A Two-Sector Model of an Open Economy

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Module 2

Outline

- Structure
  - Assumptions
  - Analytical Expression
- Numerical Implementation
  - Closure
  - Calibration
  - Validation
- Exercise

Assumptions

- A small open economy
  - Cannot influence its terms of trade with rest of the world.
- Two sectors of production:
  - Sector 1 produces an export good not sold domestically.
  - Sector 2 produces a domestic good used for both intermediate and final consumption.

Assumptions

- The demand for primary factors of production, labor and capital, is a consequence of profit maximization (or cost minimization) by firms in each sector subject to technological constraints described by Cobb-Douglas production function.
- The demand for intermediate inputs is proportional to the level of output. The intermediate good is a composite good made up of domestic good and imports.

Assumptions

- Two representative households
  - Rural household represents 60 percent of the population, and owns a fraction θ_{RL} of labor and a fraction θ_{RK} of capital
  - Urban household represents 40 percent of the population, and owns a fraction (1- θ_{RK}) of capital and a fraction (1- θ_{RL}) of labor.

Assumptions

- Two representative households
  - Each household spends its income on a composite good (made of domestic and imports) so as to maximize utility (or minimize expenditure). This income includes factor income, transfer from the government and from the rest of the world.
- Imperfect substitutability between domestic good and imports.

Assumptions

- Government revenue includes an indirect tax on domestic sales of the domestic good, and tariffs on imported final and intermediate goods. It balances the budget by re distributing its income to consumers.
- Factors and goods market are perfectly competitive. Full employment prevails.

Assumptions

- The above model is a modified version of a stylized general equilibrium model developed by Devarajan, Lewis and Robinson (1990).
- It can be interpreted as an extended version the Salter-Swan model which provides a foundation for the study of the impact of macroeconomic imbalances and adjustment policies on the real sector of a small open economy.

Analytical Expression

- Cobb-Douglas technology

$$X_i = A_i K_i^{\alpha_{ki}} L_i^{\alpha_{li}}; \quad \alpha_{ki} + \alpha_{li} = 1; \quad i = e, d.$$

- Demand for labor

$$L_i = \frac{\alpha_{li}(PVA_i X_i)}{w}; \quad i = e, d.$$

- Demand for capital

$$K_i = \frac{\alpha_{ki}(PVA_i X_i)}{r}; \quad i = e, d.$$

Analytical Expression

- Net producer price

$$PVA_i = PX_i - a_{2i} PQ_2, \quad i = e, d$$

  - a_{2i} is the amount of aggregate intermediate good (Q_2) per unit of output in sector i.

- Producer price of exports

$$PX_e = R(1 + t_e)\pi_e$$

Analytical Expression

- Armington commodity aggregation (j=1 for final and 2 for intermediate)

$$Q_j^s = B_j\left[\beta_j M_j^{-\rho_j} + (1-\beta_j)D_j^{-\rho_j}\right]^{-\frac{1}{\rho_j}}; \quad j = 1,2$$

- Import demand functions

$$M_j = B_j^{-1}\left[\beta_j^{\sigma_j} PM_j^{(1-\sigma_j)} + (1-\beta_j)^{\sigma_j} PD_j^{(1-\sigma_j)}\right]^{\frac{1}{(1-\sigma_j)}} \beta_j^{\sigma_j} PM_j^{-\sigma_j} Q_j^s; j = 1,2$$

- Demand for domestic component of the aggregation

$$D_j = B_j^{-1}\left[\beta_j^{\sigma_j} PM_j^{(1-\sigma_j)} + (1-\beta_j)^{\sigma_j} PD_j^{(1-\sigma_j)}\right]^{\frac{1}{(1-\sigma_j)}} (1-\beta_j)^{\sigma_j} PD_j^{-\sigma_j} Q_j^s; j = 1,2$$

Analytical Expression

- Elasticity of substitution

$$\sigma_j = \frac{1}{1+\rho_j}, \; j = 1, 2$$

- Domestic price of imports

$$PM_j = R(1+tm_j)\pi_j^m; \; j = 1, 2$$

- Price of domestic sales

$$PD_j = PX_d(1+tx_d); \quad j = 1, 2$$

- Producer price of domestic good

$$PX_d = \frac{\sum_{j=1}^{2} PD_j D_j}{X_d(1+tx_d)}; \quad j = 1, 2$$

Analytical Expression

- Price of composite goods

$$PQ_j = (PD_j D_j + PM_j M_j)/Q_j^s;\ j = 1, 2$$

- Income of each household includes a fraction of all labor and capital income, a fraction of government revenue, and a share of the balance of payment in local currency.

