Poverty and Distributional Outcomes of Shocks and Policies

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Foreword

"The effect of ignoring the interpersonal variations can, in fact, be deeply inegalitarian, in hiding the fact that equal consideration for all may demand very unequal treatment in favour of the disadvantaged."

Sen (1992)

Outline

1. A Stylized Framework
   - A Two-Sector Model of an Open Economy
   - The Lorenz Model of Income Distribution
     - Structure
     - Parameterization
     - Recovering the Size Distribution and Associated Measures of Inequality and Poverty
   - Base Year Data

Outline

2. Poverty and Distributional Implications of:
   - Dutch Disease
   - Deterioration in the Terms of Trade (Hands-On)
   - Fiscal Policy Reform (Hands-On)

A Two-Sector Model of an Open Economy

- A small open economy
  - Cannot influence its terms of trade with rest of the world.
- Two sectors of production:
  - Sector 1 produces an export good not sold domestically.
  - Sector 2 produces a domestic good used for both intermediate and final consumption.

A Two-Sector Model of an Open Economy

- The demand for primary factors of production, labor and capital, is a consequence of profit maximization (or cost minimization) by firms in each sector subject to technological constraints described by Cobb-Douglas production functions.

A Two-Sector Model of an Open Economy

- The intermediate good is a composite good made up of domestic good and imports.
- The demand for intermediate inputs is proportional to the level of output.
- Two representative households
  - Rural household represents 60 percent of the population, and owns a fraction θ_{RL} of labor and a fraction θ_{RK} of capital (to be determined by data in SAM).
  - Urban household represents 40 percent of the population, and owns a fraction (1- θ_{RK}) of capital and a fraction (1- θ_{RL}) of labor.

A Two-Sector Model of an Open Economy

- Each household spends its income on a composite good (made of domestic and imports) so as to maximize utility (or minimize expenditure). This income includes factor income, transfers from the government and from the rest of the world.
- Imperfect substitutability between domestic good and imports.

A Two-Sector Model of an Open Economy

- Government revenue includes an indirect tax on domestic sales of the domestic good, and tariffs on imported final and intermediate goods. It balances the budget by re-distributing all of its revenue to households.
- Factors and goods markets are perfectly competitive. Full employment prevails.

A Two-Sector Model of an Open Economy

- Above described model is an extended version of the Salter-Swan model which provides a foundation for the study of the impact of macroeconomic imbalances and adjustment policies on the real sector of a small open economy.

Structure of the Lorenz Model

Definition

- A flexible statistical model of the distribution of some welfare indicator, x, among the population.
- The Lorenz curve maps the cumulative proportion of the population (horizontal axis) against the cumulative share of welfare (vertical axis), where individuals have been ranked in ascending order of x.

