Inequality and Poverty Simulations within the Lorenz Framework

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The importance of poverty and distributional issues in policymaking creates the need for empirical tools for assessing the impact of economic shocks and policies on the distribution of economic welfare. The Lorenz curve provides an integrative framework for both the simulation and ethical evaluation of inequality and poverty. The same framework underlies the assessment of the social impact of economic growth. This paper reviews the structure and the normative underpinnings of the Lorenz curve, and demonstrates the use of numerical integration to recover inequality, poverty and pro-poor growth measures from information about the mean and the Lorenz curve. This computational approach makes it unnecessary to derive special expressions for the relevant indicators from the chosen functional form of the Lorenz curve. The approach is implemented in EViews 5.1, based on the general quadratic model and aggregate data for India and Indonesia. The source code is provided in an annex.

1. Introduction

Distributional issues are at the heart of policymaking because of *heterogeneity of participants.* Participants in the policy process may have different *tastes* over goods and services, different factor *endowments*, or different *views* on what policies would best achieve a given goal (Drazen 2000). Even when socioeconomic agents have the same preferences and initial endowments, their interests may conflict over the *distributional consequences* of the policy in question[1]. Such conflicts could throw the whole process off track as Aristotle noted long ago: "*It is when equals have or are assigned unequal shares, or people who are not equal, equal shares, that quarrels and complaints break out.*" (Quoted in Young 1994). Kanbur (1994) echoes the same observation by noting that, the likelihood of such quarrels hinges crucially on the threshold at which a gain or a loss becomes so significant that an individual or a group feels compelled to organize and fight.

Besides political economy considerations, the centrality of distributional issues to policymaking also stems from the fact that *inequality and poverty reduction* is a genuine objective of public policy. Empowerment is now emerging as the organizing concept underpinning development thought, policies and programs (Sen 1989, 1999, World Bank 2000). This vision entails expansion of the ability of an individual or group to achieve their freely chosen life plans. In this perspective, poverty is seen as the deprivation of basic capabilities to live the kind of life one has reason to value (Sen 1999). Thus the international community has declared *poverty eradication* a basic objective of development and therefore a benchmark measure of the performance of socioeconomic systems.

The importance of distributional issues in policy making creates a need for empirical tools for assessing the distributional impact of shocks and policies. It is common to think of the distribution of income (or any other welfare indicator) over a large population in terms of the distribution of a random variable. Any such distribution is basically characterized by its *size* (mean) and *relative inequality*. The Lorenz curve is

[1] Drazen (2000) refers to differences in preferences, endowments and views of the world as *ex-ante heterogeneity* while conflicts over distribution constitutes *ex-post heterogeneity*. He further explains that both types of heterogeneity are not necessarily mutually exclusive. In the case of the provision of a public good, there may be ex-ante conflict over the importance of this good relative to others that could be provided with the same resources as well as ex-post conflict as to the distribution of the cost of providing it.

arguably the workhorse of distributional analysis. Assume that individuals are ranked in increasing order of an indicator of economic welfare (income or expenditure), the Lorenz curve maps the *cumulative proportion of the population* on the horizontal axis against the *cumulative share of welfare* on the vertical axis. It is an indicator of relative inequality and a key component of a social evaluation criterion.

Consider two states of the world, one induced by the implementation of a policy and the other associated with the counterfactual. If the Lorenz curve for the policy state lies everywhere above the counterfactual one, a configuration known as *Lorenz dominance*, then inequality has decreased as a result of the policy. This is a purely descriptive statement. In order to say that social welfare has also improved, both the size and the inequality effects must be taken into consideration. The *generalized Lorenz curve* is the appropriate construct and dominance the necessary configuration for an unambiguous social welfare improvement according to the class of transfer approving social evaluation functions (Lambert, 2001).

The purpose of this paper is to show how to recover most inequality and poverty measures from the mean of a distribution of a welfare indicator and a parameterization of the associated Lorenz curve. Our point of departure is Datt (1992, 1998). This author shows how to compute members of the Foster-Greer-Thorbecke (FGT) family of poverty measures, along with associated elasticity with respect to growth and changes in the Gini coefficient. For the computation of these measures, Datt uses mathematical expressions derived from a parameterization of either the Beta Lorenz curve (Kakwani 1980) or the General Quadratic Lorenz curve (Villasenor and Arnold 1984). This approach underlies the algorithms implemented by simulation tools such as POVCAL (Chen, Datt and Ravallion 1991), and SimSIP Poverty (Ramadas, van der Mensbrugghe, and Wodon 2002). POVCAL is programmed in Microsoft Fortran 5.0, while SimSIP Poverty runs in Excel. Our simulation strategy is a modification of Datt's approach, and significantly widens its scope. From a parameterization of the Lorenz curve, we compute the associated first and second order derivatives. We then combine these results with an estimate of the mean of the distribution to recover levels of the welfare indicator (using the first order derivative) along with an estimate of the density function (based on the second order derivative). Numerical integration allows the computation of the desired measures based on their standard mathematical definitions. Thus there is no need to consider the mathematical expressions of inequality and poverty measures implied by the chosen Lorenz model.

The outline of the paper is as follows. Section 2 presents the mathematical structure of the Lorenz curve and explains the parameterization process based on the *general quadratic model*. The section also includes a discussion of the normative underpinnings of Lorenz dominance. The computation of inequality and poverty measures is discussed in section 3. In particular, we compute the following inequality measures: the extended Gini coefficient, members of the generalized entropy family of inequality measures and the Atkinson index of inequality. For poverty analysis, we show how to compute members of a class of additively separable poverty measures including: FGT, Watts and Chakravarty. We also discuss a graphical device known as the TIP curve (for the Three "I"s of Poverty, Jenkins and Lambert 1997) that provides simultaneous representation of the incidence, intensity and inequality dimension of poverty. Section 4 demonstrates the usefulness of this approach in assessing the social impact of economic growth. All computations are performed in EViews 5.1 and the computer code is presented in the appendix. Concluding remarks are presented in section 5.

2. The Lorenz Curve

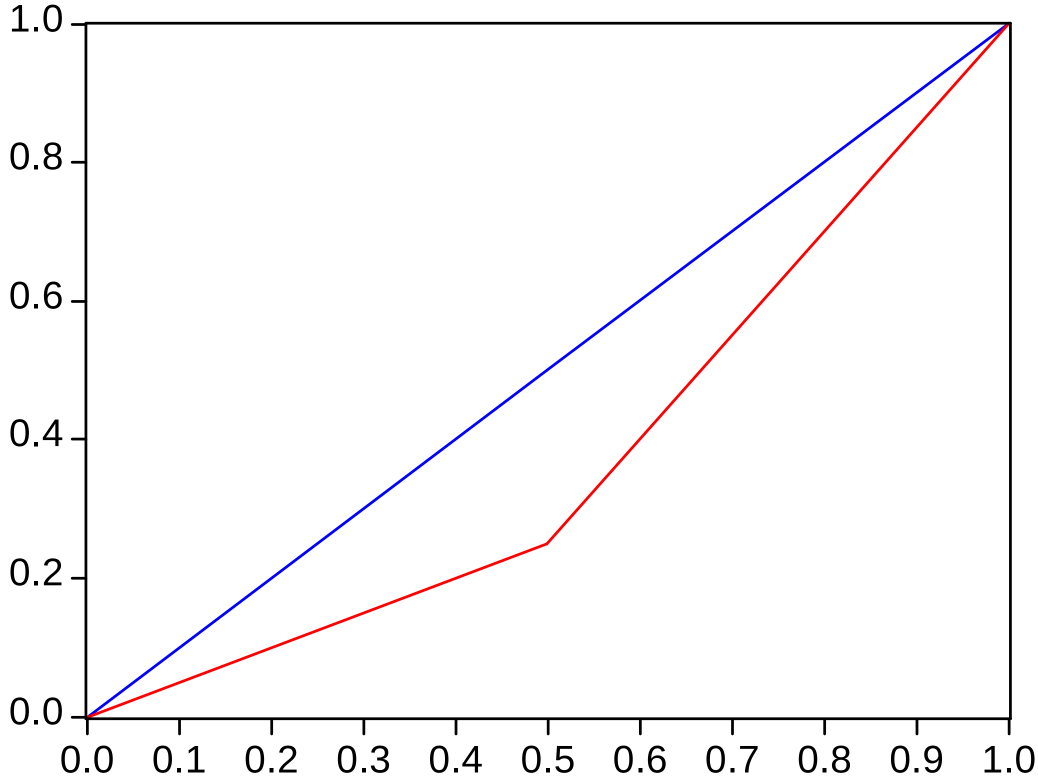
We start with a review of the *structure* of the Lorenz curve based on the simplest case of a distribution among two individuals. We then discuss *parameterization* using the General Quadratic model. Finally, we consider the *normative underpinnings* of Lorenz dominance.

2.1. Structure

The information content of a cumulative distribution function (CDF) can be transformed into a Lorenz curve. Assume that all individuals are ranked in ascending order of the welfare indicator. Let **p** stand for the poorest **100p** percent of the population, and **L(p)**, the share of total welfare going to this segment of the population. The Lorenz curve maps the cumulative proportion **p** of the population on the horizontal axis (starting from the poorest) against the cumulative share of welfare **L(p)** on the vertical. Before considering a general case, we illustrate the basic ideas with the following simplest distribution.

Suppose that there are only two individuals and the total level of income is equal to 100. One individual receives 25 and the other 75. Table 2.1 below contains the relevant data for the construction of the Lorenz curve. The corresponding Lorenz curve is shown in figure 2.1.

Figure 2.1. Lorenz Representation of a Two-Person Distribution



Cumulative Proportion of Population

Figure 2.1 reveals that the Lorenz curve associated with this two-person distribution is combination of two linear segments with a kink at the point (0.50, 0.25). The analytical expression for this curve may be written according to expression (2.1).

$$L(p) = \delta L_1(p) + (1-\delta)L_2(p); \quad p \in [0,1] \tag{2.1}$$

where δ is a dummy variable equal to 1 if p is at most equal to 0.5, and zero otherwise.

Table 2.1. A Two-Person Income Distribution

Income Level	Relative Frequency	Cumulative Frequency	Cumulative Share
0.00	0.00	0.00	0.00
25.00	0.50	0.50	0.25
75.00	0.50	1.00	1.00

Source: Made up numbers.

The first segment of the Lorenz curve is defined by:

$$L_1(p) = ap; \quad p \le 0.5 \tag{2.2}$$

The second segment is given by the following expression:

$$L_2(p) = bp + (1-b); \quad 0.5 < p \le 1.0 \tag{2.3}$$

Given that these two segments are linear, we can use the “rise over run” approach to computing the slopes **a** and **b**. The slope of the first segment is equal to the income share of the first individual divided by his share in the population. Hence,

$$a = \frac{2x_1}{(x_1 + x_2)} = \frac{x_1}{\mu} = 0.5 \tag{2.4}$$

where **μ** is the overall mean of the distribution. Similarly,

$$b = \frac{2x_2}{(x_1 + x_2)} = \frac{x_2}{\mu} = 1.5 \tag{2.5}$$

It is instructive to note from (2.4) that, if **a** is equal to one, then $\mathbf{x_1}=\mathbf{x_2}$ and **b =1**. Thus, the Lorenz curve will coincide with the 45-degree line of complete equality. Analytically, this is expressed as: $L(p) = p$. This information reveals that the coefficient of the argument **p** at a given point represents the extent to which the share of that percentile deviates from equal share. In this simple case of a two-person distribution, a measure of inequality is obtained by subtracting the income share of the poorest individual from his or her population share. In the above example, this indicator which is equal to (0.5 -0.25)=0.25. This happens to be the value of the Gini coefficient in this particular case.

The following expression defines the slope of the Lorenz curve over the entire domain of (2.1):

$$\frac{\Delta L(p)}{\Delta p} = \delta \frac{x_1}{\mu} + (1-\delta)\frac{x_2}{\mu} \tag{2.6}$$

Expression (2.6) suggests that the *slope of a Lorenz curve* at a given percentile is equal to the ratio of the corresponding income level to the overall mean income. This fact will be confirmed in the more general case that we consider next.

The rate of change of the slope of this simple Lorenz curve is given by the following expression:

$$\frac{\Delta^2 L(p)}{\Delta p^2} = \frac{1}{\mu}\left[\delta \frac{\Delta x_1}{\Delta p} + (1-\delta)\frac{\Delta x_2}{\Delta p}\right] \tag{2.7}$$

We note that each term on the right hand side of (2.7) is equal to the inverse of the rate of change of the cumulative frequency with respect to **x** (i.e. the inverse of the relative frequency at **x**). Column 3 of table 2.1 shows that the relative frequency **f(x)** is constant and equal to 0.5. Therefore, the rate of change of the slope of this Lorenz curve is equal to:

$$\frac{\Delta^2 L(p)}{\Delta p^2} = \frac{1}{\mu}\left[\delta\left(\frac{\Delta p}{\Delta x_1}\right)^{-1} + (1-\delta)\left(\frac{\Delta p}{\Delta x_2}\right)^{-1}\right] = \frac{1}{0.5\mu} = \frac{1}{\mu f(x_i)} = \frac{2}{\mu}, \; i = 1, 2. \tag{2.8}$$

In the case of discrete observations on the distribution of income among **n** people, the Lorenz curve is defined by the following expression:

$$L(p) = \frac{\sum_{k=1}^{j} x_k}{\sum_{k=1}^{n} x_k} = \frac{j\mu_j}{n\mu} = \frac{\mu_p}{\mu} p; \quad p = \frac{j}{n}; \; L(0) = 0; \; L(1) = 1 \tag{2.9}$$

All other points on the Lorenz curve are obtained by linear interpolation, just as in the simple case of figure 2.1. In this case, the local coefficient of **p** (analogous to **a** or **b** above) is equal to the mean of the poorest proportion **p** of the population divided by the overall mean of the distribution.

Expression (2.9) of the Lorenz curve also reveals the following facts. As we move from the poorest individual to the richest, the proportion of the population cumulates at a constant rate of **1/n**, while the proportion of welfare is accumulating at a variable rate of $\mathbf{x_k/n\mu}$. Therefore, the slope of the Lorenz curve at point **p** can be approximated by:

$$\frac{\Delta L(p)}{\Delta p} = \frac{x_j}{\mu} \tag{2.10}$$

This confirms that the slope of the Lorenz curve is equal to one when $\mathbf{x_j}$ is equal to the mean of the distribution. It can be shown that as **p** varies, this slope changes at a rate which is approximately equal to:

$$\frac{1}{\mu}\frac{\Delta x_j}{\Delta p} = \frac{1}{\mu\frac{\Delta p}{\Delta x_j}} = \frac{1}{\mu f(x_j)} = \frac{n}{\mu} \tag{2.11}$$

where $\mathbf{f(x_j)}$ stands for the density at $\mathbf{x_j}$. Expressions (2.10) and (2.11) thus reveal that the Lorenz curve is increasing and convex towards the south-east corner of the diagram representing it.

If we model the distribution of income by a smooth frequency density function **f(x)**, then **f(x)dx** is interpreted as the proportion of people whose income lies in the close interval **[x, dx]** for an income level **x** and an infinitesimal change **dx** (Lambert 2001). This is the bedrock of the simulation approach that we propose here.

The distribution function associated with such a smooth density is defined by the following expression:

$$F(x) = \int_0^x f(t)dt; \quad F'(x) = f(x) \tag{2.12}$$

Thus the density function is the first order derivative of the distribution function. In this case, the Lorenz curve is defined by the following (Lambert 2001):

$$p = F(x) \Rightarrow L(p) = \int_0^x \frac{tf(t)dt}{\mu} \tag{2.13}$$

The definition of the distribution function given by (2.12) implies that **dp=f(x)dx**. Therefore, the Lorenz function may also be written as:

$$L(p) = \int_0^p \frac{x(q)}{\mu}dq \tag{2.14}$$

Differentiating with respect to **p**, we find that the first order derivative is equal to:

$$L'(p) = \frac{x(p)}{\mu} \tag{2.15}$$

The second order derivative is equal to:

$$L''(p) = \frac{1}{\mu}\frac{dx}{dp} = \frac{1}{\mu \frac{dp}{dx}} = \frac{1}{\mu f(x)} \tag{2.16}$$

The standard Lorenz curve does show only relative shares, but not the size (mean) of the distribution or the total population. Multiplying the ordinary Lorenz curve by the mean of the distribution produces the *generalized Lorenz curve* defined as:

$$L(\mu, p) = \mu L(p) = \frac{1}{n}\sum_{k=1}^{j} x_k = p\mu_p;\ L(\mu,0) = 0;\ L(\mu,1) = \mu \tag{2.17}$$

In the case of a continuous distribution, we have:

$$L(\mu, p) = \int_0^x tf(t)dt = \int_0^p x(q)dq \tag{2.18}$$

2.2. Parameterization

There are two basic ways to parameterize the Lorenz curve. The first involves the use of a known expression of the Lorenz curve associated with a specific density function. For instance, if income follows a lognormal distribution, then the Lorenz curve has the form of a standard normal distribution. Similarly, in the case of a beta density the corresponding Lorenz curve can be written as an incomplete beta integral (Johnson, Kotz and Balakrishnan 1995). The underlying parameters can be either assumed or estimated form household survey data. The second approach to parameterization, is to assume directly a functional function for the Lorenz curve, and estimate structural parameters from available data. We focus here on the General Quadratic Model.

The General Quadratic Model

When household data are available, either in the form of unit records or aggregated in income or expenditure classes, one can use regression analysis to fit the data to a model such as the General Quadratic model. We follow the procedure described in Datt(1992, 1998). This calls for regressing **[L(1-L)]** on **(p^2-L)**, **L(p-1)** and **(p-L)** without an intercept, and dropping the last observation since the chosen functional form forces the curve to go through (1, 1). Here **p** stands for the abscissa (horizontal coordinate) while **L** stands for the ordinate of the Lorenz curve estimated from the initial data.

Let β_1, β_2, and β_3 be the regression coefficients. Define the following parameters:

$$e = -(\beta_1 + \beta_2 + \beta_1 + 1);\ m = (\beta_2^2 - 4\beta_1);\ n = (2\beta_2 e - 4\beta_3);\ r = (n^2 - 4me^2)^{\frac{1}{2}} \qquad (2.19)$$

The Quadratic Lorenz Curve can be written as:

$$L(p) = -\frac{1}{2}\left[\beta_2 p + e + (mp^2 + np + e^2)^{\frac{1}{2}}\right] \qquad (2.20)$$

The corresponding first derivative is equal to (Datt 1992, 1998):

$$L'(p) = -\frac{\beta_2}{2} - \frac{2mp + n}{4\sqrt{(mp^2 + np + e^2)}} \qquad (2.21)$$

The second derivative is:

$$L''(p) = \frac{r^2(mp^2 + np + e^2)^{-\frac{3}{2}}}{8} \qquad (2.22)$$

Table 2.2 shows the regression results for rural India in 1983. The underlying data come from Datt(1998). They are embedded in the computer code presented in the first annex to this paper.

