Imported Intermediate Goods and Product Innovation:
Evidence from India

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Abstract

In this study, we build a structural model of multi-product firms that illustrates how access to new foreign intermediate goods contributes to the introduction of new product varieties. We establish a stochastic dynamic model of firm evolution allowing firms to be heterogeneous in their efficiency levels. Introducing importing decision to this dynamic framework, we show that the effects of importing intermediate goods are twofold: i) it increases the revenues per each product created and ii) through the knowledge spillovers obtained from importing, firms get more likely to introduce new varieties. Calibration of the model to Indian data shows that the model can successfully explain the dynamics of product evolution and other moments related to importing and product distribution. Finally the comparison of autarky with trade equilibrium shows how liberalizing trade increases innovation performances and product growth.

JEL Classification: F12, F13, L11, O31

Keywords: Firm dynamics, heterogeneous firms, innovation, endogenous product scope, importing intermediate goods, trade liberalization, Indian manufacturing sector.

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1 Introduction

An important role of international trade is the exposure of firms to new goods. The purpose of this study is to build an analytical framework to illustrate the potential gains in an economy that international trade can generate as new products are introduced resulting from the access to new foreign intermediate goods. We illustrate this mechanism through a trade liberalization episode. We show how aggregate innovation rates as well as individual firm’s dynamics change after the liberalization period.

The role of trade expanding the set of traded goods has been the subject of recent studies. Kehoe and Ruhl (2009) find that trade liberalization episodes are characterized by a significant increase in the traded volume of goods not previously traded. They further document that such role of the extensive margin of trade seems proper of structural changes in the economy (e.g. trade liberalization) and absent from events like business cycles. Arkolakis (2009) derives a similar result within his model of heterogeneous firms and costly access to trade. In our framework, trade brings access to new intermediate goods previously unavailable to firms, provided they find it profitable to pay the fixed costs associated with international trade.

At the firm level, Goldberg et al. (2010b) establish a robust relationship between the access to new foreign intermediate inputs and the introduction of new products by domestic firms. Using data for India, they find that 31 percent of the expansion of products by firms could be explained by declining input tariffs. Similar to their empirical specification, at the center of our theoretical framework is a set of heterogeneous multi-product firms that endogenously decide on investing in the development of new product varieties. In our model access to new intermediate inputs increase firms’ revenues of existing products and they also improve firms’ innovation technologies.

The concept of innovation used throughout the paper is one of horizontal product innovation; that is, an innovation or discovery consists of the knowledge required to manufacture a new final good that does not displace existing ones. This type of framework is a natural complement to models of vertical innovation where the “creative destruction” process makes existing products obsolete. Horizontal innovation models have been extensively used in growth theory, where Romer (1990) and Grossman and Helpman (1991) represent seminal studies. Furthermore, it has been applied in trade theory to explain wage inequality, cross-country
productivity differences among other topics (see Gancia and Zilibotti (2005)). Among disaggregated firm-level models, Luttmer (2010) and Seker (2009) are relevant references. Both of these papers follow from Klette and Kortum (2004) which present a stylized model that explains the regularities in firm and industry evolution. In their model, an establishment is defined as a collection of products and each product evolves independently. Every product owned by an establishment can give rise to a new product as a result of a stochastic innovation process or can be lost to a competitor. This birth and death process of the products is the source of firm evolution. Through this model of innovation, they explain various stylized facts that relate R&D, productivity, and growth. The extension introduced by Seker (2009) allows exogenous heterogeneity in firms’ efficiency levels which provides a better fit to the data on explaining firm size distribution as well as correlations between size, age, exit and firm growth. Our basic framework, which draws extensively from Seker (2009), is meant to illustrate the role of international trade in the introduction of new products. It is also relevant to mention that in contrast to frameworks of learning externalities, ours is one in which firms fully internalize the dynamic consequences of their innovation efforts.

In our framework international trade affects both the marginal benefit and the marginal cost of innovation efforts. As in Seker (2009), each firm decides how much resource to invest into innovation, where a higher investment increases the probability of successfully introducing a new product and consequently enjoying a new stream of revenue. In the model higher variety of intermediate goods benefits a given firm with larger returns to scale (productivity) both in its existing products as well as in any new product. Within the empirical literature, Kasahara and Rodrigue (2008) find a robust and significant increase in productivity among Chilean firms that import intermediate goods. Halpern et al. (2009) find that such gain comes mostly from increased variety of intermediates and to a lesser extent from quality improvements among Hungarian firms.

Our model also assumes that the exposure to foreign intermediate goods reduces, ceteris paribus, the cost of innovation. As firms learn from the knowledge embodied in such goods, they could obtain a higher probability of success investing the same amount of resources. Empirical support for this assumption can be found in Goldberg et al. (2010b). They find that: 1) firms

\footnote{Lentz and Mortensen (2008) present a model that also introduces heterogeneity in firms’ innovation capacities. See Seker (2009) for a comparison of both models.}
that faced stronger input tariff reductions were found, ceteris paribus, more likely to introduce new products; 2) such firms were more likely to invest in R&D; and 3) the main channel of these effects was through new varieties of intermediate goods and not through the trade expansion of previously traded ones.

A central element of our analysis is the heterogeneity among firms. Besides their exogenous component of productivity level, firms also differ in their portfolio of goods currently in production. A seminal work in the area of heterogeneous firms and trade is Melitz (2003). In his framework, more productive firms are self-selected into export markets as only they find it profitable to pay the fixed costs associated with exporting. In our study, we focus on the importing side of trade rather than exporting. However the mechanism is quite similar to the one presented in Melitz (2003). Firms face fixed costs of importing. Only the efficient ones can compensate these costs and import goods. In line with Melitz (2003) our analysis highlights that trade liberalization can have a redistribution effect, where some firms could end worse-off (lower profits) after the trade liberalization episode even if they import intermediate goods. We further document a similar redistribution of innovation efforts toward most productive firms away from the least productive.

In the model, producers own multiple products. Evolution of these firms is the sum of the evolution of each of their products. In this respect, the model complements several existing models explaining product scope. Bernard, Redding, and Schott (2006a,b) provide empirical evidence on how multi-product producers dominate total production in the U.S. economy. Contribution of firms’ product margin towards output growth significantly exceeds the contribution of entry and exit. They also construct a static model of multi-product firms and analyze their behavior during trade liberalization. They introduce two margins (intensive and extensive) to expand size, and these margins are positively correlated with each other. However, their model lacks a dynamic framework of firm evolution.

The remainder of the paper is organized as follows. Section 2 describes the model economy in two main parts: 1) selection of the mix of intermediate goods and labor, as well as firm’s decision whether to import intermediate goods from abroad; 2) dynamic problem where the optimal level of innovation rate is selected. Section 3 describes the data source and section 4

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presents the quantitative model and the calibration exercise. Section 5 concludes. We further present in the appendix, detailed derivations of some of the key equilibrium conditions of the model.

2 Model

Recent empirical evidence presented in Goldberg et al. (2009, 2010b) show that imported intermediate goods lead to higher innovation rates of new products and hence faster growth. In this study, we present a structural model of firm evolution that can explain this evidence as well as other evidences on the dynamics of multi-product firms. Following Klette and Kortum (2004) and their extensions by Lentz and Mortensen (2008) and Seker (2009) we introduce a model of firm growth that accounts for the heterogeneity in producers’ efficiency levels as well as entry and exit decisions. In introducing the heterogeneity in efficiency levels, we follow Melitz (2003). There is a continuum of final good producers each of which produce a different variety. Unlike Melitz (2003), here firms produce multiple products. The dynamic nature of the model with heterogeneous firms allows us to capture the differences in the evolution of importers and non-importers and contribution of importing to the aggregate growth.

The model is presented in a general equilibrium framework. First, we discuss the demand side of the economy. Then following Kasahara and Lapham (2008), we introduce the static profit maximization problem of producers. We incorporate this static problem into a dynamic framework to discuss the growth patterns of firms distinguishing the differences between importers and non-importers.

2.1 Consumers

Consider an economy with $N + 1$ identical countries. In a continuous time setup, each country consists of a representative consumer with an intertemporal utility function given as

$$U_t = \int_t^\infty e^{-\rho(\tau-t)} \ln C_\tau \, d\tau,$$

where $\rho$ is the discount rate and $C_\tau$ is the aggregate consumption of the composite good at time $\tau$. Instantaneous utility obtained from consumption at time $t$ is $\ln C_t$. The consumer is free to
borrow or lend at the interest rate $r_t$. Aggregate expenditure at time $t$ is $E_t = P_t C_t$ where $P_t$ is the price of the composite good. The optimization problem of the consumer yields $\frac{\dot{E}}{E} = r_t - \rho$. We let total expenditure be the numeraire and set it to a constant for every period.

