\tau_k^t\), \(\tau^t\), and \(\tau^t\) respectively. In the case of labor, capital and land, the formulas use the net factor price, i.e. the after-tax factor price or the factor price perceived by the owner of the factor, not the factor price perceived by the producer. In the case of the natural resource, the aggregate income equation nets out the tax directly. Equation (Y-3) contains labor remuneration from abroad, FW. Equation (Y-4) includes the pure profit factor. It is calculated at the sectoral level. It is allocated across the different capital types using the base-year shares of capital remuneration. The terms of the share vector, \(\alpha^{pr}\), must sum to one across \(kt\) for each sector \(i\).

\[
LY_i = \sum_i NW_{i,i} L_{i,i}^d + ER FW_i \quad \text{(Y-3)}
\]

\[
KY_{kt} = \sum_i NR_{i,kt} K_{i,kt}^d + \alpha_{i,kt}^{pr} \Pi_i \quad \text{(Y-4)}
\]

\[
TY_{kt} = \sum_i NPT_{i,kt} T_{i,kt}^d \quad \text{(Y-5)}
\]

\[
RY = \sum_i \frac{PR_i NR_{i}^d}{1 + \tau_{i}^{fr}} \quad \text{(Y-6)}
\]

Distribution of profits

All of labor, land and sector-specific factor income is allocated directly to households.\(^\text{10}\) Profits (aggregated with income from the sector-specific resource), on the other hand, are distributed to three broad accounts, enterprises, households, and the rest of the world (ROW). Equation (Y-7) determines the level of profits distributed to enterprises, \(TR^E\). Equation (Y-8) represents the level of profits distributed

\(^{10}\) Depending on the structure of the final SAM, land and or income from the sector-specific resource may also pass through corporate accounts.
directly to households, \( TR^H \). And, equation (Y-9) determines the level of factor income distributed abroad, \( TR^W \). Note that the three share parameters, \( \phi^E \), \( \phi^H \), and \( \phi^W \) sum to unity.

\[
\begin{align*}
TR^E_{k,kt} &= \phi^E_{k,kt} KY_{k,t} \quad \text{(Y-7)} \\
TR^H_{k,kt} &= \phi^H_{k,kt} KY_{k,t} \quad \text{(Y-8)} \\
TR^W_{k,kt} &= \phi^W_{k,kt} KY_{k,t} \quad \text{(Y-9)}
\end{align*}
\]

**Corporate income**

Corporate income, \( CY \), less debt service payments is split into four accounts. First, the government receives its share through the corporate income tax, \( \kappa^c \). The residual is split into four: retained earnings, income distributed to households, government and the rest of the world. Equation (Y-10) determines corporate income of enterprise \( e \), \( CY_e \). It is the sum, over possible capital types, of shares of distributed profits (to corporations), plus transfers from households (presumably interest payments on privately held debt), income from abroad and interest on domestically held public debt, \( GDebt_d \), where \( \chi^\text{debt} \) represents the share of enterprise \( e \) in the stock of government domestic debt.  

Equation (Y-11) determines corporate profits, \( Prof^c \). They are equal to after tax corporate income net of foreign debt servicing. Equation (Y-12) determines retained earnings, i.e. corporate savings, \( S^c \), where the rate of retained earnings is given by \( \kappa^c \). Equations (Y-13) through (Y-15) determine the overall transfers to households, government and to ROW, respectively. Note that the share parameters, \( \phi^H \), \( \phi^G \) and \( \phi^W \) sum to unity.

\[
\begin{align*}
CY_e &= \sum_{kt} \phi^E_{k,kt} TR^E_{k,kt} + PLEV \sum_h TR^E_{h,ce} + ER \sum_TX^E_{W,ce} + r^\text{d} \chi^\text{debt} GDebt_d \quad \text{(Y-10)} \\
Prof^c_e &= \left(1 - \kappa^c_e\right) \left(CY_e - \chi^\text{debt} r^p PDebt f \cdot ER\right) \quad \text{(Y-11)} \\
S^c_e &= \kappa^c_e Prof^c_e \quad \text{(Y-12)} \\
TR^H_{c,ce} &= \phi^H_{c,ce} \left(Prof^c_e - S^c_e\right) \quad \text{(Y-13)} \\
TR^G_{c,ce} &= \phi^G_{c,ce} \left(Prof^c_e - S^c_e\right) \quad \text{(Y-14)} \\
TR^W_{c,ce} &= \phi^W_{c,ce} \left(Prof^c_e - S^c_e\right) \quad \text{(Y-15)}
\end{align*}
\]

**Household income**

Aggregate household income, \( YH \), is composed of eight elements: labor, land and sector-specific factor remuneration, distributed capital income and corporate profits, transfers from government and households, and foreign remittances, equation (Y-16).  

Government transfers, in the standard closure, are

---

11 The share parameters, \( \phi^c \), sum to unity.
12 All share parameters within the summation signs sum to unity.
fixed in real terms and are multiplied by an appropriate price index to preserve model homogeneity. Remittances, are fixed in international currency terms, and are multiplied by the exchange rate, $ER$, to convert them into local currency terms.$^{13}$

\[
\begin{align*}
YH_h &= \sum_{l \in \text{Labor}} \phi_{l,h} Y_l + \sum_{k \in \text{Capital}} \phi_{k,h} TR_{k,h} + \sum_{l \in \text{Land}} \phi_{l,h} TY_{l,h} + \phi_{h} \rho RY \\
&+ \sum_{c \in \text{Enterprise}} \phi_{c,h} TR_{c,h} + \sum_{g \in \text{Exports}} PLEV TR_{g,h} + \sum_{h \in \text{Household}} TR_{h,h'} + ER.TR_{W,h} \\
&+ \sum_{md \in \text{Other Domestic Final Demand}} \phi_{md,h} TarY_{md} \\

YD_h &= (1 - \lambda^h \kappa^h)YH_h - TR_H - PLEV \sum_{e} TR_{h,e} \\

TR_H &= \phi_{h,h} (1 - \lambda^h \kappa^h)(YH_h - PLEV\sum_{e} TR_{h,e} ) \\

TR_{h,h'} &= \phi_{h,h'} TR_{h'} \\

TR_{W} &= \phi_{W} TR_{h} \\

\end{align*}
\]  

($Y\text{-16}$)

($Y\text{-17}$)

($Y\text{-18}$)

($Y\text{-19}$)

($Y\text{-20}$)

Disposable income, $YD$, is equal to after-tax income, less household transfers, equation ($Y\text{-17}$), where the household tax rate is $\kappa^h$. It is multiplied by an adjustment factor, $\lambda^h$, which is used for model closure. In the standard closure, government savings (or deficit), is held fixed, and the household tax schedule adjusts (uniformly) to achieve the given government fiscal balance. In other words, under this closure rule, the relative tax rates across households remain constant.$^{14}$ Aggregate household transfers, $TR^H$, is a share of after-tax income, equation ($Y\text{-18}$). This is transferred to individual households and abroad, respectively $TR^E$ and $TR^W$, using constant share equations, ($Y\text{-19}$) and ($Y\text{-20}$).

**Domestic final demand**

Domestic final demand is composed of two broad agents—households and other domestic final demand. The model incorporates multiple households. Household demand has a uniform specification, however, with household-specific expenditure parameters. The other domestic final demand categories, in the standard model, include government current expenditures, $Govnt$, private and public investment expenditures, $Invst$ and $Ginvst$, exports of international trade and transport services, $TMG$, and changes in stocks, $delSt$. The other domestic final demand categories, indexed by $f$, are also assumed to have a uniform expenditure function, but with agent-specific expenditure parameters. Demand at the top-level, reflects demand for the Armington good. The latter will be decomposed into domestic and import components in a subsequent nest.

---

$^{13}$ $ER$ measures the value of local currency in terms of the international currency.

$^{14}$ An alternative would be to use an additive factor, which would adjust the average tax rates, not the marginal tax rates.
\textit{Household expenditures}

Households have a tiered demand structure, see figure 2. At the top-level, households save a constant share of disposable income, with the savings rate given by $s^h$. At the next level, residual income is allocated across goods and services, $X_A$, using the linear expenditure system (LES). Equation (D-1) represents the LES demand function. Household consumption is the sum of two components. The first, $\theta$, is referred to as the subsistence minimum, or floor consumption.\footnote{The subsistence minima are scaled by population so that they increase at the same rate as population growth.} The second is a share of supernumerary income, or discretionary income. Supernumerary income is equal to residual disposable income, subtracting savings and aggregate expenditures on the subsistence minima from disposable income. The next level is the decomposition of Armington demand, $X_A$, into its domestic and import components, see below. Equation (D-2) determines household saving, $S^h$, by residual. The consumer price index, $CPI$, is defined in equation (D-3). Note that the consumer price is a household specific Armington price, $P_A$, adjusted by a household specific consumption tax/subsidy, equation (D-4).

\begin{align*}
X_A_{k,h} &= Pop_h \theta_{k,h} + \frac{\mu_{k,h}}{PAc_{k,h}} (1 - s^h) YD_h - \sum_k PAC_{k',k} Pop_h \theta_{k',h} \\
S^h_h &= YD_h - \sum_k PAC_{k,h} X_A_{k,h} \\
CPI_h &= \frac{\sum_k PAC_{k,h} X_A_{k,h,0}}{\sum_k PAC_{k,h,0} X_A_{k,h,0}} \\
PAC_{k,h} &= PA_{k,h} (1 + \tau_{k,h}^c) \\
X_{A_{k,f}} &= \alpha_{k,f}^f \left( \frac{PF_f}{PA_{k,f}} \right)^{\sigma_f^f} XF_f \\
PF_f &= \left[ \sum_k \alpha_{k,f}^f (PA_{k,f})^{1-\sigma_f^f} \right]^{1/(1-\sigma_f^f)} \\
YF_f &= PF_f XF_f
\end{align*}
**Trade equations**

This section discusses the modeling of trade. There are three sections—import demand, and export supply and demand. The first two use a tiered structure. Import demand is decomposed in two steps. The top tier disaggregates aggregate Armington demand into two components—demand for the domestically produced good and aggregate import demand. At the second tier, the aggregate import demand is allocated across trading partners. Both of these tiers assume that goods indexed by $k$ are differentiated by region of origin, i.e. the so-called Armington assumption. A CES specification is used to model the degree of substitutability across regions of origin. The level of the elasticities will often be determined by the level of aggregation. Finely defined goods, such as wheat, would typically have a higher elasticity than more broadly defined goods, such as clothing. At the same time, non-price barriers may also inhibit the degree of substitutability, for example prohibitive transport barriers (inexistent or few transmission lines for electricity), or product and safety standards. Export supply is similarly modeled using a two-tiered constant-elasticity-of-transformation specification. This permits imperfect supply responses to changes in relative prices. Finally, the small-country assumption is relaxed for exports with the incorporation of export demand functions.

**Top-level Armington nest**

The decomposition of Armington demand between domestically produced goods and imported goods is done at the agent-level. Armington agents are indexed by $a$, and includes production activities $(i)$, households $(h)$, other final demand $(f)$ and a single domestic margin sector (‘margn’). Equations (T-1) and (T-2) decompose the Armington demand by each Armington agent $a$ into a domestic, $XD^d$, and import component, $XMT$. Aggregate (Armington) demand is determined from the specific demand functions of each Armington agent—for example the input-output matrix in the case of production activities, or the LES in the case of households. Each agent faces the same domestic price, $PD$, inclusive of the domestic trade and transport margin, $\tau^m$, but adjusted for an agent specific sales tax, $\tau^d$. The import price is agent specific. First, the price $PMT$ is agent-specific because each agent faces a different tariff structure—for example some firms may be exempt from some import taxes. Second, the sales tax may also be agent specific. Equation (T-3) defines the composite Armington.

\[
XD^d_{k,a} = \alpha^d_{k,a} \left( \frac{PA_{k,a}}{(PD_k + \tau^m_{k,a} PTMG_k) (1 + \tau^d_{k,a})} \right)^{\sigma^a_{k,a}} XA_{k,a} \tag{T-1}
\]

\[
XMT_{k,a} = \alpha^m_{k,a} \left( \frac{PA_{k,a}}{(1 + \tau^m_{k,a} PMT_{k,a})} \right)^{\sigma^a_{k,a}} XA_{k,a} \tag{T-2}
\]

\[
PA_{k,a} = \left[ \alpha^d_k \left( (PD_k + \tau^m_{k,a} PTMG_k) (1 + \tau^d_{k,a}) \right)^{1-\sigma^a_{k,a}} + \alpha^m_k (1 + \tau^m_{k,a} PMT_{k,a})^{1-\sigma^a_{k,a}} \right]^{1/(1-\sigma^a_{k,a})} \tag{T-3}
\]

---

16. This is new to the model. The previous version of the model assumed that Armington demand was aggregated across all agents and that a single national agent decomposed demand by region of origin.

17. See Ianchovichina (2004) for an implementation of duty drawbacks in the context of the global trade model known as GTAP.
Second-level Armington nest

At the second level, aggregate import demand, XMT, is allocated across trading partners using a CES specification. Equation (T-4) defines the domestic price of imports, PM. It is equal to the world price (in international currency), WPM, multiplied by the exchange rate, and adjusted for import distortions, τ, i.e. PM represents the port-price of imports, tariff-inclusive. The tariff rate is sector- and region of origin-specific, but it is also specific to a tariff regime, indexed by tr. If all domestic agents face the same tariff regime, the set tr will be of dimension 1. The largest possible dimension of the set tr will be the dimension of the set of Armington agents, a. A typical configuration may have three tariff regimes: the normal (or statutory tariff regime), tariff exempt sectors (for example firms in an export processing zone), and a special regime for investment goods. In this case, the set tr would have a dimension of three. Equation (T-5) represents the aggregate imports for each index in the set tr, XMTR, i.e. it is the sum across all Armington agents subject to tariff regime tr of their import demand. The latter is allocated across region of origin using a second CES nest, equation (T-6). This equation represents the import of commodity k under tariff regime tr from region r, XM, where the inter-regional substitution elasticity is given by σ. The aggregate price of imports by tariff regime, PMTR, is defined in equation (T-7). Finally, the aggregate import price faced by Armington agent a is the relevant aggregate import price index for the tariff regime the Armington agent is subject to, equation (T-8).

\[
PM_{tr,k,r} = ERWPM_{k,r} (1 + \sum_{md} \chi_{md} \tau_{tr,k,r,md}^w ) (1 + \tau_{tr,k,r}^i ) + \tau_{tr,k,r}^{mg,M} PTMG_k
\]  

(T-4)

\[
XMTR_{tr,k} = \sum_{a \in tr} XMT_{k,a}
\]  

(T-5)

\[
XM_{tr,k,r} = \alpha_{tr,k,r}^w \left( \frac{PM_{tr,k,r}}{PM_{tr,k,r}} \right)^{\sigma_{tr,k,r}^w} XMTR_{tr,k}
\]  

(T-6)

\[
PM_{tr,k,r} = \left[ \sum_r \alpha_{tr,k,r}^w \left( PM_{tr,k,r} \right)^{1-\sigma_{tr,k,r}^w} \right]^{1/(1-\sigma_{tr,k,r}^w)}
\]  

(T-7)

\[
PMT_{k,a} = PM_{tr,k} \text{ for } a \in tr
\]  

(T-8)