Table 2.2: Rural India 1983: Regression Output for the General Quadratic Lorenz of Household Expenditure

	Coefficient	Std. Error	t-Statistic	Prob.
BETA(1)	0.887734	0.006683	132.8389	0.0000
BETA(2)	-1.451431	0.019062	-76.14295	0.0000
BETA(3)	0.202658	0.012847	15.77521	0.0000
R-squared	0.999959	Mean dependent var		0.121975
Adjusted R-squared	0.999950	S.D. dependent var		0.087678
S.E. of regression	0.000617	Akaike info criterion		-11.73067
Sum squared resid	3.43E-06	Schwarz criterion		-11.60944
Log likelihood	73.38399	Durbin-Watson stat		0.697503

The corresponding Lorenz curve is presented in figure 2.2. We now discuss the use of the Lorenz curve in social evaluation.

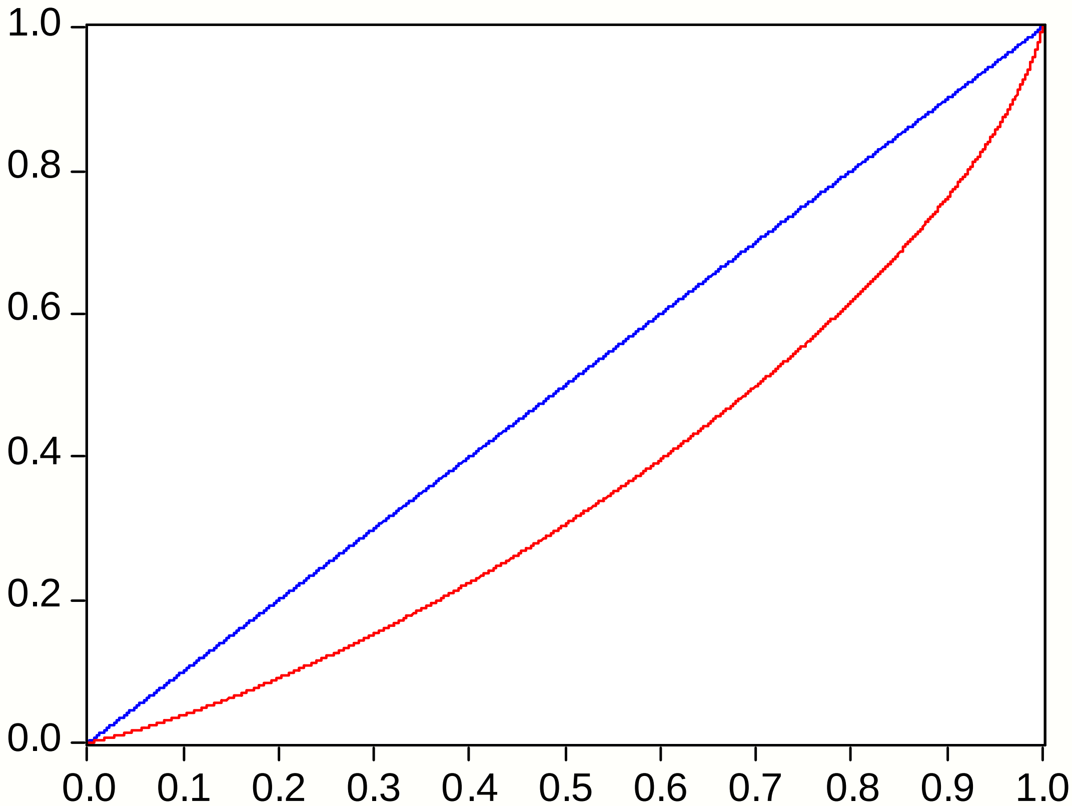
2.3. Normative Underpinnings

Lorenz dominance is a useful tool in the assessment of the inequality effect of a policy reform. The issue is whether the policy reform has brought about more or less inequality in the distribution of the living standard. Take income as an indicator of the standard of living. If the post-reform Lorenz curve $\mathbf{L_a(p)}$ lies everywhere above the pre-reform one, $\mathbf{L_b(p)}$ then for every percentile, $\mathbf{p}$, the disposable income share of the poorest **100p** percent has been increased by the reform. In other terms, inequality has decreased. This is a purely descriptive statement which also has a *normative content*, depending on the value judgments that one is willing to make in the comparison of both distributions.

To make matters simple, we assume that the population is homogeneous with respect to all other individual attributes except for income, $\mathbf{x}$. Hence non-income characteristics are either identical or considered socially irrelevant. At the individual level, we assume that *more income is preferred to less*. To investigate the normative aspects of Lorenz dominance, we need to devise a *social evaluation function* and connect it to the dominance relation. Fundamentally, a social evaluation function does two things[2]: (1) it specifies the relevant personal attributes that are directly involved in the assessment of the social desirability of a state of affairs; (2) it provides an aggregation rule for combining such attributes into an overall index of social well-being.

[2] This is a translation of Sen's (1992:73) observation that most theories of justice can be analyzed in terms of the selection of relevant personal features, and the choice of the combining characteristics they advocate. These elements constitute the informational basis of a theory of justice.

Figure 2.2: A Simulated Lorenz Curve for Rural India, 1983



Cumulative Proportion of Population

Consider the following simple social evaluation criterion:

$$W(x) = \sum_{k=1}^{n} \omega_k x_k \qquad (2.23)$$

where **x** stands for a distribution of income over the entire population and $\boldsymbol{\omega_k}$ is the social weight attached to the income of individual **k**. The change in social welfare induced by policy implementation is equal to:

$$\Delta W = \sum_{k=1}^{n} \omega_k \Delta x_k ; \quad \Delta x_k = (x_{ak} - x_{bk}) \qquad (2.24)$$

Mayshar and Yitzhaki(1995) show that the same expression may be written in terms of cumulative changes. By definition, $\Delta x_k = cmdx_k - cmdx_{k-1}$. Substituting back into

(2.24) leads to the following expression for the change in the evaluation criterion due to the change in the distribution:

$$\Delta W = \sum_{k=1}^{n} \omega_k \Delta x_k = \sum_{k=1}^{n-1} (\omega_k - \omega_{k+1}) cmdx_k + \omega_n cmdx_n; \quad cmdx_k = \sum_{i=1}^{k} \Delta x_i \qquad (2.25)$$

Social comparison of the two distributions thus hinges on the choice of the evaluative weights, $\boldsymbol{\omega_k}$. We may therefore invoke a theory of distributive justice such as the *maximin* or the *Dalton principle* to guide the specification of evaluative weights. According to the maximin principle, the desirability of a social state is determined by the situation of the worse off member of society (or group). In the context of the distribution of social resources, the Dalton version of this idea states that resource transfers from rich to poor improve social welfare. Thus we would like to construct *a transfer-approving evaluation criterion*. To implement this idea in the specification of evaluative weights, we first rank individuals according to some criterion of *social desert* in the original state. We then assign weights in such a way that of any two individuals the more deserving receives a higher weight. In our context, we consider that of any two individuals, the poorer one is the more deserving of social consideration. We therefore rank individuals in increasing order of income, choose social weights in such a way that each is *nonnegative* and *adjacent weights satisfy the condition* $\boldsymbol{(\omega_{k-1}-\omega_k)\geq 0}$.

Expression (2.25) is a sum of cross products. The first term of each product is nonnegative by assumption (choice of weights). It is therefore clear that if the second term is also nonnegative the whole sum will be nonnegative as well. The condition for an unambiguous Dalton-improvement in welfare from state **b** to state **a** may thus be stated generally as: $\mathbf{cmdx_k \geq 0}$ for all **k**. This is equivalent to the following statement:

$$\sum_{i=1}^{k} \Delta x_i \geq 0 \ \forall k \Rightarrow \sum_{i=1}^{k} x_{ai} \geq \sum_{i=1}^{k} x_{bi} \quad \forall k \in \{1, 2, \ldots, n\} \qquad (2.26)$$

Referring back to (2.17), we see that the above condition compares the numerators of the generalized Lorenz curves associated with both distributions. If both distributions cover the same population, we can normalize (2.26) by dividing throughout by **n**. Thus welfare will improve as we move from distribution **b** to distribution **a**, if the generalized Lorenz curve associated with **a** lies nowhere below that of distribution **b**. This condition is known as *second order dominance*.

We can also show that second order dominance implies welfare improvement for all social evaluation functions members of the Dalton class. Suppose the generalized Lorenz curve of distribution **a** lies nowhere below that of distribution **b**. Construct an intermediate distribution **c** with the same mean as **a,** and the same relative inequality as **b**. Specifically, we rank individuals in increasing order of their income under **b**. Change only the income of the richest individual by giving him or her, in addition to his income under distribution **b**, the difference between total income in **a,** and total income in **b**. Analytically, we define distribution **c** as follows:

$$x_{ck} = x_{bk} \quad \forall k \in \{1, 2, ..., n-1\};\ x_{cn} = x_{bn} + n(\mu_a - \mu_b) \qquad (2.27)$$

Thus **c** has the same relative inequality as **b**, but the same mean as **a** since it is obvious that $\boldsymbol{\mu_c = \mu_a}$. On the basis of (2.24) we note that $\Delta x_k = 0\ \forall k \in \{1,2,...,n-1\}$, therefore the change in social welfare as we move from distribution **b** to distribution **c** is equal to:

$$\Delta W_{bc} = n\omega_n(\mu_a - \mu_b) \geq 0 \qquad (2.28)$$

Comparing distribution **c** and **a**, on the basis of expression (2.25), we note that both distributions have the same mean. Furthermore, distribution **c** has the same relative inequality as **b.** By assumption, distribution **a** dominates **b** in the second order, this implies that as we move from **c** to **a**, all $\mathbf{cmdx_k}$ in (2.25) are nonnegative, hence welfare improves ($\Delta W_{ca} \geq 0$). We thus have the following welfare ranking $\mathbf{W_a \geq W_c \geq W_b}$. This discussion establishes that generalized Lorenz dominance is a necessary and sufficient condition for welfare dominance in terms of social evaluation criteria that respect the following two conditions: (1) each individual prefers more income is to less; (2) society prefers more equality to less. This is the essence of Shorrocks (1983) theorem. If both distributions have the same mean, then generalized Lorenz dominance is equivalently stated in terms of ordinary Lorenz curves. This is the case of Atkinson theorem (1970).

What if we had required the weights to be only nonnegative? Then expression (2.24) implies welfare would improve if $\mathbf{\Delta x_k \geq 0}$ for all **k**. In other terms, any intervention that would improve the lot of at least one individual relative to the counterfactual, without worsening that of any other individual, would be considered a social improvement. This corresponds to the *Pareto criterion*. Any set of constant weights would be acceptable as long as they are nonnegative. Thus, choosing $\mathbf{\omega_k = 1/n}$, we have the following expression for the change in welfare:

$$\Delta W = \sum_{k=1}^{n} \omega_k \Delta x_k = \frac{1}{n}\sum_{k=1}^{n}(x_{ak} - x_{bk}) = (\mu_a - \mu_b) \tag{2.29}$$

Thus, we conclude that, distribution **a** represents a Pareto improvement over distribution **b** if $\mathbf{\mu_a \geq \mu_b}$. This result is consistent with *first order stochastic dominance* criterion[3]. It is also equivalent to comparing the generalized Lorenz curve only at a single point i.e. at **p=1**. Note that Pareto improvement would be achieved if the intervention improved only the outcome of the best off and left everybody else in the situation *quo ante*. This demonstrates that the Pareto criterion is insensitive to the degree of inequality in the distribution. As an illustration of this point, consider a Pareto comparison of distributions **a**, **b** and **c** discussed above. The Pareto criterion ranks **c** ahead of **b**, because **c** has more income than **b**. But considers **c** and **a** as socially equivalent because they have the same mean. No attention is paid to the fact that inequality is less in **a** than in **c**. The Dalton principle takes this to be a good thing and therefore ranks **a** ahead of **c**.

The minimalist approach to the specification of evaluative weights underlying both the Pareto and the Dalton criteria leads to a *dominance* relation between the distribution of income in the intervention state and the distribution associated with the counterfactual. Dominance criteria provide a general framework for unambiguous ranking of social states. Since dominance does not necessarily hold among all possible

[3] Let $F_a(x)$ and $F_b(x)$ stand respectively for the CDF of **a** and **b**. Let u(x) stand for a monotone nondecreasing function of x. In terms of social evaluation, u(x) translates the idea that more is preferred to less. Then, distribution **a** first-order stochastically dominates distribution **b** if and only if the average of u(x) under **a** is at least as great as under **b**, for all such monotone nondecreasing u(x). Equivalently, **a** dominates **b** stochastically to the first order if and only if the CDF of **a** lies nowhere above that of **b** within the entire domain of definition of x (Deaton 1997).

states, we conclude that dominance is a *partial ordering*. It is clear that Dalton-improvement implies a Pareto-improvement but not the other way around. But when it comes to tests based on the configuration of curves depicting the distributions under consideration, First-order dominance implies second-order dominance, and not the other way around.

3. Recovering Inequality and Poverty Measures

Most inequality and poverty measures of interest can be computed from information on **x** (the welfare indicator), the corresponding density function **f(x)**, and a poverty line **z** (for poverty indicators). Given a parameterization of the Lorenz curve, the level of welfare is equal to the mean of the distribution times the first order derivative of the Lorenz curve [see (2.6) or 2.8) above]. Similarly, based on expression (2.14), the density function is equal to the inverse of the product of the mean by the second order derivative of the Lorenz curve. Our simulation strategy is to use these facts along with standard definitions of the relevant indicators, and the interpretation of **f(x)dx** as the proportion of people whose standard of living lies within the close interval **[x, dx]**.

The accuracy of the procedure improves with the number of points (on the distribution) simulated with the parameterized curve. The results reported below are based on 10,000 points derived from 13 data points representing a distribution of monthly expenditure in rural India in 1983. As stated earlier, this frees us from the analytical expressions for the indicators associated with the chosen functional form of the Lorenz curve. The approach also has a wider scope than the one based on analytical expressions. We are thus able to compute members of the extended Gini family of inequality indicators (including the Sen index of poverty), those of the generalized entropy family (including Atkinson), and a class of additively separable poverty measures.

3.1. The Extended Gini Family

The area between the Lorenz curve and the line of complete equality (the 45-degree line) is a measure of inequality. Indeed, the ordinary Gini coefficient is equal to that area divided by the whole area under the 45-degree line. As it turns out, the ordinary Gini is a member of a larger family defined by the extended Gini coefficient. This extended family involves a parameter **ν** that is interpreted as an indicator of the degree of *inequality aversion*. There are two methods of computing members of this family. The covariance method (Lerman Yitzhaki 1989) can be derived from the social evaluation function (2.21). The linear-segment method (Chotikapanich and Griffiths 2001) relies on a linear approximation of the Lorenz curve. We present both methods respectively before considering how to recover these measures from a parameterized Lorenz curve.

Using the definition of the covariance between two variables, the social evaluation criterion defined by (2.21) may be abbreviated as follows: $W(x) = nV(\omega)$ where **ω** is a distribution of social weights and

$$V(\omega) = \mu_\omega \mu_x + \operatorname{cov}(x, \omega)\,. \tag{3.1}$$

With no loss of generality we can normalize average social weight to one (μ_ω=1) so that:

$$V(\omega) = \mu_x + \operatorname{cov}(x, \omega) = \mu_x[1 + \frac{\operatorname{cov}(x, \omega)}{\mu_x}] = \mu_x[1 - G(\omega)]\,. \tag{3.2}$$

We interpret V(**ω**) as the *equally distributed equivalent income*[4]. If this income were given to every individual, that distribution would be socially equivalent to the observed **x**.

The term **G(ω)** turns out to be the extended Gini for a proper choice of social weights. A particular individual with income level $\mathbf{x_k}$ is considered relatively deprived if there is at least one other individual with a higher endowment. The relative rank of this individual in the parade of income is equal to $\mathbf{p_k}$ (the proportion of individuals with income less than or equal to $\mathbf{x_k}$). We represent the cumulative distribution of **x** as in section 2 above: **p=F(x)**. In the context of a large population of individuals, the likelihood of drawing at random a person with an income level higher than that of **k** is equal to $\mathbf{(1 - p_k)}$. This likelihood, also known as the *survivor function,* declines monotonically from 1 to zero as we move from the lowest to the highest ranking

[4] This is the bedrock concept in Atkinson's (1970) framework for social evaluation of inequality. When income is used as an indicator of well-being, the equally distributed equivalent income is the level of per capita income which, if enjoyed by every individual, would yield the same level of social welfare as the current distribution for some choice of utility of income.

individual[5]. This pattern is consistent with the one underlying the Dalton principle of transfers. First of all, all weights are nonnegative. Furthermore, the weights are assigned in such a way that, of any two individuals, the worse off (i.e. the poorer) is considered as more deserving and therefore gets a higher social weight. Given that the mean of the cumulative distribution is equal to 0.5, we can normalize the weights such that, $\omega_k = 2(1-p_k)$. More generally, these weights may be written as follows, where ν is the normalizing factor to be interpreted as an indicator of aversion for inequality.

$$\omega_k(\nu) = \nu(1-p_k)^{\nu-1} \qquad (3.3)$$

This choice of weights combined with expression (2.29) leads to the covariance expression of the *extended Gini coefficient*.

$$G(\nu) = -\frac{\nu}{\mu_x}\text{cov}[x,(1-p)^{\nu-1}] \qquad (3.4)$$

Alternatively, the extended Gini coefficient may be defined as (Yitzhaki 1983):

$$G(\nu) = 1-\nu(\nu-1)\int_0^1 (1-p)^{\nu-2}L(p)dp \qquad (3.5)$$

where L(p) stands for the Lorenz curve.

Chotikapanich and Giffiths (2001) propose an algorithm based on a linear approximation of the Lorenz curve in the above expression. This linear-segment estimator of the extended Gini coefficient is defined by the following expression.

$$G(\nu) = 1+\sum_{k=1}^{m}\left(\frac{\theta_k}{w_k}\right)[(1-p_k)^{\nu}-(1-p_{k-1})^{\nu}]; \quad \theta_k = \frac{w_k x_k}{\sum_{j=1}^{m} w_j x_j} \qquad (3.6)$$

where θ_k is the proportion of income held by group **k** and w_k is the population share of that group. Note that the ratio in parentheses in the above expression is also equal to: $\frac{x_k}{\sum_{j=1}^{m} w_j x_j}$ (an estimate of the slope of the Lorenz curve at x_k). Therefore the linear-segment estimator may be rewritten as:

$$G(\nu) = 1+\sum_{k=1}^{m}\frac{\Delta L(p_k)}{\Delta p_k}\left[(1-p_k)^{\nu}-(1-p_{k-1})^{\nu}\right] \qquad (3.7)$$

[5] This is so because the cumulative distribution function increases monotonically from zero to one.