In each country there is a mass of available products each of which is indexed by $j$. The total mass of products is represented by $J$. Consumers have a taste for variety and consume $y_j(t)$ units of variety $j$. Goods are substitutes with elasticity of substitution $\sigma > 1$. The composite good is determined by the following constant elasticity of substitution production function

$$C_t = \left( \int_{j \in J} y_j(j)^{\frac{\sigma+1}{\sigma}} dj \right)^{\frac{\sigma}{\sigma-1}}. \quad (1)$$

Composite good producer makes zero profit. Solution of this problem yields

$$y_j(t) = C_t \left( \frac{p_j(j)}{P_t} \right)^{-\sigma}.$$

Price of the composite good $P_t$ can be found as follows

$$P_t C_t = \int p_j(j) y_j(j) dj$$

$$P_t = \left( \int_{j \in J} p_j(j)^{1-\sigma} dj \right)^{\frac{1}{1-\sigma}}. \quad (2)$$

### 2.2 Intermediate Goods Producers

In the intermediate goods sector, firms are perfectly competitive. All firms have identical linear technologies and have the same productivities which are set as one. Labor is the only input used in the production. There is a unit continuum of domestic intermediate goods produced within a country. There is also free entry in this sector. Each intermediate good producer is a price taker hence price of these products equals to marginal cost which is the wage rate $w$. Interaction among the intermediate goods producers (and between intermediate and final goods producers, as explained below) happens through the labor market as they compete for the same resource.
2.3 Final Goods Producers: Import Decision

Producers are characterized by their efficiency level \( \varphi \), the number of products in their portfolio \( n \) and by their importing status: \( m \) if they decide to import some of their intermediate goods or \( h \) if they decide not to import intermediate goods. There are two main decisions a given firm takes, 1) whether to import or not, and 2) the amount of resources to invest in innovation, having as its ultimate goal to maximize the present discounted value of profits, \( rV_{\varphi}(n) \). We assume that firms decide whether to import when they introduce their first product and that such decision is permanent. This assumption is analogous to deciding to import in every period for every product variety produced.

In this section, we solve the producer’s decision problem regarding its importing status. Our specification of the production function follows from Kasahara and Lapham (2008) which extends Melitz (2003) by incorporating importing decision to the firm’s optimization problem. The difference in our model is that here firms produce multiple products. The final goods sector is combined of a continuum of monopolistically competitive firms producing horizontally differentiated goods. The production of each variety requires employment of labor and intermediate goods which might be either domestically produced or imported. Producers of the final goods are distinguished from each other by their efficiency levels, indexed by \( \varphi > 0 \) which is randomly drawn from a continuous cumulative distribution \( F(\varphi) \). As in Melitz (2003) higher efficiency level means producing a symmetric variety at a lower marginal cost. We assume that efficiency levels exogenously grow at rate \( g \) for all firms. Solution of the monopolistic competition model yields revenue \( r(\varphi) \) and profit \( \pi(\varphi) \) from each product \( j \) as follows\(^3\)

\[
\begin{align*}
    r(\varphi) &= p(\varphi)y(\varphi) = E \left( \frac{p(\varphi)}{P} \right)^{1-\sigma} \quad \text{for } \forall j \in J \\
    \pi(\varphi) &= \frac{r(\varphi)}{\sigma}.
\end{align*}
\]

Each product is produced by a single producer, hence we can write the equilibrium levels of revenue and profit from each product as a function of firm’s efficiency level \( \varphi \).

Firms employ labor and intermediate goods in the production of final goods. We follow

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\(^3\)Since in this section we only solve for firm’s static optimization problem, we exclude the time subscript \( t \) from the variables for brevity.
the macro-growth and trade literatures and assume that there is increasing returns to variety in intermediate goods. All intermediate goods enter symmetrically in the production function, implying that none of them is intrinsically better or worse than any other. Similar approach has been used in Ethier (1982) and Romer (1987) among many others. This specification is consistent with the empirical findings in Amiti and Konings (2008), Halpern et al. (2009), and Kasahara and Rodrigue (2008) which associate the use of foreign intermediate goods with higher productivities.

Importing firms incur a fixed cost $f_m$ and iceberg transport cost $\tau > 1$. Let $I \in \{0, 1\}$ refer to the import status of the firm. Production function for final goods is given as

$$y(\varphi, I) = \varphi l^\alpha \left[ \int_0^1 q_d(j) \frac{\gamma-1}{\gamma} \; dj + I \int_0^N q_f(j) \frac{\gamma-1}{\gamma} \; dj \right]^{\frac{(1-\alpha)\gamma}{\gamma-1}} ,$$

(4)

where $l$ shows the labor, $q_d(j)$ shows the domestic and $q_f(j)$ shows the imported intermediate goods employed in production of variety $j$ and $I$ is equal to one if the firm imports. The output elasticity of labor in production is represented by $0 < \alpha < 1$ and the elasticity of substitution between intermediate inputs is $\gamma > 1$. $N$ is the number of partner countries that are traded with.

The solution of the final good producer’s static optimization problem is given in the appendix. In the solution we get $q_d(j) = q_d$, $q_f(j) = q_f$ for $\forall j$ and $q_f = \tau^{-\gamma} q_d$. This finding simplifies the production function to

$$y(\varphi, I) = \varphi (1 + N\tau^{1-\gamma})^{\frac{1-\alpha}{\gamma-1}} l^\alpha [q_d + I N\tau q_f]^{1-\alpha} .$$

Here $\varphi (1 + N\tau^{1-\gamma})^{\frac{1-\alpha}{\gamma-1}}$ could be interpreted as a total factor productivity term. Price of a final good which only uses domestic intermediate goods is $p^h(\varphi)$ for a $\varphi$–type producer. This price is found as

$$p^h(\varphi) = \frac{\sigma}{\sigma - 1} \frac{w}{\alpha^\alpha (1-\alpha)^{1-\alpha}} \varphi .$$

If imported intermediates are used in production of the good, then the price is $p^m(\varphi) = \ldots$
\[ p^h(\varphi)/(1 + N\tau^{1-\gamma})^{1/\tau} \]. Since \((1 + N\tau^{1-\gamma})^{1/\tau} > 1\), \(p^m(\varphi) < p^h(\varphi)\) for \(\forall \varphi\). Once the prices are determined, it is straightforward to find revenues gained by importing and non-importing firms. Define \(r^h(\varphi)\) as the per-product revenue generated by a \(\varphi\)-type firm that does not import which is calculated as

\[ r^h(\varphi) = \left( \frac{p(\varphi)}{P} \right)^{1-\sigma} E = \left( \frac{\sigma}{\sigma - 1} \cdot \frac{w}{\alpha^{\alpha}(1 - \alpha)^{1-\alpha}(1 + \frac{1}{1-\alpha})} \right)^{1-\sigma} E. \quad (5) \]

Similarly, per-product revenue of an importing firm \(r^m(\varphi)\) is

\[
\begin{align*}
    r^m(\varphi) &= \left( \frac{1}{P} \left( \frac{\sigma}{\sigma - 1} \right) \left( \frac{w}{\alpha^{\alpha}(1 - \alpha)^{1-\alpha}(1 + N\tau^{1-\gamma})^{(1-\alpha)/(\gamma-1)}} \right) \right)^{1-\sigma} E \\
    &= \left( 1 + N\tau^{1-\gamma} \right)^{(1-\alpha)/(\gamma-1)} \left( \frac{\sigma}{\sigma - 1} \cdot \frac{w}{\alpha^{\alpha}(1 - \alpha)^{1-\alpha}P\varphi} \right)^{1-\sigma} E \\
    &= Z_mr^h(\varphi) \\
\end{align*}
\]

where \(Z_m = (1 + N\tau^{1-\gamma})^{(1-\alpha)/(\gamma-1)} > 1\). From equations 5 and 6 we see that revenue is proportional to the firm’s efficiency level. Defining \(\pi^h_i(\varphi)\) and \(\pi^m_i(\varphi)\) as the profit level of a non-importing and an importing firm respectively (the latter net of the fixed cost of importing \(wf_mN\)) we are now ready to describe the importing decision problem.