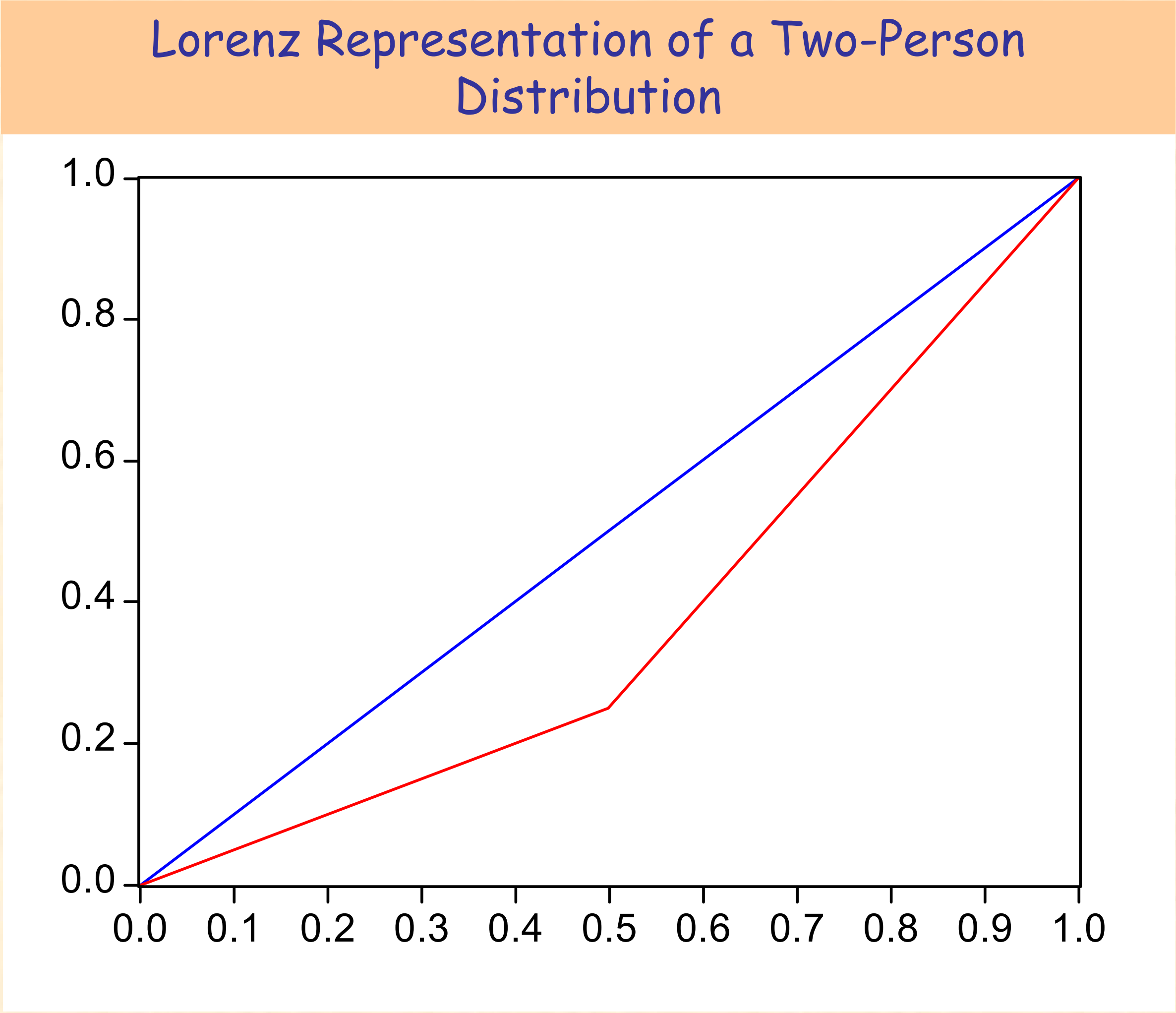
Structure of the Lorenz Model

Simple Example

- Income distribution among two individuals

Income Level	Relative Frequency	Cumulative Frequency	Cumulative Share
0.00	0.00	0.00	0.00
25.00	0.50	0.50	0.25
75.00	0.50	1.00	1.00

Structure of the Lorenz Model



Structure of the Lorenz Model

Analytical Expression

- Above example: convex combination of two linear segments with a kink at (0.50, 0.25)
  - First Segment

$$L_1(p) = ap; \quad p \le 0.5$$

$$a = \frac{2x_1}{(x_1 + x_2)} = \frac{x_1}{\mu} = 0.5$$

- μ is the overall mean of the distribution, and the slope a is computed as "rise over run".

Structure of the Lorenz Model

- Second segment

$$L_2(p) = bp + (1 - b); \quad 0.5 < p \leq 1.0$$

$$b = \frac{2x_2}{(x_1 + x_2)} = \frac{x_2}{\mu} = 1.5$$

Structure

- Combination

$$L(p)=\delta L_1(p)+(1-\delta)L_2(p);\quad p\in[0,1]$$

- δ is a dummy that is equal to 1 if p≤0.5, and 0 otherwise.
- Slope

$$\frac{\Delta L(p)}{\Delta p}=\delta\frac{x_1}{\mu}+(1-\delta)\frac{x_2}{\mu}$$

  - <u>Interpretation</u>: a local measure of inequality showing how far a given income is below or above the mean (i.e. equal share). Hence equal distribution implies slope=1 for all δ. The Lorenz curve becomes L(p)=p for all δ.

