# The Lorenz Model of the Size Distribution of Income

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# Foreword

"The effect of ignoring the interpersonal variations can, in fact, be deeply inegalitarian, in hiding the fact that equal consideration for all may demand very unequal treatment in favour of the disadvantaged."

Sen (1992)

# Outline

1. The Lorenz Curve
   - Structure
   - Normative Underpinnings
   - Parameterization
2. Recovering the Size Distribution and Associated Measures of Inequality and Poverty
   - Strategy
   - The Extended Gini Family
   - The Foster-Greer-Thorbecke (FGT) Family of Poverty Measures.
3. Decomposition of Poverty Outcomes

# The Lorenz Curve

Structure
- Definition
- Simple Example
- Analytical Expression

Normative Underpinnings
- A Social Impact Indicator
- Pigou-Dalton Principle and Second Order Dominance

Parameterization
- Approaches
- The General Quadratic Model

# Structure

**Definition**

- A flexible statistical model of the distribution of some welfare indicator, x, among the population.
- The curve maps the cumulative proportion of the population (horizontal axis) against the cumulative share of welfare (vertical axis), where individuals have been ranked in ascending order of x.

# Structure

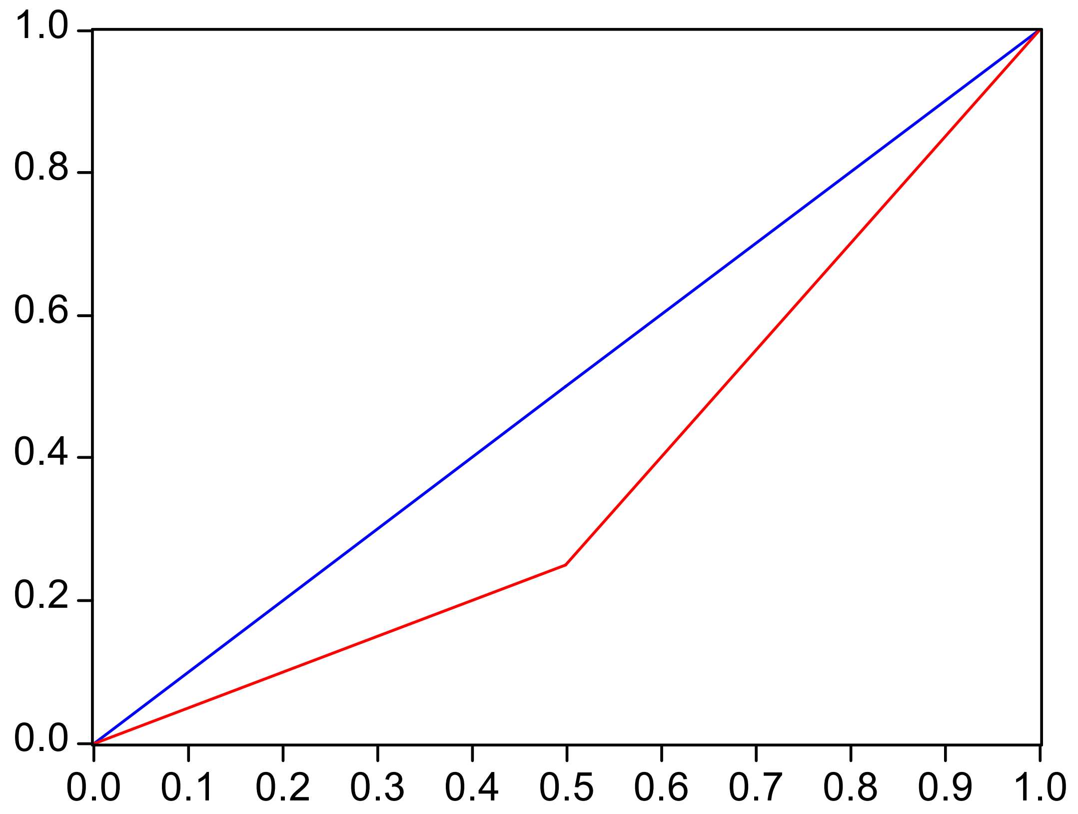
## Simple Example

- Income distribution among two individuals

| Income Level | Relative Frequency | Cumulative Frequency | Cumulative Share |
|---|---|---|---|
| 0.00 | 0.00 | 0.00 | 0.00 |
| 25.00 | 0.50 | 0.50 | 0.25 |
| 75.00 | 0.50 | 1.00 | 1.00 |

# Structure

Lorenz Representation of a Two-Person Distribution



# Structure

## Analytical Expression

- Above example: convex combination of two linear segments with a kink at (0.50, 0.25)
  - First Segment

$$L_1(p) = ap; \quad p \le 0.5$$

$$a = \frac{2x_1}{(x_1 + x_2)} = \frac{x_1}{\mu} = 0.5$$

- $\mu$ is the overall mean of the distribution, and the slope a is computed as “rise over run”.

# Structure

- Second segment

$$L_2(p) = bp + (1-b); \quad 0.5 < p \leq 1.0$$

$$b = \frac{2x_2}{(x_1 + x_2)} = \frac{x_2}{\mu} = 1.5$$

# Structure

- Combination

$$L(p)=\delta L_1(p)+(1-\delta)L_2(p);\quad p\in[0,1]$$

- δ is a dummy that is equal to 1 if p≤0.5, and 0 otherwise.
- Slope

$$\frac{\Delta L(p)}{\Delta p}=\delta\frac{x_1}{\mu}+(1-\delta)\frac{x_2}{\mu}$$

  - Interpretation: a local measure of inequality showing how far a given income is below or above the mean (i.e. equal share). Hence equal distribution implies slope=1 for all δ. The Lorenz curve becomes L(p)=p for all δ.