As mentioned earlier, besides its effects on productivity, importing intermediates also affects firms’ innovation technology. Firms basically contrast the fixed costs necessary to import intermediate, with not only the extra productivity today but also with the benefit of increasing their prospects for introducing new products. That is, both today’s and future profits would be affected by the import decision (equations 7 and 8). In the following equations \(V_\varphi(1)\) represent the value of a firm with one product and efficiency level \(\varphi\) (when characterizing the solution of the model we will also denote such value as \(v(\varphi)\) whenever there is no risk of confusion).

\[ rV_\varphi(1) = \max [rV^m_\varphi(1), rV^h_\varphi(1)] \quad (7) \]
The nature of the problem does not offer analytical solutions, but numerical simulations indicate the existence of a cutoff in terms of efficiency levels \( \varphi^*_m \) so that firms for which \( \varphi \geq \varphi^*_m \) will find it profitable to import.

### 2.4 Final Goods Producers: Innovation Decision

The dynamic optimization problem of the firm follows from Klette and Kortum (2004) and their extension in Seker (2009). The monopolistic competition in the final goods sector results in each product being produced by a single firm. Firms can produce multiple varieties. The number of products \( n \) determines the portfolio of the producer. This portfolio increases by innovating new product varieties and it decreases by destruction of the existing products\(^6\). This process determines evolution of the firm. Firm’s innovation rate depends on investment in research and development (R&D) \( R \) which is measured in labor units and knowledge capital. The innovation production function is strictly increasing and strictly concave in \( R \)\(^7\). It is strictly increasing in the knowledge capital and homogeneous of degree one in \( R \) and knowledge capital. For non-importing firms knowledge capital is measured by the total knowledge accumulated through past innovations. We use the number of products innovated \( n \) to represent this capital stock. For the importing firms, it is the product of \( (1 + N)^{\Delta} n \). Here \( (1 + N)^{\Delta} \) represents the spillover from the knowledge embodied in the imported intermediate goods. Grossman and Helpman (1991) discuss several ways in which international knowledge spillover are possible. They argue that residents of a country may find occasions to learn technical information from meeting with foreign counterparts that contributes to their stock of general knowledge. Also the use of differentiated intermediate goods that are not available in the domestic market can increase the insights that local researchers gain from inspecting and using these goods. Grossman and Helpman (1991) use the cumulative volume of trade between countries as the international

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\(^7\) The properties of this type of an innovation production function is discussed in Klette and Kortum (2004).
spillover for innovation. We use a function of the total flow of intermediate goods that are imported. We measure the magnitude of the spillover by $\zeta > 0$.

Goldberg et al. (2010b) provide empirical evidence that motivates the inclusion of imported intermediate goods in the innovation function. They show that increase in the availability of foreign intermediate products have increased the innovation capacities of firms in Indian manufacturing sector. They find that reduction of input tariffs led to imports of new varieties in the economy and this has led to an expansion of within-firm product scope by almost 8 percent.

As a result of firm’s R&D investment and knowledge capital new products arrive at a Poisson rate of $\lambda n$ where $\lambda$ is the innovation rate. The innovation function for an importing firm can be formalized as

$$R = \left( \frac{\lambda}{(1 + N)^{\zeta(1-\theta)}} \right)^{\frac{1}{\beta}} n$$

If we define $\beta = \frac{1}{\beta} > 1$ then $\frac{1-\theta}{\beta} = \beta - 1 > 0$. We can rewrite R&D investment as

$$R = \frac{\lambda^\beta}{(1 + N)^{\zeta(\beta-1)}} n.$$
products are destroyed. There is no re-entering once exit occurs.

The state of a producer is determined by its current portfolio of products \( n \). For a particular \( \varphi \)-type producer and a constant interest rate \( r \), the Bellman equation is formulated as follows\(^8\)

\[
rV^j_\varphi(n) = \max_{\lambda \geq 0} \left\{ \sum_{i=1}^{n} \pi_i^j(\varphi) - wnc^j(\lambda) + n\lambda \left[ V^j_\varphi(n+1) - V^j_\varphi(n) \right] + n\mu \left[ \sum_{i=1}^{n} \left( V^j_\varphi(n-1) - V^j_\varphi(n) \right) / n \right] \right\}, \text{ for } j = m, h
\]

(9)

This equation shows that current value of firm is equal to the sum of three terms. The first term on the right hand side shows the current profit net of R&D costs. The other two terms show the net future value of the firm. The second one is the gain in value caused by the innovation of a new variety and the last one is the expected loss associated with a loss of a randomly chosen product\(^9\).

Since the value function is linear in the number of products, it is possible to obtain an analytical solution to optimization problem. Following Lentz and Mortensen (2008), we conjecture that the value function is

\[
V^j_\varphi(n) = \sum_{i=1}^{n} \frac{\pi_i^j(\varphi)}{r + \mu} + n\Theta^j(\varphi)
\]

(10)

where \( \Theta(\varphi) \) is the type conditional continuation value of innovation. Then we incorporate this conjecture into the Bellman equation which simplifies to

\[
(r + \mu) \Theta^j(\varphi) = \max_{\lambda \geq 0} \left\{ \lambda \left( \frac{\pi_i^j(\varphi)}{r + \mu} + \Theta^j(\varphi) \right) - wnc^j(\lambda) \right\}
\]

\[
\Theta^j(\varphi) = \max_{\lambda \geq 0} \left\{ \frac{\lambda \pi_i^j(\varphi) - wnc^j(\lambda)}{r + \mu - \lambda} \right\}.
\]

(11)

In order to have the conjectured value function solve this problem, all variables have to be stationary. In the appendix we show how the economy grows on a balanced growth path.

\(^8\)Constancy of interest rate is obtained from the consumer’s optimization problem.

\(^9\)Existence of a solution for this dynamic optimization problem under heterogeneous firms is proven in Lentz and Mortensen (2005).
Solving the simplified Bellman equation, we get

\[
\frac{\pi^j(\varphi)}{r + \mu} + \Theta^j(\varphi) = wc^j(\lambda).
\]

(12)

We define \( v^j(\varphi) = \frac{\pi^j(\varphi)}{r + \mu} + \Theta^j(\varphi) \) as the expected value of a single product for a \( \varphi \)-type producer. It is the sum of the discounted stream of the profits and the innovation option value. Using the value of \( \Theta^j(\varphi) \) from equation 11 and implementing it into equation 12 we get

\[
w^j(\lambda) = \frac{\pi^j(\varphi) - wc^j(\lambda)}{r + \mu - \lambda}.
\]

(13)

Klette and Kortum (2004) show that the optimal value of \( \lambda \) is an increasing function of profit level. Since the profit level monotonically increases in \( \varphi \), this result shows firms with high efficiencies are more likely to innovate. Plugging the value of \( c^j(\lambda) = \frac{c_0 \lambda^{1+c_1}}{(1+N)^c_1} \) for an importing firm into this equation we get,

\[
w_0 (1 + c_1) \lambda^{c_1} \frac{(1 + N)^{c_1}}{(1 + N)^{c_1}} = \frac{\pi^m(\varphi) - wc_0 \lambda^{1+c_1}}{r + \mu - \lambda}
\]

for \( \varphi > \varphi_n^* \).

Similarly for the non-importing firm the solution is

\[
w_0 (1 + c_1) \lambda^{c_1} = \frac{\pi^h(\varphi) - wc_0 \lambda^{1+c_1}}{r + \mu - \lambda}
\]

for \( \varphi \leq \varphi_m^* \).

The difference between the solution for importing and non-importing firms is the spillover term which is captured by \((1 + N)^{c_1} > 1\). It enters into the equation as a positive multiplier of the profit level. Hence importing gives extra advantage to firms to innovate. To guarantee the existence of a stationary size distribution in equilibrium the condition \( \mu > \lambda(\varphi) \) for all efficiency levels \( \varphi \) must hold. If the innovation rate of a firm exceeds or becomes equal to the aggregate destruction rate, size and age of some firms diverge to infinity which precludes having a stationary size distribution\(^{10}\). This condition can be written as an upper bound on the efficiency type distribution which is presented in the appendix.