Structure of the Lorenz Model

- Rate of change of slope

$$\frac{\Delta^2 L(p)}{\Delta p^2} = \frac{1}{\mu}\left[\delta\frac{\Delta x_1}{\Delta p} + (1-\delta)\frac{\Delta x_2}{\Delta p}\right]$$

- From above table, use nearest left neighbor of x_i to compute:

$$\frac{\Delta x_i}{\Delta p} = \left(\frac{\Delta p}{\Delta x_i}\right)^{-1} = \left(\frac{1}{2}\right)^{-1} = 2 \quad \forall i$$

- Thus

$$\frac{\Delta^2 L(p)}{\Delta p^2} = \frac{2}{\mu} = \frac{1}{0.5\mu} = \frac{1}{\mu f(x_i)}$$

Structure of the Lorenz Model

- Case of n people

$$L(p) = \frac{\sum_{k=1}^{j} x_k}{\sum_{k=1}^{n} x_k} = \frac{j\mu_j}{n\mu} = \frac{\mu_p}{\mu} p; \quad p = \frac{j}{n}; L(0) = 0; L(1) = 1$$

- Rate of change of L(p) is $x_k/(n\mu)$, that of p is 1/n. Hence:

$$\frac{\Delta L(p)}{\Delta p} = \frac{x_j}{\mu}$$

$$\frac{\Delta^2 L(p)}{\Delta p^2} = \frac{1}{\mu \frac{\Delta p}{\Delta x_j}} = \frac{1}{\mu f(x_j)} = \frac{n}{\mu}$$

Structure of the Lorenz Model

- Assuming smoothness
  - Lorenz function

$$L(p) = \int_0^p \frac{x(q)}{\mu} dq$$

  - First order derivative

$$L'(p) = \frac{x(p)}{\mu}$$

  - Second order derivative

$$L''(p) = \frac{1}{\mu}\frac{dx}{dp} = \frac{1}{\mu \frac{dp}{dx}} = \frac{1}{\mu f(x)}$$

Structure of the Lorenz Model

- Generalized Lorenz Curve
  - Discrete

$$L(\mu,p)=\mu L(p)=\frac{1}{n}\sum_{k=1}^{j}x_k=p\mu_p;\ L(\mu,0)=0;\ L(\mu,1)=\mu$$

  - Continuous

$$L(\mu,p)=\int_0^x tf(t)dt=\int_0^p x(q)dq$$

Parameterization

Approaches

- Derive expression of Lorenz curve from known distribution function e.g. Lognormal or Beta. Then estimate structural parameters from data.
- Choose a functional form for the Lorenz curve.
  - Estimate its structural parameters from the data.
  - Compute the curve and associated derivatives from parameter estimates.

Parameterization

The General Quadratic Model (Datt 1992, 1998)

- Regress [L(1-L) on (p^2-L), L(p-1) and (p-L) with no intercept and dropping last observation.
- Let β_1, β_2, β_3 be the regression coefficients.

Parameterization

- Compute the following:

$$e=-(\beta_1+\beta_2+\beta_1+1); m=(\beta_2^2-4\beta_1); n=(2\beta_2 e-4\beta_3); r=(n^2-4me^2)^{\frac{1}{2}}$$