This expression clearly indicates how to use information on a Lorenz curve in order to compute members of the extended Gini family. The computer code presented in the annex relies on this algorithm. When information about the mean is available, then we can also use the covariance method by estimating the level of income at the $\mathbf{p^{th}}$ percentile. The covariance expression may thus be written as:

$$G(\nu) = -\frac{\nu}{\mu_x}\text{cov}[\mu_x L'(p), (1-p)^{\nu-1}] \tag{3.8}$$

When the aversion parameter is equal to one, the extended Gini is equal to zero. The abbreviated social evaluation function defined by (2.29) assigns equal weights to everyone regardless of their income level. This is equivalent to the Pareto criterion. Distributions are compared only on the basis of their means. When the aversion parameter is equal to 2, inequality is measured by the ordinary Gini coefficient. As the parameter tends to infinity, social evaluation focuses on the poorest individual.

Sen's measure of poverty is a close relative of the Gini coefficient. Let $\mathbf{G_p}$ stand for the Gini coefficient of the distribution of income (or expenditure) among the poor, according to Sen (1997), the following expression is an indicator of welfare among the poor:

$$w_p = \mu_p(1-G_p) \tag{3.9}$$

This expression represents the equally distributed equivalent expenditure among the poor.

Sen (1997) proposes *a poverty index that is analogous to the poverty gap* indicator (member of the Foster-Greer-Thorbecke family to be discussed below). The only difference between the two is that Sen uses $\mathbf{w_p}$ in lieu of $\boldsymbol{\mu_p}$, the mean income among the poor. Hence the expression:

$$S = H\left(1-\frac{w_p}{z}\right) \tag{3.10}$$

In the above expression for the Sen index, **z** stands for the poverty line and **H** for the proportion of individuals below the poverty line. The extended Gini coefficient may be used in the above to obtain an extended version of Sen's poverty indicator written as follows.

$$S(\nu) = H\left[1 - \frac{\mu_p[1 - G_p(\nu)]}{z}\right] \quad (3.11)$$

This is the expression we use to produce the results presented in Table 3.1. The column labeled focus shows values of the aversion parameter. The overall extended Gini and the Gini measure of inequality are presented along with Sen's index of poverty (in percentage). It is clear that the higher the aversion for inequality the more inequality and poverty is shown by these indicators.

Table 3.1. Gini Family of Indicators for Rural India in 1983

Focus	Overall Gini	Gini for Poor	Sen Index
1	0.03	0.00	12.48
2	28.89	13.54	16.89
3	38.78	20.66	19.21
4	44.22	25.12	20.67
5	47.79	28.21	21.67
6	50.35	30.48	22.41

Data source: Datt (1998)

3.2. The Generalized Entropy Measures

One class of commonly used measures of inequality is based on the notion of *entropy* or the expected information content of a set of events (Sen 1997:34-35). Let π stand for the probability that an event would occur, and let $\mathbf{h}(\pi)$ represent the information content of the event or the value we assign to the information that the event has occurred. Information theory imposes the following restrictions on $\mathbf{h}(.)$. Information that the event has occurred is more valuable the less likely the event is. If two events are statistically independent so that the occurrence of one does not affect that of the other, the likelihood of their joint occurrence is equal to the product of the respective probability. To be able to add up information content of independent events, the entropy function must be such that $\mathbf{h}(\pi_1\pi_2)=\mathbf{h}(\pi_1)+\mathbf{h}(\pi_2)$. Cowell(1995) notes that the only function satisfying this conditions is **h(π)=-log(π)=log(1/π)**. Hence the expected information content of a set of **n** events my be written as:

$$H_e = \sum_{i=1}^{n} \pi_i h(\pi_i) = \sum_{i=1}^{n} \pi_i \log(1/\pi_i) = -\sum_{i=1}^{n} \pi_i \log(\pi_i) \tag{3.12}$$

When all **n** events are equally likely so that $p_i=1/n$, H_e reaches it maximum value of **log(n)**.

Consider a distribution of income among **n** individuals. The share of individual **i** is equal to $\pi_i = \frac{x_i}{n\mu}$. This is analogous to a probability. The Theil index of inequality is obtained by subtracting the entropy measure associated with an income distribution, from its value under equal distribution. Formally, we write:

$$T_h = \log(n) + \sum_{i=1}^{n} \pi_i \log(\pi_i) = \sum_{i=1}^{n} \pi_i \left[\log(n) + \log(\pi_i)\right] = \sum_{i=1}^{n} \pi_i \log(n\pi_i) \tag{3.13}$$

Alternatively,

$$T_h = \frac{1}{n} \sum_{i=1}^{n} \frac{x_i}{\mu} \log\left(\frac{x_i}{\mu}\right) \tag{3.14}$$

As it turns out, T_h is a member of a family of inequality indices known as the generalized entropy indices defined by:

$$GE(\theta) = \frac{1}{\theta^2 - \theta}\left[\frac{1}{n}\sum_{i=1}^{n}\left(\frac{x_i}{\mu}\right)^{\theta} - 1\right] \tag{315}$$

When **θ=0**, the index becomes:

$$GE(0) = \frac{1}{n}\sum_{i=1}^{n} \log\left(\frac{\mu}{x_i}\right) \tag{3.16}$$

This is also known as the mean logarithmic deviation or Theil's second measure (Sen 1997).

When **θ=1**, **GE(1) =T_h**. Finally, when **θ=2**, the measure is equal to the square of the coefficient of variation divided by 2. That is:

$$GE(2) = \frac{1}{2n\mu^2}\sum_{i=1}^{n}(x_i - \mu)^2 \tag{3.17}$$

For empirical implementation, we note that all members of the GE class of measures are a function of the slope of the Lorenz curve, as shown by the following general expression. This fact greatly facilitates their computation when a parameterized Lorenz curve is available.

$$GE(\theta) = \frac{1}{\theta^2 - \theta}\left[\frac{1}{n}\sum_{i=1}^{n}\left(\frac{\Delta L(p_i)}{\Delta p_i}\right)^{\theta} - 1\right] \qquad (3.18)$$

The GE family can take values ranging from zero to plus infinity. Furthermore, it can be shown that when **θ<1**, every member of the Atkinson family of indicators has a corresponding, *ordinally equivalent*, member of the GE family. Letting **A(ε)** stand for the Atkinson measure of inequality with aversion parameter equal to ε, this equivalence can be expressed as follows.

$$A(\varepsilon) = 1 - \left[(\theta^2 - \theta)GE(\theta) + 1\right]^{1/\theta}, \theta = (1-\varepsilon) < 1, \theta \neq 0 \qquad (3.19)$$

When **θ=0**, we write:

$$A(\varepsilon) = 1 - \exp\left[-GE(0)\right]. \qquad (3.20)$$

In other terms, *when **θ<1**, each member of the GE family is a monotonic transformation of an Atkinson measure*. Thus, the parameter θ may be interpreted as an indicator of inequality aversion. Under this interpretation, the smaller the value of θ, the higher the degree of aversion (Sen 1997). In addition, the parameter indicates the degree of sensitivity of the GE family to transfers at different parts of the distribution. *It is known that all members of the GE family with **θ<2** favor transfers occurring at the lower end of the distribution.*

The bedrock concept in the Atkinson framework is that of *equally distributed equivalent* (EDE) income. We denote this by ξ. The EDE income represents the level of per capita income which, if enjoyed equally by every individual, would provide the same level of social welfare as the current distribution for some choice of utility function. On the basis of a utility function with a constant elasticity of marginal utility with respect to income, this definition leads to the following social evaluation function:

$$W(x) = \frac{1}{n}\sum_{i=1}^{n} u(x_i) = \frac{1}{n}\sum_{i=1}^{n} u(\xi) = u(\xi) \tag{3.21}$$

where

$$u(\xi) = \frac{\xi^{1-\varepsilon}}{1-\varepsilon} = \frac{1}{n}\sum_{i=1}^{n}\frac{x_i^{1-\varepsilon}}{1-\varepsilon} \tag{3.22}$$

The above expression implies that:

$$\xi = \left[\frac{1}{n}\sum_{i=1}^{n} x_i^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}} \tag{3.23}$$

When the aversion parameter is equal to one, the EDE is calculated from the following expression:

$$\ln(\xi) = \frac{1}{n}\sum_{i=1}^{n}\ln(x_i) \tag{3.24}$$

Therefore,

$$\xi = \exp\left[\frac{1}{n}\sum_{i=1}^{n}\ln(x_i)\right] \tag{3.25}$$

This expression reveals that when inequality aversion is set to one, the EDE income is equal to the geometric mean of income.

Within the Atkinson framework, the social cost of inequality in per capita terms is defined by:

$$c = \mu - \xi = \mu\left(1 - \frac{\xi}{\mu}\right) \tag{3.26}$$

where $\boldsymbol{\mu}$ is the mean of the distribution **x**. The indicator **c** is interpreted as the amount of per capita income that could be sacrificed with no loss of social welfare provided the rest were distributed equally (Lambert 2001). The cost of inequality per capita is equal to the mean of the distribution being assessed minus the EDE income of the distribution used as standard of comparison. This can be scaled up by multiplying the per capita amount by the population size. Obviously, the size of the cost of inequality depends on the degree of inequality aversion.

The Atkinson index of inequality, **A(ε)** is defined as a relative cost of inequality[6].

$$A(\varepsilon) = \frac{c}{\mu} = 1 - \frac{\xi}{\mu} = 1 - \left[\frac{1}{n}\sum_{i=1}^{n}\left(\frac{x_i}{\mu}\right)^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}} \tag{3.27}$$

Clearly, this can be written as a function of the first order derivative of the Lorenz curve.

$$A(\varepsilon) = 1 - \left[\frac{1}{n}\sum_{i=1}^{n}\left(\frac{\Delta L(p_i)}{\Delta p_i}\right)^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}} \tag{3.28}$$

It can be seen from expression (3.27) that the EDE income is equal to the mean multiplied by one minus the Atkinson index of inequality.

$$\xi = \mu[1 - A(\varepsilon)] \tag{3.29}$$

This is a social evaluation function or an abbreviated social welfare function that clearly combines both efficiency and equity concerns.

Table 3.2. Generalized Entropy and Atkinson Measures for Rural India, 1983

Focus	Entropy	Atkinson	EDEX
0.0	13.51	0.12	109.76
0.5	14.01	6.88	102.32
1.0	14.42	12.80	95.82
1.5	15.75	17.68	90.46
2.0	18.25	22.21	85.48

Data Source: Datt (1998)

In table 3.2 above, EDEX stands for equally distributed equivalent expenditure. When the aversion parameter ε is equal to zero, this measure of social welfare is equal to the mean of the distribution.

[6] The expression on the extreme right is obtained from the fact that $\frac{1}{\mu} = \left[\left(\frac{1}{\mu}\right)^{1-\varepsilon}\right]^{\frac{1}{1-\varepsilon}}$

3.3. A Class of Additively Separable Poverty Indices

Kakwani(1999) defines a class of additively separable poverty measures starting from the notion of deprivation. Let **z** be the poverty line, and $\mathbf{x_i}$ the level of welfare enjoyed by individual **i** in a society comprising **n** individuals. Let $\boldsymbol{\psi(z,x_i)}$ stand for an indicator of deprivation at the individual level. The following restrictions are imposed on the indicator: (i) it is equal to zero when the welfare level of the individual is greater or equal to the poverty line; (ii) the indicator is a decreasing convex function of welfare, given the poverty line.

Poverty measures of this class reflect the average deprivation suffered by the whole society and may be written, as:

$$P(z,x) = \frac{1}{n}\sum_{i=1}^{n}\psi(z,x_i). \tag{3.30}$$

The class of poverty measured defined by (3.30) is called additively separable because the deprivation felt by an individual depends only on a fixed poverty line and her/his level of welfare and not on the welfare of other individuals in society. When the population is divided exhaustively into mutually exclusive socioeconomic groups, this class of measures allows one to compute the overall poverty as a weighted average of poverty in each group. The weights here are equal to population shares. Thus such indices are also *additively decomposable*.

Particular members of this class are obtained by specification of the *deprivation function*. One prominent family of this class is due to Foster, Greer and Thorbecke (1984). The associated deprivation function may be written as follows (Jenkins and Lambert 1997).

$$\psi_{FGT}(z,x_i) = \max\{(1 - x_i / z)^{\alpha}, 0\}. \tag{3.31}$$

The parameter α is an indicator of aversion for inequality among the poor.

The first derivative of this function with respect to the welfare indicator is equal to:

$$\frac{\partial \psi_{FGT}}{\partial x_i} = \min\left\{-\frac{\alpha}{z}(1 - x_i / z)^{\alpha-1}, 0\right\} \tag{3.32}$$

and the second derivative is:

$$\frac{\partial^2 \psi_{FGT}}{\partial x_i^2} = \max\left\{-\frac{\alpha(\alpha-1)}{z^2}(1-x_i/z)^{\alpha-1},0\right\}, \quad \alpha > 1. \tag{3.33}$$

For positive values of the aversion parameter, these derivatives indicate that members of the FGT family of indicators are non-decreasing convex functions of the welfare indicator. When the aversion parameter is equal to zero, aggregate poverty in society is given by the head count index commonly denoted by **H** or $\mathbf{P_0}$. This index is totally indifferent to differences in poverty dimensions higher than incidence. When the aversion parameter is equal to one, aggregate poverty is given by the poverty gap ratio which may also be written as:

$$P_1 = H\left(1-\frac{\mu_z}{z}\right). \tag{3.34}$$

where $\boldsymbol{\mu_z}$ is the average welfare of the poor. This expression reveals the analogy between the poverty gap ratio and the extended Sen index when the aversion parameter is equal to one. One interpretation of the poverty gap indicator is based on the following consideration. Think of **x** as income, and consider a situation where **x** is observable and it is possible to give everybody a transfer of **(z-x)**. Afterwards, there would be no more poverty as every individual would have at least an income equal to **z**. Let **q** stand for the number of the poor. The total income transferred to the poor by this operation is $\mathbf{q(z-\mu_z)}$. Normalizing by the size of the population, we get $\mathbf{H(z-\mu_z)=zP_1}$. Thus in an ideal world of *incentive-preserving* transfers and perfect targeting, $\mathbf{zP_1}$ is viewed as the minimum amount of resources that must be transferred, on average, from the non-poor to the poor in order to eradicate income poverty. The poverty gap ratio does not account for inequality among the poor. To do so we must set the aversion parameter to values higher than one.

The second family in the class of additively separable measures is due to Chakravarty(1983). The corresponding deprivation function is defined as:

$$\psi_{CHK}(z, x_i) = \left[1-\left(\frac{x_i}{z}\right)^{\alpha}\right], \quad 0 < \alpha \le 1. \tag{3.35}$$

Kakwani(1999) explains that when α tends to zero, aggregate poverty is measured by the head count index. When α**=1**, aggregate poverty is measured by the poverty gap ratio.

Finally, Clark, Hemming and Ulph (1981) offer a measure closely related to that of Chakravarty. The associated index at the individual level is defined as:

$$\psi_{CHU}(z,x_i) = \frac{1}{\alpha}\left[1-\left(\frac{x_i}{z}\right)^{\alpha}\right] = \frac{1}{\alpha}\psi_{CHK}, \quad 0<\alpha\leq 1 \tag{3.36}$$

when α tends to **0**, aggregate poverty is given by the Watts (1968) measure. The Watts index of poverty is based on the following deprivation function:

$$\psi_{WAT}(z,x_i) = [\log(z)-\log(x_i)]. \tag{3.37}$$

Table 3.3. Poverty in Rural India (1983)[7]

Focus	FGT	CHK	CHU
0	45.07	45.07	15.96
1	12.48	12.48	12.48
2	4.75	20.20	10.10

Data Source: Datt (1998)

Table 3.3 shows estimates of these indicators for several values of the aversion parameter. The results are given in percentage. Thus poverty incidence in rural India was about 45 percent in 1983. When the aversion parameter is equal to zero, the Clark-Hemming-Ulph index is equal to the Watts index, about 16 percent.

Just as in the case of inequality measures, we can recover these poverty measures from information from the mean and the Lorenz curve. To see how this is done, write expression (3.30) as:

$$P(z,x) = \sum_{h=1}^{m}\psi\left(z,\mu\frac{\Delta L(p_h)}{\Delta p_h}\right)f(x_h)\Delta x_h \tag{3.38}$$

Where $\mathbf{f(x_k)}$ is an estimate of the density function. Members of the FGT family are thus defined by the following expression:

[7] Note that CHK (Chakravarty) and CHU (Clark, Hemming and Ulph) are not defined for α>**1**. We computed those values any way just out of curiosity and for symmetry in presentation.

$$P_\alpha = \sum_{h=1}^{m} \max\left[\left(1-\frac{\mu}{z}\frac{\Delta L(p_h)}{\Delta p_h}\right)^{\alpha}, \quad 0\right] f(x_h)\Delta x_h \tag{3.39}$$

The results presented in table 3.3 above were computed on the basis of these formulae. It is useful to note alternative expressions derived from the General Quadratic model of the Lorenz curve (Datt 1998). The headcount index is defined by:

$$H = -\frac{1}{2m}\left[n + r(b+2z/\mu)\{(b+2z/\mu)^2 - m\}^{-1/2}\right] \tag{3.40}$$

The poverty gap index is computed as:

$$PG = H - (\mu / z)L(H) \tag{3.41}$$

The definition the squared poverty gap relies on the following two parameters:

$$s_1 = (r-n)/(2m); \quad s_2 = -(r+n)/(2m) \tag{3.42}$$

The squared poverty gap index can then be computed as:

$$SPG = 2PG - H - \left(\frac{\mu}{z}\right)^2 \left[aH + bL(H) - \left(\frac{r}{16}\right)\ln\left(\frac{1-H/s_1}{1-H/s_2}\right)\right] \tag{3.43}$$

A poverty indicator is supposed to translate the type of concerns policy makers have about aggregate poverty. Typically, three dimensions are of interest: (i) *incidence, (ii) intensity* and *(iii) inequality* among the poor. Incidence is the proportion of the total population living below the minimum standard, while intensity (or depth) is the extent to which the well-being of the poor falls below the minimum. All the poverty indicators discussed above are designed to capture at least one of these three dimensions. There is a graphical way of capturing all those aspects at once.