\(^{10}\)The condition needed to guarantee \( \mu > \lambda(\varphi) \) for \( \forall \varphi \) is \( wc'(\mu) > \frac{\pi - wc(\mu)}{\lambda} \) which follows from equation 13.
2.5 Entrant’s Problem

From a constant potential pool of entrants, successful ones enter the economy as a result of an innovation of a new variety. Firms discover their efficiency types immediately after they enter. Entrants face the same innovation cost function as incumbent firms and they innovate at rate \( \lambda_e \). Entry rate is determined by the free entry condition given as

\[
wc' (\lambda_e) = \int v (\varphi) \phi (\varphi) d\varphi.
\]

Here \( \phi (\cdot) \) is type distribution for entrants and \( v (\varphi) \) is the value of a single product for a \( \varphi \)-type producer. Entry rate \( \eta \) is the product of mass of potential entrants \( M_e \) and innovation rate of entrants \( \lambda_e \), \( \eta = M_e \lambda_e \). From incumbent firm’s optimization problem we had \( wc' (\lambda (\varphi)) = v (\varphi) \). Incorporating this result in equation 15 we can solve for the innovation rate of entrants as a function of innovation rates of incumbent firms

\[
w_{c_0} (1 + c_1) \lambda_{c_1} e = \int wc' (\lambda (\varphi)) \phi (\varphi) d\varphi
\]

\[
\lambda_e = \left[ \int \lambda (\varphi)^{c_1} \phi (\varphi) d\varphi \right]^{1/c_1}
\]

\[
= \left[ \int_0^{1/N} \lambda (\varphi)^{c_1} \phi (\varphi) d\varphi + \int_{1/N}^{\infty} \frac{\lambda (\varphi)^{c_1}}{(1 + N)^{c_1}} \phi (\varphi) d\varphi \right]^{1/c_1}.
\]

This result shows that keeping all else constant, higher knowledge spillover from trade decrease the innovation rate of entrants, hence reduces the entry rate.

2.6 General Equilibrium

Firms evolve as a result of a birth and death process of products. We define \( M_n (\varphi) \) as total mass of \( \varphi \)-type firms with \( n \) products. \( M (\varphi) = \sum_{n=1}^{\infty} M_n (\varphi) \) is total mass of \( \varphi \)-type firms. Then \( \delta (\varphi) = M (\varphi) / M \) is defined as the steady state type distribution where \( M \) is the total mass of firms. Mass of products produced by \( \varphi \)-type firms is represented as \( \Lambda (\varphi) = \sum_{n=1}^{\infty} nM_n (\varphi) \). In steady state, the rate of product destruction \( \mu \) should be equal to the sum of the product...
creation rates of entrants and incumbent firms

\[ \mu = \eta + \int \lambda(\varphi) \Lambda(\varphi) \delta(\varphi) \, d\varphi. \]  

(16)

In equilibrium, total mass of products produced by \( \varphi \)-type firms is found as\(^{11}\)

\[ \Lambda(\varphi) = \frac{\eta \phi(\varphi)}{\mu - \lambda(\varphi)}. \]  

(17)

In the appendix, we also derive the entry type distribution \( \phi(\cdot) \) as a function of the steady state type distribution \( \delta(\cdot) \). The final equilibrium condition is labor market clearing condition. There is a fixed measure of workers in the economy which is denoted as \( L \). Labor is allocated across four activities for every \( \varphi \)-type incumbent firm: final good production \( l_f \), intermediate good production \( l_i \), and R&D investment \( c(\lambda) \), and fixed cost of importing \( N f_m \). There is also a part of the labor force allocated to research and development for potential entrants \( c(\lambda_e) \). The labor market clearing condition is stated as

\[ L = \int_0^\infty (l_f(\varphi) + l_i(\varphi) + c(\lambda(\varphi)) + I(\varphi) N f_m) \Lambda(\varphi) \delta(\varphi) \, d\varphi + M c(\lambda_e) \]  

(18)

where \( I(\varphi) = 1 \) if \( \varphi > \varphi_m^* \). Given these equilibrium conditions, a stationary equilibrium for this economy consists of aggregate destruction rate \( \mu \), wage rate \( w \), and interest rate \( r \) such that for given values of \( (\mu, w, r) \), i) any \( \varphi \)-type incumbent producer chooses the optimal innovation rate \( \lambda(\varphi) \), decides on whether to import or not, and solves equation 13 to maximize its value, ii) potential entrants choose \( \lambda_e \) in solving equation 15 and break even in expectation, iii) representative consumer maximizes utility subject to budget constraint, iv) labor market clears as in equation 18, and v) equations 16 and 17 hold. The equilibrium level of aggregate price index is derived in the appendix.

\(^{11}\)Derivation of this equation which can be found in Lentz and Mortensen (2008) is reproduced in the appendix.
3 Data

The data used for the analysis is obtained from the Prowess database which is collected by Center for Monitoring the Indian Economy (CMIE). The data is constructed as a panel of relatively large firms spanning the period from 1989 to 1997. These firms account for 60% to 70% of the all economic activity in India. We restrict our analysis to manufacturing firms. Prowess records detailed product-level information at firm level and it enables us to track firm’s product mix over time. This information is available for 2927 firms which correspond to 85% of the manufacturing firms and 90% of total output in Prowess. The data on product mix of firms allows us to test the model’s ability to explain product evolution of firms.

Definition of a product is based on the CMIE’s internal product classification which is based on the Harmonized System and National Industry Classification. This data on product-mix of firms has been used in several studies by Goldberg et al (2009, 2010a, 2010b). Goldberg et al. (2010a) in an unpublished appendix explain data cleaning process in obtaining the product level data. We follow their methodology in getting this information. They define 1886 products linked to 108 four-digit NIC industries. In the data multi-product firms account for almost half of the firms and they account for 80% of the output.

4 Quantitative Model and Calibration

The model is calibrated to match the data from Indian manufacturing sector for 1989-1997 time period. The novel contribution of the model is explaining the product evolution of firms and how this evolution is related to importing. To test the model’s ability to explain the data we choose eight data moments as presented in Table 1. The moments that relate to product distribution help identify efficiency-type distribution and innovation cost parameters. First two moments are the mean and standard deviation of product distribution\textsuperscript{12}. Since the model is capable of explaining firm evolution, we include three moments that relate to firm dynamics: mean and standard deviation of product growth distribution for surviving firms and percentage of firms that do not change their product scope in a year. Data show that 90% of firms are

\textsuperscript{12} In the data maximum number of products obtained by any firm is 35. In the calibration we use this value as the maximum number of products that any firm can produce.
in-active in the sense that they do not add or subtract products in a given year\textsuperscript{13}. In order to highlight the model’s capacity in explaining the significance of multi-product firms in economic activity, we include the contribution of their sales to aggregate output (82%). All these data moments are obtained from the Prowess database. Since the model is estimated at steady state, average values of these moments are obtained between 1989 and 1997\textsuperscript{14}.

<table>
<thead>
<tr>
<th>Table 1: Data Moments</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average number of products</td>
<td>1.9</td>
</tr>
<tr>
<td>Standard deviation of number of products</td>
<td>1.7</td>
</tr>
<tr>
<td>Average of product growth (conditional on survival)</td>
<td>1.9%</td>
</tr>
<tr>
<td>Standard deviation of product growth (conditional on survival)</td>
<td>0.17</td>
</tr>
<tr>
<td>Fraction of firms with no net annual change in products</td>
<td>90%</td>
</tr>
<tr>
<td>Contribution of multi-product firms to total sales</td>
<td>82%</td>
</tr>
<tr>
<td>Fraction of firms importing intermediates</td>
<td>19%</td>
</tr>
<tr>
<td>Import intensity (Cost of foreign inputs/Sales)</td>
<td>19%</td>
</tr>
</tbody>
</table>

Two moments that relate to the decision of importing identify fixed cost of importing as well as the elasticity of substitution between intermediate inputs. The Prowess database does not include any information about importing. Hence these moments are obtained from Annual Survey of Industries for 2001-2002 period. In India during this period roughly 19% of the firms participate in import activity. We also compute average import intensity of the importing firms as 19%. For each firm, import intensity is computed as the share of total foreign input costs to total sales.

We assume that exogenous efficiency type distribution is lognormal ($\varphi-LN(\mu_\varphi, \sigma_\varphi)$) where $\mu_\varphi$ and $\sigma_\varphi$ are the mean and standard deviation of this distribution. The parameter vector that is chosen for identification is $\Delta_1 = \{c_0, c_1, \zeta, \mu_\varphi, \sigma_\varphi, f_m, \alpha, \gamma\}$. Innovation cost has three parameters: scaling factor $c_0$, the convexity of the cost $c_1$, and spillover parameter from imports $\zeta$. There are two parameters from the production function which are the labor share of production $\alpha$ and the elasticity of substitution between intermediate products $\gamma$. We estimate $\gamma$ analytically from the model to match average import intensity of firms. Cost of imported intermediates for a product is $w \tau N q_f$. Dividing this value by the revenue obtained from a product

\textsuperscript{13} Using five year averages of firm activity instead of one year, Goldberg et al. (2010a) show that the share of in-active firms are 72%.