- $$Y_h = \theta_{hl}(wLS) + \theta_{hk}(rKS) + \theta_{hg} Y_G + \theta_{hf} RS_f;\quad h = r, u$$

Analytical Expression

- Government revenue excludes export subsidies

$$Y_G = \sum_{j=1}^{2} tm_j (R\pi_j^m M_j) + tx_d X_d - te(R\pi_e X_e)$$

- Household demand for final good

$$Q_{1h}^d = \frac{Y_h}{PQ_1}; \quad h = r, u; \quad Q_1^d = \sum_h Q_{1h}^d$$

- Total demand for intermediate good (by producers)

$$Q_2^d = \sum_i a_{2i} X_i; \quad i = e, d.$$

Analytical Expression

- Home good market equilibrium

$$X_d = \sum_{j=1}^{2} D_j;\ j = 1, 2$$

- Material balance for composite goods

$$Q_j^s = Q_j^d;\ j = 1, 2$$

- Factor market equilibrium

$$LS = \sum_i L_i;\quad KS = \sum_i K_i;\quad i = e, d$$

- Balance of payments

$$\pi_e X_e + S_f = \sum_{j=1}^{2} \pi_j^m M_j$$

Analytical Expression

- Note factor demand under cost minimization

$$L_i = \left(\frac{\alpha_{li}}{\alpha_{ki}} \frac{r}{w} \right)^{\alpha_{ki}} (X_i / A_i); \quad i = e, d$$

$$K_i = \left(\frac{\alpha_{ki}}{\alpha_{li}} \frac{w}{r} \right)^{\alpha_{li}} (X_i / A_i); \quad i = e, d$$

Numerical Implementation

- Closure
  - Use of EViews requires a one-to-one mapping between equations and endogenous variables.
  - The process of determining which variables to make endogenous and which exogenous is known generally as closure.

Numerical Implementation

- In the Walrasian framework, only relative prices matter.
  - If we multiply all prices (the price of domestic good, the exchange rate and returns to factors of production) by a constant, all nominal values will be multiplied by the same constant and real variables will remain unchanged. Hence the system is said to be homogeneous of degree zero.

Numerical Implementation

- We can therefore choose a *numéraire* good and fix its price exogenously. We select the exchange rate and make it exogenous.

- The small country assumption implies that world prices are exogenous.
- Finally, we make the balance of trade exogenous as well.

Numerical Implementation

- The model also respects Walras' law:
  - if all economic agents satisfy their budget constraints and all, but one, markets are in equilibrium, then the last market must also be in equilibrium.
  - Alternatively, if all markets are in equilibrium and all, but one, budget constraints are binding, then the last budget constraint is binding as well (Dinwiddy and Teal 1988).

Numerical Implementation

- Hence, we can drop one equilibrium condition from the system and it will still solve. We drop the balance of trade equation by assigning to it a dummy endogenous called WALRAS. This dummy must equal zero when the law is satisfied by a solution of the model.

Numerical Implementation

- Calibration
  - To make the model fully computable we resort to calibration. This process solves the relevant equations for values of the parameters that are consistent with base year data.