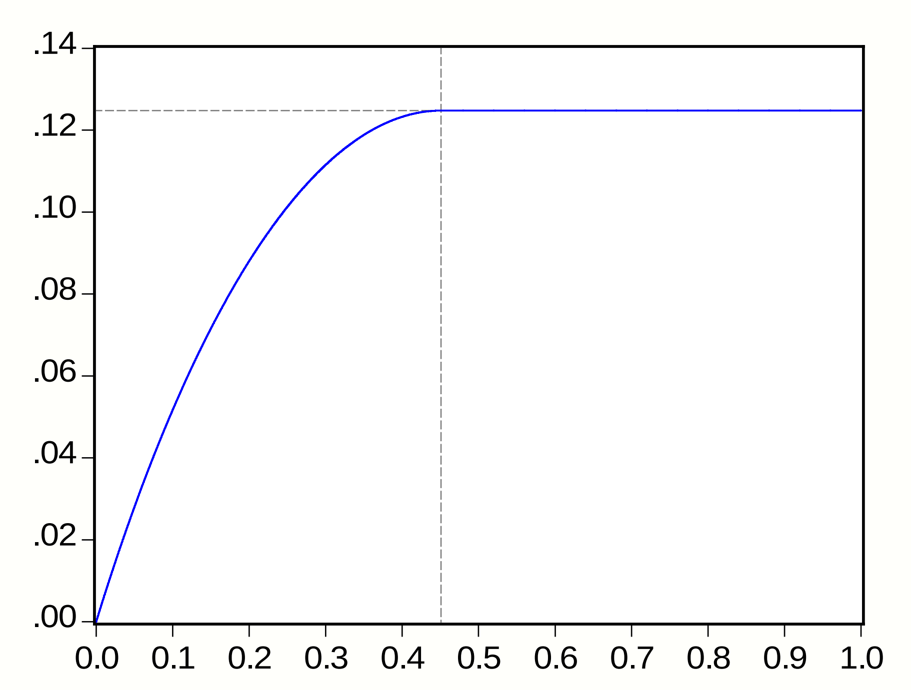
The TIP curve[8] provides a graphical summary of incidence, intensity and inequality dimensions of aggregate poverty based on the distribution of poverty gaps (Jenkins and Lambert 1997). This curve is constructed in three steps: (1) rank individuals from poorest to richest; (2) compute the relative poverty gap of individual **i** as **g_i=max{(1-x_i/z), 0}**; (3)form the cumulative sum of the relative poverty gaps divided by population size; and (4) plot the resulting cumulative sum of poverty gaps as a function of the cumulative population share. Assuming that the **n** individuals in the population are

[8] TIP stands for " three 'i's of poverty", that is incidence, intensity and inequality.

ranked from poorest to richest, then for all integers **k≤n**, the relative TIP curve may be defined as (Jenkins and Lambert 1997):

$$JL(p) = \frac{1}{n}\sum_{i=1}^{k} g_i; \quad p = \frac{k}{n}; \quad JL(0) = 0\,. \tag{3.44}$$

Figure 3.1. A TIP Curve for Rural India in 1983



Cumulative Proportion of Population

It is clear from above that the computation of the TIP curve is analogous to that of the Lorenz curve, and can also be based on normalized poverty gaps (that is absolute gaps divided by the poverty line). Figure 3.1 represents a TIP curve for the distribution of per capita monthly expenditure in rural India (the underlying data come from Datt 1998). This curve is based on normalized poverty gaps associated with the FGT measures.

The TIP curve is an increasing concave curve such that, at any given percentile, the slope is equal to the poverty gap for that percentile. The curve represents the three basic dimensions of aggregate poverty as follows: (1) the length of the non-horizontal section of the curve reveals poverty *incidence* ; (2) the *intensity* aspect of poverty is represented by the height of the curve; and (3) the degree of concavity of the non-horizontal section of the curve translates the degree of *inequality* among the poor.

4. Social Impact of Economic Growth

Inequality and Poverty indices are computed on the basis of a distribution of living standards, which is fully characterized by the mean and the degree of inequality. It is therefore reasonable to think of *a poverty indicator as a function of these two factors*. The impact of economic growth on poverty thus depends on how the growth process affects the mean and the relative inequality in the distribution of living standards. In this section, we focus on how to use the Lorenz framework to assess pro-poor growth and decompose poverty outcomes into growth and inequality components.

4.1. Assessing Pro-Poor Growth

Generally speaking, *pro-poor growth* is economic growth that is favorable to the poor. There are two basic interpretations of the term "favorable" found in the current literature. According to one view, growth is pro-poor if the change in inequality associated with the growth process is such that the incomes of the poor grow faster than those of the non-poor (Kakwani and Pernia 2000). Alternatively, growth is pro-poor if it leads to poverty reduction for some choice of a poverty measure (Kraay 2004). In this section we use the general impact indicator defined by (2.22) and its connection to the Lorenz curve to discuss the concept of pro-poor growth and illustrate its measurement.

Dominance Criteria

Consider the change in the living standard of individual $\mathbf{k}$ following a growth spell between period 0 and period 1. This can be written as**:** $\mathbf{\Delta x_k=(x_{1k} - x_{0k})}$. Based on the social evaluation criterion defined by equation (2.22), our judgment on the overall poverty impact of growth between the two periods depends on the choice of the weights $\boldsymbol{\omega_k}$. If we select the *Pareto criterion*, (i.e. requiring that the weights be only nonnegative) then Pareto improvement implies that $\mathbf{\Delta x_k \geq 0}$ for all $\mathbf{k}$. This can be restated as:

$$\Delta x_k \geq 0 \; \forall k \Leftrightarrow \frac{x_{1k}}{x_{0k}} \geq 1 \quad \forall k \tag{4.1}$$

In terms of the Lorenz curve and assuming that individuals are treated symmetrically, we rewrite the condition as follows:

$$\Delta x_k \geq 0 \; \forall k \Leftrightarrow \frac{x_{1k}}{x_{0k}} = \frac{\mu_1 L_1'(p)}{\mu_0 L_0'(p)} \geq 1 \quad \forall p \tag{4.2}$$

where $\boldsymbol{\mu_t}$ and $\mathbf{L'_t(p)}$ are respectively the mean of the distribution and the first order derivative of the Lorenz curve at time t=0, 1. Defining the growth rate at the p^{th} quantile as **g(p)**, and using the fact that the logarithm is a monotonic transformation, we get an indicator known as the *growth incidence curve* or GIC (Ravallion and Chen 2003):

$$g(p) = \gamma + \Delta \ln L'(p) \tag{4.3}$$

where $\boldsymbol{\gamma}$ is the growth rate of the overall mean of the distribution. This is a *distribution-corrected growth rate*. The adjustment factor is based on changes in the slope of the Lorenz curve. Expression (4.3) shows that **g(p)** will be greater than the growth rate $\boldsymbol{\gamma}$ only if the slope of the Lorenz curve (i.e. $\frac{x}{\mu}$) is increasing over time. When there is a Pareto criterion, **g(p)≥0** for all **p**. In other terms, the cumulative distribution function (CDF) of living standards in the second period lies nowhere above that of the first period[9]. We therefore conclude that poverty in the second period would be at most equal to the level observed in the initial period. If the growth incidence curve is strictly greater than zero, then growth is pro-poor for a wide choice of poverty measures. Thus we say that GIC dominance respects first order dominance.

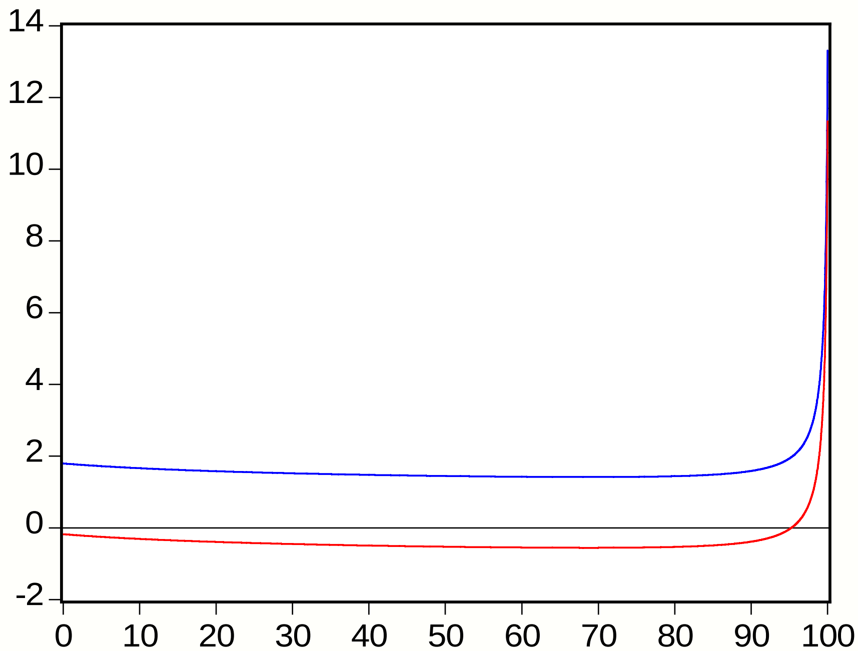
Kraay (2004) defines a relative growth incidence curve in terms of the pattern of growth in relative living standards. In terms of expression (4.3), a relative growth incidence curve plots the distribution adjustment factor against the cumulative percentage of individuals. Mathematically, we define the relative growth incidence curve (RGIC) by the following equation:

$$g_r(p) = \Delta \ln L'(p) = g(p) - \gamma \tag{4.4}$$

[9] This relation is known as first-order stochastic dominance.

Figure 4.1 shows both the GIC and the RGIC for Indonesia between 1993 and 2002. The computer code containing the underlying data is presented in annex.

Figure 4.1. Growth Incidence Curves for Indonesia: 1993-2002



Cumulative Percentage of Population

We note that the absolute (or standard) growth incidence curve lies entirely above zero, thus economic growth in Indonesia during the 1993-2002 period has been associated with a reduction in poverty.

If the GIC switches sign over the relevant range, we can no longer rely on first-order dominance to tell what happened to poverty. Additional value judgments are needed. We may, for instance, use nonnegative weights that put a premium on equality (more equality in the distribution of living standards is preferred to less). One way of implementing this idea is to select the Dalton criterion. As discussed in section 2 [see expression (2.24)], a Dalton improvement implies second order dominance which we state in terms of the generalized Lorenz curve as:

$$\sum_{i=1}^{k} \Delta x_i \geq 0 \; \forall k \Leftrightarrow \frac{\mu_{1p}}{\mu_{0p}} = \frac{\mu_1 L_1(p)}{\mu_0 L_0(p)} \geq 1 \quad \forall p \qquad (4.5)$$

Again, using the logarithm, we get an indicator known as the *poverty growth curve*[10] or PGC (Son 2004):

$$\varphi(p) = \gamma + \Delta \ln L(p) \qquad (4.6)$$

This expression says that the rate of growth of the mean income of the poorest **p** percent of the population is equal to the overall growth rate adjusted by a distribution-correction factor. In this case, the adjustment factor is based on changes in the whole Lorenz curve.

Son (2004) bases her motivation of this indicator on the poverty implications of second order dominance as stated in Atkinson's (1987) theorem. When the entire generalized Lorenz curve shifts up, all additive poverty measures would indicate a fall in poverty[11]. Thus when **φ(p)** is strictly greater than zero for all **p** (a condition we will call PGC dominance), we know that poverty has fallen as indicated by all poverty measures that reflect the depth of poverty [e.g. poverty gap or squared poverty gap (Ravallion 1994a)]. But if in addition this indicator is greater than the growth rate of the overall mean, then inequality has also fallen[12]. This is the only case that Son would declare as pro-poor. Thus, she effectively bases her assessment only on the relative component of her indicator (namely, shifts in the Lorenz curve).

Aggregate Indicators

The minimalist approach to the specification of evaluative weights underlying the Pareto and the Dalton criteria leads to a *dominance* relation between the initial distribution of the living standards and the final one. Dominance criteria provide a general framework for unambiguous assessment of pro-poor growth. Since dominance does not necessarily hold among all possible states of the world, we conclude that it is a *partial ordering*. For instance, we can no longer invoke dominance if the growth incidence or poverty growth curves switch signs within the relevant range. Further

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[10] This terminology is from Son (2004). Jean-Yves Duclos suggested that it might be more informative to call this the "cumulative income growth curve".

[11] It is known that first-order dominance implies second order dominance, but not the other way around. Also, second-order dominance poverty comparisons may be based on TIP curves (Jenkins and Lambert 1997).

[12] In the case of the growth incidence curve, Ravallion and Chen (2003) observe that if g(p) is a decreasing function of all p in its domain of definition, then all inequality measures that respect the Pigou-Dalton principle of transfers will show a decline in inequality.

restrictions need to be imposed on the evaluative weights. For instance, one can assign a unique set of values to these weights and thus uniquely determine the evaluative criterion.

Ravallion and Chen (2003) propose a measure of the *rate of pro-poor growth* based on a normalization of the area under the GIC. The measure is called the *mean growth rate of the poor*. It is equal to the area under the growth incidence curve up to the headcount index, divided by the headcount index. Formally, it can be written as:

$$g_{pr} = \frac{1}{H_t}\int_0^{H_t} g_t(p)dp = \gamma + \frac{1}{H_t}\int_0^{H_t} \Delta \ln L'(p)dp \qquad (4.7)$$

For discrete data, Ravallion and Chen normalize their measure with respect to the initial year head count index, H_{t-1}. Expression (4.7) reveals how to compute the mean growth rate of the poor by numerical integration applied to a parameterized Lorenz curve. In the case of data for Indonesia presented in 4.3 below, we find the rate of pro-poor growth equal to 1.97 percent for the 1993-2002 period and to -0.95 percent for the 1996-2002 period. These rates are very close to the observed growth rates of the mean expenditure. Accordingly, economic growth did not significantly affect inequality among the poor.

Equivalently, this measure of the rate of pro-poor growth can be computed by multiplying the ordinary growth rate in the mean by an adjustment factor that translates the extent to which distributional changes have been pro-poor. In particular, the adjustment factor is equal to the ratio of the actual change in the Watts index[13] of poverty to the change in the same index that would have occurred under distribution-neutral growth. The rate of pro-poor growth will be higher than the ordinary growth rate when the distributional shifts are poverty reducing. Otherwise, it would be equal or less than the growth rate of the overall mean.

In a manner analogous to the approach of Ravallion and Chen (2003), we can use the social evaluation framework defined by expression (2.22) and the specific weights given by (3.3) applied to all points on the growth incidence curve. This leads to the following indicator of pro-poor growth (Essama-Nssah 2004). This measure, $\theta(\nu)$, is a

[13] Ravallion and Chen (2003) show that the area under the GIC up to the headcount index is equal to minus the change in the Watts index of poverty.

function of the focal parameter, ν.

$$\theta(\nu) = \frac{1}{n}\sum_{k=1}^{n}\omega_k(\nu)\Delta\ln x_k = \frac{\nu}{n}\sum_{k=1}^{n}(1-p_k)^{\nu-1}\Delta\ln x_k \qquad (4.8)$$

Using the definition of the covariance between two variables, we can write the rate of pro-poor growth in a form analogous to the expressions presented above for points on the GIC or the PGC. It is equal to the average growth rate γ* plus *a covariance-based distribution correction factor*.

$$\theta(\nu) = \gamma^* + \nu\,\mathrm{cov}[\Delta\ln x, (1-p)^{\nu-1}] \qquad (4.9)$$

The covariance term (or distribution correction factor) would vanish, if the focal parameter ν were equal to one. This would mean that society does not care about inequality in the distribution of individual outcomes. This puts us back in the domain of Pareto evaluation.

Alternatively, the covariance term would also vanish if all incomes or expenditures grew at the same rate. In this case growth would be distribution-neutral, and the rate of prop-poor growth would be equal the average growth rate. When growth-induced distributional changes are favorable to the poor, this indicator would show a rate of pro-poor growth greater than the average growth rate[14].

To find an interpretation for this indicator, we factor out the average growth rate and express the measure in a manner analogous to (3.2).

$$\theta(\nu) = \gamma^*[1 - C_{\Delta\ln x}(\nu)] \qquad (4.10)$$

where $C_{\Delta\ln x}(\nu) = -\frac{\nu}{\gamma^*}\mathrm{cov}[\Delta\ln x, (1-p)^{\nu-1}]$ is the extended concentration coefficient of individual growth rates. Thus, we call **θ(ν)** the *equally distributed equivalent growth rate* or EDEGR. This is the growth rate that would be socially equivalent to the observed one, for some choice of the focal parameter, ν. If all expenditures or income grew at the rate of **θ(ν)**, this would lead to a change in social welfare equivalent to the one induced by the observed pattern of growth. This *distribution-corrected average rate of growth* allows

[14] If individual growth rates are declining from the poorest to the richest, then there will be a positive association with the social weights and the covariance term will be positive.

the analyst to calibrate the poverty focus of the assessment through the choice of the aversion parameter, ν.

Finally, it is possible to write this indicator as a function of the growth rate of average expenditure (or income) γ, in a manner that is analogous to the other indicators discussed above. Using the definition of the growth incidence curve given by (4.3), we get the following equivalent expression.

$$\theta(\nu) = \gamma + \frac{1}{n}\sum_{k=1}^{n}\omega_k(\nu)\Delta \ln L'(p_k) \qquad (4.11)$$

In the above expression, the distribution adjustment factor is equal to a weighted average of changes in the slope of the Lorenz curve.

Given that the EDEGR measures a change in social welfare for some choice of the degree of inequality aversion, it seems reasonable to adopt the following decision rule. Economic growth will be considered pro-poor as long as the EDGR is positive. Else, growth will be considered anti-poor. An assessment based uniquely on the *distribution adjustment factor* (DAF) in (4.9) or (4.11) would be consistent with Kakwani and Pernia (2000) interpretation of pro-poor growth. Accordingly, growth would be considered pro-poor only when the distribution adjustment factor is positive.

Table 4.1 shows an empirical illustration based on data for Indonesia drawn from the World Bank global poverty monitoring database. The data cover the 1993-2002 period. For the entire period and for the sub-period of 1996-2002, we computed the extended Gini coefficients, the equally distributed equivalent growth rates (EDEGR), and the distribution-adjustment factor (DAF) associated with the rate of pro-poor growth defined by equation (4.11). When the pattern of distributional shifts is favorable to the poor (as in 1996-2002), the adjustment factor would be positive. Otherwise, it would be negative (as for 1993-2002).

Table 4.1. Equally Distributed Equivalent Growth Rates (EDEGR) for Indonesia

	1993-2002			1996-2002			
Focal Parameter	Gini 1993	EDEGR	DAF	Gini 1996	EDEGR	DAF	Gini 2002
1.00	0.00	1.62	-0.35	0.00	-0.53	0.42	0.00
2.00	31.7	1.56	-0.41	36.5	-0.24	0.70	34.3
3.00	41.9	1.59	-0.38	46.4	-0.13	0.82	44.0
4.00	47.2	1.61	-0.36	51.4	-0.07	0.88	49.0
5.00	50.4	1.63	-0.34	54.5	-0.03	0.91	52.0
6.00	52.7	1.65	-0.32	56.6	-0.01	0.94	54.1

Source: Author's calculations

Based on the results presented in table 4.1, it appears that economic growth in Indonesia was pro-poor between 1993 and 2002 and not over the1996-2002 period. This observation is consistent with the absolute approach to evaluating pro-poor growth. The relative approach would lead to the conclusion that growth in Indonesia was pro-poor in 1996-2002 and not in 1993-2002.

