MAMS: An Economy-wide Model for Development Strategy Analysis

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DRAFT

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1 Introduction

This paper documents MAMS (Maquette for MDG Simulations), a dynamic Computable General Equilibrium (CGE) model designed to analyze strategies for medium- and long-run growth and poverty reduction in developing countries, including strategies for achieving the Millennium Development Goals (MDGs). The development of MAMS is and has been driven by a strongly felt need for an economy-wide approach to development strategy analysis (including MDG strategy analysis) that (a) considers the different effects of government interventions, not only because of their resource requirement but also through their impact on human development, including the size and educational characteristics of the labor force, and public infrastructure; and (b) is flexible enough to be linked to country-specific databases that may differ considerably in their characteristics (including disaggregation).

The starting point for MAMS is the static, standard CGE model developed at the International Food Policy Research Institute (IFPRI) (Lofgren et al., 2002). The full version of MAMS is significantly extended in two key respects: the inclusion of (recursive) dynamics (that is, a time dimension) and the addition of an MDG module that endogenizes MDG and education outcomes. Other extensions include the endogenization of factor productivity (which depends, in the basic specification, on economic openness and government capital stocks) and the tracking of assets (liabilities) of the different institutions (factor endowments, domestic government debts, and foreign debts).

In addition to the full “MDG” version (which includes all of the extensions listed above), a “core” version of MAMS has been developed. It differs from the MDG version in that it does not cover the MDG and education block, including its links to the labor

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1 The authors would like to thank Martin Cicowiez for direct contributions to the development of MAMS and many other colleagues for valuable inputs into the development of MAMS, including François Bourguignon, Maurizio Busso, Denis Medvedev, Dominique van der Mensbrugghe, and Hans Timmer. This paper has benefited greatly from detailed comments from Rob Vos and Marco V. Sánchez Cantillo on a version prepared for a joint World Bank–UNDP–UNDESA project on MDG strategies in Latin America.

2 A complementary, less comprehensive presentation of MAMS is found in Bourguignon et al. (2008). Lofgren (2008) presents a simplified mathematical statement.

3 The starting point for the MDG module is the MDG model in Bourguignon et al. (2004). All of these models are coded and solved in the General Algebraic Modeling System (GAMS). (See www.gams.com for information on this software.)
market. The advantage of the core version is that its database has a very flexible disaggregation. An aggregate database can be put together quite easily for virtually any country. The disadvantage of this version is that it can be used to analyze a more limited although still quite substantial set of issues (issues more typical of standard dynamic-recursive CGE models).

In this paper, Section 2 describes the structure of a SAM for MAMS with special emphasis on its non-standard features. Section 3, which is the bulk of this paper, presents the mathematical structure of MAMS in detail. Section 4 provides an overview of the data needs and sources. The concluding remarks, Section 5, briefly discuss how MAMS is used in scenario analysis.

2 A SAM for MAMS

The basic accounting structure and much of the underlying data of MAMS, like other CGE models, are derived from a Social Accounting Matrix (SAM). Most features of a SAM for MAMS are familiar from SAMs used for other models. However, a MAMS SAM has some unconventional features related to the explicit treatment of financial flows and, for the MDG version of MAMS, the required detailed specification of how different MDG-related services are produced and delivered. Before describing the behavioral assumptions and mathematical structure of MAMS, we first describe the features of a MAMS SAM and its key accounting identities.

A SAM is a square matrix in which each account is represented by a row and a column. It provides a comprehensive picture of the economic transactions of an economy during a time period, almost invariably one year. Each cell shows the payment from the account of its column to the account of its row. Thus, the incomes of an account appear along its row and its expenditures along its column. For each account in the SAM, total revenue (row total) should be equal to total expenditure (column total). It should be noted that SAMs almost invariably are limited to flows; additional data or assumptions are needed to define stocks. In most CGE models (including MAMS), the SAM is used to define base-year values for the bulk of the parameters in the equations that generate the corresponding payments in the model.
<table>
<thead>
<tr>
<th></th>
<th>act-prv</th>
<th>act-gov</th>
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<th>com-gov</th>
<th>f-lab</th>
<th>f-capprv</th>
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<th>row</th>
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<th>interest</th>
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<th>cap-gov</th>
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<th>inv-gov</th>
<th>dstk</th>
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<td>va</td>
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<td>trnsfr</td>
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<td>taxes interest</td>
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Note: See Table 2 for the explanation of the notation.
Table 1 shows a stylized and aggregated version of a SAM designed for MAMS, while Table 2 shows the notation that is used.\footnote{The SAMs used in applications are invariably more disaggregated in two respects: (a) the interest account is split into two, one for interest on domestic government debt and one for interest on foreign debts; and (b) the tax account is split into separate accounts for direct, import, export, value-added, and other domestic indirect taxes (of course, in some of the applications, some of these tax types may not exist). For the full MDG version, it is also necessary to disaggregate government activities, commodities and investment accounts by government function and disaggregate the labor accounts by level of education.}

### Table 2. Accounts and cell entries in Stylized Macro SAM for MAMS

<table>
<thead>
<tr>
<th>Account</th>
<th>Explanation</th>
<th>Cell entry</th>
<th>Explanation</th>
</tr>
</thead>
<tbody>
<tr>
<td>act-prv</td>
<td>activity - private production</td>
<td>bor</td>
<td>borrowing</td>
</tr>
<tr>
<td>act-gov</td>
<td>activity - government production</td>
<td>cons</td>
<td>consumption</td>
</tr>
<tr>
<td>com-prv</td>
<td>commodity - private production</td>
<td>dstk</td>
<td>stock (inventory) change</td>
</tr>
<tr>
<td>com-gov</td>
<td>commodity - government production</td>
<td>exports</td>
<td>exports</td>
</tr>
<tr>
<td>f-lab</td>
<td>factor - labor</td>
<td>imports</td>
<td>imports</td>
</tr>
<tr>
<td>f-capprv</td>
<td>factor - private capital</td>
<td>intdom</td>
<td>interest on domestic government debt</td>
</tr>
<tr>
<td>hhd</td>
<td>Household</td>
<td>intmed</td>
<td>intermediate inputs</td>
</tr>
<tr>
<td>gov</td>
<td>Government</td>
<td>introw</td>
<td>interest on foreign debt</td>
</tr>
<tr>
<td>row</td>
<td>rest of world</td>
<td>inv</td>
<td>investment (gross fixed capital formation)</td>
</tr>
<tr>
<td>taxes</td>
<td>taxes - domestic and trade interest (on domestic and foreign debt)</td>
<td>output</td>
<td>production</td>
</tr>
<tr>
<td>interest</td>
<td></td>
<td>sav</td>
<td>savings</td>
</tr>
<tr>
<td>cap-hhd</td>
<td>capital account - household</td>
<td>taxes</td>
<td>taxes (direct and indirect)</td>
</tr>
<tr>
<td>cap-gov</td>
<td>capital account - government</td>
<td>transfr</td>
<td>transfers</td>
</tr>
<tr>
<td>cap-row</td>
<td>capital account - rest of world</td>
<td>va</td>
<td>value added</td>
</tr>
<tr>
<td>inv-prv</td>
<td>investment - private capital</td>
<td>yrow</td>
<td>factor income from RoW</td>
</tr>
<tr>
<td>inv-gov</td>
<td>investment - government capital</td>
<td></td>
<td></td>
</tr>
<tr>
<td>dstk</td>
<td>stock (inventory) change</td>
<td></td>
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</tbody>
</table>
commodities. However, MAMS permits any activity to produce multiple commodities (for example, a dairy activity may produce the commodities cheese and milk), while any commodity may be produced by multiple activities (for example, activities for small-scale and large-scale maize production may both produce the same maize commodity).

The row entries of the two factor accounts in the SAM, labor and private capital, indicate that they earn value-added from domestic production activities and, for private capital, income from the rest of the world (this is less common for labor since it only applies to income from abroad for workers resident in the country of the SAM). In the factor columns, value-added is distributed to the owners of the factors. As noted, for the MDG version, labor is typically disaggregated by education into three segments with the following achievements: completed tertiary, completed secondary but not completed tertiary, and less than completed secondary. MAMS is designed to have a single factor (and SAM account) for private capital, which we define here as capital used in activities that are not part of the functions of the general government. MAMS includes one type of government capital per government activity (i.e., the activities that are part of the functions of the general government). However, typically, government capital does not earn value added and, given this, it is not represented in the SAM.

The SAM in Table 1 includes three types of institutions: households, the government, and the rest of world (row). Households may be disaggregated into various types. Each institution has a current account (its name is a shortened version of the name of the institution) and a capital account (the current-account name of the same institution

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5 The SAM (and MAMS) may also include accounts and entries representing home consumption and transactions costs associated with the commodity marketing (of imports from the border to the demander; of exports from the producer to the border; and of domestic output for domestic sales). For a more detailed discussion of the treatment of these aspects of the SAM, see Lofgren et al. (2002, pp. 3-7).

6 In addition to the current entries, it is not uncommon that the government owns part of the capital stock and earns part of operating surplus.

7 Given this assumption, it is not necessary to model how the endowments and investments of different institutions (households, government, and the rest of the world) are allocated across different private capital types (perhaps disaggregated by sector); this is a key advantage given limited knowledge of the mechanisms that determine the evolution of this distribution over time.

8 MAMS does not separate enterprises from other domestic institutions. In the SAM, these would have been linked to factors (enterprises receive factor incomes, reflecting their ownership of non-labor factors), “other” institutions (direct tax payments and transfers reflecting institutional ownership of the enterprise) and enterprise capital accounts (which spend on investments). In the country databases, these “other” institutions (primarily households) directly receive the factor transfers while assuming the savings and direct tax payments that otherwise would have been done by the enterprises. Other SAM payments are not affected.
prefixed by “cap”) linked to investment accounts and the capital accounts of other institutions. This treatment is significantly different from the more common treatment where savings and investments are handled by a unified institutional account.

In the rows of their current accounts, the domestic institutions receive their earned shares of value added, transfers from other institutions, interest income (for households and rest of world), and tax revenues (for the government), while the rest of the world receives payments for the value of goods imported by the country as well as a share of value added (profit remittances), net transfers from domestic institutions (which may be negative, for instance, reflecting workers’ remittances received by the country), and interest payments on foreign debt (introw). Along their columns, the outlays of the institutions are allocated to commodity purchases (consumption for the household and the government; and the exports of the SAM country for the rest of the world column), direct taxes (for the household), interest payments (for indebted institutions), and savings. Some MAMS applications have also included an additional institution functioning as an NGO: receiving transfers from other institutions (typically the government and/or the rest of the world) and using these resources to purchase services related to health and/or education. The tax account (which in MAMS applications is disaggregated according to type of taxation) passes on its receipts from activities, commodities, and households (along the row) to the government (along the column).9

The account for interest payments (in applications disaggregated into accounts for domestic and foreign interest) passes on payment from the (net) borrowers to the (net) lenders. Note that the SAM (and MAMS) only captures interest payments (and related debts) of domestic institutions to the rest of the world and of the government to households. It does not capture interest payments and debts linking domestic non-government institutions. In their rows, the capital accounts of the institutions record their financing sources, consisting of own savings and net borrowing from selected other institutions (for the government from the rest of the world and the household; for the household, from the rest of the world). The outlays of the institutional capital accounts include payments for fixed investments (inv) and changes in inventories (dstk) and net

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9 The SAM and MAMS may also include direct taxes levied on factor incomes (represented by payments from factor accounts to the tax account).
lending to other institutions (the counterpart of net borrowing). The payments from the capital account of the rest of the world to the private investment account refer to foreign direct investment (FDI). This structure makes it possible for MAMS to capture, in a simple way, the structure of institutional assets (different types of capital and financial claims) and liabilities (financial debt) and how the evolution of this structure differs under alternative scenarios. Other things being equal, if the database has multiple households, those with more rapid income growth will likely also have more rapid savings growth, acquiring increasing shares of private capital and government debt.

Like most other CGE models, MAMS is a “real” model in which inflation does not matter (only relative prices matter). Implicitly, in the SAM, the current account of the monetary sector is merged with service activities and commodities (typically part of some broader service activity and commodity). Its capital account is merged with the government or the non-government capital accounts. In the former case, the cells for net borrowing of the government(-monetary sector) capital account from other institutions are made up of multiple items. The cell $sam(gov,hhd)$, showing net borrowing by the government from the household is the sum of (a) net direct borrowing by government from household (net sales of government bonds on which the government pays interest); and (b) net increases in the claims of the household sector on the monetary sector (the differences between changes in broad money holdings and monetary sector credit to the household). In MAMS (but not in the SAM), the two items in this cell are treated separately, making it possible to consider the fact (a) gives rise to interest payments and a debt whereas (b) is a grant to the government, providing it with “seignorage” (as the one who spends this new money first). The second cell, $sam(gov,row)$, which shows net borrowing by government from the rest of the world, is the difference between (a) net direct borrowing by the government from the rest of the world; and (b) the increase in foreign exchange reserves.  

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10 To verify the statements in this paragraph with respect to the case of merged government and monetary sector accounts, note that the balances for government and monetary sector capital accounts can be written as $I_g = S_g + B_{gm} + B_{gh} + B_{gr}$ and $B_{gm} + B_{hm} + \Delta R = \Delta M$, respectively, where $g =$ government, $m =$ monetary sector, and $h =$ household, $I =$ investment, $B_{ij} =$ borrowing by institution $i$ from institution $j$; $R =$ foreign reserves; and $M =$ broad money. By solving the second identity for $B_{gm}$, substituting the resulting expression for $B_{gm}$ into the first identity, and rearranging, we get the merged government-monetary sector capital account: $I_g = S_g + (\Delta M - B_{pm}) + B_{gh} + (B_{gr} - \Delta R)$, where the capital account payments to the merged government-monetary sector from the household and the rest of the world are $(\Delta M - B_{pm}) + B_{gh}$, and $(B_{gr} - \Delta R)$. 

If, alternatively, the capital account of the monetary sector is part of the domestic non-government (household) capital accounts, then the cell $sam(gov,hhd)$ is the sum of (a) net direct borrowing by government from household; and (b) net government borrowing from the monetary system. In this case, the second cell, $sam(gov,row)$ simply shows net borrowing by government from the rest of the world (not considering changes in foreign exchange reserves).

In either case, while these treatments remain simple, they capture the important fact that the government, via the monetary sector, appropriates part of private savings.

Given the fact that the model does not consider effects of and private sector responses to high general inflation, MAMS should not be used for scenarios under which the resources obtained via the monetary sector are so large that inflation would accelerate. The assessment of what is a prudent upper limit for this type of borrowing should draw on expertise on the macroeconomics of each country; a few percent of GDP is often a reasonable figure.

3. The Mathematical Structure of MAMS

3.1 Introduction

Mathematically, MAMS is divided into two modules – a core CGE module and an MDG module – both of which are integrated in a simultaneous system of linear and non-linear equations. For each time period, the core CGE module gives a comprehensive and consistent account of decisions and related payments involving production (activities producing outputs using factors and intermediate inputs), consumption (by households and the government), investment (private and government), trade (both domestic and foreign), taxation, transfers between institutions (households, government, and the rest of the world), and the distribution of factor incomes to institutions (reflecting endowments). This module also considers the constraints under which the economy operates (the budget constraints of institutions and producers; macro balances; and market constraints for factors and commodities). Lastly, in addition to these standard features of a static CGE model, the core CGE module in MAMS also updates selected parameters (including $\Delta R_i$, respectively. For more details, see Agénor (2004, pp. 11-22), Rao and Nallari (2001, pp. 25-32, 176 and 168), and Barth and Hemphill (2000, pp. 71-74 and 101-106).
factor supplies, population, and factor productivity) on the basis of exogenous trends and past endogenous variables.

The MDG module captures the processes that determine MDG achievement in the human development area, most importantly the provision of services in the areas of education, health, and water and sanitation. The size and skill composition of the labor force is endogenized, in large measure depending on the evolution of education. The MDG module has feedback effects into the rest of the economy, primarily via the labor market.