$$L(p)=-\frac{1}{2}\left[\beta_2 p+e+(mp^2+np+e^2)^{\frac{1}{2}}\right]$$

$$L'(p)=-\frac{\beta_2}{2}-\frac{2mp+n}{4\sqrt{(mp^2+np+e^2)}}$$

$$L''(p)=\frac{r^2(mp^2+np+e^2)^{-\frac{3}{2}}}{8}$$

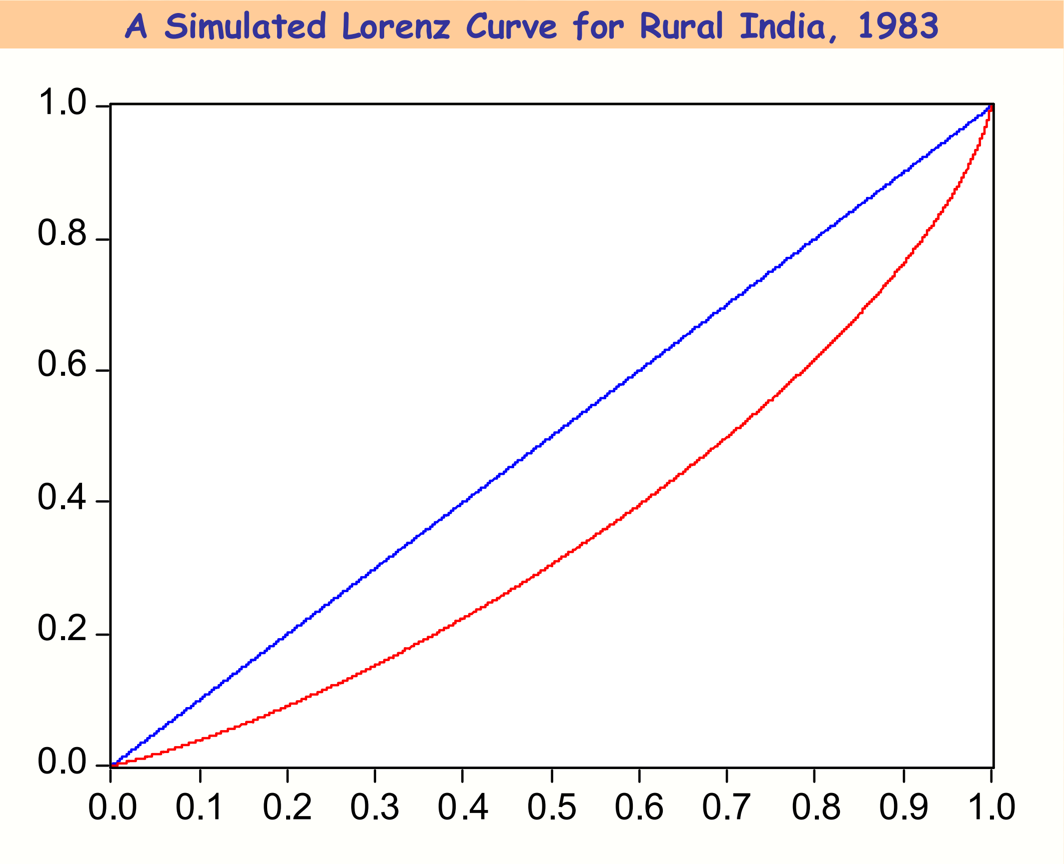
Parameterization

Rural India 1983: Regression Output for the General Quadratic Lorenz Representation of Household Expenditure

	Coefficient	Std. Error	t-Statistic	Prob.
BETA(1)	0.887734	0.006683	132.8389	0.0000
BETA(2)	-1.451431	0.019062	-76.14295	0.0000
BETA(3)	0.202658	0.012847	15.77521	0.0000
R-squared	0.999959	Mean dependent var		0.121975
Adjusted R-squared	0.999950	S.D. dependent var		0.087678
S.E. of regression	0.000617	Akaike info criterion		-11.73067
Sum squared resid	3.43E-06	Schwarz criterion		-11.60944
Log likelihood	73.38399	Durbin-Watson stat		0.697503

Data Source: Datt (1998)

Parameterization



Recovering the Size Distribution and Associated Measures of Inequality and Poverty

- Strategy
- The Extended Gini Family
- The FGT Family of Poverty Measures

Strategy

- Most, if not all, inequality and poverty measures of interest can be computed from the following basic inputs:
  - the level of income or expenditure x,
  - the associated density function f(x), and
  - a poverty line (for poverty measures).
- When relevant data are available, we can recover the first two inputs from a parameterized Lorenz function and the mean of the distribution.

Strategy

- In particular, we derive x from the mean and the first order derivative of the Lorenz function.
- An estimate of the density function is obtained from the mean and the second order derivative.
- For computational purposes, we use the fact that f(x)dx is interpreted as the proportion of the population with income or expenditure in the close interval [x, dx] for a level of x and an infinitesimal change dx (Lambert 2001)

Strategy

- Above considerations suggest applying numerical integration to the standard definitions of the measures of interest.
- This obviates the need to derive special expressions for these indicators from the chosen functional form of the Lorenz curve (an approach followed by Datt 1992, 1998 for instance).
- Focus on Gini and FGT

The Extended Gini Family

- The ordinary Gini coefficient is equal to the area between the Lorenz curve and the line of complete equality (also known as the 45-degree line) divided by the whole area under the 45-degree line.
- Ordinary Gini is a member of an extended family defined by a focal parameter that is interpreted as an indicator of inequality aversion.