4.2. Decomposition of Poverty Outcomes

Indeed, procedures have been developed for the decomposition of changes in poverty into *growth* and *inequalit*y components (Ravallion and Datt, 1992; Kakwani 1993; Shorrocks 1999). The extent of the impact of growth and distribution on poverty thus depends on the responsiveness of poverty to changes in the mean income (growth effect) and to changes in inequality. Here we focus on two approaches: the elasticity approach by Kakwani (1993) and the Shapley decomposition (Kakwani 1997 and Shorrocks 1999).

The Elasticity Approach

For small changes in poverty and under the assumption that the Lorenz curve shifts proportionately over the whole range of income distribution, Kakwani (1993) shows that the total percentage change in a poverty index is equal to the *growth elasticity of the index* times the percentage change in the mean income plus the *elasticity of the index with respect to the Gini* times the percentage change in the Gini coefficient.

The growth elasticity of the head-count index is equal to:

$$\eta_H = -\frac{zf(z)}{H} < 0 \qquad (4.12)$$

Where **f(z)** is the density function of the welfare indicator evaluated at the poverty line. In terms of the Lorenz curve, the growth elasticity of the headcount index is equal to:

$$\eta_H = -\frac{z}{\left(\mu H \frac{\Delta^2 L(H)}{\Delta p_h^2}\right)} \qquad (4.13)$$

The elasticity with respect to Gini is:

$$\varepsilon_H = -\frac{(\mu - z)}{z}\eta_H \qquad (4.14)$$

For the general class of additively separable poverty measures, the growth elasticity is given by the following expression.

$$\eta_P = \frac{1}{P}\sum_{h=1}^{m} w_h x_h \left(\frac{\partial \psi}{\partial x_h}\right) \qquad (4.15)$$

The elasticity with respect to Gini is:

$$\varepsilon_P = \eta_P - \frac{\mu}{P}\sum_{h=1}^{m} w_h \left(\frac{\partial \psi}{\partial x_h}\right) \qquad (4.16)$$

These elasticities may be used to evaluate the *potential for poverty reduction*. Indeed, a higher growth elasticity means a greater potential for poverty reduction. Focusing in particular on poverty incidence, it can be seen from (4.12) that the responsiveness of the head-count index to growth depends essentially on the *initial value* of the index and the slope of the distribution function at the poverty line. The higher the initial head-count, the lower the elasticity and the higher the actual rate of growth required to further reduce poverty (other things being equal).

In the particular case of the FGT family of indices, these elasticities are equal to:

$$\eta_{FGT} = -\frac{\alpha[P_{\alpha-1} - P_\alpha]}{P_\alpha};\ \varepsilon_{FGT} = \eta_{FGT} + \frac{\alpha \mu P_{\alpha-1}}{zP_\alpha};\ \alpha > 0. \qquad (4.17)$$

Table 4.2 shows indicators of the responsiveness of poverty in rural India with respect to growth and inequality. These indicators were computed for the FGT measures based on data found in Datt (1998).

Table 4.2. Rural India 1983: FGT Measures and Associated Elasticities

Focus	FGT	Growth	Inequality
0	45.07	-1.87	0.44
1	12.48	-2.61	1.85
2	4.75	-3.25	3.23

Data Source Datt (1998)

On the basis of expressions (4.15) and (4.16) the total impact of growth and redistribution on income poverty may be written as:

$$\frac{dP}{P} = \eta_P \frac{d\mu}{\mu} + \varepsilon_p \frac{dG}{G} \qquad (4.18)$$

Equating the proportional change in poverty to zero, Kakwani (1993) defines the marginal proportional rate of substitution between mean income and inequality as follows:

$$MPRS = \frac{\partial \mu}{\partial G}\frac{G}{\mu} = -\frac{\varepsilon_P}{\eta_P} \qquad (4.19)$$

This is a measure of the trade-off between growth and inequality to the extent that it indicates the rate at which income needs to grow to compensate for an increase of 1 percent in the Gini index.

Shapley Method

Ravallion (1994b) argues that the above decomposition may lead to large errors in the case of big discrete changes. A different approach is therefore required. Instead of summarizing inequality by the Gini index, one may use a parameterized Lorenz curve along with the mean income to decompose changes in poverty into growth and inequality components, and a residual. Ravallion and Datt (1992) note that the existence of this residual depends on whether or not the poverty index is additively separable between the mean and the Lorenz curve. The residual would vanish if the mean income or the Lorenz curve remained constant over the decomposition period. Here, we focus on a new decomposition procedure proposed by Kakwani(1997) and Shorrocks (1999) and which does not involve a residual. Shorrocks rationalizes this method on the basis of the Shapley solution of cooperative games.

The problem of the *commons* is commonly used to frame the discussion of cooperative games. A commons is a technology that is jointly owned and operated by a group of agents. Joint ventures such as law firms, farming or fishing cooperatives are good examples of commons to the extent that they require coordinated action of partners with heterogeneous ability. Partners contribute inputs (e.g. labor and or capital) and share the profit generated by the enterprise (Moulin 2003). The key issue in this context is how to assess fairly the productive contribution of each partner. The following definition of the Shapley value is given by Young(1994), in the case of cost sharing.

> "Given a cost-sharing game on a fixed set of players, let the players join the cooperative enterprise one at a time in some predetermined order. As each player joins, the number of players to be served increases. The player's *cost contribution* is his net addition to cost when he joins, that is, the incremental cost of adding him to the group of players who have already joined. The Shapley value of a player is his average cost contribution over all possible orderings of the players."

To see how the above principle translates into a decomposition procedure, consider an aggregate statistical indicator such as the overall level of poverty or inequality. Let it be a function of **m** contributory factors which together account for the value of the indicator. The decomposition approach proposed by Shorrocks (1999) is based on the marginal effect on the value of the indicator of eliminating each of the contributory factors in sequence. The method then assigns to each factor the average of its marginal contributions in all possible elimination sequences.

In the case of change in poverty over time, given a fixed poverty line, the level of poverty at time **t** is a function of the mean income and the Lorenz curve, $\mathbf{P(\mu_t, L_t)}$. The overall change in poverty from period 1 to period 2 is equal to:

$$\Delta P = P(\mu_2, L_2) - P(\mu_1, L_1) \tag{4.20}$$

With no change in inequality as measured by the first Lorenz curve, this change in poverty would be equal to:

$$\Delta_{\mu} P(\mu, L_1) = P(\mu_2, L_1) - P(\mu_1, L_1) \quad (4.21)$$

When inequality is measured by the second Lorenz curve, the expression would take the following value:

$$\Delta_{\mu} P(\mu, L_2) = P(\mu_2, L_2) - P(\mu_1, L_2) \quad (4.22)$$

Similarly, with no change in the mean income, we have the following expressions:

$$\Delta_{L} P(\mu_1, L) = P(\mu_1, L_2) - P(\mu_1, L_1) \quad (4.23)$$

and,

$$\Delta_{L} P(\mu_2, L) = P(\mu_2, L_2) - P(\mu_2, L_1) \quad (4.24)$$

If there is no change in inequality, the marginal contribution of growth to change in poverty is given by either one of the following expressions:

$$\Delta_{G1} P = \Delta P - \Delta_{L} P(\mu_1, L) = \left[P(\mu_2, L_2) - P(\mu_1, L_2)\right] \quad (4.25)$$

or

$$\Delta_{G2} = \Delta P - \Delta_{L} P(\mu_2, L) = \left[P(\mu_2, L_1) - P(\mu_1, L_1)\right] \quad (4.26)$$

The Shapley contribution of growth to change in poverty is equal to the average of the above marginal contributions.

$$S_G = \frac{1}{2}(\Delta_{G1} + \Delta_{G2}) = \frac{1}{2}\left[P(\mu_2, L_2) - P(\mu_1, L_2)\right] + \frac{1}{2}\left[P(\mu_2, L_1) - P(\mu_1, L_1)\right] \quad (4.27)$$

Similarly, it can be shown that the Shapley contribution of inequality to change in poverty is equal to:

$$S_L = \frac{1}{2}\left[P(\mu_2, L_2) - P(\mu_2, L_1\right] + \frac{1}{2}\left[P(\mu_1, L_2) - P(\mu_1, L_1)\right] \quad (4.28)$$

Shorrocks (1999) also shows that the Shapley principle leads to the decomposition of change in poverty based on subgroup contributions over times.

$$\Delta P = \sum_{k=1}^{m}\left(\frac{w_{k1} + w_{k2}}{2}\right)\Delta P_k + \sum_{k=1}^{m}\left(\frac{P_{k1} + P_{k2}}{2}\right)\Delta w_k \quad (4.29)$$

where $\mathbf{w_k}$ is the share of population in group **k**, and $\mathbf{P_k}$ is the level of poverty in that group.

The following value judgments underlie the Shapley decomposition rule: (1) *Symmetry* or anonymity: the contribution assigned to any factor should not depend on its label or the way it is listed; (2) the rule should lead to exact or *additive decomposition*;

and (3) the contribution of each factor is taken to be equal to its (first round) *marginal impact*.

To illustrate, we use the data for Indonesia presented in table 4.3 below. Each row of the table gives the mean of monthly household expenditures in 1993 Purchasing Power Parity[15] (PPP) dollars, along with expenditure share of each decile. The year 1993 (our base year) is also the year when Indonesia became a middle-income country according to World Bank's classification.

Table 4.3. Distribution of Household Expenditure in Indonesia 1993-2002

Year	Mean	Lowest Decile	2nd	3rd	4th	5th	6th	7h	8th	9th	10th
1993	68.54	3.88	4.80	5.68	6.59	7.59	8.70	10.09	11.97	15.13	25.57
1996	86.62	3.57	4.39	5.20	6.05	6.99	8.08	9.44	11.34	14.63	30.31
2002	81.84	3.64	4.77	5.57	6.35	7.20	8.20	9.53	11.45	14.78	28.51

Source: World Bank Global Poverty Monitoring Database.

Table 4.4 shows the poverty profile associated with the above data. All these estimates are based on a parameterization of the general quadratic Lorenz curve(The computer program which produced these results is presented in the annex).

Table 4.4. A Profile of Poverty in Indonesia for the 1993-2002 Period

Poverty Measures	1993	1996	2002
Headcount	61.55	50.51	52.42
Poverty Gap	21.03	15.33	15.68
Squared Poverty Gap	9.16	6.02	6.09

Source: Author's Simulations

Overall, table 4.4 shows a decrease in poverty in Indonesia between 1993 and 2002. According to the Shapley decomposition of these poverty outcomes presented in table 4.5, poverty would have decreased even more had the growth process been distribution-neutral. In particular, poverty incidence would have fallen by about 12.5 percentage points instead of the 9 percent observed.

[15] The World Bank 1993 PPP conversion factor is equal to 635.655 rupiahs to a dollar. This is what one would use to convert dollars in local currency in 1993. The local Consumer Price Index (CPI) can then be used to translate these values into values for a year other than 1993.

Table 4.5. Shapley Decomposition of Poverty Outcomes in Indonesia, 1993-2002

Measure	Total Change	Growth	Inequality
Headcount	-9.13	-12.49	3.36
Poverty Gap	-5.35	-6.87	1.52
Squared Poverty Gap	-3.07	-3.82	0.75

Source: Author's Simulations

Table 4.6. Shapley Decomposition of Poverty Outcomes in Indonesia, 1996-2002

Measure	Total Change	Growth	Inequality
Headcount	1.91	4.05	-2.14
Poverty Gap	0.35	2.04	-1.69
Squared Poverty Gap	0.07	1.07	-1.00

Source: Author's Simulations

Focusing on the 1996-2002 sub-period, we note from both table 4.3 and table 4.6 that poverty has increased most likely due to the 1997 Asian crisis. Here again, the Shapely decomposition reveals that the increase in poverty was less sever because of an improvement in inequality.

While the above results show a strong link between growth and poverty reduction in Indonesia between 1993 and 2002, they also reveal the extent to which inequality has blunted the impacts of both growth and contraction. For a more detailed discussion of these results see Essama-Nssah (2004).

5. Concluding Remarks

The importance of distributional issues in policymaking creates the need for empirical tools for assessing the impact of economic shocks and policies on the distribution of economic welfare. Assuming that individuals are ranked in ascending order of some welfare indicator, the Lorenz curve transforms the information content of the cumulative distribution by mapping the cumulative proportion of the population against the cumulative share of welfare. It also provides an integrative framework for both the simulation and ethical evaluation of inequality and poverty. The same framework underlies the assessment of the social impact of economic growth.

The structure of the Lorenz curve suggest a simulation strategy for recovering inequality and poverty measures from information about the mean of the distribution, and about the slope and its rate of change. The use of numerical integration in simulating inequality and poverty measures obviates the derivation of special expressions from the chosen functional form of the Lorenz curve.

Lorenz ranking of distribution respects the Pigou-Dalton principle of transfers underlying second-order dominance. In the context of pro-poor growth analysis, GIC dominance [i.e. $g(p) \geq 0 \ \forall p$] respects first-order dominance, while PGC dominance [i.e. $\varphi(p) \geq 0 \ \forall p$] is consistent with second-order dominance. When dominance tests fail, one can resort to aggregate indicators for inequality and poverty comparisons. All measures reviewed here depend to some extent on a focal parameter that can be interpreted as an indicator of aversion for inequality. This aversion parameter allows the analyst to calibrate the poverty focus of social impact assessment. Both the extended Gini and the Atkinson measures can be derived from the concept of equally distributed equivalent welfare.

Poverty outcomes can be decomposed into growth and inequality components using either the elasticity approach or the Shapley method. Analysis of data for Indonesia reveals that absolute and relative indicators of pro-poor growth can lead to conflicting conclusions from the same set of facts.

Annex

A1. Lerman-Yitzhaki Estimator of the Cumulative Distribution Function (CDF)

The Lorenz curve maps the cumulative distribution of the population to the cumulative distribution of an indicator of economic welfare. It is therefore useful to have a way of estimating the CDF various situations, particularly when household level data are used to parameterize the Lorenz curve.

Let **x** be a welfare indicator assumed to be continuously distributed across a given population. The cumulative distribution function (CDF) **F(x)** gives the probability of observing an individual with a living standard at most equal to **x**. The slope of this function at a particular point gives the underlying density function. There are several ways of estimating a cumulative distribution on the basis of a sample of size **n**, depending on the method of adjustment for the non-continuity of the computation. The ordinary approach is to rank the observations in increasing order of the variable of interest, and divide the rank of each observation by the total number of observations. Let $\mathbf{p_r}$ stand for the ordinary estimate of the CDF for observation with rank **r**, then we may write:

$$p_r = \frac{r}{n} \quad (1)$$

The density function is linked to the CDF by the relation:

$$f(x_i) = F(x_i) - F(x_{i-1}) \quad (2)$$

The estimate of the density function associated with (1) is therefore equal to:

$$f(x_r) = \frac{1}{n} \quad (3)$$

Household level data are usually available in the form of *a weighted sample*. A household of a given size represents a certain number of households in the population at large. In such a case, Lerman and Yitzhaki (1989) recommend the following mid-interval estimator for the CDF[16].

$$p_k = \sum_{h=0}^{k-1} w_h + \frac{w_k}{2}; \quad w_0 = 0 \quad (4)$$

[16] Deaton (1997:154) proposes a similar procedure for converting household ranks into individual ranks. Assuming that there are $w_h n_h$ people in household h (where n_h is the household size and w_h its absolute weight), then starting from $\rho_1=1$, the rank of the first person in household h+1 is given by the recursive formula: $\rho_{h+1} = \rho_h + n_h w_h$. These ranks can than be normalized relative to the overall population.

where $\mathbf{w_h}$ stands for the relative weight of each individual in household **h** in the overall population. This relative weight is equal to the household size times the absolute household weight divided by total population. Below are a few lines of code fo implementation of the Lerman-Yitzhaki estimator in EViews.

```
'LYCDF.PRG    (Progam name)
SORT  XH                        'Sort in ascending order of xh (per capita welfare indicator)
SERIES  W1=S*W                  'Inflate Household Size
SCALAR TPOP = @SUM(W1)          'Population
SERIES  WH=W1/TPOP              'Relative weight of type h household in the population
'Compute cumulative distribution  F_HAT
SMPL @FIRST  @FIRST
        SERIES FH=WH
        SMPL @FIRST+1  @LAST
        FH=FH(-1) + WH
SMPL  @ALL
SERIES F_HAT = FH – 0.5*WH
```