# Structure

- Rate of change of slope

$$\frac{\Delta^2 L(p)}{\Delta p^2} = \frac{1}{\mu}\left[\delta\frac{\Delta x_1}{\Delta p} + (1-\delta)\frac{\Delta x_2}{\Delta p}\right]$$

- From above table, use nearest left neighbor of $x_i$ to compute:

$$\frac{\Delta x_i}{\Delta p} = \left(\frac{\Delta p}{\Delta x_i}\right)^{-1} = \left(\frac{1}{2}\right)^{-1} = 2 \quad \forall i$$

- Thus

$$\frac{\Delta^2 L(p)}{\Delta p^2} = \frac{2}{\mu} = \frac{1}{0.5\mu} = \frac{1}{\mu f(x_i)}$$

# Structure

- Case of n people

$$L(p) = \frac{\sum_{k=1}^{j} x_k}{\sum_{k=1}^{n} x_k} = \frac{j\mu_j}{n\mu} = \frac{\mu_p}{\mu} p; \quad p = \frac{j}{n}; L(0) = 0; L(1) = 1$$

- Rate of change of L(p) is $x_k/(n\mu)$, that of p is 1/n. Hence:

$$\frac{\Delta L(p)}{\Delta p} = \frac{x_j}{\mu}$$

$$\frac{\Delta^2 L(p)}{\Delta p^2} = \frac{1}{\mu \frac{\Delta p}{\Delta x_j}} = \frac{1}{\mu f(x_j)} = \frac{n}{\mu}$$

# Structure

- Assuming smoothness
  - Lorenz function

$$L(p) = \int_0^p \frac{x(q)}{\mu} dq$$

  - First order derivative

$$L'(p) = \frac{x(p)}{\mu}$$

  - Second order derivative

$$L''(p) = \frac{1}{\mu}\frac{dx}{dp} = \frac{1}{\mu \frac{dp}{dx}} = \frac{1}{\mu f(x)}$$

# Structure

- Generalized Lorenz Curve
  - Discrete

$$L(\mu,p)=\mu L(p)=\frac{1}{n}\sum_{k=1}^{j}x_k=p\mu_p;\ L(\mu,0)=0;\ L(\mu,1)=\mu$$

  - Continuous

$$L(\mu,p)=\int_0^x tf(t)dt=\int_0^p x(q)dq$$

# Normative Underpinnings

A Social Impact Indicator

- Impact analysis can be framed within social evaluation since it entails a comparison of social states: observed versus counterfactual.

# Normative Underpinnings

- Assumptions
  - A homogeneous population with respect to non-income characteristics, or that such characteristics are socially irrelevant.
  - At individual level, more income is preferred to less.
  - For society, more equality is preferred to less.

# Normative Underpinnings

- Formalization

$$W(x) = \sum_{k=1}^{n} \omega_k x_k$$

$\omega_k$ is the social weight attached to the situation of k.

- Social Impact Indicator

$$\Delta W = \sum_{k=1}^{n} \omega_k \Delta x_k ; \quad \Delta x_k = (x_{ak} - x_{bk})$$

- Where **b** stands for before (or counterfactual) and **a** for after (treatment state). Assume weights do not change from one situation to the next.

# Normative Underpinnings

- Poverty-focused choice of evaluative weights
  - Rank individuals in ascending order of **x**, choose weights such that each is **nonnegative** and any adjacent weights satisfy: $(\omega_{k-1} - \omega_k) \geq 0$ . (Unequal treatment in favor of the disadvantaged).
  - Mayshar and Yitzhaki (1995) show that:

$$\Delta W = \sum_{k=1}^{n} \omega_k \Delta x_k = \sum_{k=1}^{n-1} (\omega_k - \omega_{k+1}) cmdx_k + \omega_n cmdx_n; \quad cmdx_k = \sum_{i=1}^{k} \Delta x_i$$

  - This is a sum of cross products, and the first term of each is nonnegative by choice of weights.

# Normative Underpinnings

**Pigou-Dalton Principle and Second Order Dominance**

- Lorenz-based inequality comparisons are consistent with assessments based on transfer-approving social evaluation criteria (Pigou-Dalton principle of transfers).

- A Pigou-Dalton improvement from state **b** to state **a** implies generalized Lorenz dominance, $L_a(\mu,p) \geq L_b(\mu,p)$, also known as second order dominance.
  - Pigou-Dalton improvement means $\Delta W \geq 0$. This will certainly be the case if $cmdx_k \geq 0$ for all k, implying that:

$$\sum_{i=1}^{k} \Delta x_i \geq 0 \; \forall k \Rightarrow \sum_{i=1}^{k} x_{ai} \geq \sum_{i=1}^{k} x_{bi} \quad \forall k \in \{1, 2, \ldots, n\}$$

# Normative Underpinnings

- Normalize above by population size to get equivalent statement in terms of generalized Lorenz curves.
- Use ordinary Lorenz if both distributions have same mean.
- Thus second order dominance respects the Pigou-Dalton principle of transfers.
- It can also be shown that, social evaluation criteria of the Pigou-Dalton class respect the verdict of second order dominance.

