Accounting for Intermediates: Production Sharing and Trade in Value Added*

Robert C. Johnson† Guillermo Noguera‡
Dartmouth College Columbia Business School

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Abstract

We combine input-output and bilateral trade data to compute the value added content of bilateral trade. The ratio of value added to gross exports (VAX ratio) is a measure of the intensity of production sharing. Across countries, export composition drives VAX ratios, with exporters of Manufactures having lower ratios. Across sectors, the VAX ratio for Manufactures is low relative to Services, primarily because Services are used as an intermediate to produce manufacturing exports. Across bilateral partners, VAX ratios vary widely and contain information on both bilateral and triangular production chains. We document specifically that bilateral production linkages, not variation in the composition of exports, drives variation in bilateral VAX ratios. Finally, bilateral imbalances measured in value added differ from gross trade imbalances. Most prominently, the U.S.-China imbalance in 2004 is 30-40% smaller when measured in value added.

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†Corresponding Author. Department of Economics, Dartmouth College, Hanover, NH 03755, USA. E-mail: robert.c.johnson@dartmouth.edu. Telephone: +1-847-532-0443.

‡Columbia Business School, 6-0 Uris Hall, 3022 Broadway, New York, NY 10027, USA. E-mail: gn2195@columbia.edu.
Trade in intermediate inputs accounts for as much as two thirds of international trade. By linking production processes across borders, this input trade creates two distinct measurement challenges. First, conventional gross trade statistics tally the gross value of goods at each border crossing, rather than the net value added between border crossings. This well-known “double-counting” problem means that conventional data overstate the domestic (value added) content of exports. Second, multi-country production networks imply that intermediate goods can travel to their final destination by an indirect route. For example, if Japanese intermediates are assembled in China into final goods exported to the U.S., then Chinese bilateral gross exports embody third party (Japanese) content. Together, “double-counting” and multi-country production chains imply that there is a hidden structure of trade in value added underlying gross trade flows.

In this paper, we compute and analyze the value added content of trade. To do so, we require a global bilateral input-output table that describes how particular sectors in each destination country purchase intermediates from both home and individual foreign sources, as well as how each country sources final goods. Because these bilateral final and intermediate goods linkages are not directly observed in standard trade and national accounts data sources, we construct a synthetic table by combining input-output tables and bilateral trade data for many countries. Using this table, we split each country’s gross output according to the destination in which it is ultimately absorbed in final demand. We then use value added to output ratios from the source country to compute the value added associated with the implicit output transfer to each destination. The end result is a data set of “value added exports” that describes the destination where the value added produced in each source country is absorbed.

These data on the value added content of trade have many potential uses. Most directly, we compare them to gross bilateral trade flows to quantify the scope of production sharing. This approach to measuring production sharing yields comparable figures for many countries and sectors and respects the multilateral structure of production sharing. Further, because we use the national accounts definition of intermediates, our measures are easily translated into models\(^1\). This is im-

\(^1\)This contrasts with alternative approaches, such as using data on trade in parts and components (e.g., Yeats (2001)) or trade between multinational parents and affiliates (e.g., Hanson, Mataloni, and Slaughter (2005)).
portant because the value added content of trade is a key theoretical object and calibration target in many trade and macroeconomic models. For example, value added exports can be used to calibrate “openness” and bilateral exposure to foreign shocks in international business cycle research.\(^2\) For trade research, value added flows could be used to calibrate gravity-style trade models to allow for differences in trade patterns for final and intermediate goods.\(^3\) They could also be employed to calibrate many-country models of multi-stage production and vertical specialization, as in Yi (2003, 2010). And these applications only scratch the surface.

Our approach to measuring the value added content of trade draws upon a venerable literature on input-output accounting in models with multiple regions.\(^4\) It is also intimately related to recent efforts to measure the factor content of trade with traded intermediates. Specifically, Trefler and Zhu (2010) develop a multi-country input-output framework to define a Vanek-consistent measure of the factor content of multilateral net exports.\(^5\) We elaborate on the relationship between computing value added and factor contents in our discussion of the calculation procedure. Belke and Wang (2006) and Daudin, Rifflart, and Schweisguth (forthcoming) also develop value added trade computations along the lines of those used in this paper, though our exposition and analysis differ considerably.\(^6\)

Our work is also related to an active literature on measuring vertical specialization and the domestic content of exports.\(^7\) Aggregating across sectors and export destinations for each source country, the ratio of value added to gross exports can be interpreted as a metric of the domestic content of exports.\(^8\) Our domestic content metric generalizes the work by Hummels, Ishii, and Yi (2001). Hummels et al. compute the value added content of exports under the restrictive assump-

\(^2\) See Bems, Johnson, and Yi (2010) for elaboration of this argument.
\(^3\) See Noguera (2011) for an analysis of estimated trade elasticities in gravity models with and without intermediate goods.
\(^4\) See Isard (1951), Moses (1955), Moses (1960), or Miller (1966).
\(^5\) Reimer (2006) exposits an equivalent approach in a two country case.
\(^6\) Daudin, Rifflart, and Schweisguth study the role of vertical specialization in generating regionalization in trade patterns, while Belke and Wang focus on measuring aggregate economic openness. See also more recent work by Powers, Wang, and Wei (2009) on splitting up the value chain within Asia.
\(^8\) Bilateral or sector level ratios of value added to exports do not have this domestic content interpretation.
tion that a country’s exports (whether composed of final versus intermediate goods) are entirely absorbed in final demand abroad. That is, it rules out scenarios in which a country exports intermediates that are used to produce final goods absorbed at home. By using input-output data for source and destination countries simultaneously, we are able to relax this assumption. While this generalization results in only minor adjustments in aggregate domestic content measurements in our data, we demonstrate that relaxing this assumption is critically important for generating accurate bilateral value added flows.

Turning to our empirical results, we find that the ratio of value added to gross exports (VAX ratio) varies substantially across countries and sectors. Across sectors, we show that VAX ratios are substantially higher in Agriculture, Natural Resources, and Services than in Manufactures. This is mostly due to the fact that the manufacturing sector purchases inputs from non-manufacturing sectors, and therefore contains value added generated in those sectors. Across countries, the composition of trade drives aggregate VAX ratios, with countries that export Manufactures having lower aggregate VAX ratios. Aggregate VAX ratios do not covary strongly with income per capita, however, due to two offsetting effects. While richer countries tend to export Manufactures, which lowers their aggregate VAX ratios, they also export at higher VAX ratios within the manufacturing sector.

Moving from aggregate to bilateral data, VAX ratios differ widely across partners for individual countries. For example, U.S. exports to Canada are about 40% smaller measured in value added terms than gross terms, whereas U.S. exports to France are essentially identical in gross and value added terms. These gaps arise for two main reasons. First, bilateral (“back-and-forth”) production sharing implies that value added trade is scaled down relative to gross trade. And these scaling factors differ greatly across bilateral partners. Second, multilateral (“triangular”) production sharing gives rise to indirect trade that occurs via countries that process intermediate goods. For some country pairs, bilateral VAX ratios are larger than one, as bilateral value added exports exceeds gross exports.

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9VAX ratios within Manufactures are correlated with income because richer countries tend to export in sub-sectors with relatively high VAX ratios.
These adjustments imply that bilateral trade imbalances often differ in value added and gross terms. For example, the U.S.-China imbalance is approximately 30-40% smaller when measured on a value added basis, while the U.S.-Japan imbalance is approximately 33% higher. These adjustments point to the importance of triangular production chains within Asia.

To illustrate the mechanisms at work in generating these results, we present two decompositions. In the first decomposition, we show that most of the variation in bilateral value added to export ratios arises due to production sharing, not variation in the composition of goods exported to different destinations. The second decomposition splits bilateral exports according to whether they are absorbed in the destination, embedded as intermediates in goods that are reflected back to the source country, or redirected to third countries embedded as intermediates in goods ultimately consumed there. Variation in the degree of absorption, reflection, and redirection across partners is an important driver of variation in bilateral value added to export ratios.

The rest of the paper is structured as follows. Section 1 presents the general accounting framework, defines our value added trade measures, and discusses the interpretation of value added to export ratios. Section 2 describes the data sources and assumptions we use to implement the accounting exercise. Section 3 presents our empirical results and Section 4 concludes.

1 The Value Added Content of Trade

In this section, we introduce the accounting framework and demonstrate how intermediate goods trade generates differences between gross and value added trade flows. We begin the section by presenting a general formulation of the framework with many goods and countries that we use in the calculations below. To aid intuition, we then exposit several results in stripped-down versions of this general framework. Results from these simple models carry over to the general model. We close by discussing the relationship between our framework and two related lines of work on regional input-output linkages and measurement of the factor content of trade.
1.1 The Value Added Content of Trade

Assume there are \( S \) sectors and \( N \) countries. Each country produces a single differentiated tradable good within each sector, and we define the quantity of output produced in sector \( s \) of country \( i \) to be \( q_i(s) \). This good is produced by combining local factor inputs with domestic and imported intermediate goods. It is then either used to satisfy final demand (equivalently, “consumed”) or used as an intermediate input in production.

The key feature of the global input-output framework is that it tracks bilateral shipments of this output for final and intermediate use separately. To track these flows requires four dimensional notation denoting source and destination country, as well as source and destination sectors for shipments of intermediates. Let the quantity of final goods from sector \( s \) in country \( i \) absorbed in destination \( j \) be \( q_{ij}^c(s) \) and the quantity of intermediates from sector \( s \) in country \( i \) used to produce output in sector \( t \) in country \( j \) be \( q_{ij}^m(s,t) \).