In the model, growth depends on the accumulation of production factors (labor at different educational levels, private capital, and other factors such as land, if present) and changes in factor productivity, which is influenced by the accumulation of government capital stocks and openness to foreign trade. The structure is recursive: the decisions of economic agents depend on the past and the present, not the future; in other words, the model does not consider forward-looking behavior.

Poverty and inequality analysis, as in other CGE models, can be performed in several ways. The simplest but least desirable method uses an elasticity calculation for poverty given changes in per-capita household consumption. Representative-household or survey-based microsimulation approaches are preferable. The former assume fixed distributions of income or consumption within each household group, providing welfare estimations directly from the CGE model results. The latter type of approach can be either top-down, feeding CGE simulation results to a household model, or integrated, with the household model built directly into MAMS.

The disaggregation of MAMS is data-driven and flexible in most areas: subject to computer memory constraints, there is no upper limit on the number of primary factors, households, production activities, and commodities. The government is disaggregated by function to include sectors for education (by cycle or level), health (in some applications further disaggregated by type of service or technology), water and sanitation, and other public infrastructure. To ensure that MDG achievement in education has explicit dynamic feedback effects on labor supply, the labor force is disaggregated by educational achievement, typically into three types: those who have completed tertiary, completed
secondary, or less than completed secondary. Further disaggregation of labor categories is possible in MAMS.

The applicability of the model to specific policy issues depends in large part on the degree of disaggregation. For example, the analysis of issues related to poverty requires a relatively detailed breakdown of household income sources (from factor endowments and the production activities in which they are employed). Similarly, it is likely preferable to disaggregate non-government production into multiple sectors and commodities (that is, goods and services), as it will provide more specific results of the sectoral employment and income effects of an MDG strategy, pursued on its own or in conjunction with other policies, such as trade reform.

In our discussion of the mathematical statement of MAMS, we start with the core CGE module, followed by the MDG module. Frequently, we will refer to Tables 3.1 and 3.2 (notation and equations for the core CGE module) and Tables 3.3 and 3.4 (notation and equations for the MDG Module). For ease of exposition and use, these four tables are found at the end of the chapter. The following notational conventions apply in Tables 3.1-3.4 and various parts of the main text: upper case Latin letters are used for variables; exogenous variables have a bar on top, endogenous variables do not. Parameters have Greek or lower-case Latin letters. Subscripts refer to set indices. A “0” superscript is used to refer to base-year variable values. Other superscripts are either exponents (if they correspond to a declared model parameter or variable) or part of the name of the variable or parameter; this is clear from the context. In the presence of the “0” superscript, the time subscript (t) has been suppressed. The fact that an item is a variable and not a parameter indicates that, at least under certain model assumptions, its value is endogenous. In Tables 3.2 and 3.4, the domain column, which follows the column with the equations, is an important part of the mathematical statement – it indicates the set elements to which each equation applies.

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11 For example, $\rho^{\text{v}}$ is a parameter; when it appears as a superscript, it is an exponent. In the variable $\text{SHR}^{\text{edu}}$, edu is part of the name; no separate parameter or variable is called edu.

12 For example, in Table 3.2, the domain column of equation 1 shows that this equation does not apply to all commodities; it is limited to commodities with imports.
3.2 The core CGE module

As shown in Table 3.2, the core CGE module is divided into blocks covering prices; production and trade; domestic institutions; investments; system constraints and macro variables; and stock updating and productivity. This section will describe in more technical terms the equations in each of these blocks.

Price block

The price block (equations 1-11) defines prices that can be expressed as functions of other endogenous variables (as opposed to being free variables that perform market-clearing functions). Among these prices, it is worth noting that transactions costs (the cost of moving the commodity between the border and the demanders or suppliers, or between domestic demanders and suppliers) are accounted for in the definitions of demander (domestic-currency) import prices, supplier (domestic-currency) export prices, and demander prices for domestic output sold domestically (equations 1, 2, and 4).

Whereas the transformation of output between exports and domestic sales typically is imperfect, the model also allows for the special cases of outputs exclusively produced for foreign markets (no domestic sales; see below discussion of equation 22) and of perfect transformability with zero exports as one possible outcome. Perfect transformability is useful for commodities that are relatively homogeneous; with only small differences depending on whether the demander is domestic or foreign (like grains). This case is covered by equation 3, which has three components: (a) the constraint that domestic supplier prices are larger than or equal to export prices in local currency units (LCU); (b) the constraint that exports are larger than or equal to zero (i.e., zero is a possible outcome); and (c) a complementary-slackness relationship according to which at least one of (a) and (b) has to hold as a strict equality – domestic supplier prices only exceed export prices if exports are zero or, if exports are above zero, then the two prices are equal. In terms of economics, this means that the export price is a floor price and that producers prefer to sell at the highest price that is offered. If the domestic price is above the export price, then nothing is exported. If, in the absence of exports, the price

13 Apart from the fact that variables are time indexed, most of the core CGE module is similar to the IFPRI standard, static CGE model described in Lofgren et al. (2002). That document provides more detail and references to the CGE modeling literature.
would have fallen below the export price, then exports will be positive, preventing a
decline below the export price.\textsuperscript{14}

Various aggregative prices – for composite supplies, for produced commodities,
and value-added – are derived from relationships that define total revenue or costs as the
sum of disaggregated receipts or payments (equations 5-7 and 9). The price of the
aggregative intermediate commodity for any activity depends on its commodity
composition and the prices of the commodities involved (equation 8). The model is
homogeneous of degree zero in prices, with the CPI serving as the model numéraire
(equation 10). Alternatively, the price index for non-tradables may serve as numéraire
(equation 11).\textsuperscript{15}

\textit{Production and trade block}

This block (equations 12-27) includes the first-order conditions for profit-maximizing
production and transformation decisions as well as cost-minimizing domestic demand
decisions. Given available technology and market prices (taken as given in a perfectly
competitive setting), producers maximize profits.\textsuperscript{16} The technology is defined by a
nested, two-level structure. At the top, output is a Leontief aggregation of real value-
added and a real aggregate intermediate (equations 12-13).\textsuperscript{17} At the bottom, these are
linked to a Constant Elasticity of Substitution (CES) aggregation of primary factors (a
value-added function) and a Leontief aggregation of intermediate inputs (equations 14-
16). Given that the national accounts rarely attribute value-added to government capital,

\textsuperscript{14} In GAMS, a model formulated as an MCP (mixed-complementarity program) can handle a combination
of equations that are (a) strict equalities; and (b) inequalities linked to variables with lower limits in a
mixed-complementarity relationship.

\textsuperscript{15} The GAMS code permits the user to choose either the CPI or the price index for non-tradables as
numéraire. As long as the model is homogeneous of degree zero in prices, this choice has no impact on the
equilibrium values of real variables. This homogeneity condition is not met under macro closures with
fixed savings or domestic borrowing for the government. In these cases, it is implicitly assumed that the
fixed variables are indexed to the numéraire.

\textsuperscript{16} In some applications MAMS has included a private regulated sector (typically a utility) for which
behavior deviates from the assumption of profit-maximizing output and input demand (including capital
use) given market prices and rents. Each regulated activity has its own capital stock. (Otherwise, there is
only one private capital stock, which is mobile across private activities.) For regulated activities, output
prices, investment and capital use are exogenous; production is demand-driven at fixed output prices. Their
capital stocks earn an endogenous, residual share of value-added which most likely deviates from the
market rent; other factors earn market wages.

\textsuperscript{17} MAMS also permits the alternative of a CES aggregation of the real aggregates of value-added and
intermediates. The choice does not tend to have a major impact on results. Most applications have used the
Leontief alternative.
the CES value-added functions for government production do not include capital factors. Typically, government value-added is limited to labor.\textsuperscript{18}

Each activity produces one or more outputs with fixed yield coefficients (equation 17). Any commodity may be produced and marketed by more than one activity. A CES approach, assuming profit-maximizing producer behavior, is used to aggregate market sales of any commodity from different activities (equations 18-19). Production is transformed into exports and domestic sales on the basis of a CET (Constant Elasticity of Transformation) function. The profit-maximizing, optimal ratio between the quantities of exports and domestic sales is positively related to the ratio between the corresponding supply prices (equations 20-21). A less complex relationship applies to production without exports or without domestic sales (equation 22). Government and private social services are typically non-traded, i.e. they have no exports and all of the supply is from domestic producers. For any exported commodity, two alternatives are possible for export demand: (a) exogenous prices in foreign currency units (FCU) combined with an infinitely elastic demand; or (b) price-sensitive export demands defined by constant-elasticity (CE) functions) with the FCU prices determined by domestic conditions and the exchange rate (equation 23 applies to the CE case.) Given that, in the equations, \textit{PWE} (the export world price) has a bar on top, we assume that (b) does not apply in this specific case (and that the set \textit{CED} is empty).

Domestic demanders are assumed to minimize the cost of imperfectly substitutable imports and commodities from domestic production according to an Armington (CES aggregation) function (equations 24-25). For commodities with only one supply source, the supply from this source equals the composite supply (equation 26). The transactions (trade and transport) demand for any service commodity is the sum of demands arising from domestic sales, exports, and imports, each of which is the product of the quantity traded and a fixed input coefficient (showing the quantity of the service commodity per unit of trade; equation 27).

\textsuperscript{18} Nevertheless, the model accounts for the fact that government capital stocks indeed are needed in government activities by imposing investments derived from a Leontief-relationship between government activity levels and related capital stocks, with the stocks being defined on the basis of initial stocks, investment and depreciation (see equation 45). In the exceptional cases when the SAM indicates that government capital earns value-added, this value-added is a fixed share of the total value-added of the activity (in effect equivalent to a tax on value-added), not related to any market rent.
Government service sectors typically produce single outputs and have fixed coefficients for intermediate inputs and capital (typically without any value-added payment; also see discussion in the section “Investment block”). Given this, “profit-maximization” merely involves some flexibility in terms of the composition of their labor employment as they supply the quantities that are demanded.

*Domestic institution block*

This block (equations 28-44) accounts for the receipts and expenditures of all domestic institutions, both government and non-government (households) as well as current, non-trade payment flows to and from the rest of the world; i.e., factor incomes and transfers. When they represent inflows of foreign currency, these payments tend to be fixed (in FCU). The equations are structured to accommodate databases with any number of households, one government, and one entity representing the rest of the world. The payments in this block are highly interrelated since institutions often are both at the receiving and paying ends. Transfers between any two institutions may flow in both directions; however, if so, the analyst may often find it more convenient to net these in the initial model SAM.

Turning to the equations, factor incomes are defined as a function of domestic wages (which may vary across activities) and employment levels, augmented by factor incomes from the rest of the world (equation 28) and allocated across different institutions (domestic and foreign) in value shares that depend on factor endowment shares (equations 29-30). Domestic non-government institutions: (i) earn net interest incomes, defined as the difference between net interest earnings from loans to the government and net interest payments to the rest of the world on foreign debt (equation 31); (ii) transfer fixed shares of their incomes (net of direct taxes and savings) to other institutions (domestic or foreign) (equation 32); (iii) earn total gross incomes defined as the sum of factor incomes, net interest incomes, and transfers from other institutions, where the treatment of the latter differs depending on the nature of the sending institution (government, the rest of the world, or another domestic non-government institution) and the receiving institution (household or non-household) (equation 33); (iv) pay direct taxes according to rates that are fixed unless adjusted as part of the government closure rule.
(equation 34; note that all right-hand-side terms are exogenous); and (v) save out of incomes net of direct taxes according to marginal (and average) rates that are endogenous, depending on changes in per-capita incomes if the elasticity of savings with respect to per-capita income is different from zero (equations 35-36). Alternatively, for any given institution, the savings and/or direct tax rate may be adjusted as part of the savings-investment and government closure rules. If direct tax rates are adjusted as part of the government closure rule, either they are scaled up/down “efficiently” by a factor (TINSADJ) or uniformly adjusted for selected institutions (through DTINS). As suggested by the absence of a bar above DTINS, this mathematical statement assumes that changes in direct tax payments via adjustments in DTINS clear the government budget. The savings rates can be adjusted through similar alternative mechanisms (through MPSADJ or DMPS) as part of the savings-investment rule.

For households, incomes net of direct taxes, savings, and transfers to other institutions (defined in equation 37) are allocated across different commodities according to demand functions belonging to a Linear Expenditure System (LES), defined in per-capita form with separate equations for demands from the market and from own-production (equations 38-39). If the database explicitly considers transactions costs, then market demands include these whereas demands for own production do not.

For the remaining domestic institution, the government, current incomes come from taxes (which are disaggregated into a wide range of categories), factor endowments (the government may own non-labor factors), and transfers from other domestic institutions and the rest of the world (equation 40); transfers from the rest of the world may be endogenous, clearing the government budget as part of the government closure rule. (If so, direct taxes do not perform this role.). The (re)current expenditures of the government are divided into consumption, transfers to domestic institutions (CPI-indexed) and the rest of the world (fixed in FCU), and interest payments on domestic and foreign debt (equations 41). For each period except the first, real government consumption, disaggregated by commodity (excluding consumption for infrastructure), is defined as the level in the previous year times a growth factor that consists of multiple terms (equation 42). In the mathematical statement, the right-hand side terms are all exogenous or lagged; in simulations with other rules for determining government
consumption (including simulations targeting MDGs), one of the exogenous terms is endogenous.\(^{19}\) Real government consumption of infrastructure services, also for each period except the first, is defined as the quantity of government consumption per unit of the government infrastructure capital stock times the real endowment of that capital stock by the government; i.e., the size of the capital stock determines consumption (which may represent maintenance, administration, etc.) (equation 43). Finally, government savings is simply the difference between current revenues and current expenditures (equation 44).

**Investment block**

This block (equations 45-53) covers the determination of government and private investment (including FDI) and how these are financed.

Government investment demand by capital stock (DKGOV) is defined in equation 45, which consists of three parts.\(^{20}\) Different treatments are applied to service capital (used in the production of government services) and infrastructure capital (which requires government support services) (equation 45a). For service capital, growth in service production is the driving force; investment demand is determined as the difference between (i) the anticipated capital demand next year (assuming that production growth will be the same as last year and using a fixed capital-input coefficient) and (ii) the capital stock that would remain if no investments were made.\(^{21}\) For infrastructure capital, government investment demand is determined as the difference between (i) an exogenous

\(^{19}\) The term \(q_{ggrwbar,c,t}\) defines the exogenous component of real consumption growth; the potentially endogenous part consist of the variable \(QGGRW_{c,t}\) and \(qg01_{c,c',t}\) a 0-1 parameter mapping growth in commodities \(c'\) and \(c\) (with each \(c\) being mapped to mapped to at most one \(c'\) but each \(c'\) potentially being mapped to multiple \(c\)). In the simplest case, \(QGGRW_{c,t}\) is fixed at zero for all \(c'\) and real growth for commodity \(c\) defined by \(q_{ggrwbar,c,t}\). \(QGGRW\) is flexed when consumption of one or commodities is adjusted on some other basis. For example, government consumption of \(c\) (which here may be viewed as government-produced health services) may be (i) a fixed share of GDP; if so, a side equation imposes a fixed GDP share for government consumption of \(c\); \(QGGRW_{c,t}\) is flexed; and \(qg01_{c,c',t} = 1\); or (ii) adjusted to make sure that government demand and resulting production is sufficient to meet a health MDG target; if so, the model variable for the health MDG in question is fixed while \(QGGRW_{c,t}\) is flexed; and \(qg01_{c,c,t} = 1\); or (iii) adjusted together with consumption of other commodities to clear the government budget; if so, \(QGGRW_{c,t}\) is flexed for the health commodity, \(c'\) while \(qg01_{c,c',t} = 1\) for \(c\cdot c'\) combinations when \(c = \text{one of the adjusted commodities (including health)}\) and \(c' = \text{health}\). (In the last case, the role played by health could alternatively have been played by any other commodity singled out for adjustment.)