# Normative Underpinnings

- The Pareto Criterion and the Growth Incidence Curve.
  - Require that social weights be only nonnegative.  Then Pareto improvement implies $\Delta x_k \geq 0$ for all k.
  - Alternatively

$$\Delta x_k \geq 0 \,\forall k \Leftrightarrow \frac{x_{1k}}{x_{0k}} = \frac{\mu_1 L_1'(p)}{\mu_0 L_0'(p)} \geq 1 \quad \forall p$$

# Normative Underpinnings

- Let g(p) stand for the growth rate at the p^th quantile, then the log transform of above expression yields:

$$g(p) = \gamma + \Delta \ln L'(p)$$

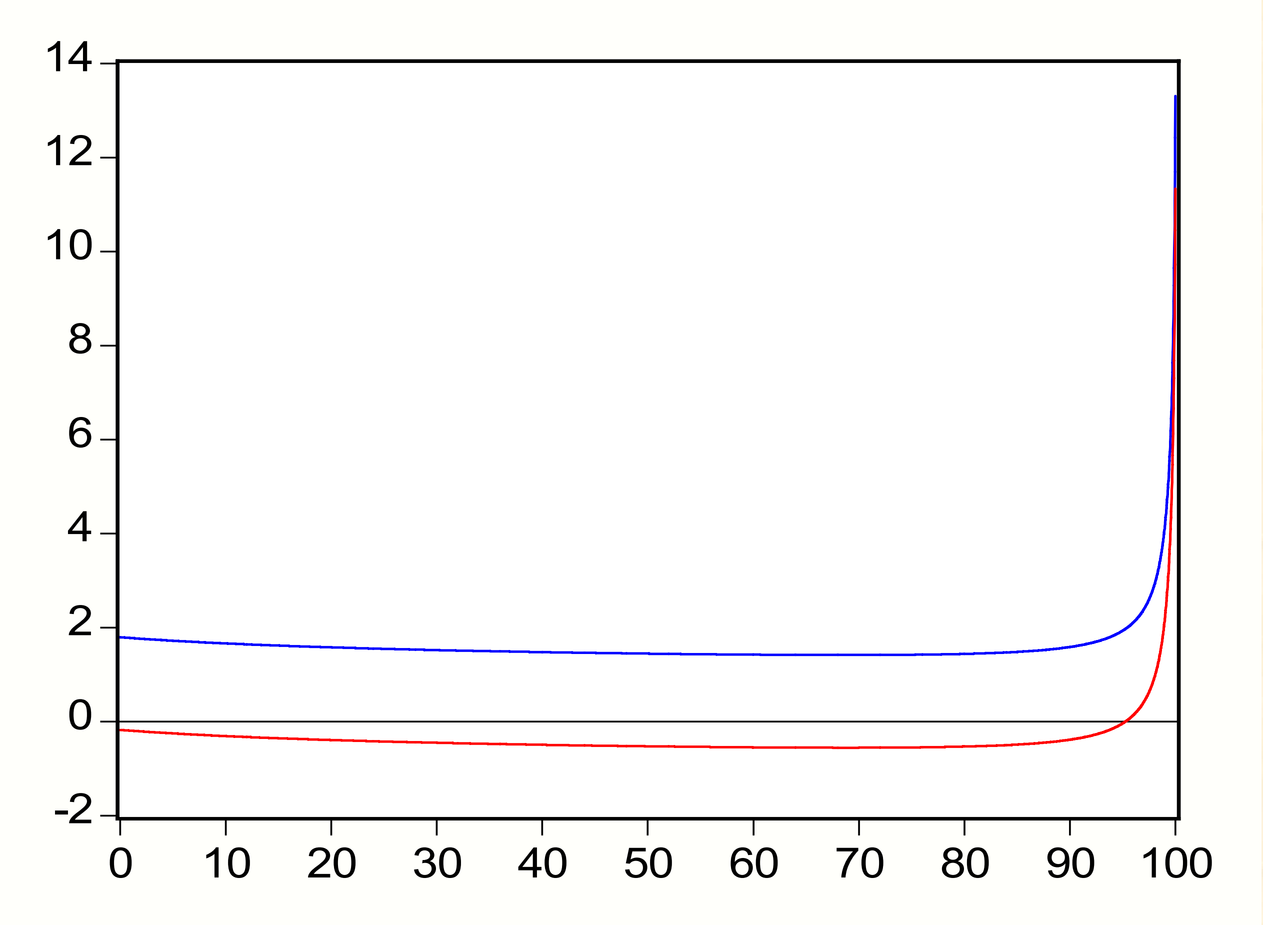
- This is equivalent to the growth incidence curve (Ravallion and Chen 2003)
- This is a distribution-adjusted, where adjustment factor is based on changes in the slope of the Lorenz curve.

# Normative Underpinnings

- If Pareto improvement, then $g(p) \geq 0$ for all p. Well-known first order dominance results indicate that this change is poverty-reducing for a wide choice of poverty measures.

# Normative Underpinnings

Growth Incidence Curves for Indonesia: 1993-2002




Cumulative Percentage of Population

# Parameterization

Approaches

- Derive expression of Lorenz curve from known distribution function e.g. Lognormal or Beta. Then estimate structural parameters from data.
- Choose a functional form for the Lorenz curve.
  - Estimate its structural parameters from the data.
  - Compute the curve and associated derivatives from parameter estimates.