The global input-output framework organizes these flows via market clearing conditions. Markets clear in quantities:

\[
q_i(s) = \sum_j q_{ij}^c(s) + \sum_j \sum_t q_{ij}^m(s,t).
\]

If we evaluate these quantity flows at a common price, say \( p_i(s) \), then we can rewrite the market clearing condition in value terms as:

\[
y_i(s) = \sum_j c_{ij}(s) + \sum_j \sum_t m_{ij}(s,t), \tag{1}
\]

where \( y_i(s) \equiv p_i(s)q_i(s) \), \( c_{ij}(s) \equiv p_i(s)q_{ij}^c(s) \), and \( m_{ij}(s,t) \equiv p_i(s)q_{ij}^m(s,t) \) are the value of production, final demand, and intermediate goods shipments. Gross bilateral exports, denoted \( x_{ij}(s) \), include goods destined for both final and intermediate use abroad: \( x_{ij}(s) = c_{ij}(s) + \sum_t m_{ij}(s,t) \).

Then (1) equivalently says that output is divided between domestic final use, domestic intermediate use, and gross exports.

To express market clearing conditions for many countries and sectors in a compact form, we define a series of matrices and vectors. Collect the total value of production in each sector in the \( S \times 1 \) vector \( y_i \) and allocate this output to final and intermediate use. Denote country \( i \)'s final demand for its own goods by \( S \times 1 \) vector \( c_{ii} \) and shipments of final goods from \( i \) to country \( j \) by
the $S \times 1$ vector $c_j$. Further, denote use of intermediate inputs from $i$ by country $j$ by $A_{ij}y_j$, where $A_{ij}$ is an $S \times S$ input-output matrix with elements $A_{ij}(s,t) = m_{ij}(s,t)/y_j(t)$. A typical element describes, for example, the value of steel ($s = \text{steel}$) imported by Canada ($j = \text{Canada}$) from the U.S. ($i = \text{U.S.}$) used in the production of automobiles ($t = \text{autos}$) as a share of total output of autos in Canada. Gross exports from $i$ to $j$ ($i \neq j$) are then $x_{ij} = c_{ij} + A_{ij}y_j$.

With this notation in hand, we collect information on intermediate goods sourcing and final goods flows in vector/matrix form:

$$A \equiv \begin{pmatrix}
A_{11} & A_{12} & \cdots & A_{1N} \\
A_{21} & A_{22} & \cdots & A_{2N} \\
\vdots & \vdots & \ddots & \vdots \\
A_{N1} & A_{N2} & \cdots & A_{NN}
\end{pmatrix}, \quad y \equiv \begin{pmatrix}
y_1 \\
y_2 \\
\vdots \\
y_N
\end{pmatrix}, \quad c_j \equiv \begin{pmatrix}
c_{1j} \\
c_{2j} \\
\vdots \\
c_{Nj}
\end{pmatrix}.$$ 

Then, we write the $S \times N$ goods market clearing conditions as:

$$y = Ay + \sum_j c_j. \tag{2}$$

This is the classic representation of an input-output system, where total output is split between intermediate and final use. Whereas a typical input-output system focuses on sectoral linkages within a single economy, this system is expanded to trace intermediate goods linkages across countries and sectors. We therefore refer to $A$ as the global bilateral input-output matrix.

Using this system, we can write output as:

$$y = \sum_j (I - A)^{-1}c_j. \tag{3}$$

To interpret this expression, $(I - A)^{-1}$ is the “Leontief inverse” of the input-output matrix. The Leontief inverse can be expressed as a geometric series: $(I - A)^{-1} = \sum_{k=0}^{\infty} A^k$. Multiplying by the final demand vector, the zero-order term $c_j$ is the direct output absorbed as final goods, the first-order term $[I + A]c_j$ is the direct output absorbed plus the intermediates used to produce that
output, the second-order term $[I + A + A^2]c_j$ includes the additional intermediates used to produce the first round of intermediates ($Ac_j$), and the sequence continues as such. Therefore, $(I - A)^{-1}c_j$ is the vector of output used both directly and indirectly to produce final goods absorbed in country $j$.

Equation (3) thus decomposes output from each source country $i$ into the amount of output from the source used to produce final goods absorbed in country $j$. To make this explicit, we define:

$$\begin{pmatrix}
y_{1j} \\
y_{2j} \\
\vdots \\
y_{Nj}
\end{pmatrix} \equiv (I - A)^{-1}c_j,$$

where $y_{ij}$ is the $S \times 1$ vector of output from $i$ used to produce final goods absorbed in $j$.

These output transfers are conceptually distinct from gross exports. Gross exports $x_{ij}(s)$ are directly observed as a bilateral shipment from sector $s$ in country $i$ to country $j$. In contrast, bilateral output transfers are not directly observed, but rather constructed using information on the global input requirements for final goods absorbed in each country. Importantly, as inputs from a particular country and sector travel through the production chain, they may be embodied in final goods of any sector or country. For example, inputs exported from country $i$ to country $j$ may be embedded in country $j$ final goods that are absorbed in a third country $k$, or inputs produced by sector $s$ may be embodied in final goods from sector $t$. These possibilities give rise to important differences in the structure of bilateral output transfers versus bilateral trade.

To calculate the value added associated with these implicit output transfers, define the ratio of value added to output for each sector within country $i$, as $r_i(t) = 1 - \sum_j \sum_s A_{ji}(s, t)$. This value added ratio, expressed here as one minus the share of domestic plus imported intermediates in total output, is equal to payments to domestic factors as a share of gross output. Put differently, this is ratio of GDP to gross output at the sector level.

With this notation in hand, we can now define value added exports and the value added to
export ratio, “VAX ratio”, as a measure of the value added content of trade.

**Definition 1 (Value Added Exports).** The total value added produced in sector $s$ in source country $i$ and absorbed in destination country $j$ is $va_{ij}(s) = r_i(s)y_{ij}(s)$. Total value added produced in $i$ and absorbed in $j$ is then $va_{ij} = \sum_s va_{ij}(s)$.

**Definition 2 (VAX Ratio).** The sector-level bilateral value added to export ratio is given by $va_{ij}(s)/x_{ij}(s)$. The aggregate bilateral value added to export ratio is $va_{ij}/\iota x_{ij}$, where $\iota$ is a $1 \times S$ vector of ones.

### 1.2 Discussion

We turn to special cases to interpret value added trade flows and the value added content of trade. We use a two country model to develop intuition for the value added content of trade calculations and link our analysis to previous work on the domestic content of exports (equivalently, vertical specialization) by Hummels, Ishii, and Yi (2001). We then use a stylized three country model to demonstrate how the framework tracks value added through the multi-country production chain, even if that value added travels to its final destination via third countries. We also discuss the interpretation of VAX ratios in multi-sector models. We conclude by setting our framework in context of related literature on regional input-output linkages and the measurement of the factor content of trade.

#### 1.2.1 Two Countries, One Sector Per Country

Suppose that there are now only two countries, and each country produces a single differentiated aggregate good. Then the analog to the output decomposition (3) is:

$$
\begin{pmatrix}
  y_1 \\
  y_2
\end{pmatrix} = \left[ I - \begin{pmatrix}
  \alpha_{11} & \alpha_{12} \\
  \alpha_{21} & \alpha_{22}
\end{pmatrix}\right]^{-1} \begin{pmatrix}
  c_{11} \\
  c_{21}
\end{pmatrix} + \left[ I - \begin{pmatrix}
  \alpha_{11} & \alpha_{12} \\
  \alpha_{21} & \alpha_{22}
\end{pmatrix}\right]^{-1} \begin{pmatrix}
  c_{12} \\
  c_{22}
\end{pmatrix}.
$$

(5)

This system describes how the gross output of each country is embodied in final consumption.
in each of the two countries. To unpack this result, we solve for the breakdown of country 1’s production:

\[ y_1 = y_{11} + y_{12} \]

with \( y_{11} = M_1 \left( c_{11} + \frac{\alpha_{12}}{1 - \alpha_{22}} c_{21} \right) \) and \( y_{12} = M_1 \left( \frac{\alpha_{12}}{1 - \alpha_{22}} c_{22} + c_{12} \right) \), \( \text{(6)} \)

where \( M_1 \equiv \left( 1 - \alpha_{11} - \frac{\alpha_{12} \alpha_{21}}{1 - \alpha_{22}} \right)^{-1} \geq 1 \) is an intermediate goods multiplier that describes the total amount of gross output from country 1 required to produce one unit of country 1’s net output. \( \text{[10]} \)

The first term \( (y_{11}) \) is the total amount of country 1’s output that is required to produce final goods absorbed in country 1. This term includes both output dedicated to satisfy country 1’s demand for its own final goods \( (M_1 c_{11}) \), as well as output needed to satisfy country 1’s demand for country 2 final goods \( (M_1 \frac{\alpha_{12}}{1 - \alpha_{22}} c_{21}) \). The second term \( (y_{12}) \) has a similar interpretation in terms of country 2’s demand. Because (6) geographically decomposes country 1’s output, we can translate this into a decomposition of value added: \( v a_i = v a_{i1} + v a_{i2} \), where \( v a_{ij} = \left[ 1 - \alpha_{11} - \alpha_{21} \right] y_{ij} \) is value added generated by country \( i \) that is absorbed in country \( j \).