Top-level CET nest

Domestic production is allocated across markets using a nested CET specification. At the top nest, producers allocate production between the domestic market and aggregate exports. At the second nest, aggregate exports are allocated across trading partners. The model allows for perfect transformation, i.e. producers perceive no difference across markets. In this case, the law-of-one-price holds. Equation (T-9)

18 PM and WPM are indexed by both commodity, k, and trading partner, r.

19 The tariff rate is also multiplied by a uniform shifter, χtm, which is normally equal to 1. The shifter can be modified exogenously for specific simulations, for example setting it to 0.5 would cut tariffs by 50 percent across the board, or it can be rendered endogenous, assuming there is an exogenous target to be achieved. For example, to calculate a uniform revenue neutral tariff rate, one could set all tariffs (or positive-rated tariffs) to 0.1 and endogenize the shift parameter, χtm. The exogenous variable is the value of initial tariff revenues (in real terms). If the calculated χtm is 1.5, this indicates that the uniform revenue neutral tariff is 15 percent.
represents the link between the domestic producer price, $PE$, and the world price, $WPE$. Export prices are both sector- and region-specific. The FOB price, $WPE$, includes domestic trade and transport margins, $\tau_{mg}^{\text{20}}$, as well as export taxes/subsidies, $\tau^{e}$. Equations (T-10) and (T-11) represent the CET optimality conditions. The first determines the share of domestic supply, $X$, allocated to the domestic market, $XD$. The second determines the supply of aggregate exports, $XET$. $PET$ represents the price of aggregate export supply. The transformation elasticity is given by $\sigma^{x}$. The model allows for perfect transformation. In this case, the optimal supply conditions are replaced by the law-of-one price conditions. Equation (T-12) represents the CET aggregation function. In the case of finite transformation, it is replaced with its equivalent, the CET dual price aggregation function. In the case of infinite transformation, the primal aggregation function is used, where the two components are summed together since there is no product differentiation.

\[
\begin{align*}
(PE_{k,r} + P T M G_{k,r}^{mg,E} (1 + \tau_{k,r}^{e} ) ) & = ERWPE_{k,r} \\
XD_{k} & = \gamma_{k}^{d} \left( \frac{PD_{k}}{P_{k}} \right)^{\sigma_{k}^{x}} X_{k} \quad \text{if} \quad \sigma_{k}^{x} \neq \infty \\
PD_{k} & = P_{k} \quad \text{if} \quad \sigma_{k}^{x} = \infty \\
XET_{k} & = \gamma_{k}^{e} \left( \frac{PET_{k}}{P_{k}} \right)^{\sigma_{k}^{x}} X_{k} \quad \text{if} \quad \sigma_{k}^{x} \neq \infty \\
PET_{k} & = P_{k} \quad \text{if} \quad \sigma_{k}^{x} = \infty \\
P_{k} & = \left[ \gamma_{k}^{d} PD_{k}^{\sigma_{k}^{x}} + \gamma_{k}^{e} PET_{k}^{\sigma_{k}^{x}} \right]^{1/(1+\sigma_{k}^{x})} \quad \text{if} \quad \sigma_{k}^{x} \neq \infty \\
X_{k} & = XD_{k} + XET_{k} \quad \text{if} \quad \sigma_{k}^{x} = \infty
\end{align*}
\]

Second-level CET nest

The second-level CET nest allocates aggregate export supply, $XET$, across the various export markets, $XE$. Equation (T-13) represents the optimal allocation decision, where $\sigma^{z}$ is the transformation elasticity. Equation (T-14) represents the CET aggregation function, where again, the CET dual price formula is used to determine the aggregate export price, $PET$. As above, the model allows the transformation elasticity to be infinite.

\[\text{Note that the domestic trade and transport margins are differentiated for three different goods: domestically produced goods sold to the domestic market, exported goods, and imported goods.}\]
\[
\begin{aligned}
XE_{k,r} &= \gamma_{k,r} \left( \frac{PE_{k,r}}{PET_k} \right)^{\sigma_k^i} XET_k \quad \text{if} \quad \sigma_k^i \neq \infty \\
PE_{k,r} &= PET_k \quad \text{if} \quad \sigma_k^i = \infty \\
PET_k &= \left[ \sum_r \gamma_{k,r} P^{1+\sigma_k^i} \right]^{1/(1+\sigma_k^i)} \quad \text{if} \quad \sigma_k^i \neq \infty \\
XET_k &= \sum_r XE_{k,r} \quad \text{if} \quad \sigma_k^i = \infty 
\end{aligned}
\]

(T-13)

(T-14)

Export demand

Export, \( ED \), demand is specified using a constant elasticity function, equation (T-15). If the elasticity, \( \eta^e \), is finite, demand decreases as the international price of exports, \( WPE \), increases. The numerator contains an exogenous export price competitive index. If the latter increases relative to the domestic export price, market share of the domestic exporter would increase. The model allows for a shift in the export demand function that is related to the income growth, \( g_w \), of the trading partners. The model allows for infinite demand elasticity. This represents the small-country assumption. In this case, the domestic price of exports (in international currency units) is constant. If the two CET elasticities are likewise infinite, then the domestic producer price is also equal to the world price of exports (adjusted for taxes and trade and transportation margins).

\[
\begin{aligned}
ED_{k,r} &= \alpha_{k,r}^e \left( \frac{WPE_{k,r}}{WPE_{k,r}} \right)^{\eta_{k,r}^e} (1 + g_w^e)^{\eta_{k,r}^e} \quad \text{if} \quad \eta_{k,r}^e \neq \infty \\
WPE_{k,r} &= WPE_{k,r} \quad \text{if} \quad \eta_{k,r}^e = \infty 
\end{aligned}
\]

(T-15)

Domestic trade and transportation margins

The marketing of each good—domestic, imports, and exports—is associated with a commodity specific trade margin.\(^{21}\) Equation (M-1) defines the volume of margin services. There are three terms. The first measures the volume of trade and transport services for each good \( k \) produced domestically and sold to domestic (Armington) agent \( a \). The second measures the volume of trade and transport services for each imported good \( k \) from region of origin \( r \) imported in regime \( tr \).\(^{22}\) The third measures the volume of trade and transport services for each good \( k \) exported to region \( r \). The assumption is that the cost structure of trade and transport services is the same across all nodes of transportation for each good \( k \).\(^{23}\) Thus equation

\(^{21}\) The model does not include international trade and transport margins. A change in the latter could be simulated by a change in the relevant world price index, \( WPM \) or \( WPE \).

\(^{22}\) It would be highly unlikely that national data would be able to distinguish the cost of margins by region of origin and trade regime, but it is a harmless assumption.

\(^{23}\) In other words, the model differentiates the margin between transporting wheat and cars and between transporting wheat for export as opposed to the domestic market. But the cost structure per unit of trade and transport volume deployed is identical—and simply scaled by the relevant overall margin. For example, if the margin of transport on domestic cars is 15 percent and that on exported cars only 10 percent (because for
(M-1) determines the national volume of demand for trade and transport services for commodity \( k \), irrespective of the transportation node. The production of the trade and transport services follows a Leontief technology. Equation (M-2) defines the demand for goods and services. In other words, to deliver commodity \( k' \) requires an input from commodity \( k \), the level of which is fixed in proportion to the overall volume of delivering commodity \( k' \) in the economy, \( XT_k^{mg} \). Equation (M-3) is the expenditure deflator, \( PT_k^{mg} \), for trade margin activities and is uniform across all transport nodes. Aggregate demand for goods and services in the trade and transport sector, i.e. the Armington demand, is given by equation (M-4). It is simply the sum of all demand for trade and transport commodities (of commodity \( k \)) across all delivered goods \( k' \). \(^{24}\)

\[
\begin{align*}
XT_k^{mg} &= \sum_a \tau_{k,a}^{mg,D} XD_{k,a}^{d} + \sum_r \sum_{r'} \tau_{r,k,r'}^{mg,M} XM_{r,k,r'} + \sum_r \tau_{k,r}^{mg,E} XE_{k,r} \\
XAmg_{k,k'} &= \alpha_{k,k'}^{mg} XT_k^{mg} \\
PT_k^{mg} &= \sum_k \alpha_{k,k}^{mg} PA_{k,Margn} \\
XA_{k,Margn} &= \sum_{k'} XAmg_{k,k'}
\end{align*}
\]

(M-1) \hspace{1cm} (M-2) \hspace{1cm} (M-3) \hspace{1cm} (M-4)

**Goods market equilibrium**

There are three fundamental commodities in the model—domestic goods sold domestically, imports (by region of origin), and exports (by region of destination). All other goods are bundles (i.e. are defined using an aggregation function) and do not require supply/demand balance. The small-country assumption holds for imports, and therefore any import demand can be met by the rest of the world with no impact on the price of imports. Therefore, there is no explicit supply/demand equation for imports. \(^{25}\) Equation (E-1) represents equilibrium on the domestic goods market, and essentially determines, \( PD \), the producer price of the domestic good. Equation (E-2) defines the equilibrium condition on the export market. With a finite export demand elasticity, the equation determines \( WPE \), the world price of exports. With an infinite export demand elasticity, the equation trivially equates export supply to the given export demand.

\[
\begin{align*}
\sum_a XD_{k,a}^{d} &= XD_k^{e} \\
ED_{k,r} &= XE_{k,r}
\end{align*}
\]

(E-1) \hspace{1cm} (E-2)

\(^{24}\) The assumption is that all transport nodes have the same decomposition of Armington demand (across domestic and import goods).

\(^{25}\) One could rather easily add an import supply equation and an equilibrium condition.
Macro closure

Macro closure involves determining the exogenous macro elements of the model. The standard closure rules are the following:

- Government fiscal balance is exogenous, achieved with an endogenous direct tax schedule
- Private investment is endogenous and is driven by available savings
- The volume of government current expenditures is exogenous
- The volume of stock changes is exogenous
- The trade balance (i.e. capital flows) is exogenous. The real exchange rate equilibrates the balance of payments.

These are detailed further below.

Government accounts

Equation (C-1) describes nominal tariff revenues, $\text{TarY}$, and equation (C-2) defines real tariff revenues, $\text{RTarY}$. There are three import distortions, indexed by md—ad valorem tariffs, import subsidies and a price wedge due to non-tariff barriers (NTBs). The price distortions are additive. Equation (C-3) defines total indirect taxes equal to the sum of sales tax on domestic and imported goods, production tax, value added tax on domestic and imported goods, taxes/subsidies on household consumption and export taxes. Equation (C-4) defines total import distortions. Equations (C-5) and (C-6) define respectively real aggregate indirect taxes and import distortions. Equation (C-7) defines total government revenues, $\text{GY}$. There are multiple components: domestic indirect and import taxes, land, resource, capital and wage taxes, corporate and household direct taxes, government share of enterprise income and transfers from the rest of the world. Equation (C-8) defines the government’s current expenditures, $\text{GEXP}$. It is the sum of five components: expenditures on goods and services, transfers to households, transfers to ROW and interest payments on domestic and foreign debt. Aggregate government savings (on current and capital operations), $S^g$, is defined in equation (C-9), as the difference between revenues and the sum of current and investment expenditures. Real government savings, $\text{RSg}$, is defined in equation (C-10). It is this latter which essentially determines the level of direct household taxation since $\text{RSg}$ is exogenous in the standard closure.
$$\sum \sum \sum A_{md}^{m} \tau_{tr,k,r,md}^{m} WPM_{k,r} XM_{tr,k,r}$$

$$RTarY_{md} = TarY_{md} / PLEV$$

$$NIPAFC^{Nom}_{inptx} = \sum \sum \sum \sum \sum \sum \sum \sum \sum \sum \sum \sum \sum \sum \sum \sum \sum \sum \sum \sum \sum \sum \sum \sum \sum \sum \sum \sum \sum \sum \sum \sum \sum \sum \sum \sum \sum \sum \sum \sum \sum \sum \sum \sum \sum \sum \sum \sum \sum \sum \sum \sum \sum \sum \sum \sum \sum \sum \sum \sum \sum \sum \sum \sum \sum \sum \sum \sum \sum \sum \sum \sum \sum \sum \sum \sum \sum \sum \sum \sum \sum \sum \sum \sum \sum \sum \sum \sum \sum \sum \sum \sum \sum \sum \sum \sum \sum \sum \sum \sum \sum \sum \sum \sum \sum \sum \sum \sum \sum \sum \sum \sum \sum \sum \sum \sum \sum \sum \sum \sum \sum \sum \sum \sum \sum \sum \sum \sum \sum \sum \sum \sum \sum \sum \sum \sum \sum \sum \sum \sum \sum \sum \sum \sum \sum \sum \sum \sum \sum \sum \sum \sum \sum \sum \sum \sum \sum \sum \sum \sum \sum \sum \sum \sum \sum \sum \sum \sum \sum \sum \sum \sum \sum \sum \sum \sum \sum \sum \sum \sum \sum \sum \sum \sum \sum \sum \sum \sum \sum \sum \sum \sum \sum \sum \sum 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There are several alternative closure rules. First, all taxes can be fixed and the government fiscal balance can be endogenous. This may lead to potential sustainability issues, but may be valid for short-term analysis. In terms of fiscal instruments, the endogenous household tax rate can be replaced by making the value added tax endogenous—either both the domestic and import rates simultaneously, or each individually (one at a time). There is also a joint adjustment factor on both the corporate and household tax rates that can be endogenous to achieve the fiscal target. The import tariff also has a uniform shifter that can also be endogenous. This has typically been used to calculate a revenue-neutral uniform tariff.