Numerical Implementation

- Base Year SAM

	Export	Domestic	Final	Intermediate	Labor	Capital	Rural	Urban	World	Total
Export									30.00	30.00
Domestic			73.00	2.00						75.00
Final							40.00	60.00		100.00
Intermediate	5.00									5.00
Labor	20.00	30.00								50.00
Capital	5.00	45.00								50.00
Rural Household					35.00	5.00				40.00
Urban Household					15.00	45.00				60.00
World			27.00	3.00						30.00
Total	30.00	75.00	100.00	5.00	50.00	50.00	40.00	60.00	30.00	

Source: Adapted from Devarajan, Lewis and Robinson (1990)

Numerical Implementation

- Calibration
  - For the parameters of the Cobb-Douglas function:

$$\alpha_{li} = \frac{wL_i}{PVA_i X_i}; \quad \alpha_{ki} = (1 - \alpha_{li}); \quad i = e, d.$$

  - and

$$A_i = \frac{X_i}{K_i^{\alpha_{ki}} L_i^{\alpha_{li}}}; \quad i = e, d.$$

Numerical Implementation

- Calibration
  - Parameters of the Armington aggregation

$$\rho_j = \frac{1-\sigma_j}{\sigma_j} \qquad \beta_j = \frac{1}{1+\frac{PD_j}{PM_j}\left(\frac{Q_j}{D_j}\right)^{-\frac{1}{\sigma_j}}} \qquad B_j = \frac{Q_j}{\left[\beta_j M_j^{-\rho_j} + (1-\beta_j) D_j^{-\rho_j}\right]^{-\frac{1}{\rho_j}}}$$

Calibrated Parameters for the Two-Sector Model

	α_L	α_K	A	β_M	β_D	B
Export	0.80	0.20	1.98			
Domestic	0.40	0.60	1.98			
Final				0.38	0.62	1.89
Intermediate				0.69	0.31	1.92

Numerical Implementation

- Calibration
  - Distribution of factor income in base year SAM: θ_{RL}=0.70, θ_{UL}=0.30, θ_{RK}=0.10, and θ_{UK}=0.90
  - Distribution of government transfers: θ_{RG}=0.60 and θ_{UG}=0.40
  - Distribution of foreign transfers: θ_{RF}=0.20 and θ_{UF}=0.80

Numerical Implementation

Source Text for Calibration

```
alphal_xpt = (WR * LD_xpt) / VA_xpt
alphak_xpt = 1 - alphal_xpt
ad_xpt = XS_xpt / (LD_xpt^alphal_xpt * KD_xpt^alphak_xpt )
alphal_dmg = (WR * LD_dmg) / VA_dmg
alphak_dmg = 1 - alphal_dmg
ad_dmg = XS_dmg / (LD_dmg^alphal_dmg * KD_dmg^alphak_dmg )
betam_fnl = 1 / (1 + ( PD_fnl / PM_fnl) * (XM_fnl / XD_fnl)^(-1 / sigma_fnl) )
betad_fnl = (1 - betam_fnl)
ces_fnl = XQ_fnl / ( betam_fnl * XM_fnl^(-rho_fnl) + betad_fnl * XD_fnl^(-rho_fnl)
)^(-1 / rho_fnl)
betam_ntr = 1 / (1 + ( PD_ntr / PM_ntr) * (XM_ntr / XD_ntr)^(-1 / sigma_ntr) )
betad_ntr = (1 - betam_ntr)
ces_ntr = XQ_ntr / ( betam_ntr * XM_ntr^(-rho_ntr) + betad_ntr * XD_ntr^(-rho_ntr)
)^(-1 / rho_ntr)
```