# Parameterization

The General Quadratic Model (Datt 1992, 1998)

- Regress [L(1-L) on ($p^2$-L), L(p-1) and (p-L) with no intercept and dropping last observation.
- Let $\beta_1$, $\beta_2$, $\beta_3$ be the regression coefficients.

# Parameterization

- Compute the following:

$$e=-(\beta_1+\beta_2+\beta_1+1);m=(\beta_2^2-4\beta_1);n=(2\beta_2 e-4\beta_3);r=(n^2-4me^2)^{\frac{1}{2}}$$

$$L(p)=-\frac{1}{2}\left[\beta_2 p+e+(mp^2+np+e^2)^{\frac{1}{2}}\right]$$

$$L'(p)=-\frac{\beta_2}{2}-\frac{2mp+n}{4\sqrt{(mp^2+np+e^2)}}$$

$$L''(p)=\frac{r^2(mp^2+np+e^2)^{-\frac{3}{2}}}{8}$$

# Parameterization

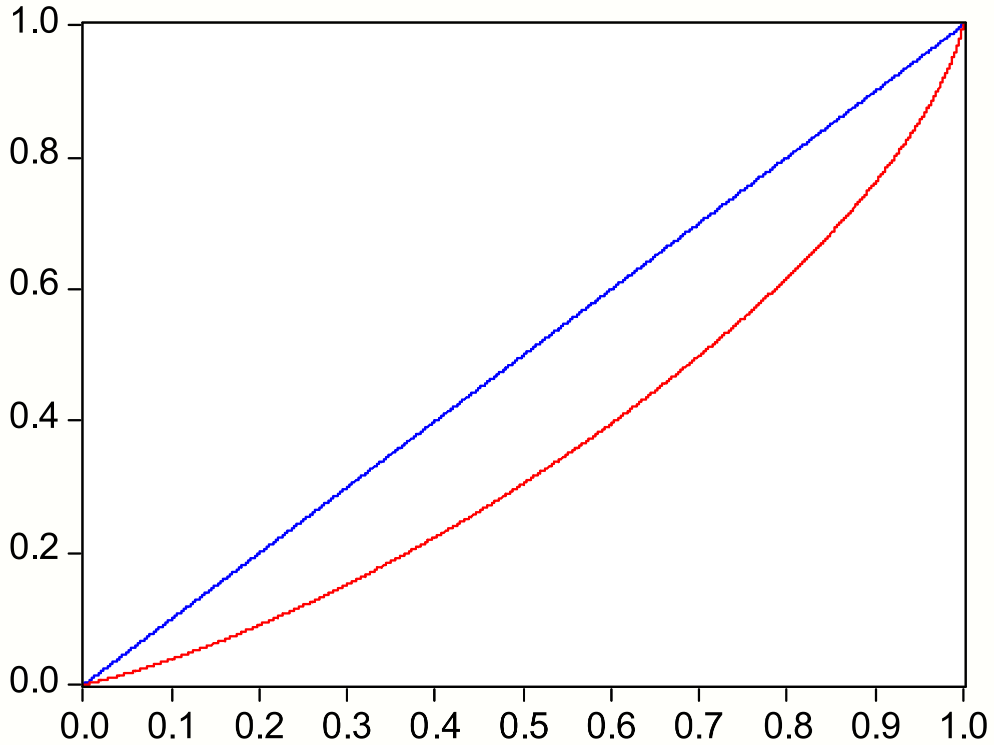
Rural India 1983: Regression Output for the General Quadratic Lorenz Representation of Household Expenditure

| | Coefficient | Std. Error | t-Statistic | Prob. |
|---|---|---|---|---|
| BETA(1) | 0.887734 | 0.006683 | 132.8389 | 0.0000 |
| BETA(2) | -1.451431 | 0.019062 | -76.14295 | 0.0000 |
| BETA(3) | 0.202658 | 0.012847 | 15.77521 | 0.0000 |
| R-squared | 0.999959 | Mean dependent var | | 0.121975 |
| Adjusted R-squared | 0.999950 | S.D. dependent var | | 0.087678 |
| S.E. of regression | 0.000617 | Akaike info criterion | | -11.73067 |
| Sum squared resid | 3.43E-06 | Schwarz criterion | | -11.60944 |
| Log likelihood | 73.38399 | Durbin-Watson stat | | 0.697503 |

Data Source: Datt (1998)

# Parameterization

A Simulated Lorenz Curve for Rural India, 1983



# Recovering the Size Distribution and Associated Measures of Inequality and Poverty

- Strategy
- The Extended Gini Family
- The FGT Family of Poverty Measures

# Strategy

- Most, if not all, inequality and poverty measures of interest can be computed from the following basic inputs:
  - the level of income or expenditure x,
  - the associated density function f(x), and
  - a poverty line (for poverty measures).
- When relevant data are available, we can recover the first two inputs from a parameterized Lorenz function and the mean of the distribution.

# Strategy

- In particular, we derive x from the mean and the first order derivative of the Lorenz function.
- An estimate of the density function is obtained from the mean and the second order derivative.
- For computational purposes, we use the fact that f(x)dx is interpreted as the proportion of the population with income or expenditure in the close interval [x, dx] for a level of x and an infinitesimal change dx (Lambert 2001)

# Strategy

- Above considerations suggest applying numerical integration to the standard definitions of the measures of interest.
- This obviates the need to derive special expressions for these indicators from the chosen functional form of the Lorenz curve (an approach followed by Datt 1992, 1998 for instance).
- Focus on Gini and FGT

# The Extended Gini Family

- The ordinary Gini coefficient is equal to the area between the Lorenz curve and the line of complete equality (also known as the 45-degree line) divided by the whole area under the 45-degree line.
- Ordinary Gini is a member of an extended family based on a focal parameter that is interpreted as an indicator of inequality aversion.