\textsuperscript{14} India went through a balance of payments crisis at the end of 1991. We exclude this year in computation of the moments.
which is derived in equation 6, we get import intensity \( In_t = \frac{\sigma^{-1}}{\sigma} (1 - \alpha) \tau^{1-\gamma} \frac{N}{(1+N\tau^{1-\gamma})} \). This equation allows us to get the value of \( \gamma \) once the other parameters are determined. The final parameter to calibrate is the fixed cost of importing \( f_m \).

Rest of the parameters \( \Delta_2 = \{g, N, r, \sigma, \tau, M_e\} \) are determined in various ways. In the model there are two components of growth: growth of efficiency levels which is exogenously set and evolution of products which is endogenously determined. The data allow us to compute the contribution of each part to the aggregate growth. Average growth rate of sales per product (the intensive margin) over the sample period is 8.9%. This value corresponds to the value of \( g \). Number of trading partner countries \( N \) is set to one as a normalization. Similarly mass of potential entrants \( M_e \) is set to unity. As in Lentz and Mortensen (2008) real interest rate is set as 5%. Elasticity of substitution between final goods \( \sigma \), is set as 2.8 which is the median value of elasticities calculated by Broda and Weinstein (2006). Tariff rate on intermediate inputs is computed as 24% which yields the iceberg cost as 1.24. Computation of input tariffs is explained in Goldberg et al. (2010b). They compute input tariffs by running industry-level tariffs through India’s input-output table for 1993-1994 period.

Solution of the model requires lengthy fixed point iterations for each choice of model parameter candidates. To avoid these iterations, we follow Lentz and Mortensen (2008) who propose directly estimating wage rate \( w \) from the data. For any given estimate of \( w \) we can always find total labor supply \( L \) that would satisfy the labor market clearing condition in equation 18 and make the \( w \) estimate consistent with the equilibrium. In our calibration we set \( w \) to 200\(^1\). Finally aggregate expenditure is normalized to 10,000.

To solve the model, we simulate a panel of 10,000 firms for two periods to obtain the moment values. Then we seek for the parameter vector that minimizes the distance between the simulated moments and the data moments. Since the model is highly nonlinear, we use down-hill simplex method (amoeba) for optimization. The steps to computationally solve the equilibrium and calibrate the model are described in the appendix.

\(^1\)The value of 200 is arbitrarily chosen. In the calibration exercise we do not match any data moment like total sales, productivity, or total compensation that relies on the scale of the economy. Thus the wage value used in estimation is not critical for the values of simulated moments chosen for calibration.
4.1 Estimation Results

The parameter estimates are presented in Table 2. The estimated parameter values are similar to those obtained from several recent studies. Kasahara and Lapham (2007) use the same production function for final goods in estimating trade premia of importing and exporting firms using data from Chilean manufacturing sector. Their estimates of the fixed importing costs vary between 0.4 and 0.8 across three industries (wearing apparel, plastic products, and structural metal). The share of labor in their production function is around 0.26. Our estimate of elasticity of substitution between intermediate goods differs from their estimates of 4.5-5. The convexity of the innovation cost $c_1$ is comparable to the estimate obtained in Lentz and Mortensen (2008) which fit their model to Danish firms. They find the value of this parameter as 3.7316. Similarly Seker (2009) estimate this parameter for five Chilean manufacturing industries with values varying between 3.8 and 5.7.

<table>
<thead>
<tr>
<th>Table 2: Parameter Values</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Calibrated Parameters</strong></td>
</tr>
<tr>
<td>Parameter</td>
</tr>
<tr>
<td>$c_0, c_1, \zeta$</td>
</tr>
<tr>
<td>$\mu_\varphi, \sigma_\varphi$</td>
</tr>
<tr>
<td>$f_m$</td>
</tr>
<tr>
<td>$\alpha$</td>
</tr>
<tr>
<td>$\gamma$</td>
</tr>
<tr>
<td><strong>Other Parameters</strong></td>
</tr>
<tr>
<td>$r$</td>
</tr>
<tr>
<td>$g$</td>
</tr>
<tr>
<td>$\tau$</td>
</tr>
<tr>
<td>$N$</td>
</tr>
<tr>
<td>$M_e$</td>
</tr>
<tr>
<td>$\sigma$</td>
</tr>
<tr>
<td>$E$</td>
</tr>
</tbody>
</table>

The calibration results are presented in Table 3. The model performs reasonably well in matching the data. It over-estimates the contribution of multi-product firms to total sales. Percentage of importers is also matched quite well. Import intensity is exactly matched as it was derived analytically from the model. In addition to these targeted moments, the model performs well in capturing several other data moments. Two of these un-targeted moments are

16The parameter $c_0$ is a scaling factor and our estimate is likely to differ from Lentz and Mortensen (2008).
presented at the bottom of the table. Goldberg et al. (2010a) show that approximately 8.5% to 11.5% of the variation in output across firms can be attributed to the variation in extensive margin which is defined as the number of products firms produce. This statistics could be produced by the model. Number of products at steady state has a logarithmic distribution which is derived in Klette and Kortum (2004). Using the formulas for the mean and variance of this distribution, we can decompose the total variation in total sales into its components of intensive and extensive margins. Total sales is $E = \int E [r (\varphi) n|\varphi] \delta (\varphi) d\varphi$ where $\delta (\varphi)$ was defined as the efficiency type distribution of firms at steady state, $r (\varphi)$ is revenue per product derived in equation 3 and $E [...]$ is the expectation operator. Total variation in size is determined as

$$Var_{Total} = \int Var [r (\varphi) n|\varphi] \delta (\varphi) d\varphi + \int (E [r (\varphi) n|\varphi] - E)^2 \delta (\varphi) d\varphi. \quad (19)$$

Using the estimated parameter values, total variation decomposition shows that almost 12% of the variation in output is attributed to the variation in the extensive margin. Another moment is decomposition of aggregate growth. Following Goldberg et al. (2010a) we decompose aggregate annual change in output of continuing firms into the changes in product mix (extensive margin), and changes in output of existing products (intensive margin). The contribution of extensive margin to aggregate growth is 15% in the data where it is 11.8% in the model. Close accordance with the data in these moments relies on a novel feature of the model. The model incorporates two forces that generate persistent differences in firm performance. In his review of models on firm evolution, Sutton (1997) lists these forces as: (i) intrinsic efficiency differences that are determined before entering the economy (ii) differences that are generated through idiosyncratic innovations that accumulate through the life of the establishment. Both forces have drawn great attention in the literature17. Our model incorporates both types of forces which allows it to produce rich firm dynamics.

Finally we look at the model’s ability to explain product distribution of multi-product firms. The model slightly underestimates the product distribution in mean and standard deviation of the product distribution when all firms are included. Restricting the comparison to multi-product firms we get the results presented in Table 4. The product distribution that emerges

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17 For a review and comparison of both types of models see Klette and Raknerud (2002) and Seker (2009).
from the simulation has a longer right tail than the one observed in the data. Although mean, median, and some of the percentiles are quiet comparable, the 99th percentile is much higher in the model.

\begin{table}[h]
\centering
\begin{tabular}{|l|c|c|}
\hline
\textbf{Targeted Moments} & Data & Model \\
\hline
Average number of products & 1.95 & 1.13 \\
Standard deviation of number of products & 1.68 & 1.19 \\
Average of product growth (conditional on survival) & 1.9% & 1.18% \\
Standard deviation of product growth (conditional on survival) & 0.17 & 0.18 \\
Fraction of firms with no net change in products & 90% & 94% \\
Contribution of multi-product firms to total sales & 82% & 92% \\
Fraction of firms importing intermediates & 19% & 17% \\
Import intensity (Cost of foreign inputs/Sales) & 19% & 19% \\
\hline
\textbf{Other Moments (Un-targeted)} & & \\
\hline
Variation of output wrt extensive margin & 8.5-11.5% & 12% \\
Between share of annual aggregate growth & 15% & 11.8% \\
\hline
\end{tabular}
\caption{Model Fit to the Data}
\end{table}

**4.2 Autarky versus Trade**

Goldberg et al. (2010a, b) show the significant contribution of trade liberalization to innovation performance of Indian firms. In order to show the success of these policy reforms in the model, we take an extreme case and compare autarky with free trade equilibrium. In order to make this comparison, we let the wage rate be determined by the labor market clearing condition. We randomly draw 10,000 efficiency values and use the same set of firms in both autarky and trade.