The Extended Gini Family

- A covariance-based expression of the extended Gini coefficient can be derived from an abbreviation of a transfer-approving social evaluation criterion defined as:

$$W(x) = \sum_{k=1}^{n} \omega_k x_k \qquad W(x) = nV(\omega)$$

$$V(\omega) = \mu_\omega \mu_x + \mathrm{cov}(x, \omega)$$

The Extended Gini Family

- With no loss of Generality, normalize average social weight to 1 (i.e. μ_ω=1)

$$V(\omega) = \mu_x + \text{cov}(x,\omega) = \mu_x[1 + \frac{\text{cov}(x,\omega)}{\mu_x}] = \mu_x[1 - G(\omega)]$$

- Analogy with equally distributed equivalent income or expenditure (see Atkinson 1970)
- A general expression of the extended Gini coefficient is therefore:

$$G(\omega) = -\frac{\text{cov}(x,\omega)}{\mu_x}$$

The Extended Gini Family

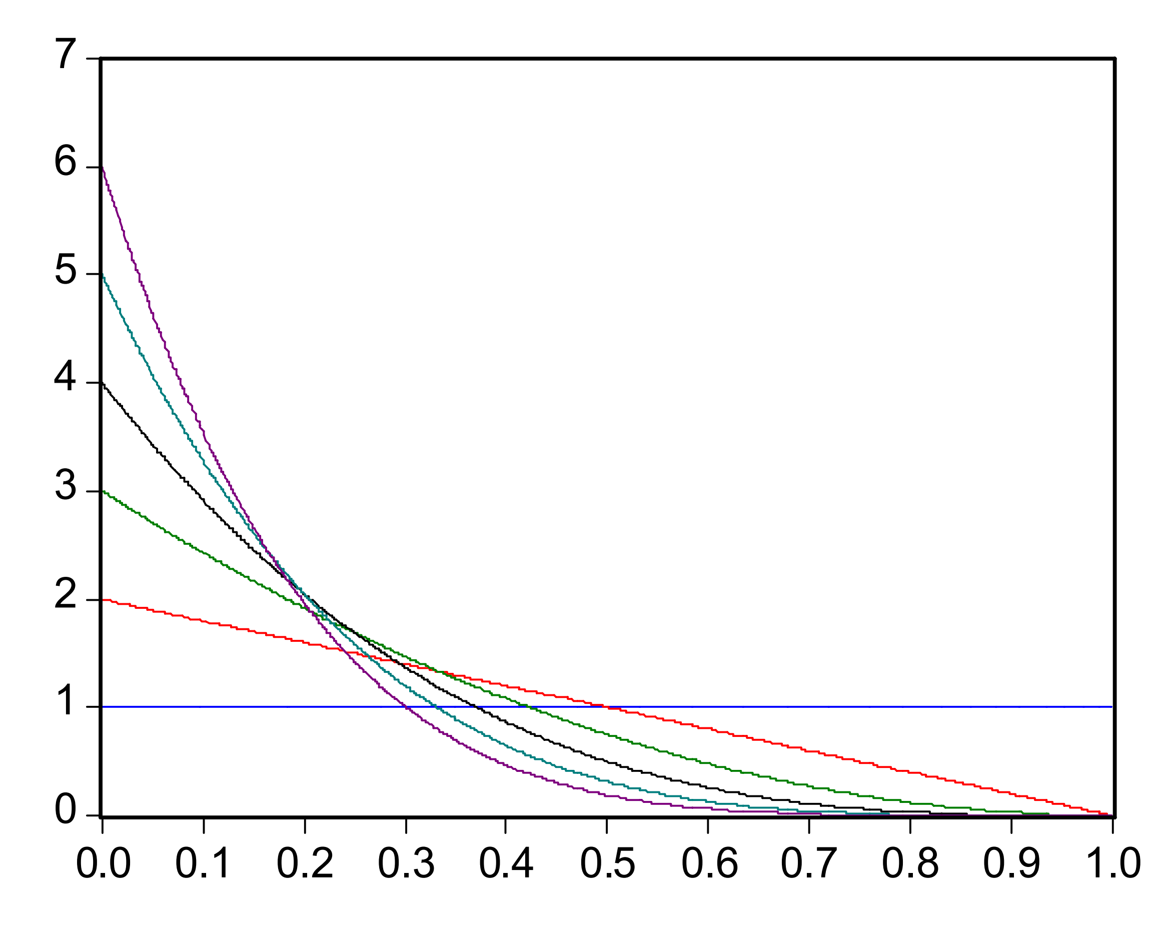
- For computational purposes, we adopt the system of weights proposed by Yitzhaki (1983)

$$\omega_k(\nu) = \nu(1 - p_k)^{\nu-1}$$

  - Where
    - p_k is the proportion of people with income less than or equal x_k, and
    - ν is the focal parameter indicating the degree of inequality aversion.