A2. Source Code for the Simulation of Inequality and Poverty Measures for Rural India 1983

```
'INDILORENZ.PRG  computes and plots a parameterized Lorenz curve  for rural India in
1983 based on data presented in Datt (1998).  It then proceedes to computing associate
inequality and poverty measures.

'Assumption about form of data available: Income classes with percentage of people and
average income in each class.

'EViews 5.1 Standard Edition, January 28, 2005.

'B. Essama-Nssah, PRMPR, The World Bank Group, Washington D.C., January 30,
2005

INCLUDE ATKINSON  'For Atkinson measures
INCLUDE ENTROPY   'For generalized entropy measures
INCLUDE KAKWANI   'For elasticty of FGT measures
INCLUDE LSEGMENT 'Linear Segment Estimator of extended Gini (arguments:
cumulative proportion of population and first order derivative of Lorenz function
INCLUDE QUADRALORENZ 'General Quadratic Lorenz curve (argument class
expenditure shares)
' -----------------------------------
'DATA PREPARATION
MODE  QUIET
DB INDIRU
WFCREATE(WF=RURALINDIA83, PAGE=INITIALDATA) U 13
SCALAR Z=89 'Poverty line in Rupis per capita per month
FOR %SR POPSHR AVGX  XSHR  XWT
        SERIES  {%SR}
NEXT
```

```
POPSHR.LABEL(d) Polulation share
POPSHR.LABEL(s) Datt(1998: 2) Table 1; Column 2
POPSHR.LABEL(u) Proportion
POPSHR.LABEL(r) Datt Gaurav.  1998. Computational Tools for Poverty Measurement and Analysis.
POPSHR.LABEL(r)  Food Consumption and Nutrition Division (FCND) Discussion Paper No. 50
POPSHR.LABEL(r)  Washington D.C:  International Food Policy Research Institute (IFPRI)
POPSHR.FILL        0.92, 2.47, 5.11, 7.90, 9.69, 15.24, 13.64, 16.99, 10.00, 9.78, 3.96, 1.81, 2.49 'Percentage
POPSHR=POPSHR/100' Proportion
AVGX.LABEL(d) Average Monthly per Capita Expenditure
AVGX.LABEL(s) Datt(1998: 2) Table 1; Column 3
AVGX.LABEL(u) In Rupis
AVGX.LABEL(r) Datt Gaurav.  1998. Computational Tools for Poverty Measurement and Analysis.
AVGX.LABEL(r)  Food Consumption and Nutrition Division (FCND) Discussion Paper No. 50
AVGX.LABEL(r)  Washington D.C:  International Food Policy Research Institute (IFPRI)
AVGX.FILL        24.84, 35.80, 45.36, 55.10, 64.92, 77.08, 91.75, 110.64, 134.90, 167.76, 215.48, 261.66, 384.97 'Rupis
XWT=POPSHR*AVGX
SCALAR MUX=@SUM(XWT)
XSHR=XWT/MUX
'-----------------------------------------------------------------
'PLOT CORRESPONDING LORENZ CURVE
SERIES PH  'Cumulative proportion of population
SERIES LPH 'Cumulative share of expenditure
SMPL @FIRST @FIRST
	PH=POPSHR
	LPH=XSHR
SMPL @FIRST+1 @LAST
	PH=PH(-1) + @NAN(POPSHR, 0)
	LPH=LPH(-1) + @NAN(XSHR,0)
SMPL @ALL
SERIES L45=PH
GROUP LCGRP PH L45 LPH
FREEZE(RURALRZ) LCGRP.XY
RURALRZ.ADDTEXT(T)  Lorenz Curve For Rural India in 1983
RURALRZ.NAME(1)  Cumulative Proportion of Population
RURALRZ.NAME(2)   Line of Complete Equality
RURALRZ.NAME(3)   Cumulative Proportion of Expenditure
'-----------------------------------------------------------------------------------------------------------
'PARAMETERIZE THE LORENZ FUNCTION BASED ON THE GENERAL QUADRATIC MODEL
CALL QUADRALORENZ(XSHR)
MODEL GQMOD  'General Quadratic Model
GQMOD.APPEND   LQ=-0.5*(B*PH + E+ @SQRT(M*PH^2 + N*PH+ E^2)   )'Lorenz function
```

```
GQMOD.APPEND   FOD=-(B/2)-( (2*M*PH+N)/(4*(@SQRT(M*PH^2+N*PH+E^2)) ))'
First Derivative
GQMOD.APPEND   SOD=((R^2)/8)*1/( (@SQRT( (M*PH^2+N*PH+E^2) ))^3   )
'Second Derivative
STORE A B CC E M N R SA SB MUX Z GQMOD RURALRZ
'-------------------------------------------------------------
'SIMULATE AND PLOT THE LORENZ CURVE
PAGECREATE(PAGE=SIMULATION) U 5000 'Choose a wide enough range to make
the estimated functions smooth
SERIES UNO=1
SERIES PH=@TREND/@OBS(UNO)
SERIES L45=PH
GROUP LCGRP PH L45
FETCH A B CC E M N R SA SB MUX Z GQMOD  'From INDIRU database
GQMOD.SCENARIO ACTUALS
GQMOD.SOLVE
SMPL @LAST @LAST
   LQ=1
SMPL @ALL
LCGRP.ADD LQ
FREEZE(GQLORENZ) LCGRP.XY
GQLORENZ.ADDTEXT(T)  Simulated Lorenz Curve for Rural India in 1983
GQLORENZ.NAME(1)  Cumulative Proportion of Population
GQLORENZ.NAME(2)   Line of Complete Equality
GQLORENZ.NAME(3)   Cumulative Proportion of Expenditure
'------------------------------------------------------------------------------------------------------------
'COMPUTE EXTENDED GINI COEFFICIENTS USING LOCAL SUBROUTINE
LSEGMENT
'DECLARE GLOBAL ARGUMENTS, NUVEC AND GINIVEC, OUTSIDE THE LOCAL
SUB ROUTINE
VECTOR(6) NUVEC
VECTOR(6) GINIVEC
CALL LSEGMENT(PH, FOD, NUVEC, GINIVEC)
VECTOR  GINIVEC=100*GINIVEC 'Scale up to give results in percentage
MATRIX(6,4) GINISEN 'The third column will be used later on to store the Sen index of
poverty
STORE GQLORENZ GINIVEC GINISEN GQMOD
'-----------------------------------------------------------------------------------
'ESTIMATE LEVEL OF EXPENDITURE AND DENSITY FUNCTION
SERIES XP=MUX*FOD
SERIES IFX=MUX*SOD
SERIES FX=1/IFX
SERIES FXDX=0
SMPL  @FIRST+1 @LAST
      FXDX=FX*D(XP)
SMPL @ALL

'---------------------------------------------------------------
'COMPUTE MEMBERS OF THE FGT FAMILY
VECTOR(3) AVRS  'Aversion for inequality among the poor
AVRS.FILL 0,1,2
```

```
VECTOR(3) FGT
SERIES POOR=(XP<=Z)
SERIES HEAD=POOR*FXDX
FGT(1)=100*@SUM(HEAD)   'Scale up by 100 to get results in percentage
SERIES NGAP=(1-XP/Z)         'Poverty gap normalized by poverty line
SERIES FGTGAP=POOR*NGAP*FXDX 'Poverty focus of the measure
FGT(2)=100*@SUM(FGTGAP)  'Scale up
SERIES FGTGPS=POOR*(NGAP^2)*FXDX
FGT(3)=100*@SUM(FGTGPS) 'Scale up
'------------------------------------------------------------
'PLOT THE CORRESPONDING TIP CURVE
SMPL  @FIRST  @FIRST
       SERIES TIPV =@NAN(FGTGAP, 0)  'Recode NA as Zero (Just in case there are
any missing data)
SMPL  @FIRST+1  @LAST
      SERIES TIPV=TIPV(-1) + @NAN(FGTGAP, 0)
SMPL @ALL
SCALAR  HDC=FGT(1)/100
SCALAR  PVG=FGT(2)/100
SCALAR  SPG=FGT(3)/100
GROUP  TIPGRP  PH   TIPV
FREEZE(FGTIP)  TIPGRP.XY
FGTIP.DRAW(DASHLINE, BOTTOM) HDC
FGTIP.DRAW(DASHLINE, LEFT)       PVG
FGTIP.ADDTEXT A TIP Representation of Rural Poverty in India in 1983
'------------------------------------------------------------------------------------------------------------------
'COMPUTE ELATICITY WITH RESPECT TO CHANGE IN MEAN CONSUMPTION AND
GINI
CALL KAKWANI  'Aglobal subroutine
'STORE POVERTY RESULTS IN A TABLE
MATRIX(3,4) INDIRUPOV
!j=1
FOR %ST AVRS FGT ETAH EPSI
       COLPLACE(INDIRUPOV, {%ST},!j)
       !j=!j+1
NEXT
FREEZE(POVTAB) INDIRUPOV
SETLINE(POVTAB,3)
!j=2
FOR %ST FOCUS FGT ETAH EPSI
        SETCELL(POVTAB, 1, !j, %ST, "C")
!j=!j+1
NEXT
POVTAB.SETFORMAT(B) G.1  'Show only one significant digit on second column
POVTAB.SETFORMAT(R4C3:R6C5)  F.2  'Fixed two decimal places from column 3 to
column 5
POVTAB.SETTEXTCOLOR(R4C2:R4C5)  RED
'--------------------------------------------
'COMPUTE THE WATTS INDEX
SERIES WATGAP=(LOG(Z/XP) )*POOR
SERIES WATSR=FXDX*WATGAP
```

```
SCALAR WATTS=@SUM(WATSR)*100  'In percent
'-------------------------------------------------------------------------------
'COMPUTE MEMBERS OF THE CHAKRAVARTY FAMILY
'Chakravarty: CHK when AVRS is 0 or 1 CHK=FGT
'Else
SERIES CHKSR= POOR*FXDX*(1 - (XP/Z)^2)
SCALAR CHK=@SUM(CHKSR)*100  'In percentage
'Clark, Hemming and Ulph: CHU when AVRS is 0  CHU=WATTS; when AVRS =1,
CHU=FGT
'Else
SERIES CHUSR=0.5*CHKSR
SCALAR CHU=@SUM(CHUSR)*100 'In percentage
STORE  FGTIP POVTAB WATTS CHK CHU
'-------------------------------------------------------
'COMPUTE GENERALIZED ENTROPY
VECTOR(5)   AVEC   'Indicator of Aversion
VECTOR(5)  ENTRO     'Generalized Entropy Measures
CALL ENTROPY(FOD, FXDX, AVEC, ENTRO) 'This is a local subroutine with four
global arguments
ENTRO=100*ENTRO 'In percent
'-----------------------------------------------------
'COMPUTE ATKINSON MEASURES
'DECLARE GLOBAL ARGUMENTS OUTSIDE THE LOCAL SUBROUTINE
VECTOR(5) ATKIN
VECTOR(5) EDEQ
CALL ATKINSON(XP, FXDX, ATKIN, EDEQ, MUX)
ATKIN=100*ATKIN 'In percent
'-----------------------------------------------------------------------------------------------------------------
'BUILD A SINGLE TABLE TO HOLD RESULTS FOR ENTROPY AND ATKINSON
MATRIX(5, 4)  GENTROPY  'Matrix of inequality indicators
!COL=1
FOR %ST AVEC ENTRO ATKIN EDEQ
         COLPLACE(GENTROPY, {%ST}, !COL)
        !COL=!COL+1
NEXT
FREEZE(ENTROP)  GENTROPY
SETLINE(ENTROP,3)
!CLN=2
 FOR %CLB  FOCUS ENTROPY ATKINSON EDEX
           SETCELL(ENTROP, 1, !CLN, %CLB, "C")
           !CLN=!CLN+1
NEXT
ENTROP.SETFORMAT(B) F.1  'Fixed one decimal place on second column
ENTROP.SETFORMAT(R4C3:R8C5)  F.2  'Fixed two decimal places from column 3 to
column 5
STORE ENTROP
'-----------------------------------------------------
'COMPUTE SEN INDEX OF POVERTY
PAGECOPY(PAGE=SENINDEX, SMPL=@ALL IF POOR) HDC GINISEN GINIVEC  XP
Z  GQMOD
DB SEN  'To store objects with same names as content of INDIRU
```

```
FOR %SR HHSZ HHWT UNO
  SERIES {%SR}=1
NEXT
GROUP RPHH  XP HHSZ  HHWT 'Representative poor household
'Transform Original Data into Cumulative shares
SERIES PCX=RPHH(1)/RPHH(2)         'Total expenditure divided by household size
SERIES POPH=RPHH(2)*RPHH(3)      'Population residing in houshold h
SERIES WH=POPH/@SUM(POPH)'Population share of household h
SERIES PCXW=POPH*PCX           'Population level expenditure in household h
SERIES  XSHARE=PCXW/@SUM(PCXW)
SORT PCX
SERIES XPWT=XP*WH
SCALAR MUPR=@SUM(XPWT)
'Compute the Cumulative Shares of Population and Corresponding Values of the Lorenz
Curve (Use Dynamic Assignment)
SMPL  @FIRST  @FIRST
        SERIES FHAT=WH
        SERIES LRNZ =@NAN(XSHARE, 0)       'To protect against missing data,
recode NA as zero
SMPL  @FIRST+1  @LAST
        SERIES FHAT=FHAT(-1) + WH
        SERIES LRNZ=LRNZ(-1) + @NAN(XSHARE, 0)
SMPL @ALL
SERIES PH=FHAT
'Create the 45-Degree Line
SERIES L45=PH
GROUP  LCPR  PH  L45 LRNZ
FREEZE(LCPOOR)  LCPR.XY
LCPOOR.SCALE(ALL) RANGE(MINMAX)
LCPOOR.ADDTEXT(T)  Lorenz Curve for  Distribution among the Poor
LCPOOR.NAME(1)  Cumulative Proportion of Population
LCPOOR.NAME(2)  Line of Complete Equality
LCPOOR.NAME(3)  Cumulative Proportion of Income
'Parameterize the Lorenz Curve for the Poor
COEF(3) BETA
   SMPL @FIRST @LAST-1
   EQUATION GQLC.LS  LRNZ*(1-LRNZ)=BETA(1)*(PH^2 - LRNZ) +
BETA(2)*LRNZ*(PH - 1) + BETA(3)*(PH - LRNZ)
   SCALAR A=BETA(1)
   SCALAR B=BETA(2)
   SCALAR CC=BETA(3)     'To avoid the reserved name c
   SCALAR E=-(A+B+CC+1)
   SCALAR M=(B^2-4*A)
   SCALAR N=(2*B*E-4*CC)
   SCALAR R=@SQRT((N^2-4*M*E^2))
   SCALAR SA=(R-N)/(2*M)
  SCALAR SB=-(R+N)/(2*M)
SMPL @ALL
STORE A B CC E M N R SA SB MUPR Z GQMOD
GQMOD.SCENARIO  ACTUALS
SOLVE GQMOD
```

```
FOR %VEC  GINIPOOR NUV  SEN
          VECTOR(6)  {%VEC}
NEXT
CALL LSEGMENT(PH,FOD,NUV,GINIPOOR)
 FOR !NU=1 TO 6
         SEN(!NU)=100*HDC*(1  - (MUPR/Z)*(1-GINIPOOR(!NU) ) ) 'In percentage
NEXT
GINIPOOR=100*GINIPOOR  'In percentage
'------------------------------------------------------------------------------------
'LOAD THE GINISEN MATRIX AND FREEZE IT IN A TABLE
!J=1
FOR %V NUV GINIVEC GINIPOOR  SEN
         COLPLACE(GINISEN,{%V},!J)
          !J=!J+1
NEXT
FREEZE(GINSENTAB)  GINISEN
SETLINE(GINSENTAB,3)
!CLN=2
FOR  %CLB FOCUS GINI GINIPOOR SEN
          SETCELL(GINSENTAB,1, !CLN, %CLB, "C")
          !CLN=!CLN+1
NEXT
GINSENTAB.SETFORMAT(B) G.1
GINSENTAB.SETFORMAT(R4C3:R9C5) F.2
GINSENTAB.SETTEXTCOLOR(R5C2:R5C5) RED
STORE GINSENTAB
'---------------------------------------------------------------------------------------
'CREATE A REPORT PAGE AND BRING IN SOME RESULTS
PAGECREATE(PAGE=REPORT) U 1
FETCH GINSENTAB MUPR
CLOSE SEN
DB INDIRU
FETCH CHK CHU ENTROP FGTIP GQLORENZ MUX POVTAB RURALRZ WATTS Z
CLOSE INDIRU

'END OF PROGRAM
```

A3.The Subroutines of INDILORENZ.PRG

ATKINSON.PRG a local subroutine to compute Atkinson measures of inequality

```
'B. Essama-Nssah, PRMPR, The World Bank Group, Washington D.C.  October 24,
2004

SUBROUTINE LOCAL ATKINSON(SERIES X, SERIES WT, VECTOR ATVEC,
VECTOR EDEV, SCALAR MU)
'Set Maximal Level of Aversion and Compute Size of Vector to Hold Results
   !MAXAV=2
   !B=2
   !CF=!B-1
   !RW=(!B*!MAXAV)+!CF
```

```
   SCALAR ATK             'Atkinson Index of Inequality
   SCALAR EDE            'Equally Distributed Equivalent Welfare
   SCALAR SCI             'Social Cost of Inequality
   SERIES YH=X
   SERIES  YHW=WT*YH
   SERIES  LGYH=LOG(YH)
   SERIES  LGYHW=WT*LGYH
    !R=1
   FOR !EPS=0 TO !MAXAV  STEP 1/!B
         IF !EPS=0 THEN
              EDE=@SUM(YHW)
              SCI=MU-EDE
                ATK=SCI/MU
                ATVEC(!R)= ATK
                EDEV(!R)=EDE
          ELSE
                   IF !EPS=1  THEN
                             EDE=EXP(@SUM(LGYHW))
                              SCI=MU-EDE
                              ATK=SCI/MU
                              ATVEC(!R)= ATK
                              EDEV(!R)=EDE
                  ELSE
                              SERIES YHWT=WT*(YH)^(1-!EPS)
                              EDE=(@SUM(YHWT))^(1/(1-!EPS))
                              SCI=MU-EDE
                              ATK=SCI/MU
                              ATVEC(!R)= ATK
                              EDEV(!R)=EDE
                  ENDIF
         ENDIF
    !R=!R+1
NEXT

ENDSUB
```

'ENTROPY.PRG a subroutine to compute generalized entropy measures of inequality
'B. Essama-Nssah, PRMPR, The World Bank Group, Washington D.C. October 24, 2004

```
SUBROUTINE LOCAL  ENTROPY(SERIES FD, SERIES WT, VECTOR AV, VECTOR
GEM)
'Set Maximal Level of Aversion and Compute Size of Vector to Hold Results
   !MAXAV=2
   !B=2
   !CF=!B-1
   !RW=(!B*!MAXAV)+!CF
   SCALAR GE
   SERIES  YHMU=FD ' FOD=XP/MUX
   SERIES  INVYM=1/YHMU
   SERIES LINVYM=LOG(INVYM)
   SERIES  LGYM=LOG(YHMU)
```

```
 !R=1
 FOR !THETA=0 TO !MAXAV  STEP 1/!B
     IF !THETA=0 THEN
          SERIES LINVYMW=WT*LINVYM
          AV(!R)=!THETA
          GE=@SUM(LINVYMW)
          GEM(!R)= GE
     ELSE
              IF !THETA=1  THEN
                      SERIES THEILW=WT*(YHMU*LGYM)
                      GE=@SUM(THEILW)
                      AV(!R)=!THETA
                      GEM(!R)=GE
              ELSE
                      SERIES GETHETA=YHMU^!THETA
                      SERIES  GETHETAW=WT*GETHETA
                      AV(!R)=!THETA
                      GE=( @SUM(GETHETAW) - 1)/((!THETA)^2 - !THETA)
                     GEM(!R)=GE
              ENDIF
     ENDIF
 !R=!R+1
NEXT

ENDSUB
```

KAKWANI.PRG is a subroutine designed to compute poverty elasticities for the FGT class of poverty measures

```
'B. Essama-Nssah, PRMPR, The World Bank Group, Washington, D.C. May 28, 2004

SUBROUTINE KAKWANI

      FOR %V  ETAH EPSI MPRS
             VECTOR(3) {%V}
      NEXT
      SCALAR SDPO=((R^2)/8)*1/( (@SQRT( (M*HDC^2+N*HDC+E^2) ))^3   )
      SCALAR ETA1=-Z/(MUX*HDC*SDPO)        'Elasticity of Headcount with respect to
MU
      SCALAR ETA2=1-(HDC/PVG)                 'Elasticity of Poverty Gap with respect
to MU
      SCALAR ETA3=-2*(PVG-SPG)/SPG           'Elasticity of Square Poverty Gap  with
respect to MU
      SCALAR EPSI1=-( (MUX-Z)/Z)*ETA1            'Elasticity of Headcount with respect
to Gini
      SCALAR EPSI2=ETA2+(MUX*HDC)/(Z*PVG)  'Elasticity of Poverty Gap with
respect to Gini
     SCALAR EPSI3=ETA3+(2*MUX*PVG)/(Z*SPG)'Elasticity of Square Poverty Gap
with respect to Gini
     FOR !J=1 to 3
           ETAH(!J)=ETA{!J}
           EPSI(!J)=EPSI{!J}
```

```
    NEXT
    FOR !J=1 TO 3
        MPRS(!J)=-EPSI(!J)/ETAH(!J) 'Marginal proportional rate of substitution
between growth and inequality
   NEXT
ENDSUB
```