First, we compare the profit levels of importing firms in autarky and trade. We define $\pi^m(\varphi), P^m, w^m$ as profit, aggregate price, and equilibrium wage in trade equilibrium. Replacing superscripts to $a$ refers to the same variables in autarky. Profit levels $\pi^m(\varphi)$ and $\pi^a(\varphi)$ are
calculated as
\[
\pi^m(\varphi) = \frac{E}{\sigma} \left[ \frac{\sigma - 1}{\sigma} \frac{P^m}{w^m} \alpha^\alpha (1 - \alpha)^{-\alpha} \left( 1 + N \tau_{m-1} \right)^{\frac{(1-\alpha)}{\gamma-1}} \varphi \right]^{\sigma-1} - w^m N f_m
\]
\[
\pi^a(\varphi) = \frac{E}{\sigma} \left[ \frac{\sigma - 1}{\sigma} \frac{P^a}{w^a} \alpha^\alpha (1 - \alpha)^{-\alpha} \varphi \right]^{\sigma-1}.
\]

For an importing firm, the comparison of these profit levels yield
\[
\pi^a(\varphi) < \pi^m(\varphi) - w^m f_m, \quad \text{iff}
\]
\[
w^m f_m < \frac{E}{\sigma} \left[ \frac{\sigma - 1}{\sigma} \alpha^\alpha (1 - \alpha)^{-\alpha} \varphi \right]^{\sigma-1} \left\{ \left( 1 + N \tau_{m-1} \right)^{\frac{(1-\alpha)}{\gamma-1}} \frac{P^m}{w^m} \right\}^{\sigma-1} - \left\{ \frac{P^a}{w^a} \right\}^{\sigma-1}.
\]

Recall that the cutoff efficiency level for the decision of importing \( \varphi^*_m \) was determined in 7. As long as \( D = \left\{ \left( 1 + N \tau_{m-1} \right)^{\frac{(1-\alpha)}{\gamma-1}} \frac{P^m}{w^m} \right\}^{\sigma-1} - \left\{ \frac{P^a}{w^a} \right\}^{\sigma-1} > 0 \) in the equation above, there is an efficiency level \( \hat{\varphi} > \varphi^*_m \) such that all firms with efficiency levels greater than \( \hat{\varphi} \) are going to gain higher profits in trade. Unlike the setup in Melitz (2003) it is not analytically possible to prove that \( D > 0 \). Hence we simulate the model to compare the autarky and trade equilibrium.

Using the parameter values obtained from the calibration exercise, we compute average profit level \( \pi(\varphi) \) per product and innovation rates \( \lambda(\varphi) \) for each efficiency level \( \varphi \). Figure 1 shows the relationship between efficiency levels and profit levels. Cutoff efficiency level \( \varphi^*_m \) is 160. The graph shows that not all importers gain from trade in profit due to higher costs of production. This finding is in accordance with Melitz (2003). He shows that among exporting firms only the most efficient ones gain from trade (\( \varphi \geq \hat{\varphi} \)). In our simulation this threshold value of efficiency levels is \( \hat{\varphi} \approx 351 \). Trade induces reallocation of resources toward the more efficient firms through competing for labor.

Next in figure 2, we look at how innovation rates are affected by trade. In trade equilibrium importing firms innovate at higher rates than they would in autarky. Importing increases firms’ profits and the knowledge spillover from using new input varieties reduce the cost of innovation. When both factors are combined we observe a faster innovation rate of importers in trade equilibrium. This finding is in accordance with the empirical evidence presented in Goldberg et al. (2010b). They show that as a result of the tariff liberalization in 1990’s, India experienced a surge in imported inputs mostly in inputs that were not available
before the liberalization. Higher usage of new input varieties has led to introduction of new products in the domestic market. Another result that can be obtained from the graph is that although some of importers with efficiency levels $\varphi$ such that $\varphi^* < \varphi < \hat{\varphi}$ observe drops in their per-product profits $\pi^m(\varphi)$ relative to autarky, they may still gain higher aggregate profits $\pi^m(\varphi)n$ in trade equilibrium because they are likely to innovate more products when trade is allowed.

Table 5 presents a comparison of autarky and trade equilibrium. In trade equilibrium average innovation rate of incumbent firms increase facilitated by the spillover from the use of imported intermediate goods and increase in profit levels. This leads to an increase in product portfolio of multi-product firms from 2.95 to 3.89. It also increases the average and aggregate innovation rates in the economy. Since prices decrease and wage rate increases in trade, innovation gets costlier which reduces the entry of new firms.

5 Conclusion

In this study we develop a general equilibrium model of multi-product firms to explain the relationship between importing, innovation, and firm growth. Following the structural model
Figure 2: Innovation rates ($\lambda$) versus efficiency levels ($\varphi$)

![Figure 2: Innovation rates (\lambda) versus efficiency levels (\varphi)](image)

Table 5: Autarky and Trade Comparison

<table>
<thead>
<tr>
<th>Moments</th>
<th>Autarky</th>
<th>Trade</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ave # of products (multi-product firms)</td>
<td>2.95</td>
<td>3.89</td>
</tr>
<tr>
<td>Ave innovation rate</td>
<td>3.3%</td>
<td>3.5%</td>
</tr>
<tr>
<td>Aggregate Innovation (by Incumbents)</td>
<td>19.2%</td>
<td>34.8%</td>
</tr>
<tr>
<td>Entry Rate</td>
<td>5.9%</td>
<td>4.6%</td>
</tr>
<tr>
<td>Average product growth rate</td>
<td>1.15%</td>
<td>1.18%</td>
</tr>
<tr>
<td>Aggregate price level</td>
<td>0.21</td>
<td>0.07</td>
</tr>
<tr>
<td>Equilibrium wage</td>
<td>150.5</td>
<td>200</td>
</tr>
</tbody>
</table>

of Klette and Kortum (2004) and their extension in Seker (2009), we present a stochastic dynamic model of firm and industry evolution. In the model we introduce heterogeneity in firms’ efficiency levels as in Melitz (2003). The novel feature of our model is that unlike many of the recent trade models that follow Melitz (2003), our model has a dynamic feature. Firms invest in R&D which results in introduction of new varieties to the economy. Each period these products face a probability of being destructed. The birth and death process of products yield the stochastic growth process of firms. Incorporating the importing decision in this setup allows us to relate trade with innovation and growth.

Firm’s efficiency is the main driver of its evolution. Only the most efficient firms can
participate in import markets as they can compensate the sunk costs of trade. These firms are also more innovative. With the learning they obtain through knowledge spillover from the use of foreign intermediates, their innovation rates increase even further relative to non-importing firms. With the additional benefits of importing on their revenues and innovation rates, these firms grow faster and exit less often.

We test the model’s ability to explain product distribution, dynamics of firm evolution, and the relationship between importing and innovation through a calibration exercise. We fit the model to Indian panel of firms for 1989-1997 time period. The model explains the targeted moments in the data relatively well. It also produces reasonable estimates of some un-targeted moments such as the variation in output with respect to extensive margin, between share of annual growth, and product distribution of multi-product firms. Finally we present a comparison of autarky and trade equilibrium. This exercise shows that as in Melitz (2003), in trade resources are reallocated to more efficient firms and this leads to an increase in average size and number of products produced by firms. Moreover, average product creation rate increases which leads to faster growth. Higher innovation rates of importing firms cause a reduction in the entry rate.
References


A Solving Firm’s Static Problem

In a symmetric equilibrium, the static profit maximization problem of a final good producer can be formalized as follows

\[
\begin{align*}
\text{Max} & \quad PC^{\frac{1}{\sigma}} y^{\frac{\sigma-1}{\sigma}} - w \int_{0}^{1} q_d (j) dj - w \tau \int_{0}^{N} q_f (j) dj - w l \\
\text{st} & \\
y & = \varphi l^\alpha \left[ \int_{0}^{1} q_d (j)^{\frac{\gamma-1}{\gamma}} dj + I \int_{0}^{N} q_f (j)^{\frac{\gamma-1}{\gamma}} dj \right]^{\frac{(1-\alpha)\gamma}{\gamma-1}}.
\end{align*}
\]
The first order conditions of this maximization problem with respect to \( q_d(j), q_f(j) \) for \( \forall j \) and \( l \) in respective order are given as follows:

\[
\begin{align*}
    w &= PC \frac{\sigma - 1}{\sigma} y^{\frac{s-1}{\sigma}} \varphi \ell^\alpha \frac{(1 - \alpha) \gamma}{\gamma - 1} \left[ \int_0^1 q_d(j)^{\frac{\gamma}{1-\gamma}} \, dj + I \int_0^N q_f(j)^{\frac{\gamma}{1-\gamma}} \, dj \right] \frac{(1-\alpha)\gamma}{\gamma - 1} q_d(j)^{\frac{\gamma}{1-\gamma} - 1} \\
    w\tau &= PC \frac{\sigma - 1}{\sigma} y^{\frac{s-1}{\sigma}} \varphi \ell^\alpha \frac{(1 - \alpha) \gamma}{\gamma - 1} \left[ \int_0^1 q_d(j)^{\frac{\gamma}{1-\gamma}} \, dj + I \int_0^N q_f(j)^{\frac{\gamma}{1-\gamma}} \, dj \right] \frac{(1-\alpha)\gamma}{\gamma - 1} q_f(j)^{\frac{\gamma}{1-\gamma} - 1} \\
    w &= C \frac{\sigma - 1}{\sigma} y^{\frac{s-1}{\sigma}} \alpha \varphi \ell^{\alpha - 1} \left[ \int_0^1 q_d(j)^{\frac{\gamma}{1-\gamma}} \, dj + I \int_0^N q_f(j)^{\frac{\gamma}{1-\gamma}} \, dj \right] \frac{(1-\alpha)\gamma}{\gamma - 1}.
\end{align*}
\]