There are four output concepts underlying flows from country 1 to country 2: (1) final goods \( c_{12} \), (2) gross exports \( x_{12} \), (3) implicit output transfers \( y_{12} \), and (4) value added exports \( v a_{12} \). We pause here to clarify the relationship between them. To begin, note that \( x_{12} = c_{12} + \alpha_{12} y_2 \), so \( c_{12} \leq x_{12} \) when there are exported intermediates. Further, using the output decomposition for country 2 \( (y_2 = y_{22} + y_{21}) \), we decompose gross exports as: \( x_{12} = \alpha_{12} y_{21} + (c_{12} + \alpha_{12} y_{22}) \).

Multiplying both sides of the expression by \( (1 - \alpha_{11})^{-1} \) then translates exports into the gross output required to produce them. \( \text{[13]} \) It is straightforward to show that \( y_{12} = (1 - \alpha_{11})^{-1} (c_{12} + \alpha_{12} y_{22}) \).

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\( \text{[10]} \) This multiplier is greater than one because output is “used up” in the production process. Without exported intermediates \( (\alpha_{12} = 0) \), this multiplier would be \( (1 - \alpha_{11})^{-1} \). The additional term reflects the fact that intermediate goods sourced from country 2 contain output produced by country 1.

\( \text{[11]} \) To export final goods \( c_{21} \) requires producing \( (1 - \alpha_{22})^{-1} c_{21} \) units of country 2 output, which itself requires \( \alpha_{12} (1 - \alpha_{22})^{-1} c_{21} \) units of country 1’s output as intermediates. To produce this country 1 output requires \( M_1 \) times \( \alpha_{12} (1 - \alpha_{22})^{-1} c_{21} \) units of country 1’s output overall, because some output is used up in the production process.

\( \text{[12]} \) To highlight how the output decomposition depends on cross-border intermediate linkages, note that if \( \alpha_{12} = 0 \) the output decomposition would be: \( y_{11} = (1 - \alpha_{11})^{-1} c_{11} \) and \( y_{12} = (1 - \alpha_{11})^{-1} c_{12} \). In this counter-factual case, output of country 1 is only used to produce final goods originating in country 1.

\( \text{[13]} \) This follows from manipulation of the market clearing condition for country 1: \( y_1 = (1 - \alpha_{11})^{-1} (c_{11} + x_{12}) \).
Therefore, \( y_{12} = (1 - \alpha_{11})^{-1}x_{12} - (1 - \alpha_{11})^{-1}\alpha_{12}y_{21} \). So the implicit output transferred from country 1 to country 2 is equal to the gross output required to produce exports minus the gross output that is reflected back embedded in country 2 goods that are absorbed by country 1.\(^{14}\) Finally, we note that \( va_{12} \leq y_{12} \), because the value added to output ratio is bounded above by one.

To directly compare value added exports to gross exports, we compute the VAX ratio:

\[
\frac{va_{12}}{x_{12}} = \frac{(1 - \alpha_{11} - \alpha_{21})y_{12}}{x_{12}} = \frac{1 - \alpha_{11} - \alpha_{21}}{1 - \alpha_{11}} \left( \frac{x_{12} - \alpha_{12}y_{21}}{x_{12}} \right),
\]

where the second line follows from the discussion in the previous paragraph. The difference \( x_{12} - \alpha_{12}y_{21} \) is exports less reflected intermediates, or equivalently the portion of exports genuinely consumed abroad. The VAX ratio will always be less than one, so value added exports are scaled down relative to gross exports.

The VAX ratio for a country can be thought of as a metric of the “domestic content of exports.” Indeed, it is closely related to previous approaches to measuring domestic content in the literature. To see this, note that the VAX ratio has two components. The first component, \( \frac{1 - \alpha_{11} - \alpha_{21}}{1 - \alpha_{11}} \), is equivalent to a metric of domestic content developed in Hummels, Ishii, and Yi (2001).\(^{15}\) This metric captures the value added associated with the gross output needed to produce exports as a fraction of total exports. The Hummels-Ishii-Yi metric is equal to the VAX ratio only when country 2 does not use imported intermediates (\( \alpha_{12} = 0 \)), and therefore country 1 exports final goods alone.\(^{16}\) In contrast, with two-way trade in intermediates the Hummels-Ishii-Yi metric overstates the amount of domestic value added that is generated per unit of exports.\(^{17}\) The second component of the

\(^{14}\)Note that if \( \alpha_{12} = 0 \), then \( y_{12} = (1 - \alpha_{11})^{-1}x_{12} \), so the gross output required to produce exports equals the actual amount of output transferred from country 1 to country 2.

\(^{15}\)Hummels et al. focus their discussion on measuring vertical specialization or the “import content of exports,” which is given by \( \alpha_{21}(1 - \alpha_{11})^{-1} \). Domestic content is then one minus the import content of exports. Though we discuss these concepts here in a scalar case, they generalize in a straightforward way to models with many sectors.

\(^{16}\)The condition \( \alpha_{12} = 0 \) is necessary and sufficient for equality between the two metrics when there is one aggregate sector, except in pathological cases. With more than one sector, restricting country 1 to export only final goods \( (\alpha_{12}(s, t) = 0 \forall s, t) \) is sufficient, but not necessary.

\(^{17}\)Footnote 18 in Trefler and Zhu (2010) provide a related discussion of how the factor content of trade differs depending on whether one assumes intermediates are traded or not.
VAX ratio allows some exports to be dedicated to producing goods that are ultimately consumed at home. That is, it allows for a portion of exports to be reflected back to the source rather than absorbed abroad.

1.2.2 Three Countries, One Sector Per Country

While the two country framework illustrates the basic discrepancy between value added and gross trade flows, additional insights emerge as one introduces a third country to the mix. We focus on a special, algebraically straightforward case that illustrates how the accounting framework tracks the final destination at which value added by a given country is consumed even if this value circulates through a multi-country production chain en route to its final destination. We construct the special case to approximate a stylized account of production chains between the U.S. and Asia.[18]

Let country 1 be the U.S., country 2 be China, and country 3 be Japan. Further, assume that China imports intermediates from the U.S. and Japan and exports only final consumption goods only to the U.S. For simplicity, we assume that the U.S. and Japan do not export any final goods and only export intermediates to China. This configuration of production can be represented as:

\[
\begin{pmatrix}
  y_1 \\
  y_2 \\
  y_3
\end{pmatrix}
= \begin{pmatrix}
  \alpha_{11} & \alpha_{12} & 0 \\
  0 & \alpha_{22} & 0 \\
  0 & \alpha_{32} & \alpha_{33}
\end{pmatrix}
\begin{pmatrix}
  y_1 \\
  y_2 \\
  y_3
\end{pmatrix}
+ \begin{pmatrix}
  c_{11} \\
  c_{22} + c_{21} \\
  c_{33}
\end{pmatrix}.
\]

(8)

[18] This example was inspired by Linden, Kraemer, and Dedrick (2007), who trace the iPod production chain. The iPod combines U.S. intellectual property from Apple with a Japanese display and disk drive, which is manufactured in China. These components are assembled in China and the iPod is shipped to the U.S.
This then can be solved to yield the following three-equation system:

\[
\begin{align*}
y_1 &= \frac{1}{1 - \alpha_{11}} c_{11} + \frac{\alpha_{12}}{(1 - \alpha_{11})(1 - \alpha_{22})} c_{21} + \frac{\alpha_{12}}{(1 - \alpha_{11})(1 - \alpha_{22})} c_{22} \\
y_2 &= \frac{1}{1 - \alpha_{22}} c_{21} + \frac{1}{1 - \alpha_{22}} c_{22} \\
y_3 &= \frac{\alpha_{32}}{(1 - \alpha_{33})(1 - \alpha_{22})} c_{21} + \frac{\alpha_{32}}{(1 - \alpha_{33})(1 - \alpha_{22})} c_{22} + \frac{1}{1 - \alpha_{33}} c_{33}.
\end{align*}
\]

This system provides the implicit output transfers needed to calculate value added flows.

Two points are interesting to note. First, as in the two-country case above, U.S. demand for U.S. output has both a direct component \(\frac{1}{(1 - \alpha_{11})} c_{11}\), and an indirect component \(\frac{\alpha_{12}}{(1 - \alpha_{11})(1 - \alpha_{22})} c_{21}\) that accounts for the fact that U.S. imports of final goods from China include embedded U.S. content. Thus, a larger share of U.S. output is ultimately absorbed at home than bilateral trade statistics would indicate. Correspondingly, Chinese bilateral exports overstate the true Chinese content shipped to the U.S. due to bilateral U.S.-China production sharing.

The second point is that, although Japan does not export directly to the U.S., the U.S. does import Japanese content embedded in Chinese exports to the U.S. This effect is the result of multi-country production chains, and was absent in the two country case analyzed above. In the equation for Japan (country 3), this effect appears as \(\frac{\alpha_{32}}{(1 - \alpha_{33})(1 - \alpha_{22})} c_{21}\).