\(^{20}\) Among these, only part (a) is an explicit equation in the GAMS code. The non-negativity constraint on DKGOV is handled via a lower limit on this variable. The complementary-slackness condition is imposed by associating the first equation (a) to the DKGOV variable in the GAMS model definition.

\(^{21}\) In GAMS, the treatment is more general, giving the user the option to assume that the rate of expected output growth is the same as the rate of simulated output growth during the last 1, 2, or 3 years.
growth term times the infrastructure capital stock in \( t \) (similar to equation 42) and (ii) the capital stock that would remain if no investments were made.\(^{22}\) A non-negativity constraint is also imposed for government investment (equation 45b). A complementary-slackness condition (equation 45c) imposes that (i) if \( DKGOV \) is positive, then equation 45a must hold as an equality; and (ii) if the right-hand side of equation 45a is negative, then \( DKGOV \) will be zero and equation 45a will hold as an inequality. This treatment is used to avoid a negative investment value (\( DKGOV < 0 \)) in the exceptional case of an anticipated production decline that is larger than the depreciation rate. Equation 46 transfers the value of \( DKGOV \) to investment by institution, \( DKINS \) (for the government), a variable that is used elsewhere in the model to represent investment across all capital stocks and institutions.

The prices of new capital stocks (disaggregated by type) depend on their composition and market prices (equation 47). The resulting fixed government investment value (defined on the basis of the price and quantity information generated in the preceding equations) is financed by some combination of government savings (net of spending on stock or inventory changes), sales of government bonds (i.e., new interest-bearing borrowing), borrowing via the monetary sector, foreign borrowing, and foreign capital grants (which is separate from current government transfers from the rest of the world) (equation 48). Returning to the equations, government bond sales and borrowing via the monetary system are allocated across households on the basis of their savings shares (equations 49-50).\(^{23}\)

Equation 48 concludes the series of equations that summarize the government budget (see also equations 40, 41, and 44). The choice of mechanism for clearing the budget (the government closure rule) is often an important part of the simulations. As noted above, changes in the variable \( DTINS \) (see equation 34) adjust direct tax payments

\(^{22}\) For public infrastructure, actual \( QG \) (government service level) is determined by the current capital stock (see equation 43). In equation 45, the exogenous growth variable \( QGRW_{c,t} \) (which is defined over \( c \), where the relevant \( c \) may be public infrastructure services) is mapped to the capital stock \( f \) associated with \( c \) and drives the expansion in the capital stock.

\(^{23}\) The savings shares are adjusted by a distortion term (\( gbdist \)) that reflects deviations between household shares of government borrowing and savings. Implicitly, the burden of monetary system borrowing is felt by other agents since it extracts real purchasing power from them by reducing the value of the old money that they hold. In the absence of an explicit treatment of money in this model, this burden is here allocated across households on the basis of their savings shares.
sufficiently to clear the budget. The other terms in the expressions for government receipts and outlays are exogenous or determined via other mechanisms. Under some other government closure that may be used, direct tax rates and transfers from the rest of the world to the government are exogenous while either government bond sales ($GBORTOT_t$) or government borrowing from the rest of the world ($FBOR_{gov.t}$) is endogenous, clearing the government budget.

Each alternative closure has specific macroeconomic repercussions. Increases in government bond sales reduces the amount of financing that is available for private investment (cf. equation 51) while increases foreign grants or foreign borrowing tend to permit more rapid growth in GDP and private final demand (consumption and investment). Reliance on foreign resources also tends to bring about real exchange rate appreciation, slower export growth, and more rapid growth in imports and production for domestic markets. The strength of these effects depends on the growth impact of the expansion in government spending as well as on whether the new spending has high or low import shares. If the country later needs to reverse the switch toward production of non-tradables (for example, because of a decline in foreign grants in the future), and its structure is rigid, it may end up suffering from “Dutch Disease.” Expansion in foreign borrowing is less favorable than grants since it drives up the foreign debt (which, in the absence of debt relief, eventually has to be repaid) and related interest payments (more or less burdensome depending on loan conditions). The alternative of raising direct taxes tends to be less favorable to growth in GDP and private final demand than reliance on foreign resources. However, given that most of the cut in household disposable income is born by consumption as opposed to savings and investment, the direct tax alternative is more favorable than domestic government borrowing for long-run growth in GDP and private final demand.

Equation 51 defines the fixed investment values for non-government institutions – all terms do not apply to each institution – as own savings, net of spending on stock (inventory) changes and lending to the government, and augmented by borrowing, capital grants and FDI from the rest of the world. For the latter, the fixed investment value is simply the value of FDI (fixed in FCU) times the exchange rate. (The FDI term is invariably fixed at zero for domestic institutions.) Implicitly, equation 51 shows a rule for
ensuring that total savings and total investment are equal: given that government and households savings, government investment, and FDI all are determined by other rules, the clearing variable is private household investment ($INVVAL_{ht}$).

For each non-government institution, real investment in different capital stocks (investment by destination) is determined by its total fixed investment values, the prices of capital goods, and exogenous value shares by capital stock; the value share is unity if the database only specifies a single private capital type (equation 52).\footnote{Typically, the model will only have one private capital stock, i.e. the value of the share parameter is unity for this capital type. If the model has more than one private capital stock, the allocation between the different stocks may be endogenized, possibly deviating from the base-level allocation in response to changes in relative profit rates, a relationship that would need to be specified in one or more additional equations.}

The final equation in this block defines total investment demand by commodity source (often referred to as investment by origin). It is defined on the basis of real gross fixed capital formation (both private and government; investment by destination) and the capital composition parameter (equation 53).

*Other system constraints: foreign exchange, factors, and commodities*

In the preceding, we discussed alternative mechanisms for clearing two of the macro constraints of the model, the government budget and the savings-investment balance. The current block (equations 54-58) includes the remaining system constraints: the balance of payments and the markets for factors and commodities.

The balance of payments (or foreign exchange constraint) (equation 54) imposes equality between foreign exchange uses (spending on imports, factor incomes and transfers to the rest of the world, and interest payments on foreign debts) and sources (export revenues, transfers, factor incomes, borrowing, capital grants, and FDI).\footnote{Implicitly, an additional system constraint, the savings-investment balance, also holds: by channeling domestic savings and the terms that make up foreign savings to investment, the model equations assure that total savings and total investment are equal.} In practice, the only plausible assumption for medium- and long-run analysis tends to be that the (real) exchange rate ($EXR_t$) clears this balance. For example, other things being equal, depreciation (an increase in $EXR_t$) will remove a deficit by raising supplies for export relative to supplies for domestic sales while reducing domestic use of imports relative to domestic use of domestic output.
The market constraint for factors (equation 55), which applies to all factors except government capital, states that total demand for any factor (the left-hand side) equals the total endowment times the employment rate (one minus the unemployment rate). The economy-wide wage ($WF_{f,t}$) clears the market. The unemployment rate is exogenous or endogenous. If it is exogenous, the economy-wide wage variable ($WF_{f,t}$) will clear the market by influencing the quantity demanded without any influence on the quantity supplied.

**Figure 1. Labor market adjustment with endogenous unemployment**

![Figure 1. Labor market adjustment with endogenous unemployment](image)

Figure 1 shows the functioning of factor markets with endogenous unemployment. The supply curve is upward-sloping, reflecting that, ceteris paribus, workers request higher wages as the labor market gets tighter. When the market reaches “full employment”, that is when the minimum unemployment rate is reached (which here is set at 5 percent but varies across applications), the supply curve turns vertical. When the factor market is below full employment, the market-clearing wage influence both the quantities demanded and supplied, the latter in association with changes in the unemployment rate ($UERAT_{f,t}$); at full employment, the market-clearing wage only influences the quantity demanded. Unemployment should be seen as broadly defined, representing the degree of underutilization of the factor (and the potential for increased
utilization), due to a combination of full or partial unemployment (i.e., also considering underemployment).

Equations 56-57 specify our treatment of the labor market. Workers have a reservation (minimum) wage ($WFRES_{f,t}$) below which they will not work (equation 56). It is defined as a function of the economy-wide wage in the base year, and the ratios between current and base-year values for the (un)employment rate, household consumption per capita (as indicator of real living standards), and the CPI. The ratio terms are raised to elasticities that determine their importance (an elasticity of zero implies that a term has no importance). Equation 57 consists of three parts: (a) the constraint that the economy-wide wage for each factor cannot fall below the endogenous reservation wage; (b) the constraint that the unemployment rate cannot fall below an exogenous minimum ($ueratmin$); and (c) a complementary slackness condition, which states that either (a) or (b) but not both are slack (non-binding). In other words: if the unemployment rate is above its minimum, then the wage must be at the reservation level; if the wage is above the reservation level, then the unemployment rate must be at its minimum.

Note that, at the activity level, the wage paid is the product of $WF_{f,t}$ and $WFDIST_{f,a,t}$ (cf. equations 15 and 28). $WFDIST_{f,a,t}$, a distortion (or differential) term that typically is exogenous, reflects relative wage differences across activities. In some cases it may be desirable to impose an exogenous time path for the employment of specific factors in selected activities (drawing on other pieces of information, for example data on the expected evolution of sectors based on the exploitation of natural resources). For the factor-activity-time combination in question, the analyst only has to flex the wage distortion variable ($WFDIST_{f,a,t}$) and fix the employment variable ($QF_{f,a,t}$). Such an assumption can coexist with factor markets with or without endogenous unemployment.

For each composite commodity, the supply is set equal to the sum of demands (equation 58). As noted earlier, composite supplies stem from two sources, imports and domestic supplies to domestic markets (cf. equation 24); for each commodity with both sources, demand is allocated between them on the basis of relative prices. The market-

26 The level of the base-year unemployment rate relative to the minimum unemployment rate indicates the potential for employment growth over and above the growth rate of the labor stock.
clearing variables are the quantity \((QM)\) for imports and, for domestic output, the price \((PDS\) for suppliers and \(PDD\) for demanders, with a wedge between the two in the presence of transactions costs).

**Asset stock updating and productivity block**

The equations in this block update institutional stocks of assets and liabilities, and TFP by activity (equations 59-66). Except for equations defining arguments for the definition of TFP, all equations in this block include lagged relationships. They do not apply to the first year, for which the values of the variables defined in this block are fixed.

Implicitly, the mathematical statement assumes that MAMS has a single representative household.\(^{27}\) For capital, the stock of any institution (household, government, and rest of world) is defined as the sum of its previous-period stock (adjusted for depreciation), new investments, and exogenous adjustments (which may reflect the impact of natural disasters or institutional changes, removing parts of the capital stock from production) (equation 59). The evolution of labor endowments in defined in equation 81. For other factors (for example agricultural land), the growth in institutional endowments \((QFINS_{i,f,t})\) is exogenous. Except for the absence of depreciation, the relationships that hold for foreign debt (equations 60) and government bonds (equations 61) are identical to those used for capital. For foreign debt, the treatment is potentially more complex since the model allows for the possibility of non-paid interest (which is added to the debt) and debt relief.

This block includes further a set of equations used to define total factor productivity (TFP) for each activity. To simplify the algebra, equations 62-63 define real GDP at market prices and the real trade-to-GDP ratio.

In equation 64, the TFP of each activity (a variable that appears in equation 14, the CES value-added function) is defined as the product of a trend term, changes due to capital accumulation, and changes due to variations in economic openness (defined by

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\(^{27}\) In applications with multiple households, it is necessary to specify how the population in each household evolves over time. Our general principle is that the household types that exist in the base-year (characterized by patterns for generation and spending of incomes) continue to exist but grow at different rates depending on the types of labor that they control. The non-labor endowments of each household type grow at the same rate as its population, scaled upwards or downwards to ensure that the total for these endowments across all household respect economy-wide constraints.
the real trade-to-GDP ratio). The effects of capital accumulation and changes in openness depend on the values of exogenous elasticities – if they are set at zero, the effect is zero and then only the trend term matters. In the definition of the trend term (equation 65), the first of the trend growth terms, $\alpha^{tag}_{a,t}$, is invariably exogenous. The second term, 

$CALTFPG$ is endogenized when a certain GDP level is targeted (a typical assumption for the base run). In this context, the parameter $tfp01$ has been used to control relative TFP growth rates across activities (with values ranging between zero and unity). However, apart from the base simulation, all right-hand terms are either exogenous or lagged while GDP is endogenous. The trade-to-GDP ratio, the most common indicator of economic openness (in terms of outcome, not policy stance) is defined in real terms (to avoid the impact of nominal changes, for example due to exchange rate depreciation) and with a potential lag to avoid unrealistically large immediate productivity effects of changes in openness: in any time period, the numerator in the last term of equation 64 is a weighted average of current and past trade-to-GDP ratios. The parameter for the length of the lag is part of the country-specific database. The final equation in this block, equation 66, defines real GDP at factor cost; it is flexible unless $CALTFPG$ is fixed.

The fact that the elasticity parameters in equation 64 are disaggregated (by activity for trade and by activity and function for capital) make it possible to specify different channels and magnitudes for the productivity effects of trade and of different types of government capital stocks. For example, if investments in roads and irrigation have their specific capital stocks and the activities include one or more agricultural and transportation activities, the productivity effects of changes in these two capital stocks could be channeled to the agricultural and transportation activities, respectively. Depending on the degree of disaggregation of these capital stocks and activities, the productivity effects can be more or less finely targeted. For example, if irrigation and

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28 When developing the model base run, $CALTFPG$ may be endogenous or exogenous. If it is endogenous, real GDP (at factor cost) should be fixed (growing exogenously over time). If so, the analyst should review the resulting economy-wide growth in TFP as well as efficiency growth in different activities ($ALPHAVA_{a,t}$) and, if needed, adjust the targeted real GDP levels. On the other hand, if $CALTFPG$ is exogenous (and real GDP endogenous), the analyst should monitor overall GDP growth and, if needed, adjust either $CALTFPG$ or $\alpha^{tag}$. The estimates of initial capital stocks and depreciation rates may also have to be revisited. For non-base runs, the determinants of trend TFP growth ($ALPHAVA2$) are typically fixed, while real GDP growth is determined by growth in factor employment and endogenous TFP changes.

29 Some MAMS applications also include a link between health-related MDG indicators and labor productivity disaggregated by labor type and activity.
road capital stocks are singled out, these could meaningfully be linked to agriculture (especially crop activities) and transportation services, respectively; other sectors would only be influenced indirectly by these productivity changes. On the other hand, if infrastructure capital is a single capital stock, the selection of targeted sectors would have to be more general (implicitly reflecting some assumed composition of this broader spending type).

3.3 The MDG module

The MDG module (equations 67-81) specifies the mechanisms that determine the values for the indicators related to the different MDGs and educational behavior as well as the size and disaggregation (by educational achievement) of the labor force. The rest of the economy, which was presented in the preceding sections, influences the evolution of the MDGs and the educational sector through variables related to household consumption, the provision of different types of MDG-related services, labor wages, and capital stocks in infrastructure. In its turn, the MDG module influences the rest of the economy through its impact on the size and composition of the labor force.\(^{30}\) In some applications, in addition, the evolution of one set of MDGs can influence other MDGs. The notation and the equations of the MDG module are respectively presented in Tables 3.3 and 3.4.

MAMS focuses on the MDGs that typically are most costly and have the greatest interactions with the rest of the economy: universal primary school completion (MDG 2; measured by the net primary completion rate), reduced under-five and maternal mortality rates (MDGs 4 and 5), and increased access to improved water sources and basic sanitation (part of MDG 7). The poverty MDG (MDG 1) is not targeted given the absence of tools (in MAMS and in most real-world, developing-country contexts) that policymakers realistically could use to fine-tune poverty outcomes.\(^ {31}\)

\(^{30}\) See footnote 29 for an additional link from the MDG module to the rest of the economy.