Investment and macro closure

Equation (C-11) defines the investment savings balance. In the standard closure, it determines the level of aggregate investment since stock changes are exogenous. These two components are financed by aggregate savings defined over corporations, households, and the government, and adjusted by foreign savings. The latter is fixed (in international currency terms). Equation (C-13) defines the investment to GDP ratio. Under the standard closure rule the ratio is endogenous. However, it is possible to fix the ratio and control either private or public investment to achieve a given ratio. Equations (C-14) and (C-15) define the volumes of public current and investment expenditures. Equation (C-15) fixes aggregate stock changes.

\[
YF_{\text{Invst}} + YF_{\text{DelSt}} = \sum_{e} S_{e}^{c} + \sum_{h} S_{h}^{h} + S_{d}^{g} - ER.S_{f}^{p} + ER.FDI
\]  
\[\text{(C-11)}\]

\[
XF_{\text{Invst}} + XF_{\text{GInvst}} = invgdp.RGDPMP
\]  
\[\text{(C-12)}\]

\[
XF_{\text{Govnt}} = \alpha_{\text{Govnt}} (RGDPMP)^{\eta_{e}}
\]  
\[\text{(C-13)}\]

\[
XF_{\text{GInvst}} = \alpha_{\text{GInvst}} (RGDPMP)^{\eta_{x}}
\]  
\[\text{(C-14)}\]

\[
XF_{\text{delSt}} = \overline{XF}_{\text{delSt}}
\]  
\[\text{(C-15)}\]

The aggregate price level, \(PLEV\), is the average absorption (Armington) price, equation (C-16). Equation (C-17) defines the aggregate CPI index. In some simulations the aggregate CPI is used as the model numéraire instead of the exchange rate.

\[\overline{YF}_{\text{Invst}} = \overline{YF}_{\text{DelSt}} = \overline{XF}_{\text{Invst}} + \overline{XF}_{\text{DelSt}} + \overline{XF}_{\text{Govnt}} + \overline{XF}_{\text{GInvst}}\]  
\[\text{(C-16)}\]

\[\overline{CPI} = \frac{\overline{YGDP} - \overline{YF}_{\text{Invst}} - \overline{YF}_{\text{DelSt}}}{\overline{YGDP}}\]  
\[\text{(C-17)}\]

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26 An alternative savings/investment closure has investment fixed in real terms. In this case, the domestic savings rate adjusts to achieve the investment target. Adjustment can occur on household savings alone, or on both corporate and household saving.
\begin{equation}
PLEV = \frac{\sum_{a} \sum_{k} PA_{k,a} XA_{k,a,0}}{\sum_{a} \sum_{k} PA_{k,a,0} XA_{k,a,0}} \tag{C-16}
\end{equation}

\begin{equation}
CPIT = \frac{\sum_{a} \sum_{h} PA_{c,k,a} XA_{k,a,0,0}}{\sum_{a} \sum_{h} PA_{c,k,a,0} XA_{k,a,0}} \tag{C-17}
\end{equation}

The next set of equations are used for closing the balance of payments, however they are the below line balancing items and have no effect on the model results itself. The first equation, (C-18) describes the desired level of the stock of foreign reserves, $RES$, (in foreign currency terms), required to cover a certain number of days of imports, where $\rho^{rq}$ reflects the number of days of import coverage. Equation (C-19) simply determines the change in foreign reserves, $delRES$. Equation (C-20) equates short-term financing (i.e. trade financing), $FRES$, to the change in the level of trade (in foreign currency terms), i.e. the stock of trade financing is equal to a share, $\chi^{fin}$, of total trade (exports plus imports). The variable $trGDPRat$, defined below, is the ratio of trade to GDP. Equation (C-21) determines the level of demand for non-debt creating foreign financing, $FTOT$. It equals short-term (trade) financing, plus the change in reserves and FDI. Equation (C-22) defines the ratio of trade to GDP. Finally, Equation (C-23) represents the balance of payments (in international currency terms). It can be shown to be redundant, and is dropped from the model specification.

\begin{equation}
RES = (\rho^{rq} / 360) \sum_{tr} \sum_{k} \sum_{r} WPM_{k,r} XM_{tr,k,r} \tag{C-18}
\end{equation}

\begin{equation}
delRES = RES - RES_{-1} \tag{C-19}
\end{equation}

\begin{equation}
FRES = \chi^{fin} (trGDPRat.GDPMP - trGDPRat_{-1}.GDPMP_{-1}) / ER \tag{C-20}
\end{equation}

\begin{equation}
FTOT = FRES + delRES + FDI \tag{C-21}
\end{equation}

\begin{equation}
trGDPRat = (NIPA^{Nom}_{Exp} - NIPA^{Nom}_{Imp}) / GDPMP \tag{C-22}
\end{equation}

\begin{equation}
BoP = \sum_{r} \sum_{k} WPE_{k,r} XE_{k,r} + \sum_{e} TR_{W,e}^{e} + \sum_{h} TR_{W,h}^{h} + TR_{W}^{g} + \sum_{l} S_{f}^{l} + \sum_{l} FW_{l}^{l} \tag{C-23}
\end{equation}

- \sum_{tr} \sum_{k} WPM_{k,r} XM_{tr,k,r} - \sum_{md} \phi^{md} TarY_{md} \quad \text{Imports}

- (1/ ER) \left[ \sum_{k} TR_{k,l}^{W} + \sum_{e} TR_{c,e}^{W} + \sum_{n} TR_{p}^{W} \right] - TR_{g}^{W} \quad \text{Outbound transfers}

\equiv 0

Model numéraire

The default numéraire is the exchange rate variable, \( ER \). This suggests that domestic prices are determined relative to the price of the basket of tradable goods that remains fixed in both international and domestic terms. Given the default closure rule of fixed foreign savings, the real exchange rate adjusts to achieve the balance of payments target. There are a number of alternative definitions of the real exchange rate. One that lends itself readily to interpretation is the GDP at factor cost deflator. This represents the average return to domestic factors—and these are truly non-tradable. It is also a good measure of the purchasing power of domestic income relative to the price of imports. An alternative numéraire could be any one of the domestic price indices—the aggregate CPI, the aggregate price of domestic absorption, or one of the GDP deflators. If one of these alternatives is chosen, the exchange rate variable, \( ER \), becomes endogenous and in some sense becomes the equilibrating variable for the balance of payments constraint. Note that an increase in \( ER \) represents a depreciation.

Factor market equilibrium

The following sections describe the standard factor market equilibrium conditions.

Labor markets

The national economy is divided into two distinct geographic zones, indexed by \( gz \). The zones define potentially separate labor markets and are designated by \( Rur \) and \( Urb \), representing respectively rural and urban areas.\(^{27}\) A third zone, \( Tot \), represents the national market. A single elasticity, \( \omega^m \), determines the nature of the labor market. If the migration elasticity is infinite, then the labor market is nationally integrated and labor is fully mobile between rural and urban activities. A single economy-wide wage rate, \( W^e \), will clear the national labor market. If the migration elasticity is finite, then there is labor market segmentation with migration.\(^{28}\) Separate market-clearing wage rates will be determined in each labor market. The decision to migrate is a function of the expected relative wages.\(^{29}\)

The equations described below are based on two indices, \( gz \) and \( gs \). The first ranges over the three zones—rural, urban and national. The second, \( gs \), is a subset of \( gz \) and only ranges over the segmented markets—rural and urban.

\[
gz \in \{ Rur, Urb, Tot \}
\]

\[
gs \in \{ Rur, Urb \} \subset gz
\]

The first three equations describe labor supply in each zone. Rural labor supply is equal to the previous period’s labor supply adjusted for (exogenous) natural growth, \( g^L \), from which is subtracted migration, \( MIGR \), equation (F-1). Equation (F-2) is a similar equation defining urban labor supply where migration augments the natural growth of urban labor supply. Equation (F-3) determines the national labor supply. It is simply the sum of rural and urban labor supplies. The national labor supply is of course independent of internal migration.

\(^{27}\) In most cases the distinction rural and urban will be blurred since it typically will represent agricultural versus non-agricultural activities. The rural sector may have significant non-agricultural activities.

\(^{28}\) Fully segmented markets can be achieved by setting the migration elasticity to 0 and fixing the base year level of migration at 0.

\(^{29}\) In the current version, expected real wages are a function of the national price level as opposed to zone-specific price indices.
The next two equations determine migration. Equation (F-4) defines the expected average wage rate, $AWAGE_{i,g}$, in both the rural and urban zones. It is the weighted average of the sector-specific (net) wage, weighted by actual sectoral labor demand within each zone. The net wage is the wage rate received by employees (net of the wage tax) and is the natural wage to use in the migration function. The average wage is adjusted by the probability of finding employment in the respective zone, i.e. one minus the rate of unemployment. Equation (F-5) determines rural to urban migration, $MIGR_i$. It is a function of the expected urban wage relative to the expected rural wage. Thus the average wage in each sector is multiplied by the probability of finding employment as measured by 1 minus the unemployment rate, $UE$. The migration function is deleted from the model specification if there is an integrated labor market, i.e. if $\omega^m$ is infinite.

The model incorporates regime-switching behavior in labor markets. The following discussion describes the basic theory and this is followed by a description of the model equations.

It is relatively straightforward to introduce semi-rigidity in wages and thus create a wedge between labor supply and demand. Let $WMIN$ represent a minimum wage, possibly 0, and $UE$ be the unemployment rate. The following orthogonality condition represents two possible regimes:

$$0 = -(W - WMIN) \times (UE - UEMIN)$$

The first regime has the prevailing wage exceeding the minimum wage. In this case unemployment must be at its minimum for the condition to hold. In the second regime, the equilibrium wage is below the minimum wage, in which case the prevailing wage, $W$, is set to the minimum wage and unemployment is greater than the minimum level given by $UEMIN$. The orthogonality condition is easily implemented using mixed complementarity programming (MCP). It is converted to the following set of conditions:

$$W \geq WMIN \quad \text{and} \quad UE \geq UEMIN$$

---

30 The absolute minimum would be a rate of 0 percent, but the model allows for the full-employment unemployment rate to be positive.
The minimum wage is driven by the following equation:

\[ WMIN = \chi^{\text{min}}(PLEV)^{\phi_p}(g_y)^{\phi_y}(UE)^{\phi_w} \]

This equation, depending on the elasticities, can represent a variety of different labor market specifications.\(^{31}\) If \(\phi_p\) is positive, wages could be linked to the overall price level, i.e. employees are targeting a real wage. If \(\phi_y\) is positive, wages could be linked to the overall GDP growth rate, i.e. employees are targeting a share of the overall growth in national income.\(^{32}\) If \(\phi_w\) is positive, this would be consistent with efficiency wages where the discipline of market forces dampens wage demands.\(^{33}\)

\[
WMIN_{1,gr} = \chi^{\text{min}}_{1,gr} PLEV^\omega_{gr} \left( \frac{1 + g_y}{1 + g_{pop}} \right)^{\omega_{gr}} (1 - UE_{1,gr})^{\omega_{gr}} \quad (F-6)
\]

\[
(W_{1,gr} - WMIN_{1,gr}).(UE_{1,gr} - UE_{\text{Min}}^{E1,gr}) = 0 
\quad (F-7)
\]

\[
UE_{1,gr} = \frac{L^d_{1,gr} - \sum_{s=gr} L^d_{s,gr}}{L^L_{1,gr}} \geq 0 \quad (F-8)
\]

Equation (F-6) determines the minimum wage for the respected labor segments.\(^{34}\) Perfect wage indexation is implied by a price elasticity of 1 and an employment elasticity of 0. Equation (F-7) is the orthogonality condition allowing for endogenous regime switching on the relevant labor market segments. Either the minimum wage is binding and unemployment is above its minimum, or unemployment is at its minimum level and the equilibrium wage is above its floor level. (F-8) determines the relevant unemployment rate, \(UE\).

Though labor is assumed to be perfectly mobile across sectors within a market segment, inter-sectoral wage differentials are allowed to co-exist reflecting specific institutional features related to the domestic labor markets.\(^{35}\) In the basic version of the model, the inter-sectoral wage differentials are assumed to be

\(^{31}\) For further discussion see Agénor et al (2002a) and Agénor et al (2002b).

\(^{32}\) In the equation formulation, the growth factor is 1 plus the per capita growth rate reflecting that wages grow in line with average incomes. It is one plus the growth rate to avoid problems with negative growth rates, and thus the elasticity should reflect this fact. An elasticity of 1 with respect to the growth rate should be higher with respect to 1 plus the growth rate.

\(^{33}\) In the equation formulation, 1 less the unemployment rate is used to avoid pushing the minimum wage down to zero. If the original elasticity—i.e. the one with respect to the rate of unemployment—is measured as \(\sigma\), the elasticity with respect to 1 less the rate of unemployment is equal to:

\[ \omega = \sigma(1 - UE_0) / UE_0 \]

If \(\sigma\) is 1, then \(\omega\) is 9 if \(UE_0\) is 10% and \(\omega\) is 19 if \(UE_0\) is 5%.

\(^{34}\) Equations (F-6) through (F-8) are coded for any possible combination of market segmentation. If the migration elasticity is finite, the equations will be specified for each of the two labor market segments, otherwise the model will assume an integrated market. The model can also handle mixed situations with segmentation for some labor skills and an integrated market for the others.

\(^{35}\) These wage differentials can represent a variety of factors—i) labor, even at the same skill level may not be totally homogeneous; ii) sectors may represent combinations of different market institutions, for example formal and informal employment, union vs. non-union; iii) other factors can influence wage differentials across sectors, for example hardship or occupational hazard premium. The implicit assumption made in the model is that all wages move in a coordinated fashion within a labor market segment keeping the differentials constant.
fixed. Equation (F-9) determines the sectoral skill-specific wage rates as a function of the base inter-sectoral wage differentials and changes in the segment-specific wage rate. The gross wage, $W$, is determined in equation (F-10).

\[
NW_{id} = \phi_{i}^{e} W_{i,gz}^{e} \quad \text{for} \quad i \in gz \\
W_{ij} = (1 + \tau_{ij}^{sff} + \tau_{ij}^{sff})NW_{id}
\]

(F-9)  
(F-10)

**Capital market**

Equilibrium on the capital market allows for both limiting cases—perfect capital mobility and perfect capital immobility, or any intermediate case. Aggregate capital, $K^s$, is allocated across sectors and type according to a nested CET system. At the top-level, the aggregate investor allocates capital across types, according to relative rates of return. Equation (F-11) determines the optimal supply decision, where $TK^s$ is the supply of capital of type $kt$, with an average return of $PTK$. $PK$ is the aggregate rate-of-return to capital. If the supply elasticity is infinite, the law-of-one-price holds. Equation (F-12) represents the top-level aggregation function, replaced by the CET dual price function in the case of a finite transformation elasticity. Perfect capital mobility is represented by setting $\omega^{kt}$ to infinity. Perfect immobility is modeled by setting the transformation elasticity to 0.

\[
\begin{align*}
TK^s_{kt} &= \gamma_{kt}^{dks} \left( \frac{PTK_{kt}}{PK} \right)^{\omega^{kt}} \quad \text{if} \quad \omega^{kt} \neq \infty \\
PTK_{kt} &= PK \quad \text{if} \quad \omega^{kt} = \infty
\end{align*}
\]

(F-11)

\[
\begin{align*}
PK &= \left[ \sum_{kt} \gamma_{kt}^{dks} PTK_{kt}^{1 + \omega^{kt}} \right]^{1/(1 + \omega^{kt})} \quad \text{if} \quad \omega^{kt} \neq \infty \\
K^s &= \sum_{kt} TK^s_{kt} \quad \text{if} \quad \omega^{kt} = \infty
\end{align*}
\]

(F-12)

At the second level, capital by type, $TK^s$, is allocated across sectors using another CET function. Equation (F-13) determines the optimal allocation of capital of type $kt$ to sector $i$, $K^i$, where the transformation elasticity is $\omega^i$. Equation (F-14) represents the CET aggregation function. The equilibrium return to capital, $R$, is determined by equation capital supply to demand, equation (F-15).\(^{36}\) Equation (F-16) determines the gross rate of return to sectoral capital.

\(^{36}\) If the transformation elasticity is infinite, equation (F-13) determines the sector- and type-specific rate of return using the law-of-one price, and equation (F-16) trivially sets capital supply equal to capital demand.
Land market equilibrium is specified in an analogous way to the capital market with a tiered CET supply system. The first tier allocates total land across types. This could have a zero transformation elasticity if for example land used for rice production could not be used to produce other commodities. Their respective prices are $PL_{\text{LAND}}$ and $PT_{\text{TT}^t}$, see equations (F-17) and (F-18).

Equations (F-19) and (F-20) determine the optimality conditions at the second and final tier, determining land supply (by type and) by sector of use. Land market equilibrium is represented by equation (F-21). And Equation (22) determines the gross land rental rate.
Natural resource market

The market for natural resources differs from the others in the sense that there is no inter-sectoral mobility, i.e. this is a sector specific resource. There is therefore a sector specific supply curve (eventually flat). Equation (F-23) describes the sector-specific supply function, or $NR_t$. Equation (F-24) then determines the equilibrium price, $PR$.

Debt Module

Debt is...