The Extended Gini Family

(Evaluative Weights as a Function of the Focal Parameter ν)



The Extended Gini Family

Cut-off Rank as a Function of the Aversion Parameter.

υ	1.1	1.2	1.5	2.0	3	6	10	20	30	50	100	200
p*	0.62	0.60	0.56	0.50	0.42	0.30	0.23	0.15	0.11	0.08	0.05	0.03

Source: Essama-Nssah (2002)

The Extended Gini Family

- The corresponding expression for the extended Gini is:

$$G(\nu) = -\frac{\nu}{\mu_x}\text{cov}[x,(1-p)^{\nu-1}]$$

- It can also be computed as:

$$G(\nu) = -\frac{\nu}{\mu_x}\text{cov}[\mu_x L'(p),(1-p)^{\nu-1}]$$

The Extended Gini Family

- Alternatively, the extended Gini coefficient can be written as:

$$G(\nu) = 1 - \nu(\nu - 1)\int_0^1 (1-p)^{\nu-2} L(p)dp$$

- Chotikapanich and Griffiths (2001) propose a linear segment estimator defined as follows:

$$G(\nu) = 1 + \sum_{k=1}^{m} \left(\frac{\theta_k}{w_k} \right) [(1-p_k)^{\nu} - (1-p_{k-1})^{\nu}]; \quad \theta_k = \frac{w_k x_k}{\sum_{j=1}^{m} w_j x_j}$$

The Extended Gini Family

- The linear segment estimator is equivalent to:

$$G(\nu) = 1 + \sum_{k=1}^{m} \frac{\Delta L(p_k)}{\Delta p_k}\left[(1-p_k)^{\nu} - (1-p_{k-1})^{\nu}\right]$$

- When the focal parameter ν is equal to 2, we get the ordinary Gini index.

The Extended Gini Family

- Sen's measure of poverty is a close relative of the Gini coefficient.

- Let μ_p and $G_p(\nu)$ be respectively the average income and the extended Gini for the poor. The extended Sen index of poverty is equal to:

$$S(\nu) = H\left[1 - \frac{\mu_p[1 - G_p(\nu)]}{z}\right]$$

The Extended Gini Family

Gini Family of Indicators for Rural India in 1983

Focus	Overall Gini	Gini for Poor	Sen Index
1	0.00	0.00	12.48
2	28.89	13.54	16.89
3	38.78	20.66	19.21
4	44.22	25.12	20.67
5	47.79	28.21	21.67
6	50.35	30.48	22.41

Source: Author's calculations

The FGT Family of Poverty Measures

- Kakwani(1999) defines a class of additively separable poverty measures starting from the notion of deprivation.
- Let $\psi(\mathbf{z},\mathbf{x}_i)$ stand for an indicator of deprivation at the individual level. A class of additively separable poverty measures can be defined as:

$$P(z,x)=\frac{1}{n}\sum_{i=1}^{n}\psi(z,x_i)=\sum_{i=1}^{n}\psi\left(z,\mu\frac{\Delta L(p_i)}{\Delta p_i}\right)f(x_i)\Delta x_i$$

The FGT Family of Poverty Measures

- Deprivation felt by an individual depends only on a fixed poverty line and her level of welfare and not on the welfare of other individuals in society.
- When population is divided exhaustively into mutually exclusive socioeconomic groups, overall poverty is a weighted average of poverty in each group.
- The weights are population shares. Hence these measures are also additively decomposable.

The FGT Family of Poverty Measures

- Specification of the deprivation function leads to particular members of the class.
- For Foster-Greer-Thorbecke (1984), expression due to Jenkins and Lambert (1997):

$$\psi_{FGT}(z, x_i) = \max\{(1 - x_i / z)^{\alpha}, 0\}.$$

The FGT Family of Poverty Measures

- α is an indicator of aversion for inequality among the poor.
  - When α=0, we get the headcount index;
  - When α=1, we get the poverty gap index; and
  - When α=2, we get the squared poverty gap index.