\(^{31}\) Implicitly, when MDGs 4 and 5 are achieved, the expansion in health services and other determinants may be sufficient to achieve MDG 6 (to halt and reverse the spread of HIV/AIDS, malaria and other diseases). MDG 3 (elimination of gender disparity in education and empowering women) was not addressed due to data issues. However, note that, if MDG 2 is achieved, gender equality is achieved in primary education.
As explained in the introductory section of this paper, MDG outcomes depend on government and private sector provision of MDG-related services as well as on demand conditions for those services. Table 4 lists the determinants that have been included in a typical country application of MAMS, identified on the basis of available evidence, preferably sector studies underpinned by econometric analysis and subject to the constraints of an economy-wide model like MAMS (including the fact that it is difficult to include finely disaggregated actions, like increasing coverage of certain types of vaccinations). Beyond per-capita real service delivery (either public or a combination of public and private), the determinants include other MDGs (for example, better access to water and sanitation may improve health outcomes – MDGs 4 and 5), as well as public infrastructure, per-capita household consumption, and wage incentives (through the ratio of labor wages of different educational levels). Other determinants should be added when evidence suggests that the effect is significant during the time frame of the analysis. One possible candidate is the impact of education on health, which may be important in long-run analyses where the educational status of the population may change significantly.

### Table 4. Determinants of non-poverty MDGs

<table>
<thead>
<tr>
<th>MDG</th>
<th>Service delivery</th>
<th>Household consumption per capita</th>
<th>Wage incentives</th>
<th>Public infrastructure</th>
<th>Other MDGs</th>
</tr>
</thead>
<tbody>
<tr>
<td>2 Primary education</td>
<td>x</td>
<td>x</td>
<td>x</td>
<td>x</td>
<td>4</td>
</tr>
<tr>
<td>4. Under-five mortality</td>
<td>x</td>
<td>x</td>
<td></td>
<td></td>
<td>7w, 7s</td>
</tr>
<tr>
<td>5. Maternal mortality</td>
<td>x</td>
<td>x</td>
<td></td>
<td></td>
<td>7w, 7s</td>
</tr>
<tr>
<td>7w. Access to safe water</td>
<td>x</td>
<td>x</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>7s. Access to basic sanitation</td>
<td>x</td>
<td>x</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

In the equations of this module, the treatment of the education MDG (2) is separate from the treatment for the remaining MDGs (4, 5, 7w, 7s) since, rather than targeting MDG 2 directly, the model defines (and may target) specific educational behavioral outcomes that jointly determine the value for MDG 2.

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32 Econometric analysis for several countries in Latin America show that the relationships between the determinants and the non-poverty MDGs in the MAMS model tend to hold from a statistical point of view. Kamaly (2006) provides examples of the literature on health and education whose findings, although sometimes contradictory, show broad support also in sub-Saharan Africa for the inclusion of the determinants referred to in Table 4.
The first three equations define arguments that enter both education and MDG functions. Equations 67-68 define aggregate human development (HD) services (which include both MDG and education services). For each service type, equation 67 separates demand into two aggregates, government and non-government, according to who is paying for the service. Typically, services paid for by the government (non-government) are also supplied by a government (non-government) activity, but this is not necessarily the case. Equation 68 generates an economy-wide aggregate (which below is fed into the determination of MDG and education outcomes), permitting two alternative assumptions: services paid for by government and non-government are perfect substitutes (simply summed) or imperfect substitutes (according to a CES function). Equation 69 defines average real household consumption per capita ($QHPC$) as total household consumption (both marketed and home commodities) at base-year prices divided by total population.

The education and labor component consists of equations 70-79. It is disaggregated by cycle (with three cycles as a typical level of disaggregation) and, with regard to student flows, also by grade (with the number of grades being cycle-specific). At the level of the cycle (not the grade), educational quality ($EDUQUAL$) is defined as the ratio between real services per student (aggregated services divided by total enrolment) in the current year and in the base year; i.e., in the base-year, educational quality is indexed to one (equation 70). Within any cycle, the model endogenizes the following aspects of student behavior (or outcomes):

- the shares of the enrolled that are promoted from their current grade, drop out, or repeat the grade next year (referred to as $prom$, $dropout$, and $rep$). The sum of these shares is unity – i.e., a student must be promoted, drop out, or become a repeater. Note that the term “prom” refers both to students who successfully complete a grade and continue to a higher grade within the cycle, and to students who successfully finish the last year of a given education cycle (and thus graduate). As a corollary, the term “dropout” refers to students (i) who are in some grade other than the last of their cycle and who do not continue to the next

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33 I.e., among students show are promoted to next grade within a cycle, the decision to start next grade is not modeled separately; students are considered as promoted only if they indeed start next grade.
grade (either dropping out during the year or between school years) or (ii) who are in the last grade of their cycle and who drop out during their last year.

- among those who are promoted from the last year of their cycle, the shares who exit the school system (grdexit) or continue to next cycle (grdcont), if there is any. The sum of these shares is also unity. For graduates from the last cycle, the share of those who exit is unity; and
- the net intake rate, i.e. the share of the cohort of the 1st year in primary school that enters school (neting1).

Drawing on the above information, we can define the number of enrolled students by cycle, grade, and year. Equation 71 defines the number of “old” enrolled students in any cycle and grade (i.e., those who were enrolled in the same cycle last year) as the sum of those who: (i) continue within the cycle after successful completion of an earlier grade; and (ii) repeat the grade they were in last year. The number of “new” enrolled students is defined in equation 72 as the sum of: (i) cohort entrants (only for the first grade of the primary cycle); (ii) non-cohort entrants starting any cycle and grade in the educational system; and (iii) for the first grade of non-primary cycles, the students promoted last year from the last grade of the preceding cycle who continue.34 The total number of enrolled students in a cycle is the sum of old and new students (equation 73).

**Figure 2. Logistic function for education**

![Logistic function for education](image)

34 This category includes non-cohort entrants to the 1st primary year of primary school (who may represent a significant number during a transitional period of primary school expansion). It may also include immigrants from other countries.
Equations 74-78 define the share variables that identify different aspects of student behavior. For each cycle, a logistic function (equation 74) defines $\textit{SHR}^{edu}$, the shares for 1\textsuperscript{st} year in-cohort entry, for students who are promoted from their current grade, and for graduates who decide to continue to next cycle (i.e. \textit{neting1}, \textit{prom}, and \textit{grdcont}, the elements of the set \textit{BLOG}). The logistic form was selected since it makes it possible to impose extreme (for education it is a maximum of one) values for the function and to incorporate extraneous information about elasticities and conditions under which target values are achieved. Another advantage is that it allows for segments of increasing and decreasing marginal returns to improvements in the determinants of educational behavior. The only endogenous variable in the logistic function ($\textit{ZEDU}$), is defined in a constant-elasticity (CE) function (equation 75) as determined by: (i) educational quality; (ii) wage incentives, defined as relative wage gains from continued schooling (i.e., the relative wage gain that students can achieve if they complete a cycle that is sufficiently high to enable them to climb to the next higher level in the labor market); (iii) the under-five mortality rate (a proxy for the health status of the school population); (iv) the size of the infrastructure capital stock; and (v) household consumption per capita. Figure 2 illustrates the logistic functional form for education. The observed base-year value for $\textit{SHR}^{edu}$ is generated at the base-year value for $\textit{ZEDU}$. The parameters of the function have to be defined such that the maximum share is one, the base-year elasticities of $\textit{SHR}^{edu}$ with respect to each determinants of $\textit{ZEDU}$ are replicated and, under values for the determinants of $\textit{ZEDU}$ identified in the database, a target level for $\textit{SHR}^{edu}$ is realized. In terms of the algebra, the parameters in equations 74-75 are selected as follows:

- the parameter $\textit{ext}^{edu}$ shows the extreme (maximum) value (here unity) to which the behavior share should converge as the value of the intermediate variable approaches infinity;
- the parameter $\alpha^{edu}$ is calibrated so that, under base-year conditions, the behavioral share replicates the base-year value;
- the parameters $\beta^{edu}$ and $\phi^{edu}$ are calibrated so that the two equations: (i) replicate the base-year elasticities of the behavioral share ($\textit{SHR}^{edu}$) with respect to the arguments of the CE function; and (ii) achieve a behavioral target between the
base-year value and the extreme value (e.g. a share close to one for neting1) under a set of values for the arguments of the CE function that have been identified by other studies; and

- the value of the parameter $\gamma^{edu}$ determines how the base-year point on the logistic function is positioned relative to the inflection point (where the curve switches from increasing to decreasing marginal returns as the determinants of educational behavior improve).

Drawing on the shares defined in the preceding equations, the shares for repeaters, dropouts, and cycle graduates exiting from the school system ($rep$, $dropout$, and $grdexit$; elements in the set $BRES$) are defined residually (equation 76). The formulation considers the fact that, as noted above, selected shares have to sum to unity. If more than one variable in $BRES$ has to be adjusted in relation to one or more elements in $BLOG$ (as is the case for the adjustment of shares for repeaters and dropouts in response to changes in the share of graduates), then all adjusted variables are scaled up or down by the same factor.\(^{35}\)

We use the net completion rate as our MDG 2 indicator. It is defined as the product of the relevant 1st-year primary school net intake rate ($neting1$) and the promotion rates ($prom$) over time for the cohort that graduate from primary school in the current year (equation 77).\(^{36}\)

The aggregate labor force participation rate (which is exogenous) is defined as the labor force ($QFINS$) divided by the population in labor force age that is not enrolled in school, with excluded labor-force-age students defined as total enrollment starting from a specified cycle and grade (equation 78). Institutional labor endowments ($QFINS$ for labor) are defined as the sum of the following components (equation 79): (i) remaining labor from the preceding year, with the variable $QFINS$ scaling the retired labor-force share to ensure that the exogenous labor force participation rate, defined in equation

\(^{35}\) The equation is formulated so that it works for cases with one or more than one term in any of the sums over related shares (defined by the mappings $MBB$ and $MBB2$) in either of the sets $BRES$ and $BLOG$.

\(^{36}\) In other words, in order for 100% of the cohort to complete the primary cycle on time, it is necessary that all of them enter at the time of their first year and then that all manage to be promoted each year (i.e., successfully complete each grade) up to the final year of the cycle. Given that we do not generate separate promotion rates for students in the relevant cohort (as opposed to students outside this cohort), we assume that the rates for in-cohort students are identical to the over-all rates for students in the cycle.
78, is satisfied; (ii) new labor force entrants among students who exited from the school system in the previous year (with separate terms for non-tertiary graduates, tertiary graduates, and dropouts); and (iii) new labor force entrants from the non-student population who reach the age at which they, to the extent that they seek work, become part of the labor force. Depending on their highest completed cycle, the new labor force entrants are allocated to a specific labor category.

The treatment underlying MDGs 4, 5, 7w and 7s is similar but less complex. For these, a logistic function directly defines the MDG indicators as a function of an intermediate variable that is defined in a related CE function (equations 80-81). The values for the parameters $ext_{mdg}$, $\alpha_{mdg}$, $\beta_{mdg}$, and $\phi_{mdg}$ are defined following the same principles as the corresponding parameters in the logistic and CE functions for education. The arguments of the CE function are similar except for that the relevant service supply is expressed in per-capita form (not per enrolled student).

4 Overview of MAMS Data Needs and Sources

The data needs of the core CGE module of MAMS are similar to those of other CGE models. Additional data (for the SAM and other parameters) is needed primarily for the MDG module but also to capture some other extensions, mostly related to the treatment of the government.

In an earlier section, we presented the structure of a SAM for the full (MDG) version of MAMS. The following aspects of this SAM give rise to data requirements that go beyond what is needed for most SAMs for CGE models:37 (i) government consumption and investment spending must be disaggregated into functions that correspond to policy tools for addressing the relevant MDGs and providing education at the three major levels (primary, secondary and tertiary); (ii) labor must be disaggregated by level of education in a manner that matches the educational system; and (iii) the SAM must include accounts for foreign and domestic interest payments. In addition, if they are important, it is also preferable to single out separate private activities and commodities in

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37 A SAM for the core (non-MDG) version of MAMS deviates from this in that may have a very aggregate treatment of sectors (activities and commodities), factors, and institutions. The minimum degree of detail is two sectors (private and government); two factors (labor and private capital), three institutions (government, household, and rest of world), two investment accounts (for private and government capital).
the MDG and education area; for example, the private sector may account for significant shares of the total supply of education, especially at higher levels. In other respects, the SAM is very similar to standard practice.

*Other data and sources*

Other than the SAM, MAMS requires data that in part coincides with those of many other models, most importantly elasticities to capture substitutability between factors in production, transformability of output between exports and domestic sales, substitutability between imports and domestic commodities in domestic demand, and responses in household consumption to income changes.

The other data requirements, specific to MAMS, are mostly due to its extensions to include MDG indicators and their determinants, an extended education module, and relatively detailed government accounts. The links between labor, education, and population necessitate consistent base-year data on employment (by activity and labor type), unemployment (by labor type), and enrolment (by level or cycle). The model also requires projections for the total population and the population of three age groups: the cohort of entrants to primary school, the cohort of entrants to labor-force age, and the broader population group in working age.

Both for education (for each of the three levels) and four of the non-education MDGs in MAMS (labeled 4, 5, 7w, and 7s), it is necessary to provide data for base-year outcomes: in education, cycle-specific rates for entry, promotion, repetition and dropout; outside education, the rates that define other MDG indicators (e.g. the under-five mortality rate). For these non-education MDGs, information on the situation in 1990 is needed to define the targets for 2015. To model how these outcomes change over time, two pieces of information are needed: (a) base-year elasticities (linking outcomes to determinants), preferably estimated using logit or probit models; and (b) a path for the evolution of these determinants that makes it possible to reach a set of future values (typically targets for 2015). This information is used to calibrate the functions so that they replicate base-year outcomes and elasticities, reach each MDG under specified conditions, and have the upper limits that were specified exogenously.
Finally, also for non-labor factors, it is necessary to define base-year stocks. For private capital, these may be defined on the basis of base-year data for rents, profit rates, and depreciation rates.\textsuperscript{38} For each type of government capital, the base-year stock is defined on the basis of historical data on service growth, investments, depreciation rates, and the assumption that the capital stock over time has grown at the same rate as real services. For other factors such as agricultural land and natural resources, base-year stocks can typically be defined so that base-year rents are normalized to unity; data on the future stock growth is also needed – as opposed to labor and capital, growth for these factors is exogenous.

The construction of this database requires the analysts to consult existing SAMs and input-output tables, other standard databases (both country-specific and those of international organizations, covering national accounts, government budgets, and the balance of payments), surveys (of households, labor, and health conditions), and relevant research on trade, production, consumption, and human development, including available MDG strategies and other analyses of the determinants of MDG outcomes. Sector-focused MDG studies (in health, education, water and sanitation, and public infrastructure), public expenditure reviews and other types of country-level economic studies are often valuable sources. It is often desirable to complement available studies with new survey-based research to better understand the determination of MDG and education outcomes.\textsuperscript{39}

As shown, MAMS does not replace other forms of sectoral research in human development; on the contrary, it draws extensively on and stimulates such research. Without sector studies that provide a strong empirical basis, the analysis of MDG strategies in an economy-wide framework (whether MAMS or any other) loses much of its power.

\textsuperscript{38} The following formula is used to define the base-year private capital stock: \(qfcap = \frac{samrent}{netprfrat + deprrat}\) where \(qfcap\) = the stock; \(samrent\) = total VA to private capital in SAM; \(netprfrat\) = the net profit rate (in decimal form); \(deprrat\) = the depreciation rate (also in decimal form).

\textsuperscript{39} The micro-simulation approach to poverty and inequality analysis requires access to a recent household survey.
5 Concluding Remarks

This paper has documented MAMS, a dynamic CGE model that is designed to analyze strategies for medium- and long-run growth and poverty reduction in developing countries, including strategies for achieving the MDGs.

In applications, MAMS is first used to generate a base or business-as-usual scenario that serves as a benchmark to which the results for other simulations are compared. The aim of this scenario is to represent a plausible projection into the future, drawing on recent trends. The length of the period analyzed has varied considerably, from 3-5 years up to 25 years. As a second step, MAMS is used to simulate a set of alternative scenarios. For each, the analyst changes one or more government policies (influencing government revenues or spending) and/or exogenous conditions (like international prices). Policy changes may be of two types: a change in a policy tool (like a tax rate) or a change in an exogenized policy target (like an MDG indicator) accompanied by endogenous adjustment in a policy tool with a sufficiently strong bearing on the target (like a government service related to the MDG in question).