# The Extended Gini Family

- A covariance-based expression of the extended Gini coefficient can be derived from an abbreviation of the transfer-approving social evaluation criterion studied above.
  - where

$$W(x) = nV(\omega)$$

$$V(\omega) = \mu_\omega \mu_x + \operatorname{cov}(x, \omega)$$

# The Extended Gini Family

- With no loss of Generality, normalize average social weight to 1 (i.e. $\mu_\omega$=1 )

$$V(\omega) = \mu_x + \text{cov}(x,\omega) = \mu_x[1 + \frac{\text{cov}(x,\omega)}{\mu_x}] = \mu_x[1 - G(\omega)]$$

- Analogy with equally distributed equivalent income or expenditure ( see Atkinson 1970)
- A general expression of the extended Gini coefficient is therefore:

$$G(\omega) = -\frac{\text{cov}(x,\omega)}{\mu_x}$$

# The Extended Gini Family

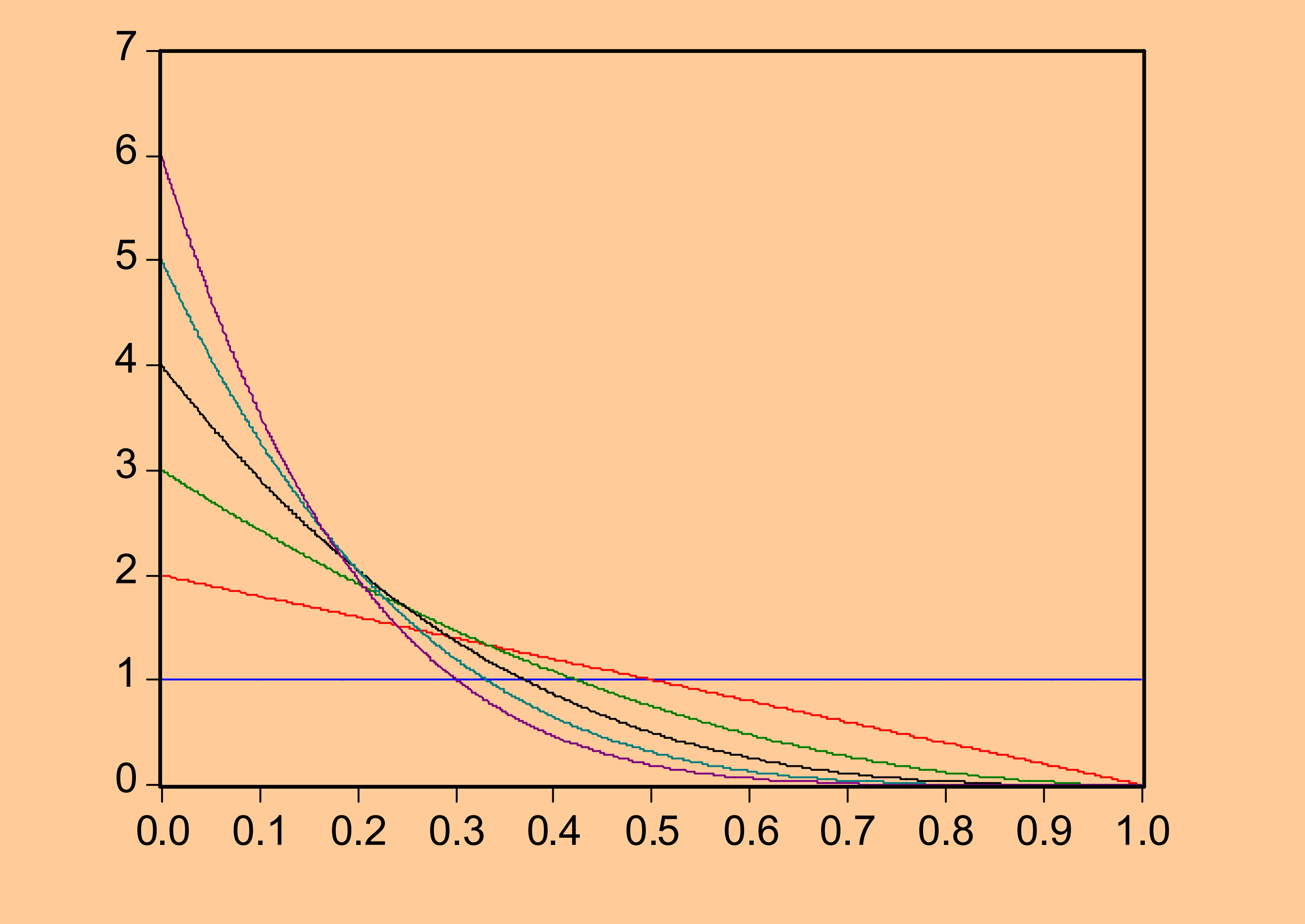
- For computational purposes, we adopt the system of weights proposed by Yitzhaki (1983)

$$\omega_k(\nu) = \nu(1 - p_k)^{\nu - 1}$$

  - Where
    - $p_k$ is the proportion of people with income less than or equal $x_k$, and
    - $\nu$ is the focal parameter indicating the degree of inequality aversion.