Because Chinese exports to the U.S. contain both U.S. and Japanese content, the bilateral VAX ratio of China-U.S. trade is:

\[
\frac{va_{21}}{x_{21}} = 1 - \left(\frac{va_{31} + \alpha_{12}y_{21}}{x_{21}}\right) < 1.
\]

This illustrates that the bilateral VAX ratio removes both the Japanese value added \((va_{31})\) and U.S. intermediate goods \((\alpha_{12}y_{21})\) from Chinese exports to the U.S.

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\(\text{\^[19\]}\) U.S. imports from China contain U.S. content because the U.S. exports intermediates to China and imports final goods from China. Thus, U.S. intermediates are reflected back to the US and constitute a portion of the value added.
value added exports to the U.S. and zero direct bilateral exports. Therefore, the bilateral VAX ratio for Japan-U.S. trade is undefined, or practically infinite for small bilateral exports. This extreme ratio illustrates another general lesson. Though the aggregate VAX ratio is bounded by one for each country, bilateral VAX ratios may be greater than one when an exporter sends intermediates abroad to be processed and delivered to a third country. Thus, bilateral VAX ratios pick up the influence of both bilateral and multilateral production sharing relationships.

When bilateral VAX ratios vary across partners, bilateral value added balances do not equal bilateral trade imbalances. To illustrate this, we define $tb_{12} \equiv x_{12} - x_{21}$ and $vab_{12} \equiv va_{12} - va_{21}$ to be bilateral U.S.-China trade and value added balances. In this special case, where the configuration of production is given by (8), these balances are related as follows:

$$tb_{12} + \alpha_{32} y_{21} = vab_{12}. \quad (11)$$

That is, $tb_{12} < vab_{12}$. So assuming the U.S. runs a trade deficit with China in this example, then it will run a smaller deficit with China in value added terms due to the fact that Chinese bilateral trade contains Japanese content ($\alpha_{32} y_{21}$). As a corollary, the U.S.’s bilateral balance with Japan will be distorted in the opposite direction.

To generalize this result, we can write any given bilateral value added balance as:

$$vab_{ij} = \frac{va_{ij}}{x_{ij}} x_{ij} - \frac{va_{ji}}{x_{ji}} x_{ji} = \frac{1}{2} (x_{ij} + x_{ji}) \left[ \frac{va_{ij}}{x_{ij}} - \frac{va_{ji}}{x_{ji}} \right] + \frac{1}{2} \left( \frac{va_{ij}}{x_{ij}} + \frac{va_{ji}}{x_{ji}} \right) [x_{ij} - x_{ji}] . \quad (12)$$

The first term adjusts the value added balance due to differences in VAX ratios between exports and imports. When the VAX ratio for exports is high relative to imports, the value added balance is naturally pushed in a positive direction. Note here that this is true even if gross trade is balanced. The second term adjusts the value added balance based on the average level of VAX ratios. Starting from an initial imbalance, the value added balance is scaled up or down relative to the trade balance, that the U.S. purchases from itself.
depending on whether VAX ratios are greater than or less than one (on average). So differences in VAX ratios between partners within a bilateral relationship and the absolute level of the VAX ratios between partners both influence the size of the adjustment in converting gross imbalances to value added terms.

1.2.3 Two Countries, Many Sectors

The interpretation of aggregate value added exports and VAX ratios developed in the one-sector examples in previous sections carries over to the many country, multi-sector framework. One important distinction between the one-sector and multi-sector frameworks is that VAX ratio at the sector level cannot be interpreted as the domestic content of exports. To explain its interpretation, we turn to an example with two countries and many sectors.\(^{20}\)

With two countries \((i, j = \{1, 2\})\) and many sectors, the VAX ratio for sector \(s\) in country 1 can be written as:

\[
\frac{v_{12}(s)}{x_{12}(s)} = \frac{r_1(s) y_{12}(s)}{x_{12}(s)}.
\]

Then the sectoral VAX ratio depends on the value added to output ratio within a given sector \((r_1(s))\) and the ratio of gross output produced in a sector that is absorbed abroad \((y_{12}(s))\) to gross exports from that sector \((x_{12}(s))\). The role of the value added to output ratio is straightforward: all else equal, sectors with low value added to output ratios (e.g., manufacturing) will have low VAX ratios relative to other sectors.

The role of differences in \(y_{12}(s)\) versus \(x_{12}(s)\) across sectors is more subtle. To sort this out, we note that we can link \(y_{12}\) and the export vector \(x_{12}\) as in Section 1.2.1. Specifically, \(x_{12} = (I - A_{11}) y_{12} + A_{12} y_{21}\). Rearranging this expression yields:

\[
y_{12} = (I - A_{11})^{-1} [x_{12} - A_{12} y_{21}].
\]

This is the many sector, matrix analog to computations embedded in Equation 7 wherein \(y_{12}\) is the gross output needed to produce exports less reflected intermediates. This decomposition points to two ways in which \(y_{12}\) could differ from \(x_{12}\).

First, suppose that \(A_{12} y_{21}\) is a vector of zeros, so that exports are 100% absorbed abroad.\(^{21}\)

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\(^{20}\)The many country version of the framework can always be collapsed to an equivalent two country framework, in which input-output linkages among countries in the rest of the world are subsumed into the “domestic” input-output structure of the rest-of-the-world composite.

\(^{21}\)If \(A_{12}\) is matrix of zeros, so that country 1 exports only final goods, this obviously holds. This can also hold for cases in which elements of \(A_{12}\) are positive, so long as the corresponding elements \(y_{21}\) are zero. For example, country 1 could export intermediates to country 2, so long as the sector purchasing those intermediates only produces output.
This implies: \( y_{12} = (I - A_{11})^{-1} x_{12} \). All that remains here separating exports and gross output for individual sectors is the domestic input-output structure. Generically, \( y_{12}(s) \neq x_{12} \), so variation in this ratio across sectors influences sector-level value added.

One important implication of this is that the sectoral VAX ratio captures information on how individual sectors engage in trade. For example, consider a situation in which producers in one sector sell intermediates to purchasers in another sector, who in turn produce goods for export. In this case, the intermediate goods suppliers engage in trade indirectly. Hence, we observe no direct exports from the intermediate goods supplier, but do observe value added exports because value added from that sector is embedded in the purchaser’s goods. Thus, value added exports from a particular sector may be physically embodied in goods exported from that sector or embodied in exports of other sectors. High ratios of value added exports to gross trade (possibly above one) at the sector level are evidence of indirect participation in trade. Low ratios instead indicate that a given sector’s gross exports embody value added produced outside that sector.

Second, suppose now that \( A_{12} \) is not composed of zeros, but rather that country 1 exports intermediates to country that are used to produce goods that are absorbed in country 1, captured by the term \( A_{12} y_{21} > 0 \). In this case, the sectoral VAX ratio is influenced by how individual sectors fit into cross-border production chains. For example, if we shut down all domestic input-output linkages, setting \( A_{11} = 0 \), then \( y_{12} = x_{12} - A_{12} y_{21} \). Then the sectoral VAX ratio depends on the sector’s connection to foreign production chains. Specifically, the VAX ratio will be depend on what share of output is absorbed abroad versus used to produce foreign goods that are ultimately absorbed at home. If exports are largely absorbed abroad (i.e., \( y_{12}(s)/x_{12}(s) \approx 1 \)), one would see a relatively high VAX ratio.

Though these influences are difficult to separate empirically in general cases, we discuss evidence below that sheds light on the relative importance of these channels.

\textsuperscript{22}For example, the “raw milk” sector in our data has near zero exports, but raw milk is sold to the “dairy products” sector, which does export. With two sectors, where 1 is the dairy products and 2 is the milk sector, this could be represented as an \( A_{11} \) matrix with one non-zero element \( a_{11}(2, 1) \) and export vector with \( x_{12}(1) > 0 \) and \( x_{12}(2) = 0 \). This structure implies \( y_{12}(1)/x_{12}(1) = 1 \) and \( y_{12}(2)/x_{12}(2) = \infty \)
1.2.4 Regional Input-Output Models and the Factor Content of Trade

The framework above is intimately related to two strands of literature in regional science and trade.

First, we draw on an extensive literature on regional input-output models. These models, outlined in seminal work by Isard (1951), Moses (1955), Moses (1960), and Miller (1966), provide frameworks for analyzing linkages across regions within countries that can be extended across borders (as above). Among this literature, Moses (1955) is the closest antecedent, as he uses proportionality assumptions to allocate inputs purchased from other regions, as we do, to build a multi-region model of the U.S. One shortcoming of this line of work is that it typically assumes that the regional system is ‘open’ vis-à-vis the rest-of-the-world, in the sense that shipments to regions not included in the model are entirely absorbed there. This assumption is a multi-region analog of the assumptions under which the Hummels-Ishii-Yi (2001) domestic content calculation is equal to the value added content of trade.

Second, the value added framework above shares a common structure with a recent parallel literature on measuring the factor content of trade. Reimer (2006) and Trefler and Zhu (2010) both outline procedures to compute the net factor content of trade when inputs are traded, and use these factor content measures to study the Vanek prediction. To draw out the similarities, note that one can think of computing both factor contents and value added contents using a two step procedure. First, one needs to compute the output transfers, specified above, that indicate how much output from each source country and sector are absorbed in final demand in a given destination. Second, one needs to use source country information on either factor contents (e.g., quantities of factors used to produce one dollar of output) or value added to output ratios to compute the factors or value added that is implicitly being traded.