Numerical Implementation

Source Code for the Main Model

```
XS_xpt = ad_xpt * ( (LD_xpt^alphal_xpt) * (KD_xpt^alphak_xpt) )
XS_dmg = ad_dmg * ( (LD_dmg^alphal_dmg) * (KD_dmg^alphak_dmg) )
LD_xpt * WR = (alphal_xpt * PVA_xpt * XS_xpt)
KD_xpt * RK = (alphak_xpt * PVA_xpt * XS_xpt)
LD_dmg * WR = (alphal_dmg * PVA_dmg * XS_dmg)
KD_dmg * RK = (alphak_dmg * PVA_dmg * XS_dmg)
XQ_ntr = io_ntr_xpt * XS_xpt + io_ntr_dmg * XS_dmg
PVA_xpt = PX_xpt - io_ntr_xpt * PQ_ntr
PVA_dmg = PX_dmg - io_ntr_dmg * PQ_ntr
PX_xpt = EXR * (1 + te_xpt) * PWE_xpt
PX_dmg * (1 + txd) * XS_dmg = PD_fnl * XD_fnl + PD_ntr * XD_ntr
XM_fnl = (XQ_fnl / ces_fnl) * (betam_fnl / PM_fnl )^sigma_fnl * (
betam_fnl^sigma_fnl * ( PM_fnl)^(1 - sigma_fnl) + betad_fnl^sigma_fnl *
(PD_fnl)^(1 - sigma_fnl) )^(sigma_fnl / (1 - sigma_fnl))
XD_fnl = (XQ_fnl / ces_fnl) * (betad_fnl / PD_fnl )^sigma_fnl * (
betam_fnl^sigma_fnl * ( PM_fnl)^(1 - sigma_fnl) + betad_fnl^sigma_fnl *
(PD_fnl)^(1 - sigma_fnl) )^(sigma_fnl / (1 - sigma_fnl))
PM_fnl = EXR * (1 + tm_fnl) * PWM_fnl
XM_ntr = (XQ_ntr / ces_ntr) * (betam_ntr / PM_ntr )^sigma_ntr * (
betam_ntr^sigma_ntr * ( PM_ntr)^(1 - sigma_ntr) + betad_ntr^sigma_ntr *
(PD_ntr)^(1 - sigma_ntr) )^(sigma_ntr / (1 - sigma_ntr))
XD_ntr = (XQ_ntr / ces_ntr) * (betad_ntr / PD_ntr )^sigma_ntr * (
betam_ntr^sigma_ntr * ( PM_ntr)^(1 - sigma_ntr) + betad_ntr^sigma_ntr *
(PD_ntr)^(1 - sigma_ntr) )^(sigma_ntr / (1 - sigma_ntr))
```

Numerical Implementation

Source Code for the Main Model

```
PM_ntr = EXR * (1 + tm_ntr) * PWM_ntr
PD_fnl = PX_dmg * (1 + txd)
PQ_fnl * XQ_fnl = PD_fnl * XD_fnl + PM_fnl * XM_fnl
PD_ntr = PX_dmg * (1 + txd)
PQ_ntr * XQ_ntr = PD_ntr * XD_ntr + PM_ntr * XM_ntr
Y_rhh = shlab_rhh * (WR * LS) + shcap_rhh * (RK * KS) + tr_rhh_gov * Y_gov
+ tr_rhh_row * SAV_row
XQ_fnl_rhh * PQ_fnl = Y_rhh
Y_uhh = shlab_uhh * (WR * LS) + shcap_uhh * (RK * KS) + tr_uhh_gov * Y_gov
+ tr_uhh_row * SAV_row
XQ_fnl_uhh * PQ_fnl = Y_uhh
Y_gov = txd * PX_dmg * XS_dmg + tm_fnl * EXR * PWM_fnl * XM_fnl +
tm_ntr * EXR * PWM_ntr * XM_ntr
WALRAS = EXR * PWM_fnl * XM_fnl + EXR * PWM_ntr * XM_ntr - EXR *
PWE_xpt * XS_xpt - EXR * SAV_row
XQ_fnl = XQ_fnl_rhh + XQ_fnl_uhh
RK * KS = RK * KD_xpt + RK * KD_dmg
WR * LS = WR * LD_xpt + WR * LD_dmg
```

Numerical Implementation

- Validation
  - Once a general equilibrium model has been made computable through the process of calibration, and be fore it is used for any policy analysis, it is important to conduct a series of validation tests designed to show that the model is behaving the way it is supposed to behave.

Numerical Implementation

- Validation
  - Model behavior depends on the underlying theory.
  - Given the principles of optimization and equilibrium underlying this simple Walrasian model, we can perform two basic tests: homogeneity and Walras' law.

Numerical Implementation

- Validation
  - Multiplying the numéraire price, here the exchange rate by 2, will double all nominal values while leaving all real values unchanged.
  - Finally, the equation that is dropped from the system because of Walras' law must hold a posteriori (i.e. after a solution). Alternatively, the dummy variable WARLAS must equal zero. This guaranties that there are no leakages.

Exercise

- De Melo and Robinson (1989)
  - Present the data in table 2 as a SAM.
  - Use the data to calibrate the following analytical expression which is an interpretation of a simple one-sector model with symmetric product differentiation for both imports and exports.