LSEGMENT.PRG is a subroutine designed to compute a vector of extended Gini coefficients based on the Linear Segment Estimator proposed by Chotikapanich and Griffiths 2001. The subroutine requires two series as arguments: P the cummulative proportion of individuals and FOD the first order derivative of the Lorenz function

```
'B. Essama-Nssah, PRMPR, The World Bank Group. Washington, D.C. October  25,
2004
'EViews 4.1, Standard Edition April 19, 2004 build

SUBROUTINE LOCAL  LSEGMENT(SERIES P, SERIES FD, VECTOR AVER,
VECTOR GINU)
            SERIES CUMPI=P
            SERIES QI=(1-CUMPI)
            SERIES QH=(1-CUMPI(-1))
            !AVMAX=6
            !B=1
            !CF=!B-1
           !RWS=(!B*!AVMAX)-!CF
           !T=1
              FOR !NU=1 TO !AVMAX  STEP 1/!B
                   SERIES QANU=QI^(!NU)
                   SERIES QBNU=QH^(!NU)
                   SERIES  QCNU=QANU-QBNU
                   SERIES  QDNU=FD*QCNU
                   SCALAR  QDSUM=@SUM(QDNU)
                   GINU(!T)=1+QDSUM
                   AVER(!T)=!NU
                  !T=!T+1
            NEXT

ENDSUB
```

'QUADRALORENZ.PRG is a subroutine designed to compute parameters of the Lorenz curve based on the General Quadratic model using percentage shares of income or expenditure (when data are organized by income class).

```
'B.Essama-Nssah, PRMPR, The World Bank Group, Washington D.C. January 29, 2005
'EViews 5.1

SUBROUTINE QUADRALORENZ(SERIES SHR)  'SHR for shares
  SMPL  @FIRST  @FIRST
      SERIES LRNZ =SHR
  SMPL  @FIRST+1  @LAST
      SERIES LRNZ=LRNZ(-1) + @NAN(SHR, 0) 'Ordinate of the Lorenz curve
```

```
   SMPL @ALL
COEF(3) BETA
   SMPL @FIRST @LAST-1
   EQUATION GQLC.LS  LRNZ*(1-LRNZ)=BETA(1)*(PH^2 - LRNZ) +
BETA(2)*LRNZ*(PH - 1) + BETA(3)*(PH - LRNZ)
   SCALAR A=BETA(1)
   SCALAR B=BETA(2)
   SCALAR CC=BETA(3)     'To avoid the reserved name c
   SCALAR E=-(A+B+CC+1)
   SCALAR M=(B^2-4*A)
   SCALAR N=(2*B*E-4*CC)
   SCALAR R=@SQRT((N^2-4*M*E^2))
   SCALAR SA=(R-N)/(2*M)
  SCALAR SB=-(R+N)/(2*M)
 SMPL  @ALL

ENDSUB
```

A4. Source Code for Pro-Poor Growth Analysis for Indonesia, 1993-2002

INDOPPG.PRG uses parameterized Lorenz curves to implement a pro-poor growth analysis for Indonesia for 1993-1996; 1996-2002.

```
'EViews 5.1 (Standard Edition January 28, 2005)

'B. Essama-Nssah, PRMPR, the World Bank Group, Washington D.C. February 06,
2005

INCLUDE  LSEGMENT
INCLUDE  QUADRALORENZ
INCLUDE PROPOORATE

MODE QUIET
'---------------------
'DATA SET UP
DB PPG   'For Pro-Poor Growth
WFCREATE(WF=INDOPROPOOR, PAGE=POVMODELS) U 11
FOR %SR UNO PH  XSHR1 XSHR2 XSHR3
        SERIES {%SR}
NEXT
UNO=1
PH=@TREND/(@OBS(UNO) -1) 'Cumulative proportions of population
XSHR1.FILL 0.00, 3.88, 4.80, 5.68, 6.59, 7.59, 8.70, 10.09, 11.97, 15.13, 25.57 'Year
1993
XSHR2.FILL  0.00, 3.57, 4.39, 5.20, 6.05, 6.99, 8.08, 9.44, 11.34, 14.63, 30.31   'Year
1996
XSHR3.FILL  0.00, 3.64, 4.77, 5.57, 6.35, 7.20, 8.20, 9.53, 11.45, 14.78, 28.51    'Year
2002
FOR !S=1 TO 3
       XSHR{!S}=XSHR{!S}/100  'To get proportions
NEXT
```

```
VECTOR(3) MUVEC
MUVEC.FILL  68.54,  86.62,  81.84 'Mean of monthly household expenditure in 1993
Purchasing Power Parity
SCALAR PPP1993=635.655
VECTOR(3) HDCT  'Published poverty incidence
HDCT.FILL 0.6155, 0.5051,  0.5242
GROUP SHRGRP XSHR1 XSHR2 XSHR3
'-----------------------------------------------
'PARAMETRIZE LORENZ CURVES
FOR !K=1 TO 3         'For 1993  1996   2002
        SCALAR MU{!K}=MUVEC(!K)  'Get  average monthly expenditure in 1993 PPP
dollars from vector MUPP
        SCALAR HD{!K}=HDCT(!K)
        CALL QUADRALORENZ(SHRGRP(!K))  'Estimate the Lorenz functions using the
subroutine QUADRALORENZ
                FOR %ST A B E M N R SA SB
                        RENAME {%ST}  {%ST}{!K}
                        STORE {%ST}{!K} MU{!K} 'HDs are the published headcount indices
we use to set the poverty lines
                NEXT
     MODEL GQMOD{!K}
     GQMOD{!K}.APPEND   LQ{!K}=-0.5*(B{!K}*PH + E{!K}+ @SQRT(M{!K}*PH^2 +
N{!K}*PH+ E{!K}^2)  )'Lorenz function
     GQMOD{!K}.APPEND   FOD{!K}=-(B{!K}/2)-(
(2*M{!K}*PH+N{!K})/(4*(@SQRT(M{!K}*PH^2+N{!K}*PH+E{!K}^2)) ))' First Derivative
     GQMOD{!K}.APPEND   SOD{!K}=((R{!K}^2)/8)*1/( (@SQRT(
(M{!K}*PH^2+N{!K}*PH+E{!K}^2) ))^3   )   'Second Derivative
     STORE GQMOD{!K}
     MODEL  FGTMOD{!K}
     FGTMOD{!K}.APPEND  HDC{!K}= -( N{!K}+R{!K}*(
(B{!K}+(2*PVLN{!K})/MUX{!K})/(@SQRT(((B{!K}+(2*PVLN{!K})/MUX{!K}) )^2 -M{!K} )) )
)/(2*M{!K})
'Share of Poor in Total Welfare, First and Second Derivatives at headcount, Between
Group Inequality
    FGTMOD{!K}.APPEND  LPO{!K}=-0.5*(B{!K}*HDC{!K} + E{!K}+
@SQRT(M{!K}*HDC{!K}^2 + N{!K}*HDC{!K}+ E{!K}^2)  )
    FGTMOD{!K}.APPEND FDPO{!K} =-(B{!K}/2)-
(2*M{!K}*HDC{!K}+N{!K})/(4*@SQRT((M{!K}*HDC{!K}^2+N{!K}*HDC{!K}+E{!K}^2)) )
    FGTMOD{!K}.APPEND SDPO{!K}=((R{!K}^2)/8)*1/( (@SQRT(
(M{!K}*HDC{!K}^2+N{!K}*HDC{!K}+E{!K}^2) ))^3   )
    FGTMOD{!K}.APPEND GBPR{!K}=HDC{!K}-LPO{!K}
'Poverty Gap
   FGTMOD{!K}.APPEND PVG{!K}=HDC{!K} - (MUX{!K}/PVLN{!K})*( -0.5*(
B{!K}*HDC{!K}+E{!K}+  @SQRT( (M{!K}*HDC{!K}^2 + N{!K}*HDC{!K} +E{!K}^2) ) )  )
'Squared Poverty Gap
   FGTMOD{!K}.APPEND SPG{!K}= 2*PVG{!K}-HDC{!K}-(
(MUX{!K}/PVLN{!K})^2)*(A{!K}*HDC{!K}+B{!K}*(  -0.5*( B{!K}*HDC{!K}+E{!K}+@SQRT(
(M{!K}*HDC{!K}^2 + N{!K}*HDC{!K} +E{!K}^2) ) ) )-(R{!K}/16)*LOG( (1-
HDC{!K}/SA{!K})/(1-HDC{!K}/SB{!K}) ) )
  SERIES TRJHDC{!K}=HD{!K} 'Set trejectory Headcount to known values
  SERIES MUX{!K}=MU{!K}
```

```
  SERIES PVLN{!K}
  FGTMOD{!K}.CONTROL PVLN{!K} HDC{!K} TRJHDC{!K}
  SCALAR Z{!K}=PVLN{!K}(1) 'Reinitialize the poverty line to minimize errors associated
with parameterization
  FGTMOD{!K}.SCENARIO ACTUALS
  FGTMOD{!K}.SOLVE
  STORE FGTMOD{!K}  HD{!K} Z{!K}
NEXT
'----------------------------------------
'NEW PAGE FOR SIMULATION
PAGECREATE(PAGE=SIMULATION) U 5000  'Choose a wide enough range to make
the estimated functions smooth (Lorenz and associated density)
VECTOR(3) EMU 'To store estimates of mean expenditure
MATRIX(6,4) INDOGINI
'---------------------------------------
'PLOT LORENZ CURVES
SERIES UNO=1
SERIES PH=@TREND/@OBS(UNO)
SERIES L45=PH
 VECTOR(6) NUVEC  'Aversion parameters in the extended Gini coefficient
FOR !K=1 TO 3
      GROUP LCGRP{!K} PH L45
      FETCH A{!K} B{!K} E{!K} M{!K} N{!K} R{!K} SA{!K} SB{!K} MU{!K} FGTMOD{!K}
GQMOD{!K} HD{!K} Z{!K}
      GQMOD{!K}.SCENARIO ACTUALS
      GQMOD{!K}.SOLVE
     SMPL @LAST @LAST
              LQ{!K}=1
    SMPL @ALL
    LCGRP{!K}.ADD LQ{!K}
    FREEZE(LORENZ{!K}) LCGRP{!K}.XY
'------------------------------------------------------------------------------------------------------------------
'COMPUTE EXTENDED GINI COEFFICIENTS USING LOCAL SUBROUTINE
LSEGMENT
'DECLARE GLOBAL ARGUMENTS OUTSIDE THE LOCAL SUBROUTINE
           VECTOR(6) GINIVEC{!K}
           CALL LSEGMENT(PH,FOD{!K}, NUVEC, GINIVEC{!K})
'------------------------------------------------------------------------------------------------------------------
'ESTIMATE LEVEL AND MEAN OF EXPENDITURE AT PH, AND THE ASSOCIATED
DENSITY FUNCTION
    SERIES MLQ{!K}=MU{!K}*LQ{!K}  'From Generalized Lorenz curve
    SERIES XP{!K}=MU{!K}*FOD{!K}   'Level of X at PH
    SERIES IFX{!K}=MU{!K}*SOD{!K}
    SERIES   FX{!K}=1/IFX{!K}         'Density around XP
    SERIES   FXDX{!K}=0
    SERIES MUP{!K}=0
    SMPL @FIRST+1 @LAST
        FXDX{!K}=FX{!K}*D(XP{!K})
        MUP{!K}=(MLQ{!K})/PH       'Mean at PH
   SMPL @ALL
   SERIES EMU{!K}=XP{!K}*FXDX{!K}
```

```
   SCALAR EMUX{!K}=@SUM(EMU{!K})
   EMU(!K)=EMUX{!K}
NEXT
'-----------------------------------------------------
'LOAD EXTENDED GINIS IN MATRIX
COLPLACE(INDOGINI,NUVEC, 1)
!COL=2
FOR !V=1 TO 3
      COLPLACE(INDOGINI,GINIVEC{!V},!COL)
      !COL=!COL+1
NEXT
INDOGINI.SETFORMAT F.2  'Two significant decimals
FREEZE(GINITAB) INDOGINI
SETLINE(GINITAB,3)
!COL=2
FOR %CLB FOCUS GINI93 GINI96  GINI02
        SETCELL(GINITAB, 1, !COL, %CLB, "C")
        !COL=!COL+1
NEXT
'-----------------------------------------------------------------------------------------------
'FORMAT GINITAB's COLUMN B CONTAINING AVERSION PARAMETER
GINITAB.SETFORMAT(B) G.1 'Show only one digit in this column
'---------------------------------------------------------------------
'CONSTRUCT GROWTH INCIDENCE CURVES
PH=100*PH
'Annualized growth rates between 1993 and 2002
SCALAR GAMMA1=(1/9)* ( LOG(MU3) - LOG(MU1))
'Compute ordinary and relative GIC
SERIES  GXP13A=100*(GAMMA1 + (1/9)*(LOG(FOD3) - LOG(FOD1) )  )
SERIES  GXP13B=100*( (1/9)*(LOG(FOD3) - LOG(FOD1) )  )
GROUP GICS13 PH GXP13A GXP13B
FREEZE(GICURVE13)  GICS13.XY
'Annualized growth rates between 1996 and 2002
SCALAR GAMMA2=(1/6)* ( LOG(MU3) - LOG(MU2))
'Compute ordinary and relative GIC
SERIES  GXP23A=100*(GAMMA2 + (1/6)*(LOG(FOD3) - LOG(FOD2) )  )
SERIES  GXP23B=100*( (1/6)*(LOG(FOD3) - LOG(FOD2) )  )
GROUP GICS23 PH GXP23A GXP23B
FREEZE(GICURVE23)  GICS23.XY
'Store annualized growth rates in a vector
VECTOR(2)   GRORATES
FOR !R=1 TO 2
       GRORATES(!R)=100*GAMMA{!R}
NEXT
'------------------------------------------------------------
'CONSTRUCT POVERTY GROWTH CURVES
SERIES GMUP13  'From 1993 to 2002
SMPL @FIRST+1 @LAST
      GMUP13=100*(GAMMA1 + (1/9)*(LOG(LQ3) - LOG(LQ1) )  )
SMPL @ALL
GROUP PGC13 PH GMUP13
```

```
FREEZE(PGCURVE13)  PGC13.XY
SERIES GMUP23 'From 1996 to 2002
SMPL @FIRST+1 @LAST
      GMUP23=100*(GAMMA2 + (1/6)*(LOG(LQ3) - LOG(LQ2) )  )
SMPL @ALL
GROUP PGC23 PH GMUP23
FREEZE(PGCURVE23)  PGC23.XY
'------------------------------------------------------------------------------------------------------------------
'COMPUTE EQUALLY DISTRIBUTED EQUIVALENT GROWTH RATES AND
ASSOCIATED DISTRIBUTION ADJUSTMENT FACTORS
MATRIX(6,3) THEMAT 'Equally Distributed Equivalent Growth Rates
MATRIX(6,3) DAFMAT 'Distribution Adjustment Factors
COLPLACE(THEMAT,NUVEC, 1)
COLPLACE(DAFMAT,NUVEC, 1)
!Q=1
PH=PH/100  'Back to proportions for the following subroutine to work properly
FOR %G1 %G2 %W GXP13A GXP13B FXDX1 GXP23A GXP23B FXDX2
        {%G1}={%G1}/100 'To make sure the subroutine works properly
        {%G2}={%G2}/100
       CALL PROPOORATE(PH,{%G1},{%W}) 'Note the change in the second argument
in order to switch from EDEGR to DAF
       VECTOR THETA{!Q}=100*THETANU 'Scale up
        COLPLACE(THEMAT,THETA{!Q},(!Q+1))
        CALL PROPOORATE(PH,{%G2},{%W})
        VECTOR DAF{!Q}=100*THETANU     'Scale up
        COLPLACE(DAFMAT,DAF{!Q},(!Q+1))
        !Q=!Q+1
NEXT
FREEZE(EDERATE) THEMAT
FREEZE(DAFTAB) DAFMAT
SETLINE(EDERATE,3)
SETLINE(DAFTAB,3)
!COL=2
FOR %CLB FOCUS PERIOD1 PERIOD2
        SETCELL(EDERATE, 1, !COL, %CLB, "C")
        SETCELL(DAFTAB, 1, !COL, %CLB, "C")
        !COL=!COL+1
NEXT
DAFTAB.SETFORMAT(B) G.1
DAFTAB.SETFORMAT(R4C3:R9C4)  F.2
EDERATE.SETFORMAT(B)  G.1
EDERATE.SETFORMAT(R4C3:R9C4)  F.2
PH=PH*100 'Back to percentages
'-----------------------------------------------------------------------------------------
'COMPUTE MEMBERS OF THE FGT FAMILY AND PLOT TIP CURVES
'Compute Poverty Measures
VECTOR(3) ALFA         'ALPHA is a reserved name in EVIEWS 5.1
MATRIX(3,4)  INDOPOV
ALFA.FILL 0,1,2
FOR !K=1 TO 3
        VECTOR(3) FGT{!K}
```

```
        SERIES POOR{!K}=(XP{!K}<=Z{!K})
        SERIES HEAD{!K}=POOR{!K}*FXDX{!K}
        FGT{!K}(1)=100*@SUM(HEAD{!K})   'Scale up by multipying by 100
        SERIES NGAP{!K}=(1-XP{!K}/Z{!K})
        SERIES FGTGAP{!K}=POOR{!K}*NGAP{!K}*FXDX{!K}
        FGT{!K}(2)=100*@SUM(FGTGAP{!K})  'Scale up
        SERIES FGTGPS{!K}=POOR{!K}*(NGAP{!K}^2)*FXDX{!K}
        FGT{!K}(3)=100*@SUM(FGTGPS{!K}) 'Scale up
NEXT
!COL=1
FOR %ST ALFA FGT1 FGT2 FGT3
         COLPLACE(INDOPOV, {%ST}, !COL)
        !COL=!COL+1
NEXT
FREEZE(FGTAB) INDOPOV
SETLINE(FGTAB,3)
  !CLN=2
  !ROW=4
  FOR %CLB  ALFA  FGT93 FGT96  FGT02
          SETCELL(FGTAB, 1, !CLN, %CLB, "C")
          !CLN=!CLN+1
NEXT
FOR %RLB HDC PVG  SPG
        SETCELL(FGTAB, !ROW, 1, %RLB, "L")
          !ROW=!ROW+1
NEXT
FGTAB.SETFORMAT(B)  G.1
FGTAB.SETFORMAT(R4C3:R6C5)  F.2
'Construct TIP Curves
 FOR !K=1 TO 3
       SMPL  @FIRST  @FIRST
       SERIES TIPV{!K} =100*@NAN(FGTGAP{!K}, 0)  'Recode NA as Zero (Just in case
there are any missing data)
       SMPL  @FIRST+1  @LAST
      SERIES TIPV{!K}=TIPV{!K}(-1) + 100*@NAN(FGTGAP{!K}, 0)
      SMPL @ALL
NEXT
GROUP  TIPGRP1  PH   TIPV1 TIPV3 'TIP for 1993 and 2002
GROUP  TIPGRP2  PH   TIPV2 TIPV3  'TIP for 1996 and 2002
FOR !j=1 TO 2
      FREEZE(FGTIP{!j})  TIPGRP{!j}.XY
      FGTIP{!j}.DRAW(DASHLINE, BOTTOM) FGT{!j}(1)
      FGTIP{!j}.DRAW(DASHLINE, LEFT)       FGT{!j}(2)
NEXT
'-----------------------------------------------------------------------------------------------------------
'COMPUTE WATTS INDICES AND ASSOCIATED RATES OF PRO-POOR GRWTH
VECTOR(3) WATTS
FOR !K=1 TO 3
         SERIES LOGZ{!K}=LOG(Z{!K})
         SERIES LOGXP{!K}=LOG(XP{!K})
         SERIES WATGAP{!K}=FXDX{!K}*(LOG(Z{!K})-LOG(XP{!K}))*POOR{!K}
```

```
        SCALAR WAT{!K}=100*@SUM(WATGAP{!K})
        WATTS(!K)=WAT{!K}
NEXT
WATTS.SETFORMAT F.2
'Mean Growth Rate of the Poor or Rate of Pro-Poor Growth for 1993-2002, and 1996-
2002
VECTOR(2) MGRP 'Mean Growth rate of the Poor
FOR !T=1 TO 2
        SERIES DAF{!T}3WT=(FXDX{!T}*GXP{!T}3B*POOR{!T})/FGT{!T}(!T)
        SCALAR WDAF{!T}3=@SUM(DAF{!T}3WT)
        SCALAR GPR{!T}3=100*GAMMA{!T}+WDAF{!T}3
        MGRP(!T)=GPR{!T}3
NEXT
MGRP.SETFORMAT F.2
'-----------------------------------------------------------------------------------------------------------
'PERFORM SUBPERIOD SHAPLEY DECOMPOSITION OF POVERTY OUTCOMES
'**********1993-2002**********
FOR !I=1 TO 3  STEP 2
       FOR !J=1 TO 3   STEP 2
       VECTOR (3)  PM{!I}L{!J}
       NEXT
NEXT
'Compute counterfactual poverty indicators members of the Foster-Greer-Thorbecke
family based on direct formulae (Datt 1998)
FOR !I=1 TO 3  STEP 2
      FOR !J=1 TO 3 STEP 2
          SCALAR  HDC{!I}{!J}= -( N{!J}+R{!J}*( (B{!J}+(2*Z{!I})/MU{!I})/(@SQRT( (
(B{!J}+(2*Z{!I})/MU{!I}) )^2 -M{!J} )) ) )/(2*M{!J}) 'Poverty Incidence
          SCALAR PVG{!I}{!J}= HDC{!I}{!J} - (MU{!I}/Z{!I})*( -0.5*( B{!J}*HDC{!I}{!J}+E{!J}+
@SQRT( (M{!J}*HDC{!I}{!J}^2 + N{!J}*HDC{!I}{!J} + E{!J}^2) ) )  )
          SCALAR SA{!J}=(R{!J} -N{!J})/(2*M{!J})
          SCALAR SB{!J}=-(R{!J}+N{!J})/(2*M{!J})
          SCALAR SPG{!I}{!J}= 2*PVG{!I}{!J}-HDC{!I}{!J}-( (MU{!I}/Z{!I})
^2)*(A{!J}*HDC{!I}{!J}+ B{!J}*(  -0.5*( B{!J}*HDC{!I}{!J}+E{!J}+@SQRT(
(M{!J}*HDC{!I}{!J}^2 + N{!J}*HDC{!I}{!J} +E{!J}^2) ) ) )-(R{!J}/16)*LOG( (1-
HDC{!I}{!J}/SA{!J})/(1-HDC{!I}{!J}/SB{!J}) ) )
          PM{!I}L{!J}(1)=100*HDC{!I}{!J} 'Scale up
          PM{!I}L{!J}(2)=100*PVG{!I}{!J}
          PM{!I}L{!J}(3)=100*SPG{!I}{!J}
          STORE PM{!I}L{!J}
   NEXT
NEXT
MATRIX(3,5)  PMLMAT1
!COL=1
FOR %ST ALFA PM1L1 PM1L3 PM3L1 PM3L3
        COLPLACE(PMLMAT1, {%ST}, !COL)
       !COL=!COL+1
NEXT
FREEZE(PMLTAB1) PMLMAT1
SETLINE(PMLTAB1,3)
  !CLN=2
```

```
  !ROW=4
  FOR %CLB  ALFA  PM1L1 PM1L3 PM3L1 PM3L3
        SETCELL(PMLTAB1, 1, !CLN, %CLB, "C")
        !CLN=!CLN+1
NEXT
FOR %RLB HDC PVG  SPG
      SETCELL(PMLTAB1, !ROW, 1, %RLB, "L")
        !ROW=!ROW+1
NEXT
'Compute the Growth and Redistribution Components of Change in Poverty
FOR  %ST  DPOV1 SHAPG1  SHAPD1
           VECTOR(3)  {%ST}
NEXT
DPOV1=(PM3L3 - PM1L1)
SHAPG1 = 0.5*( (PM3L3 - PM1L3) + (PM3L1 - PM1L1) )
SHAPD1 = 0.5*( (PM3L3 - PM3L1) + (PM1L3 - PM1L1) )
MATRIX (3,3) SHAPMAT1
!COL=1
FOR %ST DPOV1 SHAPG1  SHAPD1
       COLPLACE(SHAPMAT1, {%ST}, !COL)
      !COL=!COL+1
NEXT
FREEZE(SHAPTAB1) SHAPMAT1
SETLINE(SHAPTAB1,3)
  !CLN=2
  !ROW=4
  FOR %CLB  OVERALL  GROWTH DISTRIBUTION
        SETCELL(SHAPTAB1, 1, !CLN, %CLB, "C")
        !CLN=!CLN+1
NEXT
FOR %RLB HDC PVG  SPG
      SETCELL(SHAPTAB1, !ROW, 1, %RLB, "L")
        !ROW=!ROW+1
NEXT
'**********1996-2002**********
FOR !I=2 TO 3
      FOR !J=2 TO 3
      VECTOR (3)  PM{!I}L{!J}
      NEXT
NEXT
'Counterfactuals
FOR !I=2 TO 3
     FOR !J=2 TO 3
         SCALAR  HDC{!I}{!J}= -( N{!J}+R{!J}*( (B{!J}+(2*Z{!I})/MU{!I})/(@SQRT( (
(B{!J}+(2*Z{!I})/MU{!I}) )^2 -M{!J} )) ) )/(2*M{!J}) 'Poverty Incidence
         SCALAR PVG{!I}{!J}= HDC{!I}{!J} - (MU{!I}/Z{!I})*( -0.5*( B{!J}*HDC{!I}{!J}+E{!J}+
@SQRT( (M{!J}*HDC{!I}{!J}^2 + N{!J}*HDC{!I}{!J} + E{!J}^2) ) )  )
         SCALAR SA{!J}=(R{!J} -N{!J})/(2*M{!J})
         SCALAR SB{!J}=-(R{!J}+N{!J})/(2*M{!J})
         SCALAR SPG{!I}{!J}= 2*PVG{!I}{!J}-HDC{!I}{!J}-( (MU{!I}/Z{!I})
^2)*(A{!J}*HDC{!I}{!J}+ B{!J}*(  -0.5*( B{!J}*HDC{!I}{!J}+E{!J}+@SQRT(
```

```
(M{!J}*HDC{!I}{!J}^2 + N{!J}*HDC{!I}{!J} +E{!J}^2) ) ) )-(R{!J}/16)*LOG( (1-HDC{!I}{!J}/SA{!J})/(1-HDC{!I}{!J}/SB{!J}) ) )

            PM{!I}L{!J}(1)=100*HDC{!I}{!J}
            PM{!I}L{!J}(2)=100*PVG{!I}{!J}
            PM{!I}L{!J}(3)=100*SPG{!I}{!J}
            STORE PM{!I}L{!J}
    NEXT
NEXT
MATRIX(3,5)  PMLMAT2
!COL=1
FOR %ST ALFA PM2L2 PM2L3 PM3L2 PM3L3
        COLPLACE(PMLMAT2, {%ST}, !COL)
        !COL=!COL+1
NEXT
FREEZE(PMLTAB2) PMLMAT2
SETLINE(PMLTAB2,3)
  !CLN=2
  !ROW=4
  FOR %CLB  ALFA  PM2L2 PM2L3 PM3L2 PM3L3
          SETCELL(PMLTAB2, 1, !CLN, %CLB, "C")
          !CLN=!CLN+1
NEXT
FOR %RLB HDC PVG  SPG
        SETCELL(PMLTAB2, !ROW, 1, %RLB, "L")
          !ROW=!ROW+1
NEXT
'Growth and Redistribution Components
FOR  %ST  DPOV2 SHAPG2  SHAPD2
             VECTOR(3)  {%ST}
NEXT
DPOV2=(PM3L3 - PM2L2)
SHAPG2 = 0.5*( (PM3L3 - PM2L3) + (PM3L2 - PM2L2) )
SHAPD2 = 0.5*( (PM3L3 - PM3L2) + (PM2L3 - PM2L2) )
MATRIX (3,3) SHAPMAT2
!COL=1
FOR %ST DPOV2 SHAPG2  SHAPD2
        COLPLACE(SHAPMAT2, {%ST}, !COL)
        !COL=!COL+1
NEXT
FREEZE(SHAPTAB2) SHAPMAT2
SETLINE(SHAPTAB2,3)
  !CLN=2
  !ROW=4
  FOR %CLB  OVERALL  GROWTH DISTRIBUTION
          SETCELL(SHAPTAB2, 1, !CLN, %CLB, "C")
          !CLN=!CLN+1
NEXT
FOR %RLB HDC PVG  SPG
        SETCELL(SHAPTAB2, !ROW, 1, %RLB, "L")
          !ROW=!ROW+1
```

```
NEXT
FOR %TAB1 %TAB2 PMLTAB1 SHAPTAB1 PMLTAB2 SHAPTAB2
        {%TAB1}.SETFORMAT(B)  G.1
       {%TAB1}.SETFORMAT(R4C3:R6C6)  F.2
        {%TAB2}.SETFORMAT(R4C2:R6C4)  F.2
NEXT
'------------------------------------------------------------------------
'STORE RESULTS AND CHANGE WORKFILE PAGE
STORE DAFTAB EDERATE FGTAB FGTIP1 FGTIP2 GINITAB GRORATES LORENZ1
LORENZ2 MGRP PMLTAB1 PMLTAB2 SHAPTAB1 SHAPTAB2 WATTS
PAGECREATE(PAGE=REPORT)  U 1
FETCH  DAFTAB EDERATE FGTAB FGTIP1 FGTIP2 GINITAB GRORATES LORENZ1
LORENZ2  MGRP PMLTAB1 PMLTAB2 SHAPTAB1 SHAPTAB2 WATTS Z1 Z2  Z3
CLOSE  PPG

'END OF PROGRAM
```