From equation 20, we get \( q_d(j) = q_d \) for all \( j \), and from equation 21 we get \( q_f(j) = q_f \) for \( \forall j \). Then, taking ratios of equations 20 and 21, for all firms that import intermediate products we get

\[
\begin{align*}
    \frac{1}{\tau} &= \left( \frac{q_d}{q_f} \right)^{-\frac{1}{\gamma}} \\
    q_f &= \tau^{-\gamma} q_d \text{ for } \forall j.
\end{align*}
\]

Total output of a final good is found as

\[
\begin{align*}
    y(\varphi, I) &= \varphi \ell^\alpha \left[ q_d^{\frac{\gamma}{1-\gamma}} + IN (\tau^{-\gamma} q_d)^{\frac{\gamma}{1-\gamma}} \right] \frac{(1-\alpha)\gamma}{\gamma - 1} \\
    &= \varphi \ell^\alpha \left[ (1 + IN\tau^{1-\gamma}) q_d^{\frac{\gamma}{1-\gamma}} \right] \frac{(1-\alpha)\gamma}{\gamma - 1} \\
    &= \varphi \ell^\alpha \left( 1 + IN\tau^{1-\gamma} \right) \frac{(1-\alpha)\gamma}{\gamma - 1} q_d^{1-\alpha}. \tag{24}
\end{align*}
\]

This equation could further be simplified. Taking the ratios of equations 20 and 22 and using the results that \( q_d \) and \( q_f \) are constant, we get

\[
\begin{align*}
    \frac{l^\alpha \frac{(1-\alpha)\gamma}{\gamma - 1} \left[ q_d^{\frac{\gamma}{1-\gamma}} + INq_f^{\frac{\gamma}{1-\gamma}} \right] \frac{(1-\alpha)\gamma}{\gamma - 1} q_d^{\frac{\gamma}{1-\gamma} - 1} \frac{\gamma - 1}{\gamma} q_f^{\frac{\gamma}{1-\gamma} - 1}}{\alpha \varphi \ell^{\alpha - 1} \left[ q_d^{\frac{\gamma}{1-\gamma}} + INq_f^{\frac{\gamma}{1-\gamma}} \right] \frac{(1-\alpha)\gamma}{\gamma - 1} q_f^{\frac{\gamma}{1-\gamma} - 1}} &= 1.
\end{align*}
\]
Simplifying this ratio and using the finding \( q_f = \tau^{-\gamma} q_d \), we get
\[
\frac{l (1 - \alpha) q_d^{-\frac{1}{\gamma}}}{\alpha \left[ q_d^{\frac{1}{\gamma}} + IN q_f^{\frac{1}{\gamma}} \right]} = 1
\]
\[
\frac{l (1 - \alpha) q_d^{-\frac{1}{\gamma}}}{\alpha \left( 1 + IN \tau^{1-\gamma} q_d^{\frac{1}{\gamma}} \right)} = 1
\]
\[
(1 + IN \tau^{1-\gamma}) q_d^{\frac{1}{\gamma}} = l \left( \frac{1 - \alpha}{\alpha} \right)
\]
Now combining the result from equation 22, the finding that \( p(j) = (\frac{y}{c})^{-\frac{1}{\sigma}} P \), and the result in equation 25, we get an equation that gives the relationship between price and wage rate
\[
w = \frac{\sigma - 1}{\sigma} \frac{p \varphi l^{\alpha-1}}{P} \left[ \int_0^1 q_d (j) q^{1-\gamma} dj + I \int_0^N q_f (j) q^{1-\gamma} dj \right]^{\frac{(1-\alpha){\gamma}}{\gamma-1}}
\]
\[
p = \frac{\sigma - 1}{\sigma} \frac{w}{\varphi l^{\alpha-1} (1 + IN \tau^{1-\gamma}) q_d^{1-\alpha}}
\]
\[
p = \frac{\sigma - 1}{\sigma} \frac{\varphi (1 + IN \tau^{1-\gamma}) q_d \left( \frac{\alpha}{1 - \alpha} \right)}{w} (1 + IN \tau^{1-\gamma})^{\frac{(1-\alpha){\gamma}}{\gamma-1}} q_d^{1-\alpha}
\]
The result obtained from equation 26 shows that the final good price for a good that only uses domestic intermediate goods is \( p^b (\varphi) = \frac{\sigma - 1}{\sigma} \frac{w}{\alpha (1 - \alpha) q_d^{1-\alpha}} \varphi (1 + IN \tau^{1-\gamma}) q_d^{1-\alpha} \). If imported intermediate goods are used in production, then the price is \( p^m (\varphi) = p^b (\varphi) / (1 + N \tau^{1-\gamma})^{\frac{1-\alpha}{\gamma-1}} \).

Next we derive the equilibrium values of \( q_d \) and \( l \). From the solution of the profit maximization problem of the final good producer, replacing the equilibrium value of labor \( l \) from equation 25 into equation 24 we get
\[
y (\varphi, I) = \varphi \left( 1 + IN \tau^{1-\gamma} q_d \left( \frac{\alpha}{1 - \alpha} \right) \right)^{\alpha} (1 + IN \tau^{1-\gamma})^{\frac{(1-\alpha){\gamma}}{\gamma-1}} q_d^{1-\alpha}
\]
\[
y (\varphi, I) = \varphi \left( 1 + IN \tau^{1-\gamma} \right)^{\frac{1-\alpha}{\gamma-1}} \left( \frac{\alpha}{1 - \alpha} \right)^{\alpha} q_d.
\]
Recall that from composite good producer’s maximization problem we had

\[ y(\varphi) = p(\varphi)^{-\sigma} \frac{E}{P^{1-\sigma}}. \]  

(28)

Combining equations 27 and 28, we get

\[ p(\varphi)^{-\sigma} \frac{E}{P^{1-\sigma}} = \varphi (1 + IN^{1-\gamma})^{\frac{\gamma-\alpha}{\gamma-1}} \left( \frac{\alpha}{1-\alpha} \right)^{\alpha} q_d \]

\[ q_d = \varphi (1 + IN^{1-\gamma})^{\frac{\gamma-\alpha}{\gamma-1}} \left( \frac{\alpha}{1-\alpha} \right)^{\alpha}. \]

We can get rid of the price in this equation by plugging in the value of \( p(\varphi) \) from equation 26

\[ q_d = \frac{E}{P^{1-\sigma}} \left( \frac{\alpha}{1-\alpha} \right)^{-\alpha} \left[ \varphi (1 + IN^{1-\gamma})^{\frac{\gamma-\alpha}{\gamma-1}} \right]^{\sigma-1} \frac{w}{\sigma - 1} \frac{\alpha^{\sigma} (1-\alpha)^{1-\alpha}}{\varphi(1 + IN^{1-\gamma})^{\gamma-1}} \left( \frac{\sigma - 1}{\sigma} \frac{\alpha^{\sigma} (1-\alpha)^{1-\alpha}}{w} \right)^{\sigma}. \]

Finally labor value can be found by incorporating \( q_d \) into equation 25

\[ l = \frac{E}{P^{1-\sigma}} \left( \frac{\alpha}{1-\alpha} \right)^{1-\sigma} \varphi^{\sigma-1} (1 + IN^{1-\gamma})^{\frac{1-\sigma}{\gamma-1}} \left( \frac{\sigma - 1}{\sigma} \frac{\alpha^{\sigma} (1-\alpha)^{1-\alpha}}{w} \right)^{\sigma}. \]