More specifically, if the MDG version of MAMS is used to simulate the achievement of one or more MDGs by a specific year (typically 2015), target values for the MDG indicators are imposed for preceding years so as to gradually reach the target values while related policy tools are endogenous in the years with targets. In any year, the number of targets that are imposed exactly is equal to the number of endogenous policy tools. The selection of government closure rule is an important part of the simulation: the impact is very different if the government budget clears via changes in grant aid, foreign borrowing, domestic bonds, or domestic taxes.

References


### Tables 3.1-3.4: Mathematical Statement of MAMS

#### Table 3.1 Sets, parameters, and variables for core CGE modules of MAMS model

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Explanation</th>
<th>Symbol</th>
<th>Explanation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a \in A$</td>
<td>Activities</td>
<td>$f, f' \in F$</td>
<td>factors</td>
</tr>
<tr>
<td>$a \in ACES (\subset A)$</td>
<td>activities with CES function between Value Added and Intermediate inputs</td>
<td>$f \in FCAP(\subset F)$</td>
<td>capital factors</td>
</tr>
<tr>
<td>$a \in ALEO (\subset A)$</td>
<td>activities with Leontief fn between Value Added and Intermediate inputs</td>
<td>$f \in FCAPGOV(\subset FCAP)$</td>
<td>government capital factors</td>
</tr>
<tr>
<td>$c \in C$</td>
<td>Commodities</td>
<td>$f \in FEXOG(\subset F)$</td>
<td>factors with exogenous growth rates</td>
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<td>$f \in FLABN(\subset F)$</td>
<td>non-labor factors</td>
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<td>$c \in CDN(\subset C)$</td>
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<td>$f \in FUEND(\subset F)$</td>
<td>factors with endogenous unemployment</td>
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<td>$h \in H(\subset INSDNG)$</td>
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<td>$c \in CEN(\subset C)$</td>
<td>commodities not in $CE$</td>
<td>$i \in INS$</td>
<td>institutions (domestic and rest of world)</td>
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<td>$c \in CECETN(\subset C)$</td>
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<td>$i \in INSD(\subset INS)$</td>
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<td>infrastructure commodity</td>
<td>$i \in INSDNG(\subset INS)$</td>
<td>domestic non-government institutions</td>
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<td>$i \in INSG(\subset INS)$</td>
<td>non-government institutions</td>
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<td>$c \in CMN(\subset C)$</td>
<td>commodities not in $CM$</td>
<td>$(f, a) \in MFA$</td>
<td>mapping showing that disaggregated factor $f$ is used in activity $a$</td>
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<td>$c \in CT(\subset C)$</td>
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<td>$t \in T$</td>
<td>time periods</td>
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<tr>
<td>capcomp&lt;sub&gt;g,f,t&lt;/sub&gt;</td>
<td>quantity of commodity c per unit of new capital f</td>
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<td>interest rate on government bonds for domestic institution i</td>
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<td>ifa&lt;sub&gt;f,a&lt;/sub&gt;</td>
<td>quantity of capital f per unit of government activity a</td>
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</table>

**NOTES:**
- <sub>g</sub> = Latin letter
- <sub>c</sub> = commodity
- <sub>f</sub> = factor
- <sub>a</sub> = activity
- <sub>i</sub> = institution
- <sub>t</sub> = time
- <sub>c'</sub> = commodity c'
| \( igf_{c,f,t} \) | quantity of gov’t consumption per unit of gov’t infrastructure capital stock \( f \) | \( \text{tinsbar}_{i,t} \) | Exogenous component in direct tax rate for domestic institution \( i \) |
| \( inta_a \) | quantity of aggregate intermediate input per unit of activity \( a \) | \( \text{tm}_{c,t} \) | import tariff rate |
| \( iva_a \) | quantity of value-added per unit of activity \( a \) | \( \text{tq}_{c,t} \) | rate of sales tax |
| \( mps01_i \) | 0-1 parameter with 1 for institutions with potentially flexed direct tax rates | \( \text{trnsfr}_{i,i',t} \) | Exogenous transfer from institution \( i' \) to institution \( i \) |
| \( mpsbar_{i,t} \) | Exogenous component in savings rate for domestic institution \( i \) | \( \text{trnsfr}_{f,i',t} \) | Exogenous transfer from institution \( i' \) to factor \( f \) |
| \( poptot_i \) | total population by year | \( \text{trnsfrpc}_{c,i',t} \) | per-capita transfers from institution \( i' \) to hhd institution \( i \) |
| \( pwm_{c,t} \) | import world price of \( c \) (FCU) | \( \text{iva}_{a,t} \) | rate of value-added tax for activity \( a \) |
| \( pwse_{c,t} \) | world price for export substitutes (FCU) |

**PARAMETERS – GREEK LETTERS**

- \( \alpha^{ac}_c \): shift parameter for domestic commodity aggregation function
- \( \gamma^h_{a,c,h} \): per capita household subsistence consumption of home commodity \( c \) from activity \( a \)
- \( \alpha^{vag}_{a,a} \): exogenous component of efficiency (TFP) for activity \( a \)
- \( \gamma^m_{c,h} \): per capita household subsistence cons of marketed commodity \( c \)
- \( \alpha^q \): Armington function shift parameter
- \( \varphi_{s,f} \): elasticity of reservation wage for \( f \) w.r.t. * where * = \( qhpc \) (household per-capita consumption), \( erat \) (employment rate), or \( cpi \) (CPI)
- \( \alpha^i \): CET function shift parameter
- \( \rho^{ac}_c \): domestic commodity aggregation function exponent
- \( \beta^{h}_{a,c,h} \): marginal share of household consumption on home commodity \( c \) from activity \( a \)
- \( \rho^{q}_c \): Armington function exponent
- \( \beta^{m}_{c,h} \): marginal share of household consumption spending on marketed commodity \( c \)
- \( \rho^{sav}_i \): elasticity of savings rate with respect to per-capita income for institution (household) \( h \)
- \( \delta^{ac}_a \): share parameter for domestic commodity aggregation function
- \( \rho^i_i \): CET function exponent
- \( \delta^q_c \): Armington function share parameter
- \( \rho^{va}_a \): CES value-added function exponent
- \( \delta^i_c \): CET function share parameter
- \( \theta_{a,c} \): yield of output \( c \) per unit of activity \( a \)
<table>
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<tr>
<th>( \delta^w_f.a )</th>
<th>CES value-added function share parameter for factor ( f ) in activity ( a )</th>
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**VARIABLES**

<table>
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<tr>
<th>ALPHAVA(_{a,t} )</th>
<th>efficiency parameter in the CES value-added function</th>
</tr>
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<tr>
<td>ALPHAVA2(_{a,t} )</td>
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<td>CALTFPG(_t )</td>
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<td>consumer price index</td>
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<td>DKG0V(_{f,t} )</td>
<td>gross government investment in ( f )</td>
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<td>DKINS(_{i,f,t} )</td>
<td>gross change in capital stock (investment in) ( f ) for institution ( i )</td>
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<td>DMPS(_t )</td>
<td>uniform point change in savings rate of selected domestic institutions</td>
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<td>change in holding of government bonds for domestic institution ( i )</td>
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<td>total change in holding of government bonds</td>
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<td>GBORMS(_{i,t} )</td>
<td>implicit government Central Bank borrowing (deficit monetization) from institution ( i )</td>
</tr>
<tr>
<td>GBORMSTOT(_t )</td>
<td>total government Central Bank borrowing (deficit monetization)</td>
</tr>
</tbody>
</table>

- \( PWE_{c,t} \): export world price of \( c \) (FCU)
- \( PX_{c,t} \): aggregate producer price for commodity
- \( PXAC_{a,c,t} \): price of commodity \( c \) from activity \( a \)
- \( QA_{a,t} \): quantity (level) of activity
- \( QD_{c,t} \): quantity sold domestically of domestically produced \( c \)
- \( QE_{c,t} \): quantity of exports of commodity \( c \)
- \( QF_{f,a,t} \): quantity demanded of factor \( f \) by activity \( a \)
- \( QFINS_{i,f,a,t} \): real endowment of factor \( f \) for institution \( i \)
- \( QG_{c,t} \): quantity of government consumption of commodity \( c \)
- \( QGGRW_{c,t} \): real government consumption growth for all \( c \) in \( t \) relative to \( t-1 \)
- \( QGGRW_{c,t} \): adreal government consumption growth of \( c \) in \( t \) relative to \( t-1 \)
- \( QHA_{a,c,h,t} \): quantity consumed of home commodity \( c \) from act \( a \) by hhdd \( h \)
- \( QINTA_{a,t} \): quantity of aggregate intermediate input used by activity \( a \)
- \( QINT_{c,a,t} \): quantity of commodity \( c \) as intermediate input to activity \( a \)
- \( QINV_{c,t} \): quantity of investment demand for commodity \( c \)
- \( QM_{c,t} \): quantity of imports of commodity \( c \)
- \( QQ_{c,t} \): quantity of commodity \( c \) supplied to domestic market (composite supply)
- \( QT_{c,t} \): quantity of trade and transport demand for commodity \( c \)
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( GDEBT_{i,t} )</td>
<td>endowment of government bonds for ( i )</td>
<td>( QVA_{a,t} )</td>
<td>quantity of (aggregate) value-added</td>
</tr>
<tr>
<td>( GDPREAL_{t} )</td>
<td>real GDP at market prices</td>
<td>( QX_{c,t} )</td>
<td>aggregated quantity of domestic output of commodity</td>
</tr>
<tr>
<td>( GDPREALFC_{t} )</td>
<td>real GDP at factor cost</td>
<td>( QXAC_{c,a,s,t} )</td>
<td>quantity of output of commodity ( c ) from activity ( a )</td>
</tr>
<tr>
<td>( GSAV_{t} )</td>
<td>government savings</td>
<td>( SHIF_{i,f,t} )</td>
<td>share of institution ( i ) in income of factor ( f )</td>
</tr>
<tr>
<td>( INSSAV_{i,t} )</td>
<td>savings of domestic non-government institution ( i )</td>
<td>( TINS_{s,t} )</td>
<td>direct tax rate for domestic non-government institution ( i )</td>
</tr>
<tr>
<td>( INVVAL_{i,t} )</td>
<td>investment value for institution ( i )</td>
<td>( TINSADJ_{t} )</td>
<td>direct tax scaling factor</td>
</tr>
<tr>
<td>( MPS_{i,t} )</td>
<td>marginal propensity to save for domestic non-gov't institution ( i )</td>
<td>( TRDGDP_{t} )</td>
<td>foreign trade as share of GDP</td>
</tr>
<tr>
<td>( MPSADJ_{t} )</td>
<td>savings rate scaling factor</td>
<td>( TRII_{i,f,t} )</td>
<td>transfers from institution ( i' ) to ( i ) (both in the set INSDNG)</td>
</tr>
<tr>
<td>( PA_{a,t} )</td>
<td>activity price (unit gross revenue)</td>
<td>( TRNSFR_{ac,i,t} )</td>
<td>transfers from non-hhd inst’on ( i ) to ( i ) non-hhd inst’on or factor ac</td>
</tr>
<tr>
<td>( PDD_{c,t} )</td>
<td>demand price for commodity ( c ) produced &amp; sold domestically</td>
<td>( WF_{f,t} )</td>
<td>economywide wage of factor ( f )</td>
</tr>
<tr>
<td>( PDS_{c,t} )</td>
<td>supply price for commodity ( c ) produced &amp; sold domestically</td>
<td>( WFDIST_{f,a,t} )</td>
<td>wage distortion factor for factor ( f ) in activity ( a )</td>
</tr>
<tr>
<td>( PE_{c,t} )</td>
<td>export price (domestic currency)</td>
<td>( WFRES_{f,t} )</td>
<td>reservation wage for factor ( f )</td>
</tr>
<tr>
<td>( PINTA_{a,t} )</td>
<td>aggregate intermediate input price for activity ( a )</td>
<td>( YF_{f,t} )</td>
<td>income of factor ( f )</td>
</tr>
<tr>
<td>( PK_{f,t} )</td>
<td>price of new capital stock ( a )</td>
<td>( YG_{t} )</td>
<td>government revenue</td>
</tr>
<tr>
<td>( PM_{c,t} )</td>
<td>import price (domestic currency)</td>
<td>( YI_{c,t} )</td>
<td>income of domestic non-government institution</td>
</tr>
<tr>
<td>( POP_{i,t} )</td>
<td>population by household</td>
<td>( YIF_{i,f,t} )</td>
<td>income to domestic institution ( i ) from factor ( f )</td>
</tr>
<tr>
<td>( PQ_{c,t} )</td>
<td>composite commodity price</td>
<td>( YINT_{s,t} )</td>
<td>net interest income of institution ( i )</td>
</tr>
<tr>
<td>( PVA_{a,t} )</td>
<td>value-added price (factor income per unit of activity)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
**Table 3.2 Equations for the core CGE module of MAMS Model.**

<table>
<thead>
<tr>
<th>#</th>
<th>Equation</th>
<th>Domain</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Price Block</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(1)</td>
<td>[ PM_{c,t} = pwm_{c,t} \cdot (1 + tm_{c,t}) \cdot EXR_t + \sum_{c' \in C} \left( PQ_{c',t} \cdot icm_{c',c,t} \right) ]</td>
<td>( c \in CM ) ( t \in T )</td>
<td>Import price</td>
</tr>
<tr>
<td>(2)</td>
<td>[ PE_{c,t} = \frac{PWE_{c,t}}{1 - te_{c,t}} \cdot EXR_t - \sum_{c' \in C} \left( PQ_{c',t} \cdot ice_{c',c,t} \right) ]</td>
<td>( c \in CE ) ( t \in T )</td>
<td>Export price</td>
</tr>
<tr>
<td>(3)</td>
<td>( (a) \quad PDS_{c,t} \geq PE_{c,t} \quad (b) \quad QE_{c,t} \geq 0 )</td>
<td>( c \in (CD \cap CECETN) ) ( t \in T )</td>
<td>For non-CET exportables with domestic sales: (a) domestic floor price, (b) non-negative export quantity constraints; and (c) related complementary-slackness relationship.</td>
</tr>
<tr>
<td>(4)</td>
<td>[ PDD_{c,t} = PDS_{c,t} + \sum_{c' \in C} \left( PQ_{c',t} \cdot icd_{c',c,t} \right) ]</td>
<td>( c \in CD ) ( t \in T )</td>
<td>Domestic demander price for domestic commodity</td>
</tr>
<tr>
<td>(5)</td>
<td>[ PQ_{c,t} \cdot (1 - tq_{c,t}) \cdot QQ_{c,t} = PDD_{c,t} \cdot QD_{c,t} + PM_{c,t} \cdot QM_{c,t} ]</td>
<td>( c \in (CD \cup CM) ) ( t \in T )</td>
<td>Absorption</td>
</tr>
<tr>
<td>(6)</td>
<td>[ PX_{c,t} \cdot QX_{c,t} = PDS_{c,t} \cdot QD_{c,t} + PE_{c,t} \cdot QE_{c,t} ]</td>
<td>( c \in (CD \cup CE) ) ( t \in T )</td>
<td>Marketed output value</td>
</tr>
<tr>
<td>(7)</td>
<td>[ PA_{a,t} = \sum_{c \in C} PXAC_{a,c,t} \cdot \theta_{a,c} ]</td>
<td>( a \in A ) ( t \in T )</td>
<td>Activity price</td>
</tr>
</tbody>
</table>
\[
PINTA_{a,t} = \sum_{c \in C} PQ_{c,t} \cdot ica_{c,a} \\

\begin{align*}
\text{aggregate intermediate input price} &= \text{intermediate input cost per unit of aggregate intermediate input} \\
\end{align*}