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23 Isard (1951) suggests this technique as well, but does not pursue an empirical application himself.
24 Powers, Wang, and Wei (2009) work with a model of this type for Asia.
25 Let us trace out the calculation explicitly. Trefler and Zhu define $T_i$ to be a $(NS \times 1)$ vector of trade flows arranged as follows: $T_i = [\cdots, x_i'_{-1,1}, x_i'_{1,1}, x_i'_{i+1,1}, \cdots]'$, where $x_i = \sum_{j \neq i} x_{ij}$ is a $(S \times 1)$ vector of total exports from country $i$ to the rest of the world and $x_{j,i}$ is a $(S \times 1)$ vector of bilateral trade flows from $j \neq i$ to $i$. Further, they define $B$ to be a $F \times SN$ matrix of factor requirements for each good: $B = [B_1, \cdots, B_i, \cdots, B_N]$, where $B_i$ is the $F \times S$ matrix of factor requirements for country $i$, with $F$ denoting the number of factors. The factor content of trade for country $i$ is then: $B(I - A)^{-1}T_i$. To link this to our framework, we note that the calculation $(I - A)^{-1}T_i$ returns a vector of (signed) output transfers. In particular, $(I - A)^{-1}T_i = [\cdots, -y_{i-1,i}, y_{x,i}, -y_{i+1,i}, \cdots]'$, where
Despite this similarity in the underlying structure of value added and factor content calculations, we emphasize that there are important conceptual differences between factor contents and value added. For one, the theoretical driving forces of trade in value added may be very different than trade in factors. Costinot, Vogel, and Wang (2011) point out that differences in absolute endowments across countries influence where countries are located in the value chain, so absolute (as opposed to relative) factor endowments are a source of comparative advantage underlying trade in value added.\footnote{This is just one example of a general point: the empirical shift from factor content to value added content embodies a deeper conceptual shift in how we think about trade.}

2 Data

Our data source is the GTAP 7.1 Data Base assembled by the Global Trade Analysis Project at Purdue University. This data is compiled based on three main sources: (1) World Bank and IMF macroeconomic and Balance of Payments statistics; (2) United Nations Commodity Trade Statistics (Comtrade) Database; and (3) input-output tables based on national statistical sources. To reconcile data from these different sources, GTAP researchers adjust the input-output tables to be consistent with international data sources.\footnote{See the GTAP website at http://www.gtap.agecon.purdue.edu/ for documentation of the source data. Since raw input-output tables are based on national statistical sources, they inherit all the shortcomings of those sources. For example, import tables are often constructed using a “proportionality” assumption whereby the imported input table is assumed to be proportional to the overall aggregate input-output table.}

The GTAP data includes bilateral trade statistics and input-output tables for 94 countries plus 19 composite regions covering 57 sectors in 2004.\footnote{GTAP assigns composite regions “representative” input-output tables, constructed from input-output tables of similar countries. Composite regions do not play an important role in our results, accounting for 5% of world trade and 3% of world value added. To measure bilateral services trade, GTAP uses OECD data where available and imputes bilateral services trade elsewhere. Because services account for less than 18% of exports for the median country, our results are likely to be insensitive to moderate mismeasurement of services trade.}

Regarding sector definitions, there are 18 Agriculture and Natural Resources sectors, 24 Manufacturing sectors, 16 Mining and Energy sectors, and 19 Services sectors. The GTAP data includes bilateral trade statistics and input-output tables for 94 countries plus 19 composite regions covering 57 sectors in 2004.\footnote{Like absolute endowments, absolute productivity differences are also a source of comparative advantage in the Costinot, Vogel, and Wang model. See Johnson (2008) for an extended discussion of these calculations.}

\[ y_{xi} = \sum_{j \neq i} y_{ij} \] is total output produced in country \( i \) that is absorbed abroad and \( y_{ji} \) is output produced in country \( j \neq i \) that is absorbed in country \( i \). Thus, as suggested above, one can think of first of computing output transfers embedded in trade flows, and then computing the factor requirements needed to produce those output transfers. See Johnson (2008) for an extended discussion of these calculations.
tures sectors, and 15 Services sectors.

In the data, we have information on 6 objects for each country:

1. $y_i$ is a $57 \times 1$ vector of total gross production.

2. $c_{Di}$ is a $57 \times 1$ vector of domestic final demand.

3. $c_{Ii}$ is a $57 \times 1$ vector of domestic final import demand.

4. $A_{ii}$ is a $57 \times 57$ domestic input-output matrix, with elements $A_{ii}(s, t)$.

5. $A_{Ii}$ is a $57 \times 57$ import input-output matrix, with elements $A_{Ii}(s, t) = \sum_{j \neq i} A_{ji}(s, t)$.

6. $\{x_{ij}\}$ is a collection of $57 \times 1$ bilateral export vectors for exports from $i$ to $j$.

The definition of “final demand” is based on the national accounts, including consumption, investment, and government purchases. We value each country’s output at a single set of prices, regardless of where that output is shipped or how it is used. This ensures that the value of production revenue equals expenditure. Following input-output conventions, we use “basic prices,” defined as price received by a producer (minus tax payable or plus subsidy receivable by the producer).

Note that we do not directly observe the bilateral input-output matrices $A_{ji}$ and final demand vectors $c_{ji}$ that are needed to assemble the global input-output matrix. Rather, we need to allocate total imported intermediate use $A_{Ii}$ and imported final demand $c_{Ii}$ to individual country sources. To do so, we use bilateral trade data and a proportionality assumption. Specifically, we assume that within each sector imports from each source country are split between final and intermediate in proportion to the overall split of imports between final and intermediate use in the destination. Further, conditional on being allocated to intermediate use, we assume that imported intermediates

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29Put differently, while quantity choices may reflect price differences across destinations or uses that arise due to transport costs, tariffs, and markups, we value the resulting quantity flows at a single set of prices.

30In our framework, the level of value added differs from the one used in national accounts. We calculate value added as output at basic prices minus intermediates at basic prices, whereas the national accounts calculate value added as output at basic prices minus intermediates at purchaser’s prices.
from each source are split across purchasing sectors in proportion to overall imported intermediate use in the destination.

Formally, for goods from sector $s$ used by sector $t$, we define bilateral input-output matrices and consumption import vectors:

$$A_{ji}(s, t) = A_{Ii}(s, t) \left( \frac{x_{ji}(s)}{\sum_j x_{ji}(s)} \right)$$

and

$$c_{ji}(s) = c_{Ii}(s) \left( \frac{x_{ji}(s)}{\sum_j x_{ji}(s)} \right).$$

These assumptions imply that all variation in total bilateral intermediate and final goods flows arises due to variation in the composition of imports across partners. For example, we would find that US imports from Canada are intermediate goods intensive because most imports from Canada are goods that are on average used as intermediates (e.g., auto parts).

The proportionality assumptions above are the standard approach to dealing with the fact that data on $A_{ji}$ and $c_{ji}$ are not collected in national accounts. Initially adopted in early work on regional input-output accounts by Moses (1955), they have also been used by Belke and Wang (2006), Daudin, Rifflart, and Schweisguth (forthcoming), and Trefler and Zhu (2010) to construct global input-output tables as in this paper. Several recent papers have explored the consequences of relaxing some proportionality assumptions using alternative data sources, and appear to find that relaxing these assumptions has small effects on aggregate VAX ratios or factor contents.

In the main calculation, we also assume that production techniques and input requirements are the same for exports and domestically absorbed final goods. This assumption is problematic for countries that have large export processing sectors. These processing sectors (almost by definition)

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31Proportionality assumptions are so common in input-output accounting that many countries, including the U.S., even construct the import matrix ($A_{Ii}$) itself using a proportionality assumption in which imported inputs are allocated across sectors in the same proportion as total input use (aggregating over imported and domestic inputs). Some countries augment this data with direct surveys of input use in constructing imported input use tables. However, no countries (to our knowledge) directly collect information on bilateral sources of inputs used in particular sectors.

32Puzzello (2010) compares factor content calculations with and without the proportionality assumption using IDE-JETRO regional input-output tables for Asia. Koopman, Powers, Wang, and Wei (2010) compute value added content using disaggregate data classified under the BEC system to estimate bilateral intermediate goods flows. While relaxing proportionality seems to have small aggregate consequences, it may simultaneously have large effects on value added trade at the sector level. This remains to be explored.
produce distinct goods for foreign markets with different input requirements and lower value added to output ratios than the rest of the economy. Ignoring this fact tends to overstate the value added content of exports.

As an alternative calculation, we relax this assumption for China and Mexico, two prominent countries with large export processing sectors (roughly two thirds of exported Manufactures originates in these sectors) and key trading partners with the U.S. We present supplementary calculations below that adjust the value added content of exports using an adaptation of a procedure from Koopman, Wang, and Wei (2008). The basic idea is to measure the share of exports and imports that flow through the export processing sector, and then impute separate input-output coefficients for the processing sector so as to be consistent with these flows. Details of the procedure are presented in Appendix A. We then compute the value added content of trade using a new input-output system that includes these amended tables.

3 Empirical Results

3.1 Multilateral Value Added Exports

Table [1] reports aggregate VAX ratios for each country, grouped by region. Across countries, value added exports represent about 73% of gross exports. The magnitude of the adjustment varies both across and within regions. At the regional level, VAX ratios are lowest for Europe (broadly defined) and East Asia, and higher in the Americas, South Asia and Oceania, and the Middle East and Africa. Looking within regions, the new E.U. members (e.g., Estonia, Hungary, Slovakia, and the Czech Republic) stand out as having low VAX ratios in Central-Eastern Europe, while Japan stands out with a high VAX ratio relative to East Asia.