Exercise

- CET production possibility frontier

$$X_s = A_x \left[\gamma_e X_e^{-\phi} + \gamma_d X_d^{-\phi} \right]^{-\frac{1}{\phi}}$$

- Where the constant elasticity of transformation is: $\omega = \frac{1}{\phi + 1}$
- Export supply

$$X_e = (X_s / A_x) \left(\frac{\gamma_e}{P_e} \right)^{\omega} \left(\gamma_e^{\omega} P_e^{1-\omega} + \gamma_d^{\omega} P_d^{1-\omega} \right)^{\frac{\omega}{(1-\omega)}}$$

Exercise

- Domestic sales

$$X_d = (X_s / A_x)\left(\frac{\gamma_d}{P_d}\right)^{\omega}\left(\gamma_e^{\omega} P_e^{1-\omega} + \gamma_d^{\omega} P_d^{1-\omega}\right)^{\frac{\omega}{(1-\omega)}}$$

- Domestic price of export

$$P_e = R\pi_e$$

- GDP deflator

$$P_x X_s = (P_e X_e + P_d X_d)$$

Exercise

- Armington aggregation

$$Q_s = B_q\left[\beta_m Q_m^{-\rho} + \beta_d D_x^{-\rho}\right]^{-\frac{1}{\rho}}$$

- Demand for imports

$$Q_m = (Q_s / B_q)\left(\frac{\beta_m}{P_m}\right)^{\sigma}\left(\beta_m^{\sigma} P_m^{1-\sigma} + \beta_d^{\sigma} P_d^{1-\sigma}\right)^{\frac{\sigma}{(1-\sigma)}}$$

- Demand for domestic good

$$D_x = (Q_s / B_q)\left(\frac{\beta_d}{P_d}\right)^{\sigma}\left(\beta_m^{\sigma} P_m^{1-\sigma} + \beta_d^{\sigma} P_d^{1-\sigma}\right)^{\frac{\sigma}{(1-\sigma)}}$$

Exercise

- Domestic price of imports

$$P_m = R\pi_m$$

- Total expenditure on the composite consumption good

$$P_q Q_s = (P_m Q_m + P_d D_x)$$

- Household income

$$Y_h = P_x X_s + RS_f$$

Exercise

- Demand for composite consumption good

$$Q_d = \frac{Y_h}{P_q}$$

- Goods market equilibrium

$$X_d = D_x \qquad\qquad Q_s = Q_d$$

- Balance of trade

$$\pi_m Q_m - \pi_e X_e = S_f$$

Exercise

- Assume full employment and use the consumer price index as a *numéraire* price.

Calibration Results for the Core Model of an Open Economy

obs	OMEGA	SIGMA	BETAD	BETAM	CES	CET	GAMMAD	GAMMAE	PHI	RHO
1	-0.200	0.200	0.996	0.004	1.431	3.177	0.004	0.996	-6.00	4.00
2	-0.500	0.500	0.900	0.100	1.600	2.610	0.100	0.900	-3.00	1.00
3	-2.000	2.000	0.634	0.366	1.866	2.151	0.366	0.634	-1.50	-0.50
4	-5.000	5.000	0.555	0.445	1.944	2.059	0.445	0.555	-1.20	-0.80
5	-500.00	5.000	0.555	0.445	1.944	2.001	0.499	0.501	-1.002	-0.80

Exercise

- Simulate the welfare and structural impact of a 10 percent increase in foreign transfers

Welfare and Structural Implications of a 10% Increase in Foreign Transfers

Omega	Sigma	EXR	PD	PX	RPD	RPX	XD	XE	XM	XQ
-0.2	0.2	0.46	1.20	1.02	0.38	0.45	77.39	21.28	31.28	106.97
-0.5	0.5	0.75	1.09	1.01	0.68	0.74	77.95	21.46	31.46	108.66
-2.0	2.0	0.93	1.02	1.00	0.91	0.93	78.28	21.57	31.57	109.65
-5.0	5.0	0.97	1.01	1.00	0.96	0.97	78.35	21.59	31.59	109.86
-500.0	5.0	1.00	1.00	1.00	1.00	1.00	82.41	17.59	27.59	110.00

References

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