\text{Aggregate intermediate input price}

\[
PA_{a,t} \cdot (1-ta_{a,t}) \cdot QA_{a,t} = \\

\begin{align*}
\text{activity price (net of taxes) times activity level} &= \text{value-added price times quantity} + \text{aggregate intermediate input price times quantity} \\
\end{align*}

\text{Activity revenue and costs}

\[
\overline{CPI}_t = \sum_{c \in C} PQ_{c,t} \cdot cwts_c \\

\begin{align*}
\text{price index for non-tradables} &= \text{suppliers price for output marketed domestically times weights} \\
\end{align*}

\text{Consumer price index}

\[
DPI_t = \sum_{c \in CD} PDS_{c,t} \cdot dwts_c \\

\text{Price index for non-tradables}

\text{Production and trade block}

\[
QVA_{a,t} = iv_{a_t} \cdot QA_{a,t} \\

\begin{align*}
\text{demand for value-added} &= f \text{ activity level} \\
\end{align*}

\text{Demand for aggregate value-added}

\[
QINTA_{a,t} = int_{a_t} \cdot QA_{a,t} \\

\begin{align*}
\text{demand for aggregate intermediate input} &= f \text{ activity level} \\
\end{align*}

\text{Demand for aggregate intermediate input}

\[
QVA_{a,t} = \text{ALPHAVA}_{a,t} \cdot \left( \sum_{f \in F} \frac{\delta_{f,a} \cdot (fprd_{f,a,t} \cdot QF_{f,a,t})^{\rho^a_{f}}}{\rho^a_{f}} \right)^{\frac{1}{\rho^a_{0}}} \\

\begin{align*}
\text{quantity of aggregate value-added} &= CES \text{ factor inputs} \\
\end{align*}

\text{Value-added}

\[
WF_{f,t} \cdot WFDIST_{f,a,t} = PA_{a,t} \cdot (1-tva_{a,t}) \cdot QA_{a,t} \\

\begin{align*}
\left( \sum_{f \in F} \delta_{f,a} \cdot (fprd_{f,a,t} \cdot QF_{f,a,t})^{\rho^a_{f}} \right)^{\frac{1}{\rho^a_{0}}} \cdot \delta_{f,a} \cdot fprd_{f,a,t} \cdot QF_{f,a,t}^{\rho^a_{1}} \\
\end{align*}

\text{Factor demand}

| 43 |
\[
\begin{align*}
QINT_{c,a,t} &= i\alpha_{c,a,t} \cdot QINTA_{a,t} \\
&= \begin{cases}
\text{intermediate demand} \\
\text{from activity } a
\end{cases}
\quad c \in C
\quad a \in A
\quad t \in T
\quad \text{Disaggregated intermediate input demand}
\end{align*}
\]

\[
QXAC_{a,c,t} + \sum_{h \in H} QHA_{a,c,h,t} = \theta_{a,c} \cdot QA_{a,t}
\quad a \in A
\quad c \in C
\quad t \in T
\quad \text{Commodity production and allocation between market and home}
\]

\[
QX_{c,t} = \alpha_{c} \left( \sum_{a \in A} \delta_{a,c} \cdot QXAC_{a,c,t} \cdot \rho_{c}^{-1} \right)^{-1} \cdot \delta_{a,c} \cdot QXAC_{a,c,t} \cdot \rho_{c}^{-1}
\quad c \in (CE \cup CD)
\quad t \in T
\quad \text{Output aggregation function}
\]

\[
\frac{PXAC_{a,c,t}}{PX_{c,t}} = \frac{QX_{c,t} \cdot \sum_{a \in A} \left( \delta_{a,c} \cdot QXAC_{a,c,t} \cdot \rho_{c}^{-1} \right)^{-1} \cdot \delta_{a,c} \cdot QXAC_{a,c,t} \cdot \rho_{c}^{-1}}{\delta_{c}}
\quad a \in A
\quad c \in C
\quad t \in T
\quad \text{Ratio of prices for output aggregation function}
\]

\[
QX_{c,t} = \alpha_{c} \left( \delta_{c} \cdot QE_{c,t} \cdot \rho_{c}^{-1} + (1 - \delta_{c}) \cdot QD_{c,t} \right)^{-1}
\quad c \in (CD \cap CET)
\quad t \in T
\quad \text{Output transformation (CET) function}
\]

\[
\frac{QE_{c,t}}{QD_{c,t}} = \left( \frac{PE_{c,t} \cdot 1 - \delta_{c}}{PDS_{c,t} \cdot \delta_{c}} \right)^{-1}
\quad c \in (CD \cap CET)
\quad t \in T
\quad \text{Export-domestic supply ratio}
\]

\[
QX_{c,t} = QD_{c,t} + QE_{c,t}
\quad c \in (CD \cap CEN) \cup (CE \cap CD) \cup (CD \cap CECETN),
\quad t \in T
\quad \text{Output transformation for outputs without exports, exports without domestic sales, and non-CET exports with domestic sales}
\]

\[
\begin{align*}
\text{aggregate marketed domestic output} &\quad = \left[ \text{domestic market sales of domestic output for } c \in (CD \cap CEN) \right] + \left[ \text{exports for } c \in (CE \cap CDN) \right]
\end{align*}
\]
\[
Q_E_{c,t} = qe_{c,t} \cdot \left( \frac{PWE_{c,t}}{PWESE_{c,t}} \right)^{\nu_t}
\]

(23) \hspace{1cm} c \in CED \hspace{1cm} \text{Export demand with CE demand function}

\[
QQ_{c,t} = \alpha_t^q \left( \delta_t^q \cdot QM_{c,t}^{\cdot} + (1 - \delta_t^q) \cdot QD_{c,t}^{\cdot} \right)^{\nu_t^{\cdot}}
\]

(24) \hspace{1cm} c \in (CM \cap CD) \hspace{1cm} t \in T \hspace{1cm} \text{Composite supply (Armington) function}

\[
Q M_{c,t} = \frac{PDD_{c,t}}{PM_{c,t}} \cdot \left( \delta_t^q \cdot \left( \frac{1}{1 + \rho_t^{\cdot}} \right) \right)
\]

(25) \hspace{1cm} c \in (CM \cap CD) \hspace{1cm} t \in T \hspace{1cm} \text{Import-domestic demand ratio}

\[
Q Q_{c,t} = Q D_{c,t} + Q M_{c,t}
\]

(26) \hspace{1cm} c \in (CD \cap CMN) \cup (CM \cap CDN), \hspace{1cm} t \in T \hspace{1cm} \text{Composite supply for non-imported outputs and non-produced imports}

\[
Q T_{c,t} = \sum_{c' \in C} \left( icm_{c,c',t} \cdot Q M_{c',t}^{\cdot} + ice_{c,c',t} \cdot Q E_{c',t}^{\cdot} + icd_{c,c',t} \cdot Q D_{c',t}^{\cdot} \right)
\]

(27) \hspace{1cm} c \in CT \hspace{1cm} t \in T \hspace{1cm} \text{Demand for transaction services}

**Domestic institution block**

\[
Y F_{f,t} = \sum_{a \in A} W F_{f,a} \cdot W F D I S T_{f,a,t} \cdot Q F_{f,a,t} + \sum_{f \in F} T R N S F R_{f,raw,t} \cdot E X R_{f,t}
\]

(28) \hspace{1cm} f \in F \hspace{1cm} t \in T \hspace{1cm} \text{Factor income}

\[
SH I F_{i,f,t} = \frac{Q F A C I N S_{i,f,t}}{\sum_{i \in INS} Q F A C I N S_{i,f,t}}
\]

(29) \hspace{1cm} i \in INS \hspace{1cm} f \in F \hspace{1cm} t \in T \hspace{1cm} \text{Institutional shares in factor incomes}

\[
Y I F_{i,f,t} = SH I F_{i,f,t} \cdot \left[ (1 - f_{t,f}) \cdot Y F_{f,t} \right]
\]

(30) \hspace{1cm} i \in INS \hspace{1cm} f \in F \hspace{1cm} t \in T \hspace{1cm} \text{Institutional factor incomes}
<table>
<thead>
<tr>
<th>Equation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( YIINT_{i,t} = gintrat_{i,t} \cdot GDEBT_{i,t} - finrat_{i,t} \cdot FDEBT_{i,t} \cdot EXR )</td>
<td>( i \in \text{INDNG} ) Institutional net interest income ( t \in T )</td>
</tr>
<tr>
<td>( TRI_{i',i,t} = shii_{i',i,t} \cdot (1 - MPS_{i',t}) \cdot (1 - TINS_{i',t}) \cdot YI_{i',t} )</td>
<td>( i \in \text{INS} ) Intra-institutional transfers ( i' \in \text{INDNG} )</td>
</tr>
<tr>
<td>( YI_{i,t} = \sum_{f \in F} YIF_{i,f,t} + \sum_{i' \in \text{INDNG}^*} TRI_{i',i,t} + YIINT_{i,t} )</td>
<td>Income of domestic, non-government institutions</td>
</tr>
<tr>
<td>( + \text{TRANSFR}<em>{i,\text{gov},t} \cdot \text{CPI} + \text{TRANSFR}</em>{i,\text{gov},t} \cdot \text{POP} \cdot \text{CPI} )</td>
<td>( i \in \text{INDNG} ) ( t \in T )</td>
</tr>
<tr>
<td>( + \text{TRANSFR}<em>{i,\text{row},t} \cdot \text{EXR} + \text{TRANSFR}</em>{i,\text{row},t} \cdot \text{POP} \cdot \text{EXR} )</td>
<td></td>
</tr>
<tr>
<td>( TINS_{i,t} = tinsbar_{i,t} \cdot (1 + TINSADJ_{i} \cdot \text{tins01}<em>{i}) + DTINS</em>{i} \cdot \text{tins01}_{i} )</td>
<td>Direct tax rates for domestic non-government institutions</td>
</tr>
<tr>
<td>( MPS_{i,t} = mpsbar_{i,t} \cdot \left( \frac{1 - TINS_{i,t}}{\text{POP}<em>{i,t}} \right)^{\text{ Mormons}} \cdot (1 + \text{MPSADJ}</em>{i} \cdot \text{mps01}_{i}) )</td>
<td>Savings rates for domestic non-government institutions</td>
</tr>
<tr>
<td>( + \text{DMPS}<em>{i} \cdot \text{mps01}</em>{i} )</td>
<td></td>
</tr>
<tr>
<td>( \text{INSSAV}<em>{i,t} = MPS</em>{i,t} \cdot (1 - TINS_{i,t}) \cdot YI_{i,t} )</td>
<td>Savings for domestic non-government institutions</td>
</tr>
</tbody>
</table>
\[ EH_{h,t} = \left(1 - \sum_{i \in \text{INSNG}} shii_{i,h}\right) \cdot \left(1 - MPS_{h,t}\right) \cdot (1 - TINS_{h,t}) \cdot YI_{h,t} \quad h \in H \quad t \in T \]

Household consumption expenditure

\[ QH_{c,h,t} = \frac{POP_{h,t}}{\beta_{c,h}^{m}} \cdot \left(\frac{EH_{h,t}}{POP_{h,t}} - \sum_{c' \in C} PQ_{c',h} \cdot \gamma_{c',h}^{m} - \sum_{a \in A} \sum_{c' \in C} PXAC_{a,c',t} \cdot \gamma_{a,c',h}^{h} \right) \quad c \in C \quad h \in H \quad t \in T \]

Household consumption demand for commodities from market

\[ QHA_{a,c,h,t} = \frac{POP_{h,t}}{\beta_{a,c,h}^{h}} \cdot \left(\frac{EH_{h,t}}{POP_{h,t}} - \sum_{c' \in C} PQ_{c',h} \cdot \gamma_{c',h}^{m} - \sum_{a \in A} \sum_{c' \in C} PXAC_{a,c',t} \cdot \gamma_{a,c',h}^{h} \right) \quad a \in A \quad c \in C \quad h \in H \quad t \in T \]

Household consumption demand for own production

\[ YG_{t} = \sum_{i \in \text{INSNG}} TINS_{i,t} \cdot YI_{i,t} + \sum_{f \in F} tf_{f,t} \cdot YF_{f,t} + \sum_{a \in A} ta_{a,t} \cdot PA_{a,t} + QA_{a,t} \quad t \in T \]

Government recurrent revenue
\[
EG_t = \sum_{c \in C} PQ_{c,t} \cdot QG_{c,t} + \sum_{i \in \text{INS}D\text{NH}} \text{TRANSFR}_{i,\text{gov},t} \cdot CPI_t \]

\[
+ \sum_{h \in H} \text{TRANSFR}_{h,\text{gov},t} \cdot POP_{h,t} \cdot CPI_t + \text{TRANSFR}_{\text{ROW},\text{gov},t} \cdot EXR_t
\]

\[
+ \sum_{i \in \text{INS}} \text{gintrat}_{i,t} \cdot GDEBT_{i,t} + \text{fintrat}_{\text{gov},t} \cdot FDEBT_{\text{gov},t} \cdot EXR_t
\]

\[
\begin{array}{l}
\text{Government recurrent expenditures} \\
\end{array}
\]

\[
QG_{c,t} = QG_{c,t-1}
\]

\[
\begin{pmatrix}
\text{Real government consumption of } c \text{ in } t \\
\end{pmatrix} = \begin{pmatrix}
\text{Real government consumption of } c \text{ in } t-1 \\
\end{pmatrix} \cdot \begin{pmatrix}
1 + \text{qggrwbar}_{c,t} + \sum_{c \in C} qg01_{c,c',t} \cdot QGGRW_{c',t}
\end{pmatrix}
\]

\[
\begin{array}{l}
\text{Real government consumption of } c \text{ in } t \geq 1 \\
\end{array}
\]

\[
QG_{c,t} = \sum_{i \in \text{INS}} \sum_{f \in F} \text{igf}_{c,f,t} \cdot QFINS_{t,f,t}
\]

\[
\begin{array}{l}
\text{Real government consumption of infrastructure services} \\
\end{array}
\]

\[
GSAV_t = YG_t - EG_t
\]

\[
\begin{array}{l}
\text{Government savings} \\
\end{array}
\]

\[
\begin{array}{l}
government savings = government recurrent revenue - government recurrent expenditures \\
\end{array}
\]
### Investment block

\[(a) \quad DKGOV_{f,t} \geq \sum_{a \in A} K_{f,awMFA}^{a} \cdot QA_{a,t} \cdot \exp \left[ \frac{\ln \left( \frac{QA_{a,t}}{QA_{a,t-1}} \right)}{f \in FCAPGOVST} \right] \]

\[\text{[government investment demand for capital f]} \geq \text{[demand for government service capital in t+1: capital coefficient times expected activity level in t+1]}\]

\[+ \left( 1 + qgrrwbar_{f,t} + \sum_{c \in C} qg01_{f,c,t} \cdot QGGRW_{c,t} \cdot QFINS_{gouv, f,t} \right) \]

\[\text{[demand for government infrastructure capital in t+1: growth rate times infrastructure capital stock in t]}\]

\[- QFINS_{gouv, f,t} \cdot (1 - depr_{f,t}) \]

\[- \text{[remaining capital stock (after depreciation) in t+1 if no investment in t]}\]

\[\text{[equation 45]} \quad f \in FCAPGOV \quad t \in T \quad t > 1 \]

\[\text{Real government demand for investment in capital stock f} \]

\[\text{DKGOV}_{f,t} \geq 0 \]

\[\text{[government investment] } \geq \text{[zero]} \]

\[\text{DKGOV}_{f,t} \neq 0 \]

\[\text{DKGOV}_{f,t} = 0 \]

\[(b) \quad DKGOV_{f,t} \geq 0 \]

\[\text{[government investment] } \geq \text{[zero]} \]

\[(c) \quad (DKGOV_{f,t} - DKGOVDEM_{f,t}) \cdot (DKGOV_{f,t} - 0) = 0 \]

where \(DKGOVDEM_{f,t} = \text{right-hand of part (a) of Equation 45}\)

\[\text{Complementary slackness relationship:} \]

1. If government investment exceeds its demand then this investment level is zero.
2. If the government investment level is above zero, then it equals its demand.