33 For Mexico, we classify exports originating from maquiladoras as processing exports. For China, we use estimates from Koopman, Wang, and Wei (2008) constructed from Chinese trade statistics, obtained from Zhi Wang.

34 We perform this calculation at a higher level of aggregation than our baseline calculation, with three composite sectors. We believe the results are not very sensitive to aggregation, as aggregate value added flows are nearly identical in the original, unadjusted data whether computed using 57 sectors or 3 composite sectors.

35 We omit ratios for composite regions from the table.
For China and Mexico, we report two separate calculations of the VAX ratio in the table, one computed without adjusting for processing trade and a second adjusted for processing trade.\textsuperscript{36} VAX ratios for both China and Mexico fall substantially when we adjust for export processing trade, from 0.70 to 0.59 for China and from 0.67 to 0.52 for Mexico. This brings the ratios for China and Mexico in line with other emerging markets such as South Korea or Hungary, and is evidence of the low value added to export ratios within each country’s processing sector.\textsuperscript{37}

Moving down a level of disaggregation, we report VAX ratios for three composite sectors by country in Table \[\text{1}\] as well. The three sectors are: Agriculture and Natural Resources, Manufacturing, and Services. VAX ratios are typically greater than or equal to one in the Agriculture and Natural Resources and Services sectors, and markedly less than one in Manufacturing. This cross-sector variation is primarily due differences in the manner in which each sector engages in trade, rather than differences across sectors in the degree of participation in cross-border production sharing. Further, differences in value added to output ratios across sectors are also an important source of variation.

To sort through these influences, we refer back to Section \[\text{1.2.3}\]. Recall sectoral VAX ratios would tend to be low when exports are used to produce foreign goods that are ultimately absorbed at home. If we assume that all output was absorbed abroad, then the output needed to produce exports would be: 
\[\tilde{y}_{ix} = (I-A_{ii})^{-1}\left(\sum_{j\neq i} x_{ij}\right)\], where \(\tilde{y}\) is used to signify that this is a counter-factual value and \(\tilde{y}_{ix} = \sum_{j\neq i} \tilde{y}_{ij}\). Then the counter-factual sectoral value added to export ratios would be: 
\[r_{i(s)}\tilde{y}_{ix} / x_{i(s)}\], with \(x_{i(s)} = \sum_{j\neq i} x_{ij}\). In our data, this counter-factual calculation yields ratios that are very close to the actual VAX ratios. As such, differences across sectors in the degree of foreign absorption of exports does not appear to drive the VAX ratios. Further, we note that differences in value added to output ratios also cannot explain the full variation in VAX ratios.

\textsuperscript{36} In the calculation adjusted for processing trade in China and Mexico, VAX ratios in all countries change relative to the unadjusted benchmark calculation. The absolute size of the changes in aggregate VAX ratios is very small, with a median of 0.016 and 90\% of changes less than 0.053. Therefore, we report only one set of ratios for all countries other than China and Mexico.

\textsuperscript{37} For the processing sector, we estimate that China’s VAX ratios is 0.13, while Mexico’s VAX ratio is 0.08. These ratios measure the value added produced within the processing sector as a share of processing exports. These ratios represent a lower bound on the domestic content of processing exports, since the processing sector purchases intermediates from other domestic sectors.
across sectors. In the data, the value added to output ratio in Manufactures is roughly 0.25 lower than in Agriculture and Natural Resources and Services sectors. This goes part of the way toward explaining differences in VAX ratios across sectors, but falls substantially short.

The remaining driver of variation in VAX ratios across sectors is cross-sector variation in the extent to which sectoral output is directly exported versus indirectly exported, embodied in other sectors’ goods that are then exported. Recall that we observe gross exports from a given sector \( i.e., \sum_j x_{ij}(s) > 0 \) only if output from that sector crosses an international border with no further processing. With this in mind, it is obvious that sector-level VAX ratios are greater than one when a sector exports value added embodied in another sector’s gross output and exports. In the data, it appears that Manufactures, which are directly exported, embody substantial value added from the other sectors. One implication of this fact is that the composition of aggregate value added flows differs from that of gross trade. Figure 1 summarizes this fact by plotting the share of Manufactures and Services in both types of trade for the 10 largest exporters. The role of Manufactures in value added trade is diminished, while Services is increased by a roughly equivalent amount. The upshot is that Services are far more exposed to international commerce than one would think based on gross trade statistics.

To organize the cross-country variation in the data, we construct a “between-within” decomposition of the aggregate VAX ratio. The decomposition is constructed relative to a reference country as follows:

\[
VAX_i - \overline{VAX} = \sum_s \left[ VAX_i(s) - \overline{VAX}(s) \right] \frac{\omega_i(s) + \overline{\omega}(s)}{2} + \sum_s \left[ \omega_i(s) - \overline{\omega}(s) \right] \frac{VAX_i(s) + \overline{VAX}(s)}{2}, \tag{13}
\]

where \( s \) denotes sector, \( i \) denotes country, and \( \omega(s) \) and \( VAX(s) \) are the export share and VAX

\[38\] Agriculture and Natural Resources constitutes a roughly equal share of value added and gross trade.
ratio in sector $s$. Bars denote reference country variables, which are constructed based on global composites. In this decomposition, the Within Term varies primarily due to differences in VAX ratios within sectors across countries, while the Between Term is influenced mainly by differences in the sector composition of trade. To isolate compositional shifts between Manufactures and non-Manufactures, we calculate the decomposition using two composite sectors, pooling Services plus Agriculture and Natural Resources into a single composite non-manufacturing sector.

Cross-country variation in aggregate VAX ratios is to a large extent driven by variation in the composition of exports. To illustrate this, we plot VAX deviations ($VAX_i - \overline{VAX}$) against the Between and Within Terms separately in Figure 2. In the top panel, the Between Term is a strong and tight predictor of a country’s aggregate VAX ratio. In contrast, the Within Term is actually weakly negatively correlated with the aggregate VAX ratio in the bottom panel, and this relationship is relatively noisy. This visual impression is naturally confirmed by a simple variance decomposition. If we split the covariance of the Between and Within Terms equally, the Between Term “accounts for” nearly all the variation in the aggregate VAX ratio. The Between Term is dominant because of the large differences in VAX ratios across sectors. Countries that export predominantly Manufactures, the sector with the lowest VAX ratio, tend to have low aggregate VAX ratios as well.

Despite this strong composition effect, aggregate VAX ratios are only weakly related to the overall level of economic development. Panel A in Table 2 reports that a one log point increase in income per capita is associated with a fall in domestic content of 0.8 percentage points, though this correlation is not quite significantly different from zero at conventional significance levels.

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39Reference country VAX ratios for each sector are the ratios of value added exports to gross exports for the world as a whole. Export shares are the share of each sector in total world exports.

40The regression line in the top panel is $VAX_i - \overline{VAX} = .26 \times$ Between Term, with robust standard error .04 and $R^2 = .36$. The regression line in the bottom panel is $VAX_i - \overline{VAX} = -.11 \times$ Within Term, with robust standard error .06 and $R^2 = .04$.

41Specifically, the variance breaks down as follows: $\text{var(\text{Agg. VAX})} = 0.01$, $\text{var(\text{Within})} = 0.03$, $\text{var(\text{Between})} = 0.04$, and $\text{cov(Within, Between)} = -0.03$. Due to the negative covariance between the two terms, the variance decomposition is sensitive to how one chooses to assign the covariance. The scatter plots above can be thought of as representing a situation in which one assigns the covariance equally to the two terms.

42The p-value for a two-sided test that the correlation does not equal zero is 14%. In this regression, we omit outliers Belgium, Luxembourg, and Singapore. If these three countries are included, the correlation roughly doubles in size and becomes highly significant.
This weak aggregate correlation is a manifestation of two offsetting effects. First, richer countries tend to have exports concentrated in Manufactures, which has a relatively low VAX ratio. Second, richer countries tend to export with higher VAX ratios than poorer countries within composite sectors, particularly within Manufactures.

To illustrate these offsetting effects, we project the Between Term and the Within Term separately on exporter income to quantify the relative contribution of each to the overall correlation. In Panel A of Table 2 we see that there is a strong negative correlation of the Between Term with exporter income. That is, countries systematically shift toward manufacturing (which has lower value added to output on average) as they grow richer and this depresses the aggregate VAX ratios. The effect of this on overall VAX ratios is obscured because the Within Term is significantly positively correlated with exporter income. This positive correlation is mostly due to the fact that rich countries have higher VAX ratios within Manufactures. Panel B of Table 2 reports the correlation of VAX ratios for Manufactures with income per capita and splits this into Between and Within Terms as above. The positive correlation between Manufactures VAX ratios and income is itself driven by a positive composition (“between”) effect, wherein richer countries tend to specialize in manufacturing sectors with high VAX ratios.

### 3.2 Bilateral Value Added Exports and Balances

For a particular exporter, bilateral VAX ratios differ widely across destinations. For concreteness, we graphically present bilateral value added to trade ratios for the two largest exporters, the U.S. and Germany, in Figure 3. In the figure, value added to import ratios are VAX ratios for each country exporting to the U.S./Germany, while value added to export ratios are recorded for U.S./German exports to each country.