'PROPOORATE.PRG is a subroutine designed to compute an indicator of the rate of pro-poor growth as a weighted average of points on a Growth Incidence Curve (GIC). The indicator is sensitive to the degree of inequality aversion.

'EViews 4.1, Standard Edition April 19, 2004 build.

'B. Essama-Nssah, PRMPR, The World Bank Group, Washington D.C. May 25, 2004

```
SUBROUTINE  PROPOORATE(SERIES P, SERIES  GX, SERIES WHT) 'Percentile,
Growth rate of X, Inflation factor
                SERIES SURVIVE=(1-P)
                 !AVMAX=6
                 !B=1
                 !CF=!B-1
                !RWS=(!B*!AVMAX)-!CF
                VECTOR(!rws)   NUV 'Indicator of Aversion
                VECTOR(!rws) THETANU 'Rate of propoor growth
               !T=1
                    FOR !NU=1 TO !AVMAX  STEP 1/!B
                          SERIES OMEGA=(!NU)*SURVIVE^(!NU-1)
                          SERIES GXWT=GX*WHT
                          SERIES  OGXWT=OMEGA*GXWT
                         THETANU(!T)=@SUM(OGXWT)
                          NUV(!T)=!NU
                         !T=!T+1
                  NEXT

ENDSUB
```

References

Atkinson, A. B. 1970. On the Measurement of Inequality. *Journal of Economic Theory*, 2, 244-263.

Chotikapanich, Duangkamon and Griffiths, Willianm. 2001. On the Calculation of the of the Extended Gini Coefficient. Reviwe of Income and Wealth. Series 47, No. 4: 541-547.

Cowell, Frank A. 1995. Measuring Inequality. London: Prentice Hall

Chakravarty, S. R. 1983. A New Index of Poverty. *Mathematical Social Sciences*, Vol. 6.

Clark, S.R., Hemming R., Ulph D. 1981. On Indices for Poverty Measurement. Economic Journal.

Datt, Gaurav. 1998. Computational Tools for Poverty Measurement and Analysis. Washington D.C.: International Food Policy Research Institute (IFPRI) Discussion Paper No.50 (Food Consumption and Nutrition Division).

Datt, Gaurav. 1992. Computational Tools for Poverty Measurement and Analysis. Washington D.C.: The World Bank.

Deaton, Angus. 1997. The Analysis of Household Surveys: A Microeconometric Approach to Development Policy. Baltimore: The John Hopkins University Press.

Dixit, Avinash K. 1996. The Making of Economic Policy: A Transaction Cost Politics Perspective. Cambridge, Massachusetts: The MIT Press.

Drazen Allan. 2000. Political Economy in Macroeconomics. Princeton: Princeton University Press.

Essama-Nssah, B. 2004. A Unified Framework for Pro-Poor Growth Analysis. Policy Research Working Paper No. 3397. Washington, D.C.: The World Bank.

______________. 2002. Assessing the Distributional Impact of Public Policy. Policy Research Working Paper No. 2883. Washington, D.C.: The World Bank.

Fields, Gary S. 2001. Distribution and Development: A New Look at the Developing World. New York: Russell Sage Foundation.

Foster J., Greer, J. and Thorbecke, E. 1984. A Class of Decomposable Poverty Measures. *Econometrica*, Vol. 52, No.3 , 761-766. (May).

Jenkins, S. and Lambert, Peter J. . 1997. Three ‘I’s of Poverty Curves, with Analysis of UK Poverty Trends. Oxford Economic Papers, 49: 317-327.

Kakwani, Nanak. 2000. Growth an Poverty Reduction: An Empirical Analysis. Asian Development Review, Vol.18, No.2: 74-84.

______________. 1999. Inequality, Welfare and Poverty: Three Interrelated Phenomena. In Jacques Silber (ed.) “*Handbook of Income Inequality Measurement*”. Boston: Kluwer Academic Publishers.

______________. 1997. On Measuring Growth and Inequality Components of Changes in Poverty with Application to Thailand. Discussion Paper No. 97/16. School of Economics: The University of New South Wales.

______________. 1993. Poverty and Economic Growth with Application to Côte d’Ivoire. Review of Income and Wealth, Series 39, No. 2: 121-139.

Kakwani, Nanak and Pernia, Ernesto M. 2000. What is Pro-Poor Growth? Asian Development Review, Vol.18, No.1: 1-16.

Kolenikov, Stanislav and Shorrocks, Anthony. 2003. A Decomposition Analysis of Reginal Poverty in Russia. World Institute for Development Economics Research (WIDER) Discussion Paper No. 2003/74.

Kraay, Aart. 2004. When is Growth Pro-Poor? Cross-Country Evidence. World Bank Policy Research Working Paper No. 3225. Washington, D.C.: The World Bank.

___________. 2002. Growth is Good for the Poor. Journal of Economic Growth, 7: 195-225.

Lambert, Peter J. 2001. The Distribution and Redistribution of Income. Manchester: Manchester University Press.

_____________. 1993. Evaluating Impact Effect of Tax Reforms. *Journal of Economic Surveys*, Vol. 7, No.3: 205-242.

Lerman, Robert I. and Yitzhaki, Shlomo. 1989. Improving the Accuracy of Estimates of Gini Coefficients. *Journal of Econometrics*, 42, 43-47.

Mayshar, Joram and Yitzhaki, Shlomo. 1995. Dalton-Improving Indirect Tax Reform. The American Economic Review, Vol. 85, No. 4: 793-807.

Moulin, Hervé. 2003. Fair Division and Collective Welfare. Cambridge, Massachusetts: The MIT Press.

Ramadas, K., van der Mensbrugghe and Wodon, Quentin. 2002. SimSIP Poverty: Poverty and Inequality Comparisons Using Group Data. Washington, D.C.: The World Bank (mimeo).

Ravallion, Martin and Chen Shaohua. 2003. Measuring Pro-Poor Growth. Economics Letters, **78**: 93-99.

Ravallion, Martin. 1994a. Poverty Comparisons. Chur: Harwood Academic.

_______________. 1994b. Decomposing Changes in Poverty Measures: A Comment on Kakwani. Washington, D.C.: The World Bank (mimeo).

Ravallion, Martin and Datt, G. 1992. Growth and Redistribution Components of Changes in Poverty Measures: A Decomposition with Applications to Brazil and India in the 1980s. *Journal of Development Economics* 38, 275-295

Ravallion, Martin and Huppi, Monika. 1991. Measuring Changes in Poverty: A Methodological Case Study of Indonesia during an Adjustment Period. The World Bank Economic Review, Vol. 5, No.1: 57-82.

Sen, Amartya. 1999. *Development as Freedom*. New York: Alfred A. Knopf.

___________. 1997. On Economic Inequality: Expanded Edition with a substantial Annexe by James E. Foster and Amartya Sen. Oxford: Clarendon Press.

___________. 1989. The Concept of Development. In Hollis Chenery and T.N. Srinivasan (eds), *Handbook of Development Economics*, Vol. 1. Amsterdam: Elsevier.

Shorrocks, A. F. 1999. Decomposition Procedures for Distributional Analysis: A Unified Framework Based on the Shapley Value. Mimeo, University of Essex.

_____________. 1983. Ranking Income Distributions. *Economica* 50, 1-17.

Watts, H.M. 1968. An Economic Definition of Poverty. In D.P. Moynihan (Ed), *On Understanding Poverty*. New York: Basic Books.

World Bank. 2000. World Development Report 2000/2001: Attacking Poverty. Washington, D.C.: The World Bank.

Yitzhaki, Shlomo. 1983. On an Extension of the Gini Index of Inequality. International Economic Review, Vol. 24, No 3: 617-628.

Young, H. Peyton. 1994. *Equity: In Theory and Practice*. Princeton: Princeton University Press.

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