\[\text{DKINS}_{gouv, f,t} = DKGOV_{f,t} \]

\[\text{[gross investment in f of institution ins (here "ins" = gov)]} = \text{[gross government investment demand for capital]} \]

\[\text{DKINS}_{gouv, f,t} \text{ where} \]

\[\text{DKINS}_{gouv, f,t} = DKGOV_{f,t} \]

\[\text{[equation 46]} \quad f \in FCAPGOV \quad t \in T \quad t > 1 \]

\[\text{Real government investment in capital stock f (investment by destination)} \]

\[PK_{f,t} = \sum_{c \in C} \text{capcomp}_{c,f,t} \cdot PQ_{c,t} \]

\[\text{[price of new capital stock]} = \text{[total value of commodities c per unit of new capital]} \]

\[\text{[equation 47]} \quad f \in FCAP \quad t \in T \]

\[\text{Price of new capital stock} \]
$$\sum_{f \in FCAPGOV} PK_{f,t} \cdot DKINS_{gov,f,t} = GSAV_{i,t} - \sum_{c \in C} PQ_{c,t} \cdot qdst_{c,gov,t} + GBORTOT_{t}$$  
(48)  

$$t \in T$$  
Government investment value and financing  

$$GBORMS_{i,t} = \left( GBORMSTOT_{t} \right) \cdot GBORTOT_{t}$$  
(49)  

$$i \in INSNG \quad t \in T$$  
Allocation of government bond borrowing across domestic non-government institutions  

$$GBORMS_{i,t} = \sum_{i' \in INSNG} \frac{gbdist_{i'} \cdot INSSAV_{i',t}}{gbdist_{i} \cdot INSSAV_{i,t}} \cdot GBORMSTOT_{t}$$  
(50)  

$$i \in INSNG \quad t \in T$$  
Allocation of the burden of Central Bank borrowing across domestic non-government institutions  

$$INVVAL_{i,t} = INSSAV_{i,t} - \sum_{c \in C} PQ_{c,t} \cdot qdst_{c,t} - GBOR_{i,t}$$  
(51)  

$$i \in INSNG \quad t \in T$$  
Investment financing for non-government institutions  

$$PK_{f,t} \cdot DKINS_{c,f,t} = gfcfshr_{f,t} \cdot INVVAL_{i,t}$$  
(52)  

$$i \in INSNG \quad f \in FCAP \quad t \in T$$  
Non-government investment by capital stock (investment by destination)  

$$QINV_{c,t} = \sum_{f \in FCAP} \left( capcomp_{c,f,t} \cdot \sum_{i \in INS} DKINS_{i,t} \right)$$  
(53)  

$$c \in C \quad t \in T$$  
Total real investment demand by commodity (investment by origin or source)
Constraints for foreign exchange, factors, and commodities

\[
\sum_{c=CM} pwm_{c,t} \cdot QM_{c,t} + \sum_{f \in F} YIF_{row,f,t} \cdot EXR_{t} + \sum_{i \in INSID} TRII_{row,i,t}
\]
\[
\text{import spending} + \text{factor income to Rest of World} + \text{transfers from domestic non-gov institutions to RoW}
\]
\[
+ \text{TRANSFR}_{row,gov,t} + \sum_{i \in INSID} \text{finrat}_{i,t} \cdot FDEBT_{i,t,2}
\]
\[
+ \text{transfers from government to RoW} + \text{interest payment on foreign debt}
\]
\[
= \sum_{c \in CE} PWE_{c,t} \cdot QE_{c,t} + \sum_{i \in INSID} \text{TRANSFR}_{i,row,t} + \sum_{h \in H} \text{transfrpc}_{h,row,t} \cdot POP_{h,t}
\]
\[
= \text{export revenue} + \text{transfers from RoW to domestic non-household institutions} + \text{transfers from RoW to domestic households}
\]
\[
+ \sum_{f \in F} \text{TRANSFR}_{f,row,t} + \sum_{i \in INSID} \text{FBOR}_{i,t} + fdi_{row,t}
\]
\[
+ \text{factor income from RoW} + \text{borrowing from RoW} + \text{grants from RoW} + \text{foreign direct investment}
\]
\[
\sum_{a \in A} QF_{a,f,a,t} = (1 - \text{UERAT}_{f,t}) \cdot \sum_{i \in INS} \text{QFIN}_{i,f,t}
\]
\[
\text{demand for market factor } f \text{ (i.e., employment rate)} \cdot \text{sum of all institutional endowments of factor } f
\]
\[
WFRES_{f,t} = WF_{f,0} \cdot \left( \frac{QHC}{QHC^0} \right)^{q_{f,t}} \cdot \left( \frac{1 - \text{UERAT}^0}{1 - \text{UERAT}_{f,t}} \right)^{\theta_{f,t}} \cdot \left( \frac{CPI}{CPI^0} \right)^{\phi_{f,t}}
\]
\[
\text{reservation wage for factor } f \text{ in year } t = \text{economy-wide wage for factor } f \text{ in the base year} \cdot \text{adjustment due to per-capita household consumption; employment rate; and CPI (all relative to base year values)}
\]
\[
WF_{f,t} \geq WFRES_{f,t} \quad \text{(a)} \quad \text{Wage and (b) unemployment constraints; and (c) related complementary-slackness relationship}
\]
\[
(1 - \text{UERAT}_{f,t}) \cdot \text{UERAT}_{f,t} \geq \text{ueratmin}_{f,t}
\]
\[
\sum_{c=CM} pwm_{c,t} \cdot QM_{c,t} \geq \sum_{f \in F} YIF_{row,f,t} \cdot EXR_{t} + \sum_{i \in INSID} TRII_{row,i,t}
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\[
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\]
\[
+ \text{TRANSFR}_{row,gov,t} + \sum_{i \in INSID} \text{finrat}_{i,t} \cdot FDEBT_{i,t,2}
\]
\[
+ \text{transfers from government to RoW} + \text{interest payment on foreign debt}
\]
\[
= \sum_{c \in CE} PWE_{c,t} \cdot QE_{c,t} + \sum_{i \in INSID} \text{TRANSFR}_{i,row,t} + \sum_{h \in H} \text{transfrpc}_{h,row,t} \cdot POP_{h,t}
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\]
\[
+ \sum_{f \in F} \text{TRANSFR}_{f,row,t} + \sum_{i \in INSID} \text{FBOR}_{i,t} + fdi_{row,t}
\]
\[
+ \text{factor income from RoW} + \text{borrowing from RoW} + \text{grants from RoW} + \text{foreign direct investment}
\]
\[
\sum_{a \in A} QF_{a,f,a,t} = (1 - \text{UERAT}_{f,t}) \cdot \sum_{i \in INS} \text{QFIN}_{i,f,t}
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\text{demand for market factor } f \text{ (i.e., employment rate)} \cdot \text{sum of all institutional endowments of factor } f
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\[
WFRES_{f,t} = WF_{f,0} \cdot \left( \frac{QHC}{QHC^0} \right)^{q_{f,t}} \cdot \left( \frac{1 - \text{UERAT}^0}{1 - \text{UERAT}_{f,t}} \right)^{\theta_{f,t}} \cdot \left( \frac{CPI}{CPI^0} \right)^{\phi_{f,t}}
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\]
\[
WF_{f,t} \geq WFRES_{f,t} \quad \text{(a)} \quad \text{Wage and (b) unemployment constraints; and (c) related complementary-slackness relationship}
\]
\[ QQ_{c,t} = \sum_{a \in A} QINT_{c,a,d} + \sum_{h \in H} QH_{c,h,t} + QG_{c,t} \]

\[ \text{composite supply} = \text{intermediate use} + \text{household consumption} + \text{government consumption} \]

\[ + QINV_{c,t} + \sum_{t \in INS} qdst_{c,i,t} + QT_{c,t} \]

\[ + \text{fixed investment} + \text{stock change} + \text{trade and transport} \]

(58) \[ c \in C \]

\[ t \in T \]

Composite commodity markets

---

Asset stock updating and productivity block

\[ QFINS_{i,f,t} = (1 - depr_{f,t}) \cdot QFINS_{i,f,t-1} + DKINS_{i,f,t-1} + aqfins_{i,f,t-1} \]

\[ \text{stock of capital type } f \text{ held by institution } i \]

\[ \text{non-depreciated capital stock} + \text{fixed investment in } t-1 + \text{exogenous adjustment in capital stock} \]

(59) \[ i \in INS \]

\[ f \in FCAP \]

\[ t \in T, t > 1 \]

Capital stocks by institution

\[ FDEBT_{i,t} = FDEBT_{i,t-1} + FBOR_{i,t-1} \]

\[ + \left( \text{finratdue}_{i,t-1} - \text{finrat}_{i,t-1} \right) \cdot FDEBT_{i,t-1} - \text{fdebtrelief}_{i,t-1} \]

\[ \text{foreign debt in } t \]

\[ \text{foreign debt in } t-1 \]

\[ \text{unpaid interest on foreign debt in } t-1 \]

\[ \text{foreign debt relief in } t-1 \]

(60) \[ i \in INS \]

\[ t \in T \]

\[ t > 1 \]

Foreign debt of domestic institutions

\[ GDEBT_{i,t} = GDEBT_{i,t-1} + GBOR_{i,t-1} \]

\[ \text{stock of government bond held by institution } i \]

\[ \text{redistributed holdings of government bond held by institution } i \text{ in } t-1 \]

\[ \text{government borrowing from } i \text{ in } t-1 \]

(61) \[ i \in INS \]

\[ t \in T \]

\[ t > 1 \]

Government bond holdings of domestic institutions

\[ GDPREAL_t = \sum_{c \in C} \sum_{h \in H} PQ_t \cdot QH_{c,h,t} + \sum_{a \in A} \sum_{c \in C} \sum_{h \in H} PXAC_{a,c,h} \cdot QHA_{a,c,h,t} \]

\[ \text{real GDP} = \text{household market consumption} + \text{household production consumption} \]

\[ + \sum_{c \in C} PQ_t \cdot QG_{c,t} + \sum_{c \in C} PQ_t \cdot QINV_{c,t} + \sum_{c \in C} \sum_{t \in INS} PQ_t \cdot qdst_{c,i,t} \]

\[ + \text{government consumption} + \text{fixed investment} + \text{stock change} \]

\[ + \sum_{c \in CE} EXR^0 \cdot PWE^0 \cdot QE_{c,t} - \sum_{c \in CM} EXR^0 \cdot PWM^0 \cdot QM_{c,t} \]

\[ + \text{exports} - \text{imports} \]

(62) \[ t \in T \]

Real GDP at market prices

\[ TRDGP_t = \frac{\sum_{c \in CE} EXR^0 \cdot PWE^0 \cdot QE_{c,t} + \sum_{c \in CM} EXR^0 \cdot PWM^0 \cdot QM_{c,t}}{GDPREAL_t} \]

\[ \text{ratio of trade to GDP} = \frac{\text{real trade}}{\text{real GDP}} \]

(63) \[ t \in T \]

Real Trade-GDP ratio
### Efficiency (TFP) by activity

**Formula:**

\[
\alpha_A = \alpha_{A1} \cdot \prod_{f \in \text{FCAP}} \left( \frac{\sum_{i \in \text{INS}} QFINS_{i,f,t}}{\sum_{i \in \text{INS}} \sum_{f \in \text{FCAP}} QFINS_{i,f}} \right) \cdot \left( \sum_{f \in T} tfptrdwt_{t,f,t} \cdot TRDGDP_{t} \right) \cdot \left( \frac{\text{TRDGDP}^0}{tfpelastr_{t}} \right)
\]

- **Efficiency:**
  - **Trend term for activity:**
    - **Product of:** ratio of all current real capital endowment \( f \) to initial value, raised to the relevant elasticity
    - **Weighted avg. (over time):** ratios of openness to initial value, raised to the relevant elasticity

### TFP trend term by activity

**Formula:**

\[
\alpha_{A2} = \alpha_{A21} \cdot \left( 1 + \alpha_{A2}^{\text{var}} + \text{CALTFPG}_{t} \cdot tfp01_{t} \right)
\]

- **Trend term for activity:**
  - **Growth adjustment factor:**

### Real GDP at factor cost

**Formula:**

\[
GDP_{\text{REALFC},t} = \sum_{a \in A} PVA_{a,t} \cdot (1 - \text{tva}^0_{a,t}) \cdot QVA_{a,t}
\]

- **Real GDP:**
  - **Value-added net of taxes:**

### Table

<table>
<thead>
<tr>
<th>Equation</th>
<th>Description</th>
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<tr>
<td>(64)</td>
<td>Efficiency (TFP) by activity</td>
</tr>
<tr>
<td>(65)</td>
<td>TFP trend term by activity</td>
</tr>
<tr>
<td>(66)</td>
<td>Real GDP at factor cost</td>
</tr>
</tbody>
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### Table 3.3  Notation for MDG module of MAMS model

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Explanation</th>
<th>Symbol</th>
<th>Explanation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a \in A$</td>
<td>Activities</td>
<td>$i \in INSNGAGG$</td>
<td>aggregate (domestic) non-government institution</td>
</tr>
<tr>
<td>$b \in B$</td>
<td>student behavioral characteristics = {$rep = \text{repeater}; , dropout = \text{dropout}; , prom = \text{promotion}; , grdcont = \text{continuing graduate}; , grdexit = \text{exiting graduate}; , neting1 = \text{net \text{(in-cohort) intake to grade 1}}$</td>
<td>$b, b' \in MBB$</td>
<td>mapping between $b$ (in $BRES$) and $b'$ (in $BLOG$): =${(rep, \text{dropout}).grd, , grdexit.grdcont}$</td>
</tr>
<tr>
<td>$b \in BLOG \quad (\subseteq B)$</td>
<td>student behavior determined by logistic function = {$prom, , grdcont, , neting1$}</td>
<td>$b, b' \in MBB2$</td>
<td>mapping between $b$ (in $BRES$) and all elements $b'$ (also in $BRES$) that are related to the same element(s) in $BLOG$: =${rep.(rep, \text{dropout}), , dropout.(rep, \text{dropout}), , grdexit.grdexit}$</td>
</tr>
</tbody>
</table>
| $b \in BRES \quad (\subseteq B)$ | student behavior determined by residual scaling = {$rep = \text{repeater}; \, dropout = \text{dropout}; \, grdexit = \text{exiting graduate}$} | $c, c' \in MCE$ | mapping private and public education into 1 education commodity, by cycle = {$c\text{-edup}.(c\text{-edup}, \, c\text{-edupng})$} where $c\text{-edupng}$ is private primary; similarly for $c\text{-edup2}$, $c\text{-edus}$, and $c\text{-edut}$.