# The Extended Gini Family

(Evaluative Weights as a Function of the Focal Parameter $\nu$)



# The Extended Gini Family

Cut-off Rank as a Function of the Aversion Parameter.

| | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $\upsilon$ | 1.1 | 1.2 | 1.5 | 2.0 | 3 | 6 | 10 | 20 | 30 | 50 | 100 | 200 |
| p* | 0.62 | 0.60 | 0.56 | 0.50 | 0.42 | 0.30 | 0.23 | 0.15 | 0.11 | 0.08 | 0.05 | 0.03 |

Source: Essama-Nssah (2002)

# The Extended Gini Family

- The corresponding expression for the extended Gini is:

$$G(\nu) = -\frac{\nu}{\mu_x}\operatorname{cov}[x, (1-p)^{\nu-1}]$$

- It can also be computed as:

$$G(\nu) = -\frac{\nu}{\mu_x}\operatorname{cov}[\mu_x L'(p), (1-p)^{\nu-1}]$$

# The Extended Gini Family

- Alternatively, the extended Gini coefficient can be written as:

$$G(\nu) = 1 - \nu(\nu - 1)\int_0^1 (1-p)^{\nu-2} L(p)\,dp$$

- Chotikapanich and Griffiths (2001) propose a linear segment estimator defined as follows:

$$G(\nu) = 1 + \sum_{k=1}^{m}\left(\frac{\theta_k}{w_k}\right)[(1-p_k)^{\nu} - (1-p_{k-1})^{\nu}]; \quad \theta_k = \frac{w_k x_k}{\sum_{j=1}^{m} w_j x_j}$$

# The Extended Gini Family

- The linear segment estimator is equivalent to:

$$G(\nu) = 1 + \sum_{k=1}^{m} \frac{\Delta L(p_k)}{\Delta p_k} \left[ (1 - p_k)^{\nu} - (1 - p_{k-1})^{\nu} \right]$$

- When the focal parameter ν is equal to 2, we get the ordinary Gini index.

# The Extended Gini Family

- Sen's measure of poverty is a close relative of the Gini coefficient.

- Let $\mu_p$ and $G_p(\nu)$ be respectively the average income and the extended Gini for the poor. The extended Sen index of poverty is equal to:

$$S(\nu) = H\left[1 - \frac{\mu_p[1 - G_p(\nu)]}{z}\right]$$

# The Extended Gini Family

Gini Family of Indicators for Rural India in 1983

| Focus | Overall Gini | Gini for Poor | Sen Index |
|---|---|---|---|
| 1 | 0.00 | 0.00 | 12.48 |
| 2 | 28.89 | 13.54 | 16.89 |
| 3 | 38.78 | 20.66 | 19.21 |
| 4 | 44.22 | 25.12 | 20.67 |
| 5 | 47.79 | 28.21 | 21.67 |
| 6 | 50.35 | 30.48 | 22.41 |

Source: Author's calculations

# The FGT Family of Poverty Measures

- Kakwani(1999) defines a class of additively separable poverty measures starting from the notion of deprivation.
- Let $\psi$**(z,x$_i$)** stand for an indicator of deprivation at the individual level. A class of additively separable poverty measures can be defined as:

$$P(z,x)=\frac{1}{n}\sum_{i=1}^{n}\psi(z,x_i)=\sum_{i=1}^{n}\psi\left(z,\mu\frac{\Delta L(p_i)}{\Delta p_i}\right)f(x_i)\Delta x_i$$

# The FGT Family of Poverty Measures

- Deprivation felt by an individual depends only on a fixed poverty line and her level of welfare and not on the welfare of other individuals in society.

- When population is divided exhaustively into mutually exclusive socioeconomic groups, overall poverty is a weighted average of poverty in each group.

- The weights are population shares. Hence these measures are also additively decomposable.

# The FGT Family of Poverty Measures

- Specification of the deprivation function leads to particular members of the class.
- For Foster-Greer-Thorbecke (1984), expression due to Jenkins and Lambert (1997):

$$\psi_{FGT}(z, x_i) = \max\{(1 - x_i / z)^{\alpha}, 0\}.$$

# The FGT Family of Poverty Measures

- $\alpha$ is an indicator of aversion for inequality among the poor.
  - When $\alpha$=0, we get the headcount index;
  - When $\alpha$=1, we get the poverty gap index; and
  - When $\alpha$=2, we get the squared poverty gap index.

# The FGT Family of Poverty Measures

TIP Representation of Poverty (Jenkins and Lambert 1997)

- TIP stands for the Three “I”s of Poverty:
  - incidence,
  - intensity, and
  - inequality (among the poor).
- The curve provides a graphical summary of those three dimensions of poverty.
- Construction analogous to that of Lorenz curve.