The FGT Family of Poverty Measures

TIP Representation of Poverty (Jenkins and Lambert 1997)

- TIP stands for the Three "I"s of Poverty:
  - incidence,
  - intensity, and
  - inequality (among the poor).
- The curve provides a graphical summary of those three dimensions of poverty.
- Construction analogous to that of Lorenz curve.

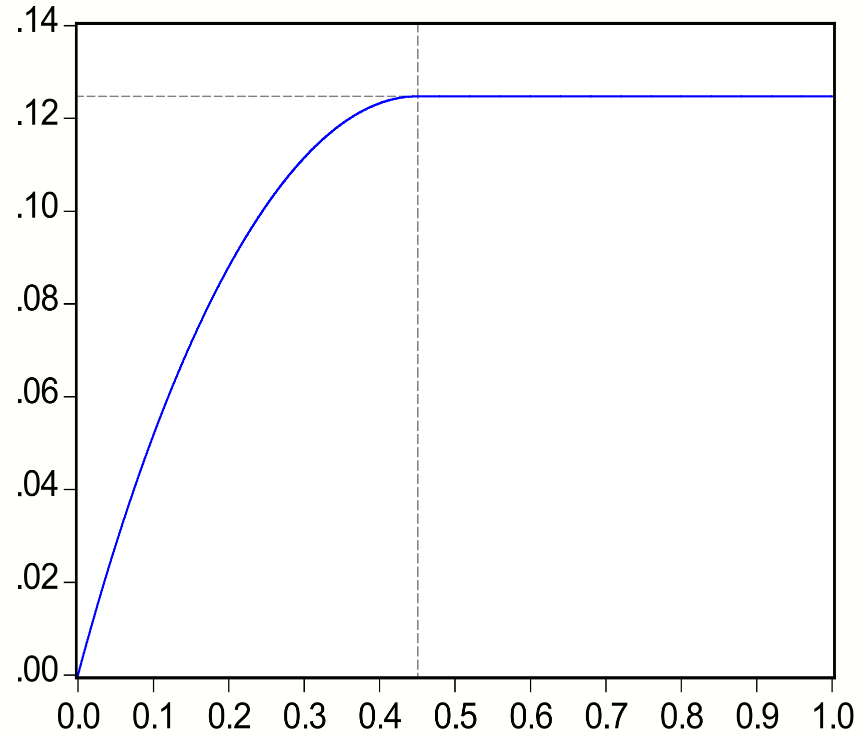
The FGT Family of Poverty Measures

- Step 1: rank individuals from poorest to richest.
- Step 2: compute relative poverty gaps, **$g_i = \max\{(1 - x_i/z),\ 0\}$**.
- Step 3: form cumulative sum of poverty gaps normalized by population size.
- Step 4: plot result as function of cumulative population shares:

$$JL(p) = \frac{1}{n}\sum_{i=1}^{k} g_i; \quad p = \frac{k}{n}; \quad JL(0) = 0$$

The FGT Family of Poverty Measures

A TIP Curve for Rural India in 1983



The FGT Family of Poverty Measures

Properties of the TIP Curve:

- An increasing concave curve such that the slope is equal to the poverty gap at the given percentile.
- The length of the non-horizontal section shows poverty incidence.
- The height of the curve reveals intensity.
- The degree of concavity of the non-horizontal section translates the degree of inequality among the poor.
- Respects Second-Order Dominance

Data

- Sam

	Export	Domestic	Final	Intermediate	Labor	Capital	Rural	Urban	World	Total
Export									30.00	30.00
Domestic			73.00	2.00						75.00
Final							40.00	60.00		100.00
Intermediate	5.00									5.00
Labor	20.00	30.00								50.00
Capital	5.00	45.00								50.00
Rural Household					35.00	5.00				40.00
Urban Household					15.00	45.00				60.00
World			27.00	3.00						30.00
Total	30.00	75.00	100.00	5.00	50.00	50.00	40.00	60.00	30.00	

Source: Adapted from Devarajan, Lewis and Robinson (1990)

Data

- Calibrated Parameters

Calibrated Parameters for the Two-Sector Model

	α_L	α_K	A	β_M	β_D	B
Export	0.80	0.20	1.98			
Domestic	0.40	0.60	1.98			
Final				0.38	0.62	1.89
Intermediate				0.69	0.31	1.92