<p>| $c \in C$ | Commodities | $c, c' \in MCHDC$ | human development service $c$ is aggregated to $c'$ |
| $c \in CEDU \quad (\subseteq C)$ | education services = {$c\text{-edup} = \text{primary cycle}; , c\text{-edus} = \text{secondary cycle}; , c\text{-edut} = \text{tertiary cycle}$}; may include both private and public education | $c, c' \in MCM$ | mapping between aggregate (CMDG) and disaggregated MDG service commodities (CHLTH and CWTSN) = {$c\text{-hlt}1g, , c\text{-hlt2g}, , c\text{-hlt3g}, , c\text{-hlt1ng}, , c\text{-hlt2ng}, , c\text{-hlt3ng}$} and {$c\text{-wtsn}.(c\text{-wtsn})$} |
| $c \in CEDUP \quad (\subseteq C)$ | primary education services = {$c\text{-edup}$} | $mdg \in MDG$ | selected MDG indicators ={$mdg2, , mdg4, , mdg5, , mdg7w, , mdg7s$} |
| $c \in CEDUT \quad (\subseteq C)$ | tertiary education services = {$c\text{-edut}$} | $c, b, t', t' \in MCYC$ | MDG2 in $t$ is defined as the product over selected combinations of $b$ and $t'$ (where $t' \in T11$) = {$prom, , neting1$} |
| $c \in CHLTH \quad (\subseteq C)$ | health services (public) = {$c\text{-hlt1g} = \text{low-tech}; , c\text{-hlt2g} = \text{medium-tech}; , c\text{-hlt3g} = \text{high-tech}$}; corresponding private health services labeled with &quot;ng&quot; | $mdg \in MDGSTD$ | MDG indicators ={$mdg4 = \text{under-5 mortality rate}; , mdg5 = \text{maternal mortality rate}; , mdg7w = \text{access to safe water}; , mdg7s = \text{access to basic sanitation}$ |</p>
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>cmdg ∈ CMDG</td>
<td>aggregate MDG (non-education) service commodities = {c-hlt = aggregate health in MDG functions, not in C; c-wtsn = water-sanitation services}</td>
<td></td>
</tr>
<tr>
<td>c ∈ CWTSN (⊂ C)</td>
<td>water-sanitation service commodities {c-wtsn = water-sanitation services}</td>
<td></td>
</tr>
<tr>
<td>eduarg ∈ EDUARG</td>
<td>arguments in CE function for educational behavior ={edu-qual = qty of services per student; w-prem = skilled-unskilled wage ratio; w-prem2 = superskilled-skilled wage ratio; mdg4 = under-five mortality rate; fcapinf = infrastructure capital stocks; qhpc = per-capita hhd consumption}</td>
<td></td>
</tr>
<tr>
<td>f ∈ FEXOG</td>
<td>factors with exogenous growth</td>
<td></td>
</tr>
<tr>
<td>f ∈ FLAB</td>
<td>labor factors {f-labn = less than 12 yrs of education; f-labs = 12-14 yrs of education (secondary education or 2 years of tertiary); f-labt = more than 14 yrs of education (at least 3 years of tertiary)}</td>
<td></td>
</tr>
<tr>
<td>g ∈ G</td>
<td>grade (in a cycle of schooling)</td>
<td></td>
</tr>
<tr>
<td>c, g ∈ MCGLAB</td>
<td>students in cycle c, grade g are in labor force age</td>
<td></td>
</tr>
<tr>
<td>c, g ∈ MCGLABENT</td>
<td>students in cycle c, grade g are in labor force entry age</td>
<td></td>
</tr>
<tr>
<td>c, g ∈ MCGMAX</td>
<td>grade g is the last (maximum) grade in cycle c</td>
<td></td>
</tr>
<tr>
<td>c, g ∈ MCGMIN</td>
<td>grade g is the first (minimum) grade in cycle c</td>
<td></td>
</tr>
<tr>
<td>mdgarg ∈ MDGARG</td>
<td>arguments in CE function for MDGs ={cmdg = agg commodities; mdg = different MDGs; fcapinf = infrastructure capital stocks; qhpc = per-capita hhd consumption}</td>
<td></td>
</tr>
<tr>
<td>h ∈ H</td>
<td>households (excl. NGOs) ={h = the single household}</td>
<td></td>
</tr>
<tr>
<td>i ∈ INSG</td>
<td>government institution</td>
<td></td>
</tr>
<tr>
<td>t ∈ T</td>
<td>time periods including preceding years for MDG2 calculation</td>
<td></td>
</tr>
<tr>
<td>t ∈ T11</td>
<td>time periods</td>
<td></td>
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</tbody>
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**PARAMETERS**

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<td>constant in logistic function for educational behavior</td>
<td></td>
</tr>
<tr>
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<td>constant in CE function for educational behavior</td>
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<tr>
<td>( \alpha_{mdg}^{mdg} )</td>
<td>constant in logistic function for MDG achievement</td>
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</tr>
<tr>
<td>( \alpha_{mdgce}^{mdg} )</td>
<td>constant in CE function for intermediate MDG variable</td>
<td></td>
</tr>
<tr>
<td>( \alpha_{h}^{hd} )</td>
<td>efficiency term in CES aggregation function for human development</td>
<td></td>
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<tr>
<td>( \beta_{mdg}^{log} )</td>
<td>constant in logistic function for MDG achievement</td>
<td></td>
</tr>
<tr>
<td>$\delta_{c,i}^{hd}$</td>
<td>share parameter for HD CES function</td>
<td></td>
</tr>
<tr>
<td>------------------</td>
<td>-----------------------------------</td>
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</tr>
<tr>
<td>$\varphi_{b,c,edu}^{edu}$</td>
<td>elasticity of behavior $b$ in cycle $c$ w.r.t. argument $edu$ in educational CE function</td>
<td></td>
</tr>
<tr>
<td>$\varphi_{mdg,mdgarg}^{mdg}$</td>
<td>elasticity of $mdg$ w.r.t. argument $mdgarg$ in CE function for MDG</td>
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</tr>
<tr>
<td>$\gamma_{b,c}$</td>
<td>parameter in logistic function for education</td>
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</tr>
<tr>
<td>$\gamma_{mdg}$</td>
<td>parameter in logistic function for non-education MDGs</td>
<td></td>
</tr>
<tr>
<td>$\rho_{c}^{hd}$</td>
<td>exponent in CES aggregation function for human development</td>
<td></td>
</tr>
<tr>
<td>$demot_{c,c'}$</td>
<td>cycle $c$ is the cycle preceding $c'$</td>
<td></td>
</tr>
<tr>
<td>$depr_{f,t}$</td>
<td>depreciation rate for factor $f$</td>
<td></td>
</tr>
</tbody>
</table>

### VARIABLES

<table>
<thead>
<tr>
<th>$EDUQUAL_{c,t}$</th>
<th>educational quality in cycle $c$ in year $t$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$QFACINS_{c,f,t}$</td>
<td>endowment of labor type $f$ for institution $i$ in $t$</td>
</tr>
<tr>
<td>$EG_{t}$</td>
<td>government expenditures</td>
</tr>
<tr>
<td>$INVVAL_{i,t}$</td>
<td>investment value for institution $i$</td>
</tr>
<tr>
<td>$QH_{c,b,h,t}$</td>
<td>consumption of commodity $c$ in $t$ by household $h$</td>
</tr>
<tr>
<td>$MDGVAL_{mdg,t}$</td>
<td>value for MDG indicator $mdg$ in $t$</td>
</tr>
<tr>
<td>$QHA_{a,c,h,t}$</td>
<td>quantity consumed of home commodity $c$ from activity $a$ by household $h$</td>
</tr>
<tr>
<td>$QHPC_{i,t}$</td>
<td>Per-capita household consumption in $t$</td>
</tr>
<tr>
<td>$PQ_{c,t}$</td>
<td>price of commodity $c$ in $t$</td>
</tr>
<tr>
<td>$QQ_{c,t}$</td>
<td>quantity of goods supplied to domestic market (composite supply)</td>
</tr>
<tr>
<td>$PXAC_{a,c,t}$</td>
<td>price of commodity $c$ from activity $a$</td>
</tr>
<tr>
<td>$SHR_{b,c,t}^{edu}$</td>
<td>share of students in cycle $c$ with behavior $b$ in $t$</td>
</tr>
<tr>
<td>$QENR_{c,t}$</td>
<td>total number of students enrolled in cycle $c$ in year $t$</td>
</tr>
<tr>
<td>$WF_{f,t}$</td>
<td>economy-wide wage for factor $f$ in $t$</td>
</tr>
<tr>
<td>$QENR_{c,t}^{old}$</td>
<td>number of old students enrolled in cycle $c$ in year $t$</td>
</tr>
<tr>
<td>$ZEDU_{b,c,t}$</td>
<td>intermediate variable for educational outcome (defined by CE function; entering logistic function)</td>
</tr>
<tr>
<td>$QENR_{c,t}^{new}$</td>
<td>number of new students enrolled in cycle $c$ in year $t$</td>
</tr>
<tr>
<td>$ZMDG_{mdg,t}$</td>
<td>intermediate variable for standard MDGs (4-5-7w-7s) (defined by CE function; entering logistic function)</td>
</tr>
</tbody>
</table>
Table 3.4 Equations for MDG module of MAMS model

<table>
<thead>
<tr>
<th>#</th>
<th>Equation</th>
<th>Domain</th>
<th>Description</th>
</tr>
</thead>
</table>
| 67 | \[
QHD_{c,i,j} = \sum_{c}^{i} \left( \frac{QG_{c,i,j}}{c} \right) + \sum_{c}^{i} \left( \frac{\left( QQ_{c,i,j} \right)}{c} \right) \] |        | Separation of human development (HD services into government and non-government) |
| 68 | \[
QHDAGG_{c,d} = \alpha^h_{c,d} \sum_{i}^{C} \left( QHD_{c,i,t} \right) \] |        | Aggregation of human development (HD) services (i.e., MDG and education)     |
| 69 | \[
QHPC_t = \frac{\sum_{c} \left( PQ^0_{c} \cdot QH_{c,h,t} \right) + \sum_{a} \sum_{c} \sum_{h} \sum_{t} \left( PXAC^0_{a,c,h,t} \cdot \frac{QHA_{a,c,h,t}}{pop} \right) }{t} \] |        | Real household consumption per capita                                      |
| 70 | \[
EDUQUAL_{c,t} = \frac{QHDAGG_{c,t}}{\sum_{c} QENR_{c,g,t}} / \sum_{c} QENR_{c,g} \] |        | Educational quality                                                        |
| 71 | \[
QENR_{c,g,t} = \sum_{g} \left( \sum_{g'} \left( \frac{QENR_{c,g,t}}{g'g} \right) \right) + \sum_{g} \left( SHR_{pass,c,g,t-1} \cdot QENR_{c,g,t-1} \right) \] |        | Old enrolled students                                                      |
The $\alpha$ and $\beta$ parameters in the logistic functions (equations 74 and 82) have been calibrated so that (i) under base-year conditions, the left-hand side variables (showing student behavior shares or MDG values) will replicate base-year values; and (ii) under conditions derived from supporting studies, the left-hand side variables will take on values indicative of or compatible with MDG achievement.
\[ ZEDU_{b,c,t} = \alpha_{b,c}^{edu} \cdot \left( \text{EDUQUAL}_{c,t} \right)^{\phi_{b,c,edu-qual}^{edu}} \cdot \left( \frac{\text{WF}_{f-labs,t}}{\text{WF}_{f-lab, b,t}} \right)^{\phi_{b,c,w-prem}^{edu}} \cdot \left( \frac{\text{WF}_{f-lab,t}}{\text{WF}_{f-labs,t}} \right)^{\phi_{b,c,w-prem}^{edu}} \cdot \text{MDGVAL}_{mdg4,t}^{b,c} \]

\[ \prod_{f \in \text{FCAPGOVINF} \setminus \text{INS}} \left( \sum_{i \in \text{INS}} QFINS_{i,f,t}^{b,c} \right) \cdot \text{QHPC}_{t}^{b,c} \cdot \text{MDGVAL}_{mdg4,t}^{b,c} \]

Student behavior (CE function defining intermediate variable)\(^{41}\)

\[ \text{SHR}_{b,c,t}^{edu} = \left( 1 - \sum_{b' \in \text{BLOG}} \sum_{(b,b') \in \text{MBB}} \text{SHR}_{b,c,t}^{edu} \right) \left( \sum_{b' \in \text{BRES}} \text{SHR}_{b,c,t}^{edu} \right) \]

\[ \text{MDGVAL}_{mdg2,t} = \prod_{b,c \in \text{CEDU} \setminus \text{BLOG}} \text{SHR}_{b,c,t}^{edu} \]

MDG 2

\[ \text{labparrat}_{t} = \frac{\sum_{i \in \text{INS} \setminus \text{FLAB}} QFINS_{i,f,t}}{\sum_{c \in \text{CEDU} \setminus \text{BLOG}} \sum_{g \in \text{QLAB}} QENR_{c,g,t}} \]

Labor Force Participation Rate

\[^{41}\] In the computer program, equations 75 and 81 (constant-elasticity functions defining intermediate variables for educational behavior or MDG achievement) are more complex in two respects. First, the terms that are raised to exponents, which represent elasticities, are all divided by base-year values. This formulation was preferred given our desire to simulate scenarios with changes in elasticities but without any changes in simulated base-year values for left-hand side variables. Second, for the element gridcont ∈ BLOG, the decision to continue to the next education cycle depends on the values for the right-hand side variables that correspond to the next cycle.
\[
QFINS_{i,f,t} = shif_{i,f,t}^{d}
\]

\[
\text{endowment of labor type } f \text{ for institution } i \text{ in } t = \text{share of } i \text{ in labor type } f
\]

\[
\cdot \left(1 - \text{depr}_{f,t-1} \cdot QFLADJ_T \right) \cdot \sum_{i \in \text{INS}} QFINS_{i,f,t-1}
\]

\[
\cdot \left[ \text{non-retired labor from previous year} \right]
\]

\[
+ \sum_{c,c' \in C, f \in \text{FLAB}, t \in \text{T}} \text{demot}_{c,c'} \cdot shr_{c,t}^{labent}
\]

\[
\cdot \sum_{c \in \text{CEDUT}} SHR_{\text{edu}, \text{c,t-1}} \cdot \sum_{c \in \text{CEDUT}} SHR_{\text{edu}, \text{prom,c,t-1}} \cdot QENR_{c,g,t-1}
\]

\[
\cdot \left[ \text{enrolled in non-tertiary cycle in } t-1, \text{who pass their last grade, exit the school system, and enter labor force in } t \right]
\]

\[
+ \sum_{c \in \text{CEDUT}} \left( shr_{c,t}^{labent} \cdot \sum_{c \in \text{CEDUT}} SHR_{\text{educ}, \text{c,t-1}} \cdot QENR_{c,g,t-1} \right)
\]

\[
+ \sum_{c \in \text{CEDUT}} \text{demot}_{c,c'} \cdot shr_{c't}^{labent} \cdot \sum_{c \in \text{CEDUT}} SHR_{\text{edu}, \text{dropout,c,t-1}} \cdot QENR_{c',g,t-1}
\]

\[
+ \left[ \text{enrolled in tertiary cycle in } t-1, \text{who graduate and enter the labor force in } t \right]
\]

\[
+ shr_{f,t}^{labent2} \cdot \sum_{c \in \text{CEDUT}} \sum_{g \in \text{GRD}} \left( QENR_{c,g,t} \right)
\]

\[
+ \left[ \text{enrolled in school in } t-1, \text{who drop out and enter labor force in } t \right]
\]

\[
+ \left[ \text{entrants from outside educational system who are of labor-force-age} \right]
\]

\[
\text{MDGVAL}_{mdg,t} = \text{ext}_{mdg}^{t} + \frac{\alpha_{mdg}^{t}}{1 + \text{EXP} \left( \gamma_{mdg}^{t} + \beta_{mdg}^{t} \cdot ZMDG_{mdg}^{t} \right)}
\]

\[
\begin{bmatrix}
\text{MDG}_{t} \\
\text{value}
\end{bmatrix} = \left[ \text{logistic function of intermediate} \right]
\]

\[
\text{MDG value} \left( ZMDG_{mdg,t} \right)
\]

\[
\text{MDGs 4, 5, 7w, and 7s}
\]

\[
\text{(logistic function)}
\]

60
\[
Z_{MDG}^{mdg,t} = \alpha_{mdg} \cdot \Pi_{cmdg \in CMDG} \left( \sum_{c \in C_{cmdg,c} \in MCM} \frac{QQ_{c,t}^{cmdg}}{pop_{t}^{tot}} \right) ^{\phi_{mdg,cmdg}} \\
\cdot \Pi_{FCA\times GOV\times INF} \left( \sum_{i \in INS} Q_{FINS}^{i,f,t} \right) ^{\phi_{mdg,f}} \\
\cdot \Pi_{MDGSTD}^{mdg} MDGVAL^{\phi_{mdg,mdg'}}^{mdg'} \cdot Q_{HPC}^{\phi_{mdg,bbd\times ccpc}}^{t}
\]

[intermediate variable for MDGs 4 and 5] = [exogenous parameter]

influence of: real value for services per capita; level of infrastructure; water and sanitation MDGs; household consumption per capita

MDGs 4, 5, 7w, and 7s (CE function defining intermediate variable)