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37 More realistic models allow for kinked supply curves. It is typically easier to take resources out of production than to bring them online—the latter requiring new investments and/or new exploration. Thus a so-called down supply elasticity would be higher than a so-called up supply elasticity.
Macroeconomic identities

The macroeconomic identities are not normally needed for the model specification, i.e. they could be calculated at the end of a simulation. In the case of dynamic scenarios, one or more of them could be used to calibrate dynamic parameters to a given set of exogenous assumptions. For example, the growth of GDP could be made exogenous. In this case, a growth parameter, typically a productivity factor, would be endogenous and set to target the given growth path of GDP.

The macro accounts are stored in two separate vectors—NIPA and NIPAFC. The former defines the national income and product accounts (NIPA) at market price—consumption, investment, etc. The latter defines the macro accounts at factor cost. Both are calculated in nominal and real terms.

The first set of equations define nominal GDP at market price. Equations (I-1) through (I-5) define respectively nominal private consumption, government current expenditures, aggregate investment, exports and (the negative) of imports. Equation (I-6) defines nominal GDP at market price as the sum across all of the components (indexed by na).
\[
\text{NIPA}_{\text{Cons}}^{\text{Nom}} = \sum_k \sum_h PAc_{k,h} XA_{k,h} \]  \quad (I-1)

\[
\text{NIPA}_{\text{Gov}}^{\text{Nom}} = \sum_k PA_{\text{Govnt}} XA_{\text{Govnt}} \]  \quad (I-2)

\[
\text{NIPA}_{\text{Inv}}^{\text{Nom}} = \sum_k [PA_{\text{Invst}} XA_{\text{Invst}} + PA_{\text{Ginvst}} XA_{\text{Ginvst}} + PA_{\text{delSt}} XA_{\text{delSt}}] \]  \quad (I-3)

\[
\text{NIPA}_{\text{Exp}}^{\text{Nom}} = ER \sum_k \sum r WPE_{k,r} XE_{k,r} \]  \quad (I-4)

\[
\text{NIPA}_{\text{Imp}}^{\text{Nom}} = -ER \sum_k \sum r \sum \text{tr} WPM_{k,r,\text{tr}} XM_{\text{tr},k,r} \]  \quad (I-5)

\[
\text{GDPMP} = \sum_{\text{na}} \text{NIPA}_{\text{na}}^{\text{Nom}} \]  \quad (I-6)

Equations (I-7) through (I-12) define the same identities but for the volume components of GDP at market price. The only difference in the formulas is the use of base year prices rather than current year prices in the definitions. Equation (I-13) defines the GDP at market price deflator.

\[
\text{NIPA}_{\text{Cons}}^{\text{Real}} = \sum_k \sum h PAc_{k,h,0} XA_{k,h} \]  \quad (I-7)

\[
\text{NIPA}_{\text{Gov}}^{\text{Real}} = \sum_k PA_{\text{Govnt},0} XA_{\text{Govnt}} \]  \quad (I-8)

\[
\text{NIPA}_{\text{Inv}}^{\text{Real}} = \sum_k [PA_{\text{Invst},0} XA_{\text{Invst},0} + PA_{\text{Ginvst},0} XA_{\text{Ginvst},0} + PA_{\text{delSt},0} XA_{\text{delSt},0}] \]  \quad (I-9)

\[
\text{NIPA}_{\text{Exp}}^{\text{Real}} = ER \sum_k \sum r WPE_{k,r,0} XE_{k,r} \]  \quad (I-10)

\[
\text{NIPA}_{\text{Imp}}^{\text{Real}} = -ER \sum_k \sum r \sum \text{tr} WPM_{k,r,\text{tr},0} XM_{\text{tr},k,r,0} \]  \quad (I-11)

\[
\text{RGDPMP} = \sum_{\text{na}} \text{NIPA}_{\text{na}}^{\text{Real}} \]  \quad (I-12)

\[
\text{PGDPMP} = \text{GDPMP} / \text{RGDPMP} \]  \quad (I-13)

Equations (I-14) and (I-15) define the two main components of nominal GDP at factor cost—labor remuneration and non-wage income. Equations (I-16) and (I-17) define the same in volume terms. Note
that real GDP at factor cost is evaluated in efficiency units. Equations (I-18) and (I-19) define respectively nominal and real GDP at factor cost. Equation (I-20) defines the GDP at factor cost deflator.

\[
NIPAFC_{\text{Nom}}^{\text{Lab}} = \sum_l \sum_i W_{i,l} L_{i,l}^d
\]  
(I-14)

\[
NIPAFC_{\text{Nom}}^{\text{Cap}} = \sum_i \left( \sum_{kt} R_{i,kt} K_{i,kt}^d + \sum_{lt} P_{i,lt} T_{i,lt}^d + PR_{i} NR_{i}^d \right)
\]  
(I-15)

\[
NIPAFC_{\text{Real}}^{\text{Lab}} = \sum_l \sum_i W_{i,l,0} \lambda_{i,l} L_{i,l}^d
\]  
(I-16)

\[
NIPAFC_{\text{Real}}^{\text{Cap}} = \sum_i \left( \sum_{kt} R_{i,kt,0} \lambda_{i,kt} K_{i,kt}^d + \sum_{lt} P_{i,lt,0} \lambda_{i,lt} T_{i,lt}^d + PR_{i,0} \lambda_{i} NR_{i}^d \right)
\]  
(I-17)

\[
GDPFC = NIPAFC_{\text{Nom}}^{\text{Lab}} + NIPAFC_{\text{Nom}}^{\text{Cap}}
\]  
(I-18)

\[
RGDPFC = NIPAFC_{\text{Real}}^{\text{Lab}} + NIPAFC_{\text{Real}}^{\text{Cap}}
\]  
(I-19)

\[
PGDPFC = GDPGFC / RGDPFC
\]  
(I-20)

The next set of equations define total GDP at factor cost, i.e. these definitions include the indirect taxes such that the nominal definition of GDP at factor cost lines up with nominal GDP at market price. Thus equation (I-21) defines total nominal GDP at factor cost. Equation (I-22) defines total real GDP at factor cost. Given growth dynamics, it will be equal to real GDP at market price, i.e. both are assumed to grow at the same rate. The total GDP at factor cost deflator is defined in equation (I-23) using the standard deflator equation. In essence, the GDP deflator will be defined implicitly by the fact that total real GDP at factor cost is assumed to grow at the same rate as real GDP at market price.

\[
TGDPFC = \sum_{fc} NIPAFC_{fc}^{\text{Nom}}
\]  
(I-21)

\[
TRGDPFC = (1 + g^y)TRGDPFC_{-1}
\]  
(I-20)

\[
TPGDPFC = TGDPGFC / TRGDPFC
\]  
(I-21)

---

38 So is nominal GDP at factor cost, but the efficiency factors cancel out in the equation since the nominal wage is divided by the efficiency factor to derive the efficiency wage.
Growth equations

Model equations

In a simple dynamic framework, equation (G-1) defines the growth rate of GDP at market price. Equation (G-2) determines the growth rate of labor productivity. The growth rate has two components, a uniform factor applied in all sectors to all types of labor, \( \gamma \), and a sector- and skill-specific factor, \( \chi \). In defining a baseline, the growth rate of GDP is exogenous. In this case, equation (G-1) is used to calibrate the \( \gamma \) parameter. In policy simulations, \( \gamma \) is given, and equation (G-1) defines the growth rate of GDP. Other elements of simple dynamics include exogenous growth of labor supply, exogenous growth rates of capital and land productivity (typically 0), and investment driven capital accumulation.\(^{39}\)

\[
\frac{RGDP}{MP} = (1 + g^y) \frac{RGDP_{MP}}{l}
\]  
\( (G-1) \)

\[
\lambda_{ip,l} = (1 + \gamma^l + \chi_{ip,l}^l) \lambda_{ip,l-1}
\]  
\( (G-2) \)

Equations external to the model

The remaining growth equations are external to the model. They involve only exogenous variables which can be determined outside of the model specification. There are four elements driving model dynamics—labor growth, capital accumulation, growth of natural resources, and productivity.

Equation (G-3) determines labor supply growth. It simply applies an exogenous assumption about the growth of labor supply, \( g^l \), to the labor supply shift parameter. If the supply curve is vertical, it will simply move the vertical supply curve by the growth rate. In the absence of independent growth rates for labor, the growth rate of the population tranche of persons aged between 15 and 65 is sometimes used as an approximate growth rate for labor supply. Equation (G-4) updates population (by household). Equations (G-5) and (G-6) are similar growth equations for land and the sector-specific resource, respectively.\(^{40}\)

\[
\alpha^h = (1 + g^l_h) \alpha^h_{l-1}
\]  
\( (G-3) \)

\[
Pop_h = (1 + g^{pop}_h) Pop_{h,-1}
\]  
\( (G-4) \)

\[
Land = (1 + \gamma^l) Land_{l-1}
\]  
\( (G-5) \)

\[
\gamma^nr = (1 + g^\gamma^nr) \gamma^nr_{l-1}
\]  
\( (G-6) \)

Capital accumulation is based on the level of investment of the previous period less depreciation. Equation (G-7) represents the motion equation for capital growth, where \( \delta \) is the rate of depreciation and \( KAP \) is the capital stock. The variable \( KAP \) differs from the capital stock described in the model, \( K^s \) (see

\(^{39}\) Note that public investment, in this version of the model, has no impact on production technology.

\(^{40}\) If the sector specific resource is a renewable or non-renewable natural resource, the growth equation should normally be replaced by equations determining the underlying supply of the resource. For example, a depletion module could be used for a non-renewable resource such as crude oil.
Equations (F-4) and (F-5)). \( KAP \) represents the true volume of the capital stock, the so-called non-normalized value. The variable \( K' \) is a capital stock index, which may be equal to the true value of the capital stock, but is often set equal to the normalized value of the capital stock. The distinction is important in the accumulation equation but is of no consequence for the model specification, i.e. the normalization of the capital stock value does not affect model results. An example may help clarify the distinction. Start with an economy with a GDP of 100 and a 40 percent capital share, i.e. 40 percent of GDP is composed of profits. The normalized value of the capital stock is 40, i.e. it is the value of the capital stock consistent with a rental rate of capital of 1. Assume the rate of return on capital is 20 percent. Then the non-normalized value of the capital stock is 200, i.e. investors receive a return of 40 because 20 percent of 200 is 40. Next assume investment is 30 percent of GDP in this economy, and the rate of depreciation is 8 percent. The capital stock in the following period is 214 (= 0.92*200+30), i.e. an increase of 7 percent. The investment, 30, must be added to the non-normalized value of the capital stock because the units matter in the capital accumulation function. Equation (G-8) determines the capital stock index which simply assumes that the rate of the capital stock index to the non-normalized capital stock remains constant. In other words, the growth rate of the normalized capital stock is equal to the growth rate of the non-normalized capital stock.

\[
KAP = (1 - \delta)KAP_{-1} + XF_{Zlp,-1} \quad \text{(G-7)}
\]

\[
K' = \left( K'_0 / KAP_0 \right) KAP \quad \text{(G-8)}
\]

Equation (G-2) determines labor productivity growth in a subset of sectors, indexed by \( ip \). In all other sectors, labor productivity growth is exogenous. The complementary subset is indexed by \( np \). Equation (G-9) represents the increase in labor productivity in sectors not subject to the uniform productivity shift factor \( \gamma_l \). Equations (G-10) through (G-12) update productivity of capital, land and the sector specific factor, respectively. The updating of productivity of these factors, unlike labor, is always assumed to be exogenous. One standard assumption is to isolate agricultural sectors from the others, i.e. to make the subset \( ag \) a subset of \( np \). If agricultural productivity is assumed to be uniform across all factors of production, then the same growth parameter will be applied in formulas (G-9) through (G-12) for all sectors indexed by \( ag \). Equation (G-13) determines the change in efficiency in the trade and transport sector. If the parameter \( \gamma_{mg}^{\text{eff}} \) is negative, for example -1 percent, then efficiency is improving.

\[
\lambda'_{np, l} = (1 + \chi'_{np, l}) \lambda'_{np, l, -1} \quad \text{(G-9)}
\]

\[
\lambda'_{i, k} = (1 + \chi'_{i, k}) \lambda'_{i, k, -1} \quad \text{(G-10)}
\]

\[
\lambda'_{i, l} = (1 + \chi'_{i, l}) \lambda'_{i, l, -1} \quad \text{(G-11)}
\]

\[
\lambda'_{i} = (1 + \chi'_{i}) \lambda'_{i, -1} \quad \text{(G-12)}
\]

\[
\tau_{k, m}^{\text{mg}} = (1 + \gamma_{k, m}^{\text{mg}}) \tau_{k, m, -1} \quad \text{(G-13)}
\]

The assumption that productivity growth is only labor-augmenting may not be appropriate in all situations. There are two possible alternatives. The first assumes that productivity growth is uniform between capital and labor. In this case equations (G-2) and (G-10) would be replaced with:
\[ \lambda_{ip,l}^l = (1 + \gamma + \lambda_{ip,l}^l) \lambda_{ip,l,-1} \]

\[ \lambda_{ip,kt}^k = (1 + \gamma + \lambda_{ip,kt}^k) \lambda_{ip,kt,-1} \]

(Equation G-10 would still hold for the sectors indexed by \( n_p \).) Thus in the baseline scenario, with GDP growth fixed, a common productivity factor, \( \gamma \), would apply to both labor and capital in sectors indexed by \( ip \). A third alternative is to introduce an additional target to determine a capital-specific productivity factor. In some applications, the additional target is some formula which expresses so-called balanced growth. One version of balanced growth is that the capital per worker, in efficiency units, remains constant over time. In this alternative, equation (G-2) is maintained, with the uniform factor, \( \gamma \), still determined by the GDP growth rate. The additional equation (target) is the balanced growth expression given by:

\[
\frac{\sum \sum \lambda_{i,kt}^k K_{i,kt}^d}{\sum \sum \lambda_{i,lt}^l L_{i,lt}^d} = \frac{\sum \sum \lambda_{i,lt,0}^l K_{i,lt,0}^d}{\sum \sum \lambda_{i,lt,0}^l L_{i,lt,0}^d} = \lambda_0^k
\]

This expression represents the ratio of capital to labor in efficiency units. The capital productivity equation is replaced by:

\[ \lambda_{ip,kt}^k = (1 + \gamma^k + \lambda_{ip,kt}^k) \lambda_{ip,kt,-1} \]

The expression holds only over sectors indexed by \( ip \) and includes a productivity factor, \( \gamma^k \), uniform over all \( ip \) sectors, but different from \( \gamma \).

Other exogenous variables may require updating for the baseline. One obvious one is government expenditure. This is typically assumed to grow at the same rate as GDP:

\[ XFGov = (1 + g) XFGov,-1 \]

Other variables that may need updating include the various transfer variables, foreign savings, exogenous world prices (i.e. the terms of trade), and fiscal policies.
References

On social accounting matrices and data:


On general equilibrium models:


On production:


On demand:


On trade theory:


*On labor markets and migration:*


Annex A: Model variables and parameters

Tables 2-5 provide a complete list of model variables and parameters. Tables 2 and 3 list respectively endogenous and exogenous model variables. Table 4 shows the standard macro closure rules and their alternatives. Table 5 provides a list of the key model parameters, mostly substitution, demand and supply elasticities. Table 6 provides a list of the model’s calibrated parameters. Each table has three columns. The first column represents the symbol of the respective variable or parameter as it is used in this document. The second column shows the equivalent GAMS name with the appropriate indices. The third column provides a brief description.