**B Balanced Growth in the Economy**

We assume that efficiency levels grow at rate \( g (\varphi/\varphi = g) \). Since there is no population growth, \( L \) is constant. Then from equation 4 we get \( \dot{y}(\varphi, I)/y(\varphi, I) = g \). Equation 1 gives

\[ \dot{C} = \frac{\sigma}{\sigma - 1} \left( \int_{j \in J} y_t(j)^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}} \left( \int_{j \in J} \frac{\sigma - 1}{\sigma} y_t(j)^{\frac{\sigma-1}{\sigma}-1} y(j) \frac{\dot{y}(j)}{y(j)} dj \right) \]

\[ \dot{C} = \frac{\sigma}{\sigma - 1} \sigma_{\sigma-1} C g \]

\[ \frac{\dot{C}}{C} = g. \]
Since we normalize aggregate expenditure to a constant in the model $E = P_t C_t$ implies that $P$ and $p(j)$ for $\forall j$ decreases at rate $g$.

\[
\begin{align*}
P_t &= \left( \int_{j \in J} p_t(j)^{1-\sigma} \, dj \right)^{\frac{1}{\sigma}} \\
\dot{P} &= \frac{1}{1-\sigma} \left( \int_{j \in J} p_t(j)^{1-\sigma} \, dj \right)^{\frac{1}{1-\sigma} - 1} \left( \int (1-\sigma) p(j)^{-\sigma} \frac{\dot{p}(j)}{p(j)} p(j) \, dj \right) \\
\frac{\dot{p}}{P} &= -g = \frac{\dot{p}(j)}{p(j)} \text{ for } \forall j.
\end{align*}
\]

From the final good producer’s optimization problem we found that

\[
p(\varphi) = \frac{\sigma}{\sigma - 1} \frac{w}{\varphi}.
\]

Since $p(\varphi)$ decreases at rate $g$ and $\varphi$ grows at rate $g$, wage is constant. Then profit per product $\pi(\varphi) = \left( \frac{p(\varphi)}{P} \right)^{1-\sigma} E$ is constant. This allows us to get the stationary solution in the Bellman equation. Note that although wage is constant, real wage which is $w/P_t$ and real output $r(\varphi)$ grow at rate $g$.

## C Steady State Size Distribution of Firms


\[
M_n(\varphi) = \frac{\eta \phi(\varphi)}{\mu n} \left( \frac{\lambda(\varphi)}{\mu} \right)^{n-1}.
\]

Using this equation, we can derive total mass of firms of type $\varphi$ as

\[
M(\varphi) = \sum_{n=1}^{\infty} M_n(\varphi) = \frac{\eta \phi(\varphi)}{\mu} \sum_{n=1}^{\infty} \frac{1}{n} \left( \frac{\lambda(\varphi)}{\mu} \right)^{n-1} = \frac{\eta \phi(\varphi)}{\lambda(\varphi)} \ln \left( \frac{\mu}{\mu - \lambda(\varphi)} \right). \tag{30}
\]

Here, convergence to a stationary size distribution requires $\mu > \lambda(\varphi)$ for all $\varphi$. Taking the ratio of equations 29 and 30 gives the steady state size distribution for the number of products

\[
\frac{M_n(\varphi)}{M(\varphi)} = \frac{1}{n} \left( \frac{\lambda(\varphi)}{\mu} \right)^n \ln \left( \frac{\mu}{\mu - \lambda(\varphi)} \right).
\]

30
This is probability distribution for the logarithmic distribution with parameter value of $\lambda(\varphi)/\mu$. Finally total mass of products produced by $\varphi$-type firms is found as

$$\Lambda(\varphi) = \sum_{n=1}^{\infty} n M_n(\varphi) = \sum_{n=1}^{\infty} \eta^n(\varphi) \left( \frac{\lambda(\varphi)}{\mu} \right)^{n-1} = \frac{\eta\lambda(\varphi)}{\mu - \lambda(\varphi)}. \quad (31)$$

D Deriving Entry Type Distribution in Steady State

From equation 30 we get

$$\eta\phi(\varphi) = \frac{M(\varphi) \lambda(\varphi)}{\ln \left( \frac{\mu}{\mu - \lambda(\varphi)} \right)} = \frac{M\lambda(\varphi) \delta(\varphi)}{\ln \left( \frac{\mu}{\mu - \lambda(\varphi)} \right)}.$$

Taking the integrals of both sides and using the fact that $\int \phi(\varphi) \, d\varphi = 1$, we get

$$\eta = \int \eta\phi(\varphi) \, d\varphi = M \int \frac{\lambda(\varphi) \delta(\varphi)}{\ln \left( \frac{\mu}{\mu - \lambda(\varphi)} \right)} \, d\varphi$$

$$\phi(\varphi) = \frac{\int \frac{\lambda(\varphi) \delta(\varphi)}{\ln \left( \frac{\mu}{\mu - \lambda(\varphi)} \right)} \, d\varphi}{\int \frac{\lambda(\varphi) \delta(\varphi)}{\ln \left( \frac{\mu}{\mu - \lambda(\varphi)} \right)} \, d\varphi}. \quad (32)$$

E Deriving Aggregate Price Index

We defined $\Lambda(\varphi)$ as total mass of products produced by $\varphi$-type firms. Plugging the value of $p(\varphi)$ from equation 26 into aggregate price index from equation 2, we get

$$P^{1-\sigma} = \int_{0}^{\varphi_m^*} p^h(\varphi)^{1-\sigma} \Lambda(\varphi) \, d\varphi + \int_{\varphi_m^*}^{\infty} p^m(\varphi)^{1-\sigma} \Lambda(\varphi) \, d\varphi$$

$$P = \left( \frac{\sigma}{\sigma - 1} \frac{w}{\alpha^\alpha (1 - \alpha)^{1-\alpha}} \right) \left( \int_{0}^{\varphi_m^*} \varphi^{\sigma-1} \Lambda(\varphi) \, d\varphi + \int_{\varphi_m^*}^{\infty} \left( \varphi (1 + N \tau^{1-\gamma}) \frac{1}{\gamma} \right)^{\sigma-1} \Lambda(\varphi) \, d\varphi \right)^{-\frac{1}{\sigma-1}}.$$

In the simulation exercise, aggregate price index can be computed by replacing $\Lambda(\varphi)$ by its value in equation 31 and $\phi(\varphi)$ by its value in equation 32 which yields to

$$P^{1-\sigma} = \int_{0}^{\infty} p(\varphi)^{1-\sigma} \Lambda(\varphi) \, d\varphi$$

$$= \int_{0}^{\infty} p(\varphi)^{1-\sigma} \frac{\lambda(\varphi) \delta(\varphi)}{\ln \left( \frac{\mu}{\mu - \lambda(\varphi)} \right)} \, d\varphi.$$
F Maximum Attainable Level of Efficiency

Let’s define \( \pi^{\text{max}} \) as the maximum attainable level of profit that would satisfy \( \lambda(\varphi) < \mu \) for all \( \varphi \).

\[
\begin{align*}
\pi^{\text{max}} &= \frac{w c_0 (1 + c_1) \mu c_1}{(1 + N)^{c_1}} (r + c_1 + \mu).
\end{align*}
\]

The efficiency level that would correspond to this profit level can be found as follows

\[
\begin{align*}
\bar{\pi} &= \pi^{\text{max}} - f_m \\
&= \frac{E}{\sigma} \left[ \frac{\sigma - 1}{\sigma} P \alpha^a (1 - \alpha)^{1-a} \varphi (1 + N \tau^{-1-\gamma})^{-\gamma} \right]^{\sigma-1} - f_m \\
\varphi^{\text{max}} &= \left[ (\bar{\pi} + f_m) \frac{\sigma}{E} \right]^{\frac{1}{\sigma-1}} \frac{w}{\sigma - 1} P \alpha^a (1 - \alpha)^{1-a} (1 + N \tau^{-1-\gamma})^{-\gamma}. \end{align*}
\]

G Algorithm Steps for the Model Solution

For a given set of model parameters \( \Delta = \{ \Delta_1, \Delta_2 \} \) we simulate a panel of 10,000 firms which are identified by their unique efficiency levels. Using this panel, we compute the simulated moments and seek for the parameter vector that minimizes the distance between the simulated moments and the data moments. The steps to compute the equilibrium are described as follows:

1. The parameter vector is initialized and the vertices of the simplex are determined.
2. For each parameter vector, equilibrium level of aggregate price index \( P \) and aggregate destruction rate \( \mu \) are computed.
3. For each of the 10,000 efficiency level draws, Bellman equation is solved and innovation rates are found.
4. Using these values, moment are computed.
5. The value of the criterion function is checked and using the amoeba routine, the simplex of parameter vectors is updated.
6. The system is iterated until either the value of the criterion function or the parameter vector converges.