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43 VAX ratios for the non-Manufactures composite are positively correlated with income per capita, but the correlation is not significant. Therefore, we do not report these results separately.

44 We display data for the 15 largest trade partners for each country plus additional countries selected for illustration purposes, including adjusted and unadjusted bilateral VAX ratios for China and Mexico. In line with the aggregate results, adjusting for processing trade lowers bilateral VAX ratios vis-a-vis these countries but has only modest effects on ratios for other countries.
Looking at the U.S., there is wide variation in VAX ratios. For some partners, value added exports are quite close to gross exports. For example, the difference between gross and value added exports to the U.K. amounts to only 3% of gross exports. For others, gross trade either overstates or understates the bilateral exchange of value added. Value added exports to Canada are $77 billion (40%) smaller than gross exports, and value added exports to Mexico are $40-$50 billion (36-44%) smaller. Value added trade falls by a similar proportional amount, between 30-50%, relative to gross trade for countries like Ireland, Korea, and Taiwan, which are well-cited examples of production sharing partners. At the other end of the spectrum, several countries have VAX ratios toward the U.S. above one. For example, countries on Europe’s Eastern periphery (see Russia) have bilateral VAX ratios above one mainly because they supply intermediates to Western European countries that then end up being consumed in the U.S. Further, commodity producers (see Australia) also often have ratios above one.

The U.S. data are representative of general patterns in the data. Looking at Germany, discrepancies between value added and gross trade also vary in meaningful ways across partners. Value added trade is scaled down quite substantially for the vast majority of its large European partners, in contrast to the U.S. This surely is an indication of the integrated structure of production within the European Union and its neighbors. Consistent with anecdotal evidence, this is most pronounced for the Czech Republic and Hungary. Geography appears to play a substantial role, as trade with partners of similar income levels such as the U.S. and Japan is relatively less distorted.

One consequence of these trade adjustments is that bilateral trade balances differ when measured in gross versus value added terms. Figure 4 displays three measures of bilateral balances for the U.S.: the bilateral trade balance, the bilateral value added balance, and the bilateral value added balance adjusted for processing exports in China and Mexico. In interpreting this figure, it is important to keep in mind that multilateral trade balances equal the multilateral value added balance for each country. Therefore, a decline in the bilateral value added balance relative to the gross trade balance for one country necessarily implies an increase for some other country.

---

The median bilateral VAX ratio in the data is 0.91, and the $10^{th} - 90^{th}$ percentile range is 0.59 to 2.07. Approximately 40% of the bilateral VAX ratios are greater than one.
Comparing these alternate measures, there are large shifts in bilateral balances in Asia. Most prominently, the U.S. deficit with China falls by roughly 30-40% ($35-50 billion), while the deficit with Japan rises by around 33% ($17-18 billion). The end result is that the value added balances (adjusted for processing trade) are nearly equal for Japan and China. Looking elsewhere within Emerging Asia, U.S. deficits with Taiwan and South Korea also rise and U.S. surpluses with Australia and Singapore fall. Together, adjustments in these five countries (Australia, Japan, Singapore, South Korea, and Taiwan) nearly exactly add up to the fall in the U.S.-China deficit, which points to triangular production sharing within Asia with these countries feeding intermediates to China that are then embodied in Chinese exports to the U.S.

To understand these adjustments, we focus on the U.S.-China and U.S.-Japan balances with reference to the decomposition of the value added balance in Equation (12). First, looking at China, the VAX ratio for U.S. exports to China exceeds the VAX ratio for imports by about 8% in the unadjusted calculation and 4% in the adjusted calculation. This tends to raise the value added balance relative to the trade balance, though only modestly (by $10 billion without adjustment and $5 billion with adjustment). Second, the value added content of both bilateral U.S. exports and imports to/from China are well below one. The simple average VAX ratio across exports and imports is 0.80 without adjustment and 0.66 with adjustment. If VAX ratios for both exports and imports were equal to this average level, this would imply value added deficits 20% or 34% smaller than the gross deficits. This second “level effect” accounts for most of the adjustment from gross to value added balances for China (between $25-$44 billion of the total change). In contrast, for Japan, this level effect is virtually nil, as the simple average VAX ratio is near one (literally, 0.98 without adjustment and 1.00 with adjustment). The U.S. deficit with Japan rises in value added terms mainly because the ratio of value added imports to gross imports is high relative to the ratio of value added exports to gross exports (the VAX ratio for imports is 0.16 higher than for exports in both calculations).

46If gross trade were (counterfactually) balanced between the U.S. and China, the value added balance would show a surplus due to this force alone.
3.3 Inspecting the Mechanism: Bilateral Decompositions

To demonstrate that production sharing drives variation in bilateral VAX ratios, we construct two decompositions in the data. The first decomposition splits variation in bilateral VAX ratios into components arising from differences in the composition of exports across destinations and differences in bilateral production sharing relations. The second decomposition looks directly at how output circulates within cross-border production chains by (approximately) splitting bilateral exports into components absorbed and consumed in the destination, reflected back and ultimately consumed in the source, and redirected and ultimately consumed in a third destination.

To construct the first decomposition, we express the bilateral VAX ratio as:

\[
\frac{va_{ij}}{l_x_{ij}} = \frac{\ell (I - A_{ii} - A_{fi}) y_{ij}}{l_x_{ij}} + \frac{\ell (I - A_{ii} - A_{fi}) (y_{ij} - (I - A_{ii})^{-1} x_{ij})}{l_x_{ij}}
\]  

The first term is equivalent to the Hummels-Ishii-Yi measure of the domestic content of exports calculated using bilateral exports. For a given source country, it varies only due to variation in the composition of the export basket across destinations.

The second term is a production sharing adjustment. This adjustment depends on the difference between the amount of country \(i\) output consumed in \(j\), \(y_{ij}\), and the gross output from \(i\) required to produce bilateral exports to \(j\), \((I - A_{ii})^{-1} x_{ij}\). When \(y_{ij} < (I - A_{ii})^{-1} x_{ij}\), the VAX ratio is smaller than the bilateral HIY benchmark. This situation arises when country \(i\)’s intermediate goods shipped to country \(j\) are either reflected back to itself embedded in foreign produced final goods or intermediate goods used to produce domestic final goods, or redirected to third destinations embedded in country \(j\)’s goods. When \(y_{ij} > (I - A_{ii})^{-1} x_{ij}\), the VAX ratio is larger than the HIY benchmark. This situation arises when country \(i\) ships intermediates to some third country that then (directly or indirectly) embeds those goods in final goods absorbed in country \(j\).

To quantify the role of each term in explaining bilateral VAX Ratios, we decompose the vari-
ance of the bilateral VAX Ratio for each exporter across destinations, \( \text{var}_i \left( \frac{v_{ij}}{\ell x_{ij}} \right) \), into variation due to the BHIY Term versus the PSA Term. Table 3 reports the share of the total variance accounted for by the BHIY and PSA terms for representative exporters. The production sharing adjustment (PSA Term) evidently dominates the decomposition. This implies that variation in production sharing relations across partners, not export composition across destinations, drives the bilateral VAX ratio. Put differently, bilateral VAX ratios are determined not by what an exporter sends to any given destination, but rather how those goods are used abroad. In concrete terms, even though the U.S. sends automobile parts to both Canada and Germany, the U.S. VAX ratio with Canada is lower than with Germany because Canada is part of a cross-border production chain with the U.S.

To look at production chains more directly, we construct a second decomposition that splits bilateral exports according to whether they are absorbed, reflected, or redirected by the destination to which they are sent. We construct the decomposition using the division of bilateral exports into final and intermediate goods along with the output decomposition for the foreign destination:

\[
\ell x_{ij} = \ell \left( c_{ij} + A_{ij}y_j \right)
\]

\[
= \ell \left( c_{ij} \right) + \ell A_{ij}y_j + \sum_{k \neq j,i} \ell A_{ij}y_{jk}
\]

(15)

The first term captures the portion of bilateral exports absorbed and consumed in destination \( j \), including both final goods from country \( i \) and intermediates from \( i \) embodied in country \( j \)'s consumption of its own goods. The second term captures the reflection of country \( i \)'s intermediates back to itself embodied in country \( j \) goods. The third term is the summation of country \( i \)'s intermediates embodied in \( j \)'s goods that are consumed in all other destinations, i.e., redirected to third destinations.

47 In the table, we split the covariance equally between the BHIY and PSA Terms. Because the covariance is small, our conclusions are not sensitive to how we split the covariance.