### Table 2: Endogenous variables

#### Production

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( ND_i )</td>
<td>Demand for aggregate intermediate demand bundle</td>
</tr>
<tr>
<td>( VA_i )</td>
<td>Demand for value added bundle</td>
</tr>
<tr>
<td>( VC_i )</td>
<td>Unit variable cost of production</td>
</tr>
<tr>
<td>( FC_i )</td>
<td>Unit fixed cost of production</td>
</tr>
<tr>
<td>( TC_i )</td>
<td>Unit total cost of production</td>
</tr>
<tr>
<td>( CDR_i )</td>
<td>Cost disadvantage ratio</td>
</tr>
<tr>
<td>( PX_i )</td>
<td>Producer price net of production tax</td>
</tr>
<tr>
<td>( PP_i )</td>
<td>Producer price</td>
</tr>
<tr>
<td>( XA_{k,j} )</td>
<td>Intermediate demand for goods and services</td>
</tr>
<tr>
<td>( PND_i )</td>
<td>Price of aggregate intermediate demand bundle</td>
</tr>
<tr>
<td>( KL_i )</td>
<td>Demand for capital-labor bundle</td>
</tr>
<tr>
<td>( TT_i )</td>
<td>Demand for aggregate land bundle</td>
</tr>
<tr>
<td>( NR_i )</td>
<td>Demand for sector-specific resource</td>
</tr>
<tr>
<td>( PVA_i )</td>
<td>Price of value added bundle</td>
</tr>
<tr>
<td>( UL_i )</td>
<td>Demand for aggregate unskilled labor bundle</td>
</tr>
<tr>
<td>( KSK_i )</td>
<td>Demand for capital/skilled labor bundle</td>
</tr>
<tr>
<td>( PKL_i )</td>
<td>Price of capital-labor bundle</td>
</tr>
<tr>
<td>( SKL_i )</td>
<td>Demand for aggregate unskilled labor bundle</td>
</tr>
<tr>
<td>( KT_i )</td>
<td>Demand for aggregate capital bundle</td>
</tr>
<tr>
<td>( PKSK_i )</td>
<td>Price of capital/skilled labor bundle</td>
</tr>
<tr>
<td>( LV_{i,l} )</td>
<td>Sectoral variable demand for labor by labor type</td>
</tr>
<tr>
<td>( PUL_i )</td>
<td>Sectoral demand for unskilled labor bundle</td>
</tr>
<tr>
<td>( PSKL_i )</td>
<td>Price of aggregate skilled labor bundle</td>
</tr>
<tr>
<td>( L_{i,l} )</td>
<td>Sectoral total demand for labor by labor type</td>
</tr>
<tr>
<td>( KV_{i,kt} )</td>
<td>Sectoral variable demand for capital by capital type</td>
</tr>
<tr>
<td>( PKT_i )</td>
<td>Price of aggregate capital demand bundle</td>
</tr>
<tr>
<td>( K'_{i,kt} )</td>
<td>Sectoral total demand for capital by capital type</td>
</tr>
<tr>
<td>( T_{l,lt} )</td>
<td>Sectoral demand for land by land type</td>
</tr>
</tbody>
</table>
\( P_{IT_i} \) \( \text{pttd}(i) \) Price of aggregate land demand bundle
\( XS_{i,k} \) \( \text{xs}(i,k) \) Supply of good \( k \) produced by activity \( i \)
\( XP_i \) \( \text{xp}(i) \) Aggregate output from activity \( i \).
\( PS_{i,k} \) \( \text{ps}(i,k) \) Price of commodity \( k \) produced by activity \( i \).
\( P_k \) \( \text{p}(k) \) Aggregate producer price of commodity \( k \)

**Income distribution**

\( \text{Vatdy}_i \) \( \text{vatdy}(i) \) Revenues generated by domestic value added tax
\( \Pi_i \) \( \text{profit}(i) \) Pure profit
\( LY_i \) \( \text{ly}(i) \) Aggregate net labor remuneration
\( KY_{kt} \) \( \text{ky}(kt) \) Aggregate after-tax capital income
\( TY_{lt} \) \( \text{ty}(lt) \) Aggregate after-tax land income
\( RY \) \( \text{ry} \) Aggregate after-tax income from sector-specific resource
\( TR^E_{kt} \) \( \text{ktre}(kt) \) Capital income transferred to enterprises
\( TR^H_{kt} \) \( \text{ktrh}(kt) \) Capital income transferred to households
\( TR^W_{kt} \) \( \text{ktrw}(kt) \) Capital income transferred abroad
\( CY_e \) \( \text{cy}(e) \) Corporate income
\( S^c_e \) \( \text{savc}(e) \) Corporate retained earnings
\( TR^H_{c,e} \) \( \text{ctrh}(e) \) Corporate earnings transferred to households
\( TR^G_{c,e} \) \( \text{ctrg}(e) \) Government share of corporate earnings
\( TR^W_{c,e} \) \( \text{ctrw}(e) \) Corporate earnings transferred abroad
\( YH_h \) \( \text{yh}(h) \) Aggregate household income
\( YD_h \) \( \text{yd}(h) \) Disposable income net of taxes and transfers
\( TR^H_h \) \( \text{htr}(h) \) Aggregate transfers by households
\( TR^W_{h,h',h'} \) \( \text{htrw}(h,h,h) \) Intra-household transfers
\( TR^W_h \) \( \text{htrw}(h) \) Household transfers abroad

**Domestic demand variables**

\( XA_{k,h} \) \( \text{xa}(k,h) \) Household demand for goods and services
\( S^h_h \) \( \text{savh}(h) \) Household savings
\( CPI_h \) \( \text{cpi}(h) \) Household-specific consumer price index
\( PAC_{k,h} \) \( \text{pac}(k,h) \) Consumer prices
\( XAF_{k,f} \) \( \text{xaf}(k,f) \) Other domestic final demand for goods and services
\( PF_f \) \( \text{pf}(f) \) Other domestic final demand price deflator
\( YF_f \) \( \text{yf}(f) \) Other domestic final demand aggregate expenditure level

**Trade variables**

\( XD^d_{k,a} \) \( \text{xdd}(k,a) \) Domestic demand for domestic production
\( XMT_{k,a} \) \( \text{xmt}(k,a) \) Domestic demand for aggregate imports
\( PA_{k,a} \) \( \text{pa}(k,a) \) Price of Armington good
\[
PM_{tr,k,r} = pm(tr,k,r) \quad \text{Domestic tariff-inclusive price of imports by region of origin}
\]
\[
XMTR_{tr,k} = xmr(tr,k) \quad \text{Aggregate import demand by tariff regime}
\]
\[
XM_{tr,k,r} = xm(tr,k,r) \quad \text{Import demand by region of origin and tariff regime}
\]
\[
PMTR_{tr,k} = pmt(tr,k) \quad \text{Price of aggregate import bundle by tariff regime}
\]
\[
PMT_{a} = pmt(k,a) \quad \text{Price of imports by Armington agent}
\]
\[
PE_{k,r} = pe(k,r) \quad \text{Producer price of exports by region of destination}
\]
\[
XD_{k} = xds(k) \quad \text{Domestic output sold domestically}
\]
\[
XET = xet(k) \quad \text{Aggregate export supply}
\]
\[
X_{k} = x(k) \quad \text{Aggregate output}
\]
\[
XE_{k,r} = xe(k,r) \quad \text{Export supply by region of destination}
\]
\[
PET = pet(k) \quad \text{Price of aggregate exports}
\]
\[
ED_{k,r} = ed(k,r) \quad \text{Demand for exports by region of destination}
\]

**Domestic trade and transportation margins**

\[
XT_{k}^{mg} = xtmg(k) \quad \text{Aggregate trade and transport volumes for commodity } k
\]
\[
XAmg_{k,kk}^{mg} = xamg(k,kk) \quad \text{Demand for good } k \text{ used to transport commodity } kk
\]
\[
PT_{k}^{mg} = ptmg(k,mg) \quad \text{Aggregate trade and transport price to transport commodity } k
\]
\[
XA_{k,\text{Margn}}^{mx} = xa(k, '\text{margn}') \quad \text{Aggregate Armington demand in margins sector}
\]

**Goods price equilibrium**

\[
\pi_{i} = \text{markup}(i) \quad \text{Price markup (exogenous for the moment)}
\]
\[
PD_{k} = pd(k) \quad \text{Price of domestic goods sold domestically}
\]
\[
WPE_{k,r} = wpe(k,r) \quad \text{World price of exports by region of destination}
\]

**Macro variables**

\[
TarY_{md} = Tary(md) \quad \text{Nominal tariff revenues}
\]
\[
RTarY_{md} = Rtary(md) \quad \text{Real tariff revenues}
\]
\[
GY = gy \quad \text{Government revenues}
\]
\[
GEXP = gexp \quad \text{Total government current expenditures}
\]
\[
S^{\delta} = savg \quad \text{Nominal government savings}
\]
\[
\lambda^{\delta} = dirtxadj \quad \text{Household direct tax schedule shifter}
\]
\[
XF_{\text{invst}} = xf("\text{invst}") \quad \text{Volume of private investment}
\]
\[
PLEV = Plev \quad \text{Absorption price deflator}
\]
\[
CPIT = Cpit \quad \text{Aggregate consumer price deflator}
\]

**Factor market variables**

\[
L_{l,gl}^{l} = ls(l,gl) \quad \text{Labor supply}
\]
\[
AWAGE_{l,gl}^{l} = awage(l,gl) \quad \text{Expected average wage rate}
\]
\[
MIGR_{l} = migr(l) \quad \text{Rural to urban migration}
\]
\[
WMIN_{l,gl}^{l} = wmin(l,gl) \quad \text{Minimum wage}
\]
\[
W_{l,gl}^{e} = ewage(l,gl) \quad \text{Equilibrium wage rate}
\]
\[
\begin{align*}
UE_{t,gz} & \quad \text{Unemployment rate} \\
NW_{t,i} & \quad \text{Sector specific wage rate net of wage tax} \\
W_{t,i} & \quad \text{Sector specific wage rate} \\
TK_{i,t} & \quad \text{Aggregate capital supply by type} \\
PK & \quad \text{Economy-wide aggregate rate of return to capital} \\
K_{i,kt} & \quad \text{Sectoral capital supply by type} \\
PTK_{i,kt} & \quad \text{Economy-wide aggregate rate of return to capital by type} \\
NR_{i,kt} & \quad \text{Sectoral rate of return to capital by type net of tax} \\
R_{i,kt} & \quad \text{Sectoral rate of return to capital by type} \\
TT_{i,t} & \quad \text{Aggregate land supply by type} \\
PLAND & \quad \text{Economy-wide aggregate rate of return to land} \\
T_{i,lt} & \quad \text{Sectoral land supply by type} \\
PTT_{i,lt} & \quad \text{Economy-wide aggregate rate of return to land by type} \\
NPT_{i,lt} & \quad \text{Sectoral rate of return to land by type net of tax} \\
PT_{i,lt} & \quad \text{Sectoral rate of return to land by type} \\
NR_{i,lt} & \quad \text{Sectoral rate of return to capital by type} \\
PR_{i} & \quad \text{Price of sector-specific factor} \\
\end{align*}
\]

**Macroeconomic variables**

- \(GDPMP\) \(gdpmp\) \quad \text{Nominal GDP at market price}
- \(RGDPMP\) \(rgdpmp\) \quad \text{Real GDP at market price}
- \(PGDPMP\) \(pgdpmp\) \quad \text{GDP at market price deflator}
- \(GDPFC\) \(gdpfc\) \quad \text{Nominal GDP at factor cost}
- \(RGDPFC\) \(rgdpfc\) \quad \text{Real GDP at factor cost}
- \(PGDPFC\) \(pgdpfc\) \quad \text{GDP at factor cost deflator}

**Growth variables**

- \(g\) \(ggdp\) \quad \text{Growth rate of real GDP}
- \(\lambda_{ip,l}\) \(lambdal(ip,l)\) \quad \text{Sector- and labor-specific growth factor}
Table 3: Exogenous variables

**Growth factors**

- $\gamma_l$: Economy-wide labor productivity growth
- $\lambda_{kt}$: Capital productivity factor
- $\lambda_{lt}$: Land productivity factor
- $\lambda_{i}$: Sector-specific factor productivity
- $K^s$: Aggregate (normalized) capital stock
- $\text{LAND}$: Aggregate land supply

**Trade prices**

- $WPM_{k,r}$: World price of imports (CIF)
- $WPE_{k,r}$: Export price index of competitors
- $\text{ER}$: Exchange rate and model numéraire

**Fiscal variables**

- $RS_g$: Government fiscal target (in real terms)
- $XF_{Govnt}$: Volume of government expenditures on goods and services
- $\tau^p$: Production tax
- $\tau^{cd}$: Sales tax on domestic goods
- $\tau^{cm}$: Sales tax on import goods
- $\tau^{i,D}$: Value added tax on domestic goods
- $\tau^{x,M}$: Value added tax on imported goods
- $\text{Hldts}(k,h)$: Subsidies on household consumption
- $K^h$: Initial marginal direct tax rates
- $TR^H_{k,h}$: Transfers from government to households
- $K^c$: Corporate tax rates
- $\lambda_{md}$: Uniform tariff adjustment factor
- $\tau^{tr,k,r}$: Sectoral tariffs by region of origin and tariff regime
- $\tau^e$: Sectoral export taxes by region of destination
- $\tau^{i,l}$: Wage tax by sector and labor type
- $\tau^{i,kt}$: Capital tax by sector and capital type
- $\tau^{i,lt}$: Land tax by sector and land type
- $\tau^{i}$: Tax on natural resource
- $\tau^{i,l}$: Wage subsidy by sector and labor type
- $\tau^{i,kt}$: Capital subsidy by sector and capital type
- $\tau^{i,lt}$: Land subsidy by sector and land type
- $\tau^{i}$: Subsidy on natural resource
**Table 3, continued: Exogenous variables**

**Increasing returns to scale variables and imperfect competition**

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N_i$</td>
<td>Number of firms</td>
</tr>
<tr>
<td>$LF_{i,l}^d$</td>
<td>Fixed labor demand</td>
</tr>
<tr>
<td>$KF_{i,kt}^d$</td>
<td>Fixed capital demand</td>
</tr>
</tbody>
</table>

**Miscellaneous exogenous variables**

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$TR_{W,h}^r$</td>
<td>Remittances from abroad</td>
</tr>
<tr>
<td>$TR_{G}^r$</td>
<td>Foreign transfers (and grants) to government</td>
</tr>
<tr>
<td>$TR_{W}^r$</td>
<td>Transfers from government to rest of the world</td>
</tr>
<tr>
<td>$XF_{DST}$</td>
<td>Volume of stock building</td>
</tr>
<tr>
<td>$S_f^c$</td>
<td>Net capital flows, i.e. capital account balance</td>
</tr>
<tr>
<td>$r_{k,a}^{mg,D}$</td>
<td>Domestic trade margins on domestic goods</td>
</tr>
<tr>
<td>$r_{tr,k,r}^{mg,M}$</td>
<td>Domestic trade margins on imported goods</td>
</tr>
<tr>
<td>$r_{k,r}^{mg,E}$</td>
<td>Domestic trade margins on exported goods</td>
</tr>
</tbody>
</table>