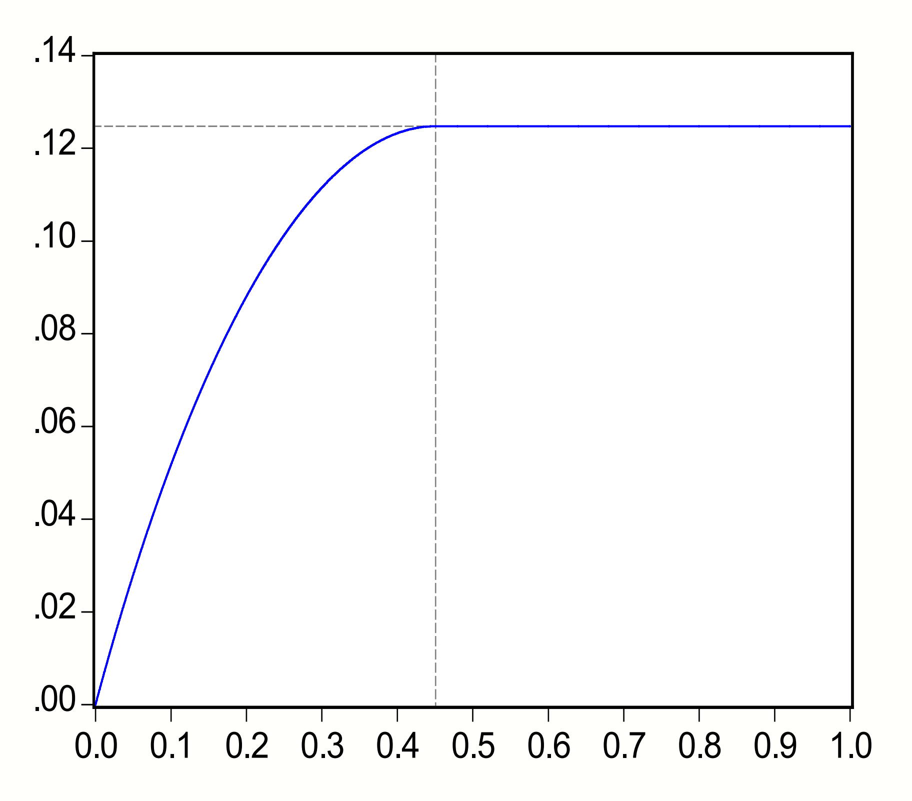
# The FGT Family of Poverty Measures

- Step 1: rank individuals from poorest to richest.
- Step 2: compute relative poverty gaps, $g_i = \max\{(1 - x_i/z),\ 0\}$.
- Step 3: form cumulative sum of poverty gaps normalized by population size.
- Step 4: plot result as function of cumulative population shares:

$$JL(p) = \frac{1}{n}\sum_{i=1}^{k} g_i; \quad p = \frac{k}{n}; \quad JL(0) = 0$$

# The FGT Family of Poverty Measures

A TIP Curve for Rural India in 1983



# The FGT Family of Poverty Measures

Properties of the TIP Curve:

- An increasing concave curve such that the slope is equal to the poverty gap at the given percentile.
- The length of the non-horizontal section shows poverty incidence.
- The height of the curve reveals intensity.
- The degree of concavity of the non-horizontal section translates the degree of inequality among the poor.
- Respects Second-Order Dominance

# Decomposition of Poverty Outcomes

- Poverty measures are computed on the basis of a distribution of an indicator of the living standard. Such a distribution is fully characterized by the mean and the degree of relative inequality.
- It is therefore reasonable to think of poverty outcomes as determined by these two factors.
- We focus on the elasticity approach and the Shapley method of decomposing poverty outcomes.

# Decomposition of Poverty Outcomes

The Elasticity Approach (Kakwani 1993)

- Assumptions:
  - "Small" or infinitesimal changes.
  - The Lorenz curve shifts proportionately over the whole range of the distribution.
- Total percentage change in poverty

$$\frac{dP}{P} = \eta_P \frac{d\mu}{\mu} + \varepsilon_p \frac{dG}{G}$$

# Decomposition of Poverty Outcomes

Where

- $\eta_p$ stands for the growth elasticity of the poverty index,
- $\mu$ is the mean of the distribution,
- $\varepsilon_p$ represents the elasticity of the poverty index with respect to the ordinary Gini coefficient, G.

- Setting the proportional change in poverty equal to zero leads to a indicator of the trade-off between mean income growth and inequality.

# Decomposition of Poverty Outcomes

- The indicator is known as the marginal proportional rate of substitution between mean income and inequality (Kakwani 1993).

$$MPRS = \frac{\partial \mu}{\partial G}\frac{G}{\mu} = -\frac{\varepsilon_P}{\eta_P}$$

- This is a measure of the extent to which income needs to grow to compensate for an increase of 1 percent in the Gini index.

# Decomposition of Poverty Outcomes

- For the class of additively separable poverty measures, the general formula for the growth elasticity is:

$$\eta_P = \frac{1}{P}\sum_{h=1}^{m} w_h x_h \left(\frac{\partial \psi}{\partial x_h}\right)$$

- The elasticity with respect to Gini is:

$$\varepsilon_P = \eta_P - \frac{\mu}{P}\sum_{h=1}^{m} w_h \left(\frac{\partial \psi}{\partial x_h}\right)$$

  - $w_h$ stands for the relative frequency of observation $x_h$.