48 This decomposition is only approximate, because the output split used in constructing the decomposition is influenced by the entire structure of cross-border linkages. Nonetheless, this decomposition is informative as it returns shares that are consistent with the zero order and first round effects of the Leontief matrix inversion (i.e., \([I + A]\))
We report the results of this decomposition for informative bilateral pairs in Table 4. Looking at the upper left portion of the table, we see that Japan’s exports to China are primarily either absorbed in China or redirected to the U.S. Comparing Japan’s trade with China to that with the U.S., we see that Japanese exports to the U.S. are nearly exclusively absorbed by the U.S., indicating minimal bilateral U.S.-Japan production sharing. In contrast, looking at the upper right panel, we see that large portions of U.S. exports to Canada and Mexico are reflected back to the U.S. for final consumption. Looking at the lower left panel, we see that sharing a common border with two different countries does not necessarily imply tight bilateral production sharing relationships. German exports to France are primarily absorbed there, while nearly half of exports to the Czech Republic are reflected or redirected. Finally, in the lower right corner, we see that Korea is engaged in triangular trade with the U.S. and other destinations via China. In contrast, a larger share of Korean exports to Japan are eventually consumed there. These results are consistent with our priors regarding the role of China as a production sharing hub in Asia.\footnote{These decompositions are computed without adjusting for processing trade in China. Adjusting for processing trade tends to amplify reflection and redirection effects. Thus, our table understates the amount of redirection within Asia and reflection in U.S.-Mexico trade.}

4 Concluding Remarks

Intermediate goods trade is a large and growing feature of the international economy. Quantification of cross-border production linkages is therefore central to answering a range of important empirical questions in international trade and international macroeconomics. This requires going beyond specific examples or country/regional studies to develop a complete, global portrait of production sharing patterns. This paper provides such a portrait using input-output and trade data to compute bilateral trade in value added. We document significant differences between value added and gross trade flows, differences that reflect heterogeneity in production sharing relationships. We look forward to applying this data in future work to deepen our understanding of the consequences describing how final goods absorbed in each destination are produced. We prefer the decomposition in the text to this alternative “first-order approximation” of the production structure because it adds up to bilateral exports.\footnote{These decompositions are computed without adjusting for processing trade in China. Adjusting for processing trade tends to amplify reflection and redirection effects. Thus, our table understates the amount of redirection within Asia and reflection in U.S.-Mexico trade.}
of production sharing.

References


Powers, William, Zhi Wang, and Shang-Jin Wei. “Value Chains in East Asian Production Net-
works – An International Input-Output Model Based Analysis.” USTIC Working Paper No. 2009-10-C.


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<th>Manuf.</th>
<th>Services</th>
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Source: Authors’ calculations based on GTAP Database Version 7.1. Data is for 2004.
Table 2: Aggregate and Manufacturing VAX Decompositions

**Panel A: Aggregate VAX Decomposition**

<table>
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<tr>
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<th>( VAX_i - \overline{VAX} )</th>
<th>Within Term</th>
<th>Between Term</th>
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<tr>
<td>Log Income Per Capita</td>
<td>-0.008</td>
<td>0.028**</td>
<td>-0.036***</td>
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<td>(0.005)</td>
<td>(0.011)</td>
<td>(0.013)</td>
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<tr>
<td>( R^2 )</td>
<td>0.02</td>
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<td>0.08</td>
</tr>
<tr>
<td>N</td>
<td>90</td>
<td>90</td>
<td>90</td>
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**Panel B: Manufacturing VAX Decomposition**

<table>
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<tr>
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<th>( VAX_i - \overline{VAX} )</th>
<th>Within Term</th>
<th>Between Term</th>
</tr>
</thead>
<tbody>
<tr>
<td>Log Income Per Capita</td>
<td>0.018***</td>
<td>-0.007</td>
<td>0.025***</td>
</tr>
<tr>
<td></td>
<td>(0.006)</td>
<td>(0.009)</td>
<td>(0.008)</td>
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<tr>
<td>( R^2 )</td>
<td>0.11</td>
<td>0.01</td>
<td>0.12</td>
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<tr>
<td>N</td>
<td>89</td>
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</table>

Robust standard errors are in parentheses. Significance levels: * \( p < .1 \), ** \( p < .05 \), *** \( p < .01 \). Constants included in all regressions. Income per capita equals exporter value added per capita, where value added is calculated using our data and population is from the GTAP 7.1 database. Belgium, Luxembourg, and Singapore excluded in Panel A and Botswana, Hong Kong, Paraguay, and Peru excluded in Panel B as outliers.

Table 3: Bilateral VAX Ratio: Bilateral HIY vs. Production Sharing Adjustment

**Variance Decomposition**

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<th>PSA Term</th>
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<td>95%</td>
</tr>
<tr>
<td>Germany</td>
<td>5%</td>
<td>95%</td>
</tr>
<tr>
<td>Japan</td>
<td>1%</td>
<td>99%</td>
</tr>
<tr>
<td>China</td>
<td>9%</td>
<td>91%</td>
</tr>
<tr>
<td>Argentina</td>
<td>1%</td>
<td>99%</td>
</tr>
<tr>
<td>France</td>
<td>8%</td>
<td>92%</td>
</tr>
<tr>
<td>Hungary</td>
<td>5%</td>
<td>95%</td>
</tr>
<tr>
<td>India</td>
<td>7%</td>
<td>93%</td>
</tr>
<tr>
<td>Portugal</td>
<td>9%</td>
<td>91%</td>
</tr>
<tr>
<td>Median Country</td>
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<td>97%</td>
</tr>
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See the text for details regarding the decomposition. The Median Country is the median statistic for all 93 countries in the data.
Table 4: Decomposing Trade: Absorption, Reflection, and Redirection

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<tr>
<td>China</td>
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<tr>
<td>U.S.</td>
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<td>Japan</td>
<td>4.3%</td>
</tr>
<tr>
<td>Germany</td>
<td>2.5%</td>
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</table>

<table>
<thead>
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<th>Germany exports to:</th>
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<th>Japan</th>
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<td>France</td>
<td>74.8%</td>
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<tr>
<td>U.S.</td>
<td>2.6%</td>
<td>2.6%</td>
<td>2.7%</td>
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</table>

See the text for details regarding the decomposition. The entries in the table describe the approximate share of bilateral exports to each destination that are ultimately consumed in that destination. Shares do not sum to one because we include only the top four destinations for each bilateral pair. Data is for 2004.

Figure 1: Composite Sector Shares of Gross Exports and Value Added Exports, by Country (2004)
Figure 2: Between-Within Decomposition of Aggregate VAX Ratios, by Country (2004)

Figure 3: Value Added to Gross Trade Ratios for the United States and Germany, by Partner (2004)
Figure 4: Bilateral Trade and Value Added Balances for the United States, by Partner (2004)
Appendix A

The basic idea behind the adjustment for processing trade is to split the aggregate economy into separate processing and non-processing units, each with its own input-output structure. Both sectors use domestic and imported intermediates, but they differ in terms of intermediate input intensity and the source (domestic versus imported) of intermediates. Further, all output in the export processing sector is exported.

From the input-output data, we observe the domestic intermediate use matrix \( m_{ii} \) and import use matrix as \( m_{Ii} \) for the economy as a whole. From trade data, we observe total exports originating from and imported intermediates used by the processing sector, denoted \( x_{i}^{P} \) and \( \bar{m}_{Ii}^{P} \) respectively. Output in the non-processing sector, denoted \( y_{i}^{N} \), is calculated by subtracting \( x_{i}^{P} \) from total output in the input-output accounts. We seek separate intermediate use matrices for the two sectors \( \{ m_{ii}^{N}, m_{ii}^{P}, m_{Ii}^{N}, m_{Ii}^{P} \} \) and value added by sector \( \{ va_{i}^{N}, va_{i}^{P} \} \) that satisfy:

\[
\begin{align*}
  m_{ii} &= m_{ii}^{N} + m_{ii}^{P} \quad \text{(A1)} \\
  m_{Ii} &= m_{Ii}^{N} + m_{Ii}^{P} \quad \text{(A2)} \\
  y_{i}^{N} &= va_{i}^{N} + \iota \left[ m_{ii}^{N} + m_{Ii}^{N} \right] \quad \text{(A3)} \\
  x_{i}^{P} &= va_{i}^{P} + \iota \left[ m_{ii}^{P} + m_{Ii}^{P} \right] \quad \text{(A4)} \\
  \bar{m}_{Ii}^{P} &= m_{Ii}^{P} \iota', \quad \text{(A5)}
\end{align*}
\]

where \( \iota \) is a conformable row vector of ones and \( \iota' \) is its transpose\(^{50} \).

If there are \( N \) sectors, then there are \( 4(N \times N) + 2N \) unknowns and only \( 2(N \times N) + 3N \) constraints so we cannot solve directly for the unknown coefficients. We therefore follow Koopman, Wang, and Wei (2008) and use a constrained minimization routine to impute the unknown coefficients, where the objective function minimizes squared deviations between imputed values and target values. Target values are set by splitting intermediate use and value added across processing

\(^{50}\)These constraints differ from those used by Koopman, Wang, and Wei (2008) in that we use the domestic and import intermediate use matrices separately, whereas they pool this information into a single overall use matrix.
and non-processing sectors according to their shares in total output.

With the resulting split tables, we use bilateral trade data as in the main text to construct bilateral sourcing matrices and the global input-output table. In performing the calculation, we use processing trade shares from Koopman, Wang, and Wei (2008) for China. For Mexico, we obtain trade data for the maquiladora sector from the Bank of Mexico. Due to concerns about the quality of disaggregate data and the accuracy of the imputation procedure for individual sectors, we aggregate the data to 3 composite sectors prior to imputing coefficients. Because bilateral value added trade results are essentially identical in the main data when computed with 57 sectors or 3 composite sectors, we believe aggregation does not result in diminished accuracy.

\footnote{In the resulting system, China and Mexico effectively have \(2N\) sectors, where each of the \(N\) sectors is separated into processing and non-processing sub-sectors.}

\footnote{Data is available at: http://www.banxico.org.mx/polmoneinflacion/estadisticas/balanzaPagos/balanzaPagos.html.}