  - A is tfp (total factor productivity) parameter in the Cobb-Douglas production function
  - α's are factor shares (exponents in the production function).
  - β's are shares in the Armington aggregation function and B is a scale factor.
  - Distribution of factor income in base year SAM: θ_{RL}=0.70, θ_{UL}=0.30, θ_{RK}=0.10, and θ_{UK}=0.90
  - Distribution of government transfers: θ_{RG}=0.60 and θ_{UG}=0.40
  - Distribution of foreign transfers: θ_{RF}=0.20 and θ_{UF}=0.80

Data

- Base Year Income Distribution

Size Distribution of Income within the Two Socioeconomic Groups

Group	Mean	Poorest Decile	2nd	3rd	4th	5th	6th	7th	8th	9th	10th
National	1.00	0.01	0.03	0.04	0.06	0.07	0.09	0.11	0.14	0.18	0.28
Rural	0.66	0.02	0.03	0.05	0.07	0.08	0.10	0.12	0.14	0.17	0.21
Urban	1.50	0.00	0.04	0.06	0.07	0.09	0.10	0.12	0.14	0.16	0.23

Source: Author's calculations

Baseline Inequality

Focus	National	Rural	Urban
1	0.00	0.00	0.00
2	0.41	0.32	0.32
3	0.57	0.47	0.47
4	0.65	0.56	0.57
5	0.71	0.62	0.64
6	0.74	0.66	0.69

Source: Author's Calculations

Data

- Parameters underlying the General Quadratic Lorenz Model

Parameterization of the Lorenz Model

Parameter	**National**	**Rural**	**Urban**
β_1	**1.52**	**2.16**	**1.46**
β_2	**-0.89**	**-1.42**	**-1.83**
β_3	**0.02**	**0.08**	**-0.15**
e	**-1.65**	**-1.82**	**-0.48**
m	**-5.29**	**-6.60**	**-2.51**
n	**2.86**	**4.84**	**2.37**
r	**8.11**	**10.50**	**2.82**

Source: Author's calculations

Implications of Dutch Disease

- Simulation: An increase of the balance of trade from 0 to 10.

Structural and Poverty Implications of Dutch Disease

	Base	Dutch
Exports	30.0	78.63
Domestic Good	75.0	106.98
Final Imports	27.0	115.59
Intermediate Imports	3.0	79.19
Total Consumption	100.0	109.87
Rural Consumption	40.0	101.50
Urban consumption	60.0	115.48
Total Poverty Incidence	59.2	96.61
Rural Poverty Incidence	78.3	98.72
Urban Poverty Incidence	30.5	83.33
Overall Poverty Gap	29.7	96.67
Rural Poverty Gap	38.7	97.44
Urban Poverty Gap	16.3	87.50

Source: Author's calculations.
Note: second column expressed as percentage of baseline

Implications of Dutch Disease

- Production of exports falls by about 21 percent while production of domestic good increases by about 7 percent, which induces the observed decline in intermediate import. Why?
  - Foreign capital inflow is distributed to households.
  - Increase in household income implies increased demand for both domestic good and imports (due to assumed imperfect substitutability).
  - Increased demand for non-tradable (domestic good) induces an increase in its price relative to that of exports (real appreciation).
  - Real appreciation represents a change in the configuration of incentives which causes resources to move out of the export sector into the non-tradable sector to meet increased demand for domestic good.

Implications of Dutch Disease

- Note sequence of events in above explanation:
  - Foreign capital inflow changes the circumstances for households.
  - Change in behavior that was previously desirable but unfeasible now becomes possible.
  - Compatibility breaks down.
  - Incentive structure changes to induce firms to meet the demand expressed by households.
  - When that happens, compatibility is restored.

Implications of Dutch Disease

- Aggregate consumption increases by almost 10 percent.
- Urban consumption increases by about 15 percent while rural consumption increases by 1.5 percent (recall: underlying distributional mechanisms assign 80 percent of the transfer to urban households).
- Overall poverty incidence declines by about 3.4 percent.
- Urban bias in the redistribution of foreign transfers causes urban poverty to fall by about 17 percent, while rural poverty falls only by 1.3 percent.

Hands-On

- Deterioration in the Terms of Trade (Hands-On)
- Fiscal Policy Reform (Hands-On)
- [See Essama-Nssah (2005) for results]

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The End.