# Decomposition of Poverty Outcomes

- For the headcount index:
  - the growth elasticity is:

$$\eta_H = -\frac{zf(z)}{H} < 0$$

  - The elasticity with respect to Gini is:

$$\varepsilon_H = -\frac{(\mu - z)}{z}\eta_H$$

# Decomposition of Poverty Outcomes

- For the rest of the FGT family, these parameters have the following expressions:

$$\eta_{FGT} = -\frac{\alpha[P_{\alpha-1} - P_{\alpha}]}{P_{\alpha}};$$

$$\varepsilon_{FGT} = \eta_{FGT} + \frac{\alpha\mu P_{\alpha-1}}{zP_{\alpha}};$$

$$\alpha > 0.$$

# Decomposition of Poverty Outcomes

**FGT and Associated Elasticity Measures for Rural India,1983**

| Focus | FGT | Growth | Inequality |
|---|---|---|---|
| 0 | 45.07 | -1.87 | 0.44 |
| 1 | 12.48 | -2.61 | 1.85 |
| 2 | 4.75 | -3.25 | 3.23 |

**Source: Author's calculations**

# Decomposition of Poverty Outcomes

- Ravallion (1994b) argues that the elasticity approach may lead to large errors in the case of big discrete changes.
- In these cases, it is preferable to use the mean and the Lorenz curve to decompose changes in poverty into growth and inequality components.
- Ravallion and Datt (1992) propose a method that leaves a residual (interpreted as an interaction term).

# Decomposition of Poverty Outcomes

- Kakwani (1997) and Shorrocks (1999) offer another approach to the decomposition that does not involve a residual. We focus on this one.
- Shorrocks (1999) rationalizes the approach with reference to the Shapley value of a **cooperative game**. In such a game, partners join forces to produce a good that must be shared. This raises the issue of fair division.

# Example of Division of Common Property

Birhor Rule for Sharing the Kill after a Monkey Hunt

| Claimants | Share |
|---|---|
| Spirits of the Chase | A bit of meat roasted by Chief |
| Tribal Chief | Neck and half of back meat of each animal in addition to his hunter's share |
| Beater | Equal share of entrails, tails and feet based on number of beaters |
| Owner of Net | Hind leg |
| Flanker | Hind leg |
| Chief's Hunter | [not specified in source document] |

Source: Young(1994). Note: "Any left over is divided into as many portions as there are eligible persons, plus an extra share for the chief". The Birhors are a tribe in east-central India.

# Decomposition of Poverty Outcomes

- Example: Cost Sharing (Young, 1994)

  "Given a cost-sharing game on a fixed set of players, let the players join the cooperative enterprise one at the time in some predetermined order. As each player joins, the number of players to be served increases. The player's cost contribution is his net addition to the cost when he joins, that is the incremental cost of adding him to the group of players who have already joined. The Shapley value of a player is his average cost contribution over all possible orderings of players"

# Decomposition of Poverty Outcomes

- Interpretation of the above principle in the context of poverty decomposition leads to the following expressions.
  - The Shapley contribution of growth to change in poverty:

$$S_G = \frac{1}{2}\left[P(\mu_2, L_2) - P(\mu_1, L_2)\right] + \frac{1}{2}\left[P(\mu_2, L_1) - P(\mu_1, L_1)\right]$$

# Decomposition of Poverty Outcomes

- The Shapley contribution of inequality is:

$$S_L = \frac{1}{2}\left[P(\mu_2, L_2) - P(\mu_2, L_1\right] + \frac{1}{2}\left[P(\mu_1, L_2) - P(\mu_1, L_1)\right]$$

- The following value judgments underpin the Shapley method:
  - Symmetry or anonymity (the identity or label of a factor is irrelevant)
  - Rule should lead to exact decomposition
  - Contribution of each factor is equal to its first round marginal impact.

# Decomposition of Poverty Outcomes

**A Profile of Poverty in Indonesia, 1993-2002**

| Poverty Measures | 1993 | 1996 | 2002 |
|---|---|---|---|
| Headcount | 61.55 | 50.51 | 52.42 |
| Poverty Gap | 21.03 | 15.33 | 15.68 |
| Squared Poverty Gap | 9.16 | 6.02 | 6.09 |

**Source: Author's simulations (poverty line about $2/day)**

# Decomposition of Poverty Outcomes

**Shapley Decomposition of Poverty Outcomes in Indonesia, 1993-2002**

| Measure | Total Change | Growth | Inequality |
|---|---|---|---|
| Headcount | -9.13 | -12.49 | 3.36 |
| Poverty Gap | -5.35 | -6.87 | 1.52 |
| Squared Poverty Gap | -3.07 | -3.82 | 0.75 |

Source: Author's simulations

# Decomposition of Poverty Outcomes

**Shapley Decomposition of Poverty Outcomes in Indonesia, 1996-2002**

| Measure | Total Change | Growth | Inequality |
|---|---|---|---|
| Headcount | 1.91 | 4.05 | -2.14 |
| Poverty Gap | 0.35 | 2.04 | -1.69 |
| Squared Poverty Gap | 0.07 | 1.07 | -1.00 |

Source: Author's Simulations

# Summary

- The use of numerical integration in simulating inequality and poverty measures obviates the need to derive special expressions from the chosen functional form of the Lorenz curve.
- Lorenz ranking of distributions respects the Pigou-Dalton principle of transfers underlying Second-Order Dominance.
- Ranking by the GIC is consistent with the Pareto criterion underlying First-Order Dominance
- When dominance tests fail, one can resort to aggregate indicators for inequality and poverty comparisons.

# Summary

- All measures reviewed here depend to some extent on a focal parameter that can be interpreted as an indicator of aversion for inequality.
- The aversion parameter allows the analyst to calibrate the poverty focus of social impact assessment.
- The use of the Lorenz curve facilitates the decomposition of poverty outcomes in terms of changes in the mean and in relative inequality.

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The End.