**Table 4: Closure alternatives**

<table>
<thead>
<tr>
<th></th>
<th>Exogenous</th>
<th>Endogenous</th>
<th>Alternative instruments</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Fiscal</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Default</td>
<td>RSg</td>
<td>dirtxhadj</td>
<td>dirtxadj, vattxadj, vatdtxadj, vatmtxadj</td>
</tr>
<tr>
<td>Alternative</td>
<td>Tax rates</td>
<td>RSG</td>
<td></td>
</tr>
<tr>
<td><strong>Investment</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Default</td>
<td>Savings rates</td>
<td>xf(&quot;invst&quot;)</td>
<td></td>
</tr>
<tr>
<td>Alternative</td>
<td>xf(&quot;invst&quot;)</td>
<td>Savings rates</td>
<td>savadj, savadj</td>
</tr>
<tr>
<td><strong>Balance of payments</strong></td>
<td>savf</td>
<td>Real exchange rate</td>
<td>ER if the numéraire is CPIT</td>
</tr>
<tr>
<td>Alternative</td>
<td>pgdpfc</td>
<td>savf</td>
<td>ER if the numéraire is CPIT</td>
</tr>
</tbody>
</table>
Table 5: Key model elasticities

**Production elasticities**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma^p_i$</td>
<td>Substitution elasticity between ND and VA bundles</td>
</tr>
<tr>
<td>$\sigma^a_i$</td>
<td>Substitution elasticity across intermediate goods</td>
</tr>
<tr>
<td>$\sigma^v_i$</td>
<td>Substitution elasticity between KL, TT and ND bundles</td>
</tr>
<tr>
<td>$\sigma^b_i$</td>
<td>Substitution elasticity between unskilled labor and capital/skilled labor</td>
</tr>
<tr>
<td>$\sigma^s_i$</td>
<td>Substitution elasticity between capital and skilled labor</td>
</tr>
<tr>
<td>$\sigma^u_i$</td>
<td>Substitution across unskilled labor categories</td>
</tr>
<tr>
<td>$\sigma^u_i$</td>
<td>Substitution across skilled labor categories</td>
</tr>
<tr>
<td>$\sigma^k_i$</td>
<td>Substitution across types of capital</td>
</tr>
<tr>
<td>$\omega^e_i$</td>
<td>Product aggregation transformation elasticity for produced goods</td>
</tr>
<tr>
<td>$\omega^e_i$</td>
<td>Product aggregation elasticity (converting produced goods to consumed goods)</td>
</tr>
</tbody>
</table>

**Demand elasticities**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\eta_{k,h}$</td>
<td>Base household income elasticities</td>
</tr>
<tr>
<td>$\sigma^f_i$</td>
<td>Other final demand substitution elasticity</td>
</tr>
</tbody>
</table>

**Trade elasticities**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma^{m,a}_{k,a}$</td>
<td>Armington elasticity between domestic and aggregate import demand</td>
</tr>
<tr>
<td>$\sigma^{m,ab}_{k,a}$</td>
<td>Armington elasticity for import demand across regions</td>
</tr>
<tr>
<td>$\sigma^w_{k,k}$</td>
<td>Top-level transformation elasticity between the domestic market and aggregate exports</td>
</tr>
<tr>
<td>$\sigma^w_{k}$</td>
<td>Transformation elasticity of exports across regions of destination</td>
</tr>
<tr>
<td>$\eta^r_{k,r}$</td>
<td>Transformation elasticity of exports across regions of destination</td>
</tr>
</tbody>
</table>

**Factor market elasticities**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\omega^m_i$</td>
<td>Elasticity of migration with respect to relative wages</td>
</tr>
<tr>
<td>$\omega^{m,pc}_{l,pc}$</td>
<td>Elasticity of minimum wage with respect to price level</td>
</tr>
<tr>
<td>$\omega^{m,uc}_{l,uc}$</td>
<td>Elasticity of minimum wage with respect rate of unemployment</td>
</tr>
<tr>
<td>$\omega^v_i$</td>
<td>Transformation of capital across types</td>
</tr>
<tr>
<td>$\omega^v_i$</td>
<td>Transformation of capital by type across sectors</td>
</tr>
<tr>
<td>$\omega^l_i$</td>
<td>Transformation of land across types</td>
</tr>
<tr>
<td>$\omega^l_i$</td>
<td>Transformation of capital by type across sectors</td>
</tr>
<tr>
<td>$\omega^s_i$</td>
<td>Supply elasticity of sector specific factor</td>
</tr>
</tbody>
</table>
Table 6: Calibrated parameters

**Production**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha_{i,nd}^{a(i)}$</td>
<td>CES share parameter for ND bundle</td>
</tr>
<tr>
<td>$\alpha_{i,va}^{a(i)}$</td>
<td>CES share parameter for VA bundle</td>
</tr>
<tr>
<td>$a_{k,j}$</td>
<td>Leontief coefficients for intermediate demand</td>
</tr>
<tr>
<td>$\alpha_{i,kl}^{akl(i)}$</td>
<td>CES share parameter for KL bundle</td>
</tr>
<tr>
<td>$\alpha_{i,tt}^{att(i)}$</td>
<td>CES share parameter for TT bundle</td>
</tr>
<tr>
<td>$\alpha_{i,ar}^{ar(i)}$</td>
<td>CES share parameter for NR bundle</td>
</tr>
<tr>
<td>$\alpha_{i,au}^{au(i)}$</td>
<td>CES share parameter for UL bundle</td>
</tr>
<tr>
<td>$\alpha_{i,aksk}^{aks(i)}$</td>
<td>CES share parameter for KSK bundle</td>
</tr>
<tr>
<td>$\alpha_{i,as}^{as(i)}$</td>
<td>CES share parameter for SKL bundle</td>
</tr>
<tr>
<td>$\alpha_{i,akt}^{akt(i)}$</td>
<td>CES share parameter for KT bundle</td>
</tr>
<tr>
<td>$\alpha_{i,al}^{al(i,ul)}$</td>
<td>CES share parameter for unskilled labor demand</td>
</tr>
<tr>
<td>$\alpha_{i,sl}^{al(i,sl)}$</td>
<td>CES share parameter for skilled labor demand</td>
</tr>
<tr>
<td>$\alpha_{i,ak}^{ak(i,kt)}$</td>
<td>CES share parameter for capital demand</td>
</tr>
<tr>
<td>$\alpha_{i,at}^{at(i,lt)}$</td>
<td>CES share parameter for land demand</td>
</tr>
<tr>
<td>$\gamma_{i,k}^{gp(i,k)}$</td>
<td>CET share parameter for commodity aggregation</td>
</tr>
<tr>
<td>$\alpha_{i,k}^{ac(i,k)}$</td>
<td>CES share parameter for commodity aggregation</td>
</tr>
</tbody>
</table>
### Income distribution parameters

- \( \phi_{kt}^{E} \)
  - Enterprise share of after-tax capital income
- \( \phi_{kt}^{H} \)
  - Household share of after-tax capital income
- \( \phi_{kt}^{W} \)
  - Rest of the world share of after-tax capital income
- \( \phi_{kt}^{x} \)
  - Distribution of capital income across enterprises
- \( \phi_{kt}^{H} \)
  - Household share of after-tax corporate income
- \( \phi_{kt}^{W} \)
  - Rest of world share of after-tax corporate income
- \( \phi_{ht}^{l} \)
  - Distribution of wage income across households
- \( \phi_{kt}^{b} \)
  - Distribution of capital income across households
- \( \phi_{kt}^{h} \)
  - Distribution of land income across households
- \( \phi_{kt}^{h} \)
  - Distribution of sector-specific income across households
- \( \phi_{ht}^{b} \)
  - Distribution of corporate income across households
- \( \phi_{ht}^{H} \)
  - Transfer share of household after-tax income
- \( \phi_{ht}^{W} \)
  - Rest of world share of household transfers

### Demand parameters

- \( \delta_{h} \)
  - Household savings rate
- \( \theta_{k,h} \)
  - Household consumption floor parameter
- \( \mu_{k,h} \)
  - Household marginal consumption (out of discretionary income) parameter
- \( \alpha_{k,f}^{a} \)
  - Other final demand CES share parameters

### Trade parameters

- \( \alpha_{k}^{a} \)
  - Domestic share parameter in top-level Armington CES
- \( \alpha_{k}^{e} \)
  - Import share parameter in top-level Armington CES
- \( \alpha_{k,r}^{w} \)
  - Regional import share parameter in second-level Armington CES
- \( \gamma_{k}^{d} \)
  - Domestic share parameter in top-level CET
- \( \gamma_{k}^{e} \)
  - Export share parameter in top-level CET
- \( \gamma_{k}^{r} \)
  - Regional export share parameter in second-level CET
- \( \alpha_{k,r}^{e} \)
  - Export demand shift parameter

### Domestic trade and transport parameters

- \( \alpha_{k,k'}^{mg} \)
  - Leontief coefficients for transporting good \( k' \)
Table 6, continued: Calibrated parameters

<table>
<thead>
<tr>
<th>Factor market parameters</th>
<th>( \chi_{\text{migr}} )</th>
<th>chim(l)</th>
<th>Migration function shift parameter</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \chi_{\text{wmin}} )</td>
<td>chiwmin(l)</td>
<td>Minimum wage function shift parameter</td>
<td></td>
</tr>
<tr>
<td>( \phi_{i,l} )</td>
<td>phil(i,l)</td>
<td>Inter-sectoral wage differential parameter</td>
<td></td>
</tr>
<tr>
<td>( \gamma_{ks} )</td>
<td>akst(kt)</td>
<td>Top-level CET capital allocation share parameters</td>
<td></td>
</tr>
<tr>
<td>( \gamma_{i,kt} )</td>
<td>aks(i,kt)</td>
<td>Second-level CET capital allocation share parameters</td>
<td></td>
</tr>
<tr>
<td>( \gamma_{lt} )</td>
<td>atts(lt)</td>
<td>Top-level CET land allocation share parameters</td>
<td></td>
</tr>
<tr>
<td>( \gamma_{i,lt} )</td>
<td>ats(i,lt)</td>
<td>Second-level CET land allocation share parameters</td>
<td></td>
</tr>
<tr>
<td>( \eta_{i} )</td>
<td>ars(i)</td>
<td>Sector-specific factor supply shifter</td>
<td></td>
</tr>
</tbody>
</table>
Annex B: The CES and CET Functions

The CES Function

Because of the frequent use of the constant elasticity of substitution (CES) function, this appendix will develop some of the properties of the CES, including some of its special cases. The CES function can be formulated as a cost minimization problem, subject to a technology constraint:

$$\min \sum_i P_i X_i$$

subject to

$$V = \left[ \sum_i a_i (\lambda_i X_i)^\rho \right]^{1/\rho}$$

where $V$ is the aggregate volume (of production, for example), $X$ are the individual components (“inputs”) of the production function, $P$ are the corresponding prices, and $a$ and $\lambda$ are technological parameters. The $a$ parameters are most often called the share parameters. The $\lambda$ parameters are technology shifters. The parameter $\rho$ is the CES exponent, which is related to the CES elasticity of substitution, which will be defined below.

A bit of algebra produces the following derived demand for the inputs, assuming $V$ and the prices are fixed:

$$X_i = a_i (\lambda_i)^\sigma \left( \frac{P_j}{P_i} \right)^\sigma V$$

where we define the following relationships:

$$\rho = \frac{\sigma - 1}{\sigma} \iff \sigma = \frac{1}{1 - \rho} \quad \text{and} \quad \sigma \geq 0$$

$$a_i = a_i^\sigma \iff a_i = a_i^{1/\sigma}$$

and

$$P = \left[ \sum_j a_j \left( \frac{P_j}{\lambda_j} \right)^{1 - \sigma} \right]^{1/(1 - \sigma)}$$

$P$ is called the CES dual price, it is the aggregate price of the CES components. The parameter $\sigma$, is called the substitution elasticity. This term comes from the following relationship, which is easy to derive from Equation (1):

$$\frac{\partial (X_i / X_j)}{\partial (P_i / P_j)} \left( \frac{P_j}{P_i} \right) \left( X_i / X_j \right) = -\sigma$$

In other words, the elasticity of substitution between two inputs, with respect to their relative prices, is constant. (Note, we are assuming that the substitution elasticity is a positive number). For example, if the price of input $i$ increases by 10 per cent with respect to input $j$, the ratio of input $i$ to input $j$ will decrease by (around) $\sigma$ times 10 per cent.
The Leontief and Cobb-Douglas functions are special cases of the CES function. In the case of the Leontief function, the substitution elasticity is zero, in other words, there is no substitution between inputs, no matter what the input prices are. Equations (1) and (2) become:

\[(1') \quad X_i = \frac{\alpha X}{\lambda_i}\]

\[(2') \quad P = \sum \alpha_i \frac{P_i}{\lambda_i}\]

The aggregate price is the weighted sum of the input prices. The Cobb-Douglas function is for the special case when \(\sigma\) is equal to one. It should be clear from Equation (2) that this case needs special handling. The following equations provide the relevant equations for the Cobb-Douglas:

\[(1'') \quad X_i = \alpha_i \frac{P_i}{P} V\]

\[(2'') \quad P = A^{-\sigma} \prod_i \left( \frac{P_i}{\alpha_i \lambda_i} \right)^{\alpha_i}\]

where the production function is given by:

\[V = A \prod_i (\lambda_i X_i)^{\alpha_i}\]

and

\[\sum \alpha_i = 1\]

Note that in Equation (1'') the value share is constant, and does not depend directly on technology change.

Another extreme case is when the substitution elasticity is infinite. In this case, the law-of-one price must hold, i.e. all components are priced identically, and the aggregate is equated to the sum of the components. Equation (1) is replaced with the following condition:

\[P_i = P\]

And equation (2) with the following:

\[V = \sum X_i\]

**Calibration**

Typically, the base data set along with a given substitution elasticity are used to calibrate the CES share parameters. Equation (1) can be inverted to yield:

\[\alpha_i = \left( \frac{P_i}{P} \right)^{\sigma} \frac{X_i}{V}\]
assuming the technology shifters have unit value in the base year. Moreover, the base year prices are often normalized to 1, simplifying the above expression to a true value share. Let’s take the Armington assumption for example. Assume aggregate imports are 20, domestic demand for domestic production is 80, and prices are normalized to 1. The Armington aggregate volume is 100, and the respective share parameters are 0.2 and 0.8. (Note that the model always uses the share parameters represented by $\alpha$, not the share parameters represented by $a$. This saves on compute time since the $a$ parameters never appear explicitly in any equation, whereas $a$ raised to the power of the substitution elasticity, i.e. $\alpha$, occurs frequently.)

The CET Function

With less detail, the following describes the relevant formulas for the CET function, which is similar to the CES specification.

$$\max \sum_i P_i X_i$$

subject to

$$V = \left[ \sum_i g_i X_i^\nu \right]^{1/\nu}$$

where $V$ is the aggregate volume (e.g. aggregate supply), $X$ are the relevant components (sector-specific supply), $P$ are the corresponding prices, $g$ are the CET (primal) share parameters, and $\nu$ is the CET exponent. The CET exponent is related to the CET transformation elasticity, $\omega$ via the following relation:

$$\nu = \frac{\omega + 1}{\omega} \iff \omega = \frac{1}{\nu - 1}$$

Solution of this maximization problem leads to the following first order conditions:

$$X_i = g_i \left( \frac{P_i}{P} \right)^{\omega} V$$

$$P = \left[ \sum_i g_i P_i^{1+\omega} \right]^{1/(1+\omega)}$$

where the $\gamma$ parameters are related to the primal share parameters, $g$, by the following formula:

$$\gamma_i = g_i^{-\omega} \iff g_i = \left( \frac{1}{\gamma_i} \right)^{1/\omega}$$

The special case of infinite transformation implies the following conditions:

$$P_i = P$$

and

$$V = \sum_i X_i$$
Annex C: Increasing returns to scale and imperfect competition

This annex is divided into two sections. The first describes the introduction of increasing returns to scale and imperfect competition (IRTS) in a simple model. This section is intended to provide a theoretical underpinning for the introduction of imperfect competition in the prototype CGE model without the full complexity of the full model. It also introduces notation which will be used subsequently. The second section describes how to use the implementation of increasing returns to scale.

Scale economies and increasing returns to scale in a simple framework

The purpose of this section is to indicate how to introduce elements of increasing returns to scale and imperfect competition into a simple single-country CGE model. The first part presents a canonical comparative static open-economy model with constant returns to scale, and perfectly competitive markets.

Multi-sector open economy model with constant returns to scale and perfect competition

Table C-1 describes implementation of a simple multi-sector open economy model of a single country. Equations (1)-(3) represent a simple production structure, where production consists of combining labor, $L$, and capital, $K$, to produce good $X$, using a CES specification. Equations (2) and (3) represent the reduced forms for labor and capital demand, respectively, where $\sigma$ is the substitution elasticity between $K$ and $L$, $W$ is the uniform economy-wide wage rate, and $R$ is the sector-specific rental rate. Equation (1) represents the unit variable cost of production, $VC$, and under the assumption of constant returns to scale, $VC$ also represents the total unit cost of production, i.e. fixed costs are null.

Equation (4) determines household income, $Y$. It asserts that all factor income flows to a single representative household. The domestic level of private saving, $S_h$, is a constant share of after tax income (see equation (5)), where $s$ is the propensity to save, and $\tau$ is the household direct tax rate. Household consumption, $C$, is based on a Cobb-Douglas utility function, implying that budget shares out of disposable income are constant, where $P_a$ is the vector of consumer (and Armington) prices, and the budget shares are given by the $\mu$ parameters.

Equations (7) and (8) are two macro accounting identities. The first equates government saving, $S_g$, (possibly negative) to total government revenues less government expenditures. Revenues include direct household taxes and tariff income. The government expenditure function is fixed coefficients, with the coefficients represented by the $\gamma$ parameters. Equation (8) posits that the aggregate value of investment is equal to aggregate saving, i.e. the sum of private saving, $S_h$, government saving, $S_g$, and foreign saving, $S_f$. The last is expressed in foreign currency and therefore is multiplied by the exchange rate. The investment expenditure function is also fixed coefficients with the shares determined by the $\iota$ parameters.

Trade is modeled using the ubiquitous Armington assumption on the demand side, and the constant elasticity of transformation (CET) function on the supply side. Equation (9) determines aggregate Armington demand, $A$, summing across all economic agents. Aggregate Armington demand is subsequently divided into two components, demand for a domestically produced good, $D^d$, and demand for an import good, $M$. Equations (10) and (11) determine this decomposition for respectively $D^d$ and $M$, with the Armington substitution elasticity given by $\sigma_m$. The price of the domestic good is $P^d$, and the price of the import good is $P^m$. The Armington price, $P^\alpha$, is given by the CES dual price function of $P^d$ and $P^m$, see equation (12).

A symmetric assumption is made on the supply side. Producers are assumed to differentiate between domestic and export markets, and will produce for both along a transformation frontier. Assuming the
transformation frontier takes the CET functional form, equations (13) and (14) represent the supply to the domestic, \( D^* \), and foreign markets, \( E^* \), respectively. Equation (15) determines aggregate output, which will be a CET combination of \( D^* \) and \( E^* \). (It can be written in equivalent form by using the CET dual price function.) The transformation elasticity is given by \( \omega \). If the elasticity is infinite, producers perceive no difference in the two markets, than the CET first order conditions are replaced by the law of one price rule.

Equations (16) and (17) determine the domestic price of imports, \( P^m \), and exports, \( P^e \), respectively. Equation (18) represents the balance of payments equation (in foreign currency). Equation (19) represents an export demand curve by the rest of the world. If the demand elasticity, \( \eta \), is infinite, than domestic producers can sell any quantity of their production at the given world price. With a finite elasticity, the export demand is a negative function of the domestic export price (relative to a fixed international price), leads to a reduction in export demand.

Equilibrium is represented by three equations. Equation (20) determines equilibrium on the domestic market for domestic production, i.e. it essentially determines the price \( P^d \). Equation (21) determines equilibrium on the export market. If the export demand curve is flat, this equation will simply equate foreign demand to the domestic supply of exports. If the export demand curve is not flat, it will determine the equilibrium export price, \( P^e \). Equation (22) determines the economy-wide equilibrium exchange rate, \( W \). It is assumed that labor is perfectly mobile across sectors. Capital is assumed to be completely sector specific, hence the rate of return on capital is likewise sector specific. Equation (23) determines the consumer price index.

In a perfectly competitive environment, producers have no market power, hence there is no markup between their average cost and the market price. This is reflected in equation (24), where the unit cost of production, \( VC \), is equated to the market price, \( P \).

Table C-1: A Simple n-Sector Model of an Open Economy

<table>
<thead>
<tr>
<th>Production</th>
</tr>
</thead>
<tbody>
<tr>
<td>( VC_i = \left[ \alpha_{i,j} W^{1-\sigma_i} + \alpha_{i,k} R_i^{1-\sigma_i} \right]^{1/(1-\sigma_i)} )</td>
</tr>
<tr>
<td>( L_i = \alpha_{i,j} \left( \frac{VC_i}{W} \right)^{\sigma_i} X_i )</td>
</tr>
<tr>
<td>( K_i = \alpha_{i,k} \left( \frac{VC_i}{R_i} \right)^{\sigma_i} X_i )</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Income Distribution and Household Consumption</th>
</tr>
</thead>
<tbody>
<tr>
<td>( Y = \sum_{i=1}^{n} [WL_i + R_i K_i] )</td>
</tr>
<tr>
<td>( S_h = sY (1 - \tau) )</td>
</tr>
<tr>
<td>( P_i^a C_i = \mu_i [Y (1 - \tau) - S_h] )</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Closure</th>
</tr>
</thead>
<tbody>
<tr>
<td>( S_g = \tau Y + ER \sum_{i=1}^{n} \tau_{i}^m P_i^m M_i - \sum_{i=1}^{n} \gamma_i P_i^a G )</td>
</tr>
</tbody>
</table>
\[ \sum_{i=1}^{n} I_i P_i^* = S_h + S_g + ER S_f \]

**Trade**

\[ A_i = C_i + \gamma_i G + t_i I \]

\[ D_i^d = \beta_{d,i} \left( \frac{P_i^a}{P_i^d} \right)^{\alpha_i^d} A_i \]

\[ M_i = \beta_{m,i} \left( \frac{P_i^a}{P_i^m} \right)^{\gamma_i^m} A_i \]

\[ P_i^a A_i = P_i^d D_i^d + P_i^m M_i = \left[ \beta_{d,i} \left( \frac{P_i^d}{P_i} \right)^{-\sigma_i^d} + \beta_{m,i} \left( \frac{P_i^m}{P_i} \right)^{-\sigma_i^m} \right]^{\frac{1}{1-\sigma_i}} \]

\[ D_i^s = \delta_{d,i} \left( \frac{P_i^d}{P_i} \right)^{\omega_i} X_i \quad \text{or} \quad P_i^d = P_i \quad \text{if} \quad \omega_i = \infty \]

\[ E_i^s = \delta_{e,i} \left( \frac{P_i^c}{P_i} \right)^{\omega_i} X_i \quad \text{or} \quad P_i^c = P_i \quad \text{if} \quad \omega_i = \infty \]

\[ P_i X_i = P_i^d D_i^s + P_i^c E_i^s \]

\[ P_i^m = \text{ER} P_i^*, \left( 1 + \tau_i^m \right) \]

\[ P_i^c = \text{ER} P_i^*, \text{c} \]

\[ \sum_{i=1}^{n} P_i^s E_i^s + S_f = \sum_{i=1}^{n} P_i^s M_i \]

\[ E_i^d = \chi_i \left( \frac{\text{WPI}}{P_i^c} \right)^{\eta_i} \quad \text{or} \quad P_i^s, c = \overline{P_i^c} \quad \text{if} \quad \eta_i = \infty \]

**Equilibrium**

\[ D_i^s = D_i^d \]

\[ E_i^s = E_i^d \]

\[ L' = \sum_{i=1}^{n} L_i \]

**Price Index**

\[ 
\text{CPI} = \sum_{i=1}^{n} P_i C_{i,0} / \sum_{i=1}^{n} P_{i,0} C_{i,0} 
\]

**Domestic Pricing Rule**

\[ V C_i = P_i \]

The canonical closure rules are to fix government and investment expenditures in real terms, respectively \( G \) and \( I \). The direct household tax rate is endogenous in order to achieve the desired level of investment,
while the government surplus/deficit, $S$, is endogenous. Aggregate labor supply, $L'$, is exogenous, as are the sector specific capital stock variables, $K_i$. The world price of imports, $P_m^w$, are exogenous (i.e. the small country assumption is made on the import side), as are the world price of competitors on the export markets, WPI. Finally, the trade balance is fixed, and set equal to the level of foreign saving, $S_f$.

The number of equations is $17n+7$, and the number of endogenous variables is $17n+6$. One equation can be dropped, for example, equation (18), which can be derived from a combination of others.

**Increasing Returns to Scale and Imperfect Competition**

*Increasing returns to scale*

Typically, increasing returns to scale are introduced by splitting the total cost of production into fixed costs and variable costs:

$$TC = FC + VC$$

The assumption is that firms wishing to produce even one unit need to expend a fixed amount represented by $FC$. Fixed cost could also be interpreted as an entry cost. Average cost is represented by the following equation:

$$AC = \frac{TC}{X} = \frac{FC}{X} + \frac{VC}{X} = \frac{FC}{X} + MC$$

If marginal costs are constant, than average cost is a declining function of output, and at the limit, average cost converges towards marginal cost. This is shown in the figure below.

![Cost Graph](image)

The cost disadvantage ratio, $CDR$, represents the distance between the average cost and the marginal cost, and can be defined by:

$$CDR = \frac{AC - MC}{AC}$$

The following describes the changes needed for the canonical model to incorporate increasing returns to scale. First, it will be assumed that fixed costs represent a fixed combination of labor and capital. Hence, the labor and capital variables are divided into fixed and variable components. Assume that there are $N$ identical firms in each sector, each requiring $LF_i$ units of labor and $KF_i$ units of capital. Aggregate fixed costs will be determined by the following equation:

$$FC_i = N_i \left( W_i LF_i + R_i KF_i \right)$$
Equation (1) does not change, but equations (2) and (3) now concern the demand for variable labor and capital. Hence they are written as:

\[(2') \quad LV_i = \alpha_{i,j} \left( \frac{VC_i}{W} \right)^{\sigma_j} X_i\]

\[(3') \quad KV_i = \alpha_{k,j} \left( \frac{VC_i}{R_i} \right)^{\sigma_j} X_i\]

In these equations, \(LV\) and \(KV\) represent respectively the variable demand for labor and capital. (N.B. The calibration of the various parameters will need to be changed, and this will be described below.) Two new identities are introduced into the model representing the sectoral aggregate demands for labor and capital:

\[(26) \quad L_i = N_i L F_i + LV_i\]

\[(27) \quad K_i = N_i K F_i + KV_i\]

The addition of increasing returns to scale and imperfect competition can lead to a price wedge between the market price, \(P\), and the average cost, i.e. pure profits. Pure profits, \(\Pi\), can be defined by the following equation:

\[(28) \quad \Pi_i = P_i X_i - FC_i - VC_i X_i\]

The income equation needs to be re-defined to include pure profits:

\[(4') \quad Y = \sum_i [WL_i + R_i K_i + \Pi_i]\]

This completes the description of scale economies. The next section describes non-competitive markets.

**Imperfect Competition**

*Contestable Markets*

The closest equivalent to perfectly competitive markets with increasing returns to scale is the assumption of contestable markets. Under this assumption, entry and exit costs are reasonably low so that firms are forced to price at average cost. Under this assumption, equation (24) is replaced by:

\[(24') \quad P_i = \frac{FC_i}{X_i} + VC_i\]

Note that this does not change equation (24) substantially since the variable \(FC\) is simply 0 under constant returns to scale. Also note that pure profits are automatically zero under this assumption.

*Endogenous Markups*

While many markets can be characterized by zero pure profit conditions, at least in the short to medium run, there are sectors which have been identified as having considerable market power, and therefore able to influence the price wedge between marginal costs and the market price. Assume in each sector that there are \(N\) identical firms, each assumed to face a downward sloping demand curve. Each firm maximizes its profits by choosing an optimal output level. Let output by firm \(f\) in sector \(i\) be represented by \(X_{i,f}\). Aggregate output, \(X_i\) is simply \(N_i X_{i,f}\). The market elasticity of demand is given by:
\[ \varepsilon_i = -\frac{dX_i}{dP_i} \frac{P_i}{X_i} \]

(In the rest of this section, we will assume that producers do not differentiate supply between the home and foreign markets, i.e. the law of one price holds.) The conjectural variation represents the change in market output with respect to a change in firm’s output as perceived by firm \( f \):

\[ \Omega_{i,f} = \frac{dX_i}{dX_{i,f}} \]

Since we are assuming all firms are symmetric, we have \( \Omega_{i,f} = \Omega_i \) for all \( f \). Profits are defined by the following equation:

\[ \Pi_{i,f} = P_i(X_i)X_{i,f} - TC_{i,f} \]

The first order condition for profit maximization is:

\[ \frac{d\Pi_{i,f}}{dX_{i,f}} = P_i(X_i) + X_{i,f} \frac{dP_i}{dX_i} \frac{dX_i}{dX_{i,f}} - \frac{dTC_{i,f}}{dX_{i,f}} = 0 \]

Re-arranging and inserting the definitions from above yields:

\[ P_i = VC_i - X_{i,f} \frac{dP_i}{dX_i} \frac{X_i}{P_i} \frac{P_i}{X_i} \Omega_i = VC_i + \frac{X_i}{N_i} \frac{1}{\varepsilon_i} \frac{P_i}{X_i} \Omega_i \]

Finally, the percentage markup over marginal cost pricing is given by:

\[ \frac{P_i - VC_i}{P_i} = \frac{\Omega_i}{N_i \varepsilon_i} \]

There are three special cases to be noted. The first is for \( \Omega_i \) equal to 0. In this case the firm conjectures that an increase in its output will not increase aggregate sectoral output in which case firms undertake marginal cost pricing. The second special case is known as Cournot conjectures, this is for \( \Omega_i \) equal to 1, i.e. an increase in one unit of its output increases sectoral output by one unit. In the case of Cournot conjectures, firms assume that increases in their output will not change the output decisions of other firms in the sector. The final case is for \( \Omega_i \) equal to \( N_i \). In this case, the industry is either identified by a single monopoly, or there is perfect collusion among the firms in the industry. The markup in this case is equal to the inverse of the market demand elasticity.

We now proceed to determine the market demand elasticity. It is defined by:

\[ \varepsilon_i = -\frac{dX_i}{dP_i} \frac{P_i}{X_i} = -\frac{dD_i}{dP_i} \frac{P_i}{D_i} \frac{D_i}{X_i} - \frac{dE_i^d}{dP_i} \frac{P_i}{E_i^d} \frac{E_i^d}{X_i} = \varepsilon_i^d \frac{D_i}{X_i} + \varepsilon_i^e \frac{E_i^d}{X_i} \]

The market demand elasticity is a weighted average of the domestic demand elasticity and the export demand elasticity, using their respective volume shares as weights. Derivation of the domestic demand elasticity is somewhat tedious. Let’s start with the domestic demand function:
\[
\frac{dD_i}{dP_i} = \frac{d}{dP_i} \left[ \beta_{d,j} \left( \frac{P_i^a}{P_i^m} \right)^{\sigma_i^m} A_i \right]
\]
\[
= -\sigma_i^m \frac{D_i}{P_i^m} + \sigma_i^m \frac{D_i}{P_i^m} \frac{dP_i^a}{dP_i^a} - \frac{D_i}{P_i^m} \frac{dA_i}{dP_i^a}
\]
By multiplying by the ratio \(-P/D\) and re-arranging, we have:
\[
\epsilon_i^d = \sigma_i^m - \sigma_i^m \frac{P_i^a}{P_i^m} \frac{dP_i^a}{dP_i^m} - \frac{dA_i}{dP_i^a} \frac{P_i^a}{P_i^m} \frac{dP_i^a}{dP_i^m}
\]
Next, we need to calculate the elasticity of the Armington price with respect to the price of its domestic component:
\[
\frac{dP_i^a}{dP_i} = \frac{d}{dP_i} \left[ \beta_{d,j} \left( P_i^a \right)^{\gamma_i} + \beta_{m,j} \left( P_i^m \right)^{\gamma_i} \right]^{1/(1-\gamma_i)}
\]
\[
= \beta_{d,j} \left( P_i^a \right)^{\gamma_i}
\]
\[
= \frac{D_i}{A_i}
\]
Multiplying by the ratio \(P/P^a\) yields the following:
\[
\frac{dP_i^a}{dP_i} \frac{P_i^m}{P_i^a} = \frac{P_i^m D_i}{P_i^a A_i} = s_i^d
\]
Re-inserting this into the equation defining the domestic market demand elasticity yields:
\[
\epsilon_i^d = \sigma_i^m - \sigma_i^m s_i^d + s_i^d \epsilon_i^a = (1 - s_i^d) \sigma_i^m + s_i^d \epsilon_i^a
\]
where
\[
\epsilon_i^a = -\frac{dA_i}{dP_i} \frac{P_i^a}{A_i}
\]
is the Armington demand elasticity. We see that the domestic market demand elasticity is a weighted average of the Armington substitution elasticity and the Armington demand elasticity where the former is weighted by the import value share, and the latter is weighted by the domestic value share.

The final step is to calculate the Armington price demand elasticity. The Armington price demand elasticity is the weighted average of the separate Armington demands: intermediate, household, and other final demand. In the simple model, intermediate demand is absent, and the other final demand expenditure functions are fixed coefficients. Hence, the price elasticity of the latter is zero. The Armington price demand elasticity is therefore:
\[
\epsilon_i^a = -\frac{dC_i + \gamma_i G + t_i I}{dP_i} \frac{P_i^a}{A_i} = -\frac{dC_i}{dP_i} \frac{P_i^a}{C_i} \frac{C_i}{A_i} = \frac{C_i}{A_i}
\]
since the consumer price demand elasticity is \(-1\). Finally, the domestic market demand elasticity is:

\[ \varepsilon_i^d = (1 - s_i^d) \sigma_i^m + s_i^d \frac{C_i}{A_i} \]

Assuming firms have market power abroad, the export price demand elasticity is \(\eta\). The total market demand elasticity is therefore:

\[
(30) \quad \varepsilon_i = \left( (1 - s_i^d) \sigma_i^m + s_i^d \frac{C_i}{A_i} \right) \frac{D_i}{X_i} + \eta_i \frac{E_i^d}{X_i}
\]

**Endogenous Conjectures**

Conjectural variations can either be held fixed or else be allowed to vary with the number of firms. If the numbers of firms increase, one would anticipate more competitive pricing. This leads to the following equation:

\[
(31) \quad \Omega_i = \frac{\Omega_i^0}{N_i / N_i^0}
\]

**Summary**

The following table summarizes the different possibilities:

<table>
<thead>
<tr>
<th></th>
<th>Perfect competition</th>
<th>Contestable markets</th>
<th>No entry/exit</th>
<th>Entry/exit</th>
<th>Entry/exit with endogenous conjectures</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fixed costs</td>
<td>0</td>
<td>FC</td>
<td>FC</td>
<td>FC</td>
<td>FC</td>
</tr>
<tr>
<td>CDR</td>
<td>0</td>
<td>Endogenous</td>
<td>Endogenous</td>
<td>Endogenous</td>
<td>Endogenous</td>
</tr>
<tr>
<td>Markup</td>
<td>0</td>
<td>Endogenous</td>
<td>Endogenous</td>
<td>Endogenous</td>
<td>Endogenous</td>
</tr>
<tr>
<td>Profits</td>
<td>0</td>
<td>0</td>
<td>Endogenous</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Number of firms</td>
<td>..</td>
<td>..</td>
<td>Exogenous</td>
<td>Endogenous</td>
<td>Endogenous</td>
</tr>
<tr>
<td>Conjectures</td>
<td>..</td>
<td>..</td>
<td>Exogenous</td>
<td>Exogenous</td>
<td>Endogenous</td>
</tr>
<tr>
<td>Equations</td>
<td>24'</td>
<td>24'</td>
<td>29</td>
<td>29</td>
<td>29, 31</td>
</tr>
</tbody>
</table>

In the modeling of IRTS, the following equations are valid for all different market structures:

- Equation (25), the definition of fixed costs. This is simply 0 under perfect competition.
- Equations (26) and (27), equating fixed and variable factor demands to total sectoral factor demand. Again, a harmless assumption under perfect competition.
- Equation (28), which defines sectoral profits.
- Equation (30), which defines the market demand elasticity.

**Calibration**

Calibration typically requires knowledge of the CDR, the price markup, profits, conjectural variations and/or number of firms. Let’s start with the assumption that we know the CDR and the price markup, respectively CDR and \(\pi\):
The CDR represents the gap between average cost and marginal cost, whereas the price markup represents the gap between the market price and the marginal cost. Profits are defined by the following relation:

\[
\Pi_j = P_j X_j - TC_j = P_j X_j - AC_j, X_j = P_j X_j - X_j \frac{MC_j}{1 - CDR_j} = P_j X_j - X_j \frac{P_i}{(1 + \pi_j)(1 - CDR_j)}
\]

So that we have:

\[
\Pi_j = P_j X_j \left[1 - \frac{1}{(1 + \pi_j)(1 - CDR_j)} \right]
\]

Given initial values for \( \pi \) and CDR, it is possible to determine aggregate sectoral profits. Note that the zero initial profit condition is:

\[
\Pi = \frac{CDR}{1 - CDR} = \frac{AC - MC}{MC} \Leftrightarrow P = AC
\]

Once we have determined profits, they need to be extracted from the gross operating surplus of the base SAM. Assuming gross operating surplus has been initialized in the aggregate capital stock variables, this entails the following equation:

\[
K_i^0 = \left( R_i^0 K_i^0 - \Pi_j \right) / R_i^0
\]

The next step is to determine which proportion of total sectoral labor and capital volumes are allocated to the fixed proportion. This is based on the CDR estimate and leads to the following equations:

\[
KF_i^0 = CDR_i K_i^0 / N_i^0 \quad \text{and} \quad LF_i^0 = CDR_i L_i^0 / N_i^0
\]

The variable components are calculated by residual:

\[
KV_i^0 = K_i^0 - N_i^0 KF_i^0 \quad \text{and} \quad LV_i^0 = L_i^0 - N_i^0 LF_i^0
\]

The next step is to calibrate the conjectural variation, assuming we know the number of firms. We need estimates of the market demand elasticity. The following three equations show the sequential procedure:

\[
s_i^d = \frac{P_i^d,0 D_i^0}{P_i^{n,0} A_i^0}
\]

\[
\varepsilon_i = \left(1 - s_i^d\right) \sigma_i^m + s_i^d \frac{C_i}{A_i}
\]

\[
\Omega_i = \left(\frac{P_i^0 - VC_i^0}{P_i^0}\right) N_i^0 \varepsilon_i
\]

An alternative would be to calibrate the model based on a given value for the conjectural variation, for example the Cournot conjecture (i.e. 1), and use the last equation to solve for the initial number of firms. However, this would have to be done prior to the other adjustments above, since some of the calibration equations rely on the number of firms.
Implementation in the prototype CGE

The specification defined above is largely reflected in the model specification described in the body of the document. The IRTS components are reflected in equations (P-4), (P-5), (P-6), (P-25) and (P-28). The imperfect competition components are reflected in equations (P-7), (Y-2), (Y-4) (and others, to be done, linked to endogenous markups and/or conjectural variations).

Using IRTS and imperfect competition

[to be completed].

The user enters three parameters in the input data set—the initial CDR, profits and the markup. They are not all independent. The user must enter the initial CDR and either the initial profit (typically 0) or the initial markup. If the profit is given, the markup will be calibrated consistent with the model equations, the initial profit level and the base data. If the markup is given, the profit level will be calibrated. In either case, the CDR will be used to extract the fixed costs (from labor and capital), and profits will be taken out of the variable capital costs. Firms are initialized to 1 at the moment.

[In future releases, the markup and conjectures will also feature in the model code. At the current time, the markup is exogenous.]
Annex D: Country-specific features—Tunisia

This annex provides a description of the specific features of the dynamic version of the prototype model and its adaptation to the Tunisian version of the model. The first section describes the standard and alternative version of the dynamic calibration. The second section describes the calibration of the model results during the validation period (2001-2005). The final section describes the long-term assumptions of the dynamic model.

**Dynamic calibration**

There are two options for calibrating the dynamic baseline. The default for the Tunisia model is to use a variant of the so-called ‘balanced’ growth assumption. Under this assumption, the capital/labor ratio—in efficiency units—is exogenous and is given a time trend. In the default baseline, the ratio is assumed to decline by 1 percent annually, thus we have the following identity:

\[
\frac{K_t}{L_t} = (1 + g_t^{bl}) \chi_{t-1}^{bl} = (1 + g_t^{bl})^{t-t_0} \chi_{t_0}^{bl}
\]

Where \( g_t^{bl} \) is equal to -0.01. In effect, this determines capital productivity growth in sectors with endogenous productivity in the baseline, i.e. those indexed by \( ip \). In these sectors, both the labor and capital productivity factors are assumed to differ across sectors using a sector- (and factor-) specific shift parameter. Thus we have the following equations:

\[
\chi_{ip, t}^l = (1 + \gamma^l + \chi_{ip, t}^l) \chi_{ip, t-1}^l
\]

\[
\chi_{ip, k, t}^k = (1 + \gamma^k + \chi_{ip, k}^k) \chi_{ip, k, t-1}^k
\]

The default assumption is that the shift parameter is 2 percentage points higher in the manufacturing sector than in services. Thus if the economy-wide productivity growth is 2% in services, it will be 4% in manufacturing. The economy-wide productivity growth factor, \( g \), will be determined by the exogenous assumption on per capita growth.

Finally, in other sectors, productivity is fixed. For agriculture, it is assumed to be 2 percent and uniform over both labor and capita.

In the alternative scenario, all productivity growth is assumed to be labor-augmenting only (except in agriculture). In this case, the capital productivity factor (and its shifter) are both set to 0. The capital/labor ratio becomes endogenous and the labor productivity parameter is calibrated to the exogenous assumption on per capita growth. The following table summarizes the two approaches:
**Agricultural productivity**

Fixed, and uniform for both labor and capital  
Default assumption—2 percent per year

**Non-agricultural productivity**

**Balanced growth**

Capital/labor ratio and per capita income growth are fixed  
Capital and labor specific productivity factors are calibrated, albeit with differentiation between manufacturing and services. Capital/labor ratio declines by 1 percent per annum, and manufacturing productivity is 2 percentage points higher than in services.

**Labor-augmenting growth**

Per capita income is fixed, as is capital productivity  
The capital/labor ratio is endogenous and labor productivity growth is calibrated, albeit with differentiation between manufacturing and services. Manufacturing productivity is 2 percentage points higher than in services.

**Historical validation**

Observed (or estimated) data between 2001 and 2005 on the key macro aggregates are used to validate the model over the historical period. Hence, GDP growth and its main components are exogenized to line up with the historical data. The following table shows how the GDP components are exogenized:

<table>
<thead>
<tr>
<th>Component</th>
<th>Validation period</th>
<th>Variable</th>
<th>Post-validation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Private consumption</td>
<td>$s, s^h$</td>
<td>savadj</td>
<td>savadj takes its 2005 value</td>
</tr>
<tr>
<td>Government current expenditures</td>
<td>$\alpha_{Govnt}$</td>
<td>agovnt</td>
<td>agovnt takes its 2005 value</td>
</tr>
<tr>
<td>Government investment expenditure</td>
<td>$\alpha_{Ginvst}$</td>
<td>aginvst</td>
<td>aginvst takes its 2005 value</td>
</tr>
<tr>
<td>Private investment expenditure</td>
<td>$s^h$</td>
<td>savhadj</td>
<td>By default, household saving always adjusts to achieve a given aggregate investment to GDP ratio. During the validation period, the investment to GDP ratio is lined up to reflect the aggregate investment growth rate. Following the validation period, the investment to GDP ratio is fixed at its base year level.</td>
</tr>
<tr>
<td>Change in stocks</td>
<td>$XF_{\text{delst}}$</td>
<td>xf(<em>delst</em>)</td>
<td>The change in stocks is lined up with historical values (given as a share of GDP). Afterwards, it is assumed to grow in line with GDP.</td>
</tr>
<tr>
<td>Exports</td>
<td>WPE</td>
<td>expshft</td>
<td>The world competitive export price (in foreign currency terms) is allowed to adjust. It takes its last year value in 2006 and beyond.</td>
</tr>
</tbody>
</table>

Note, no adjustments are needed for the aggregate import component, since by residual, it will line up given that all of the other components are fixed as is overall GDP growth.
Figures

Figure 1: Nested structure of production
Figure 2: Nested structure of consumer demand

Disposable income ($Y_D$)  
Household saving ($S^h$)  
Expenditure on goods and services

LES

(Armington) demand for goods and services ($X_{Ac}$)

Aggregate demand for imports ($X_{MT}$)

Demand for domestic production ($X_D$)

Import demand by region of origin ($X_M$)
Labor supply (in a given time period and region) is fixed at $L^S$. There is a minimum wage given by $W_{Min}$. (The latter may fluctuate based on the level of unemployment.) The actual wage level will depend on demand conditions. At sufficiently high demand, for example curve $D^H$, wages will be set by equilibrium conditions and be given by $W^H$. Should demand drop below $L^S$, for example curve $D^U$, the prevailing wage is the minimum wage, $W_{Min}$, and supply will be rationed. In this case, labor demand is $L^U$ and $(L^S-L^U)$ is the number of unemployed workers. The curve $D$ represents the point of regime shift from full employment to rationed supply. The regime shift will be determined endogenously in the model using mixed complementarity programming (MCP) and the orthogonality conditions described in the model documentation.