Partial and General Equilibrium Analysis

Frank van Tongeren
Overview of the session

- Trade tools in a partial equilibrium setting
- Geometry of PE analysis
  - Assignment 1: first analysis of a policy
- Mathematics of a perfect substitutes PE model
- Spreadsheet implementation
  - Assignment 2: quantitative analysis of a policy
- Imperfect substitutes: The Armington model
Tools for analyzing trade

- **analytical tools:**
  - stylized partial equilibrium models
  - stylized general equilibrium models

- **statistical tools:**
  - gravity models
  - growth regressions

- **numerical tools:**
  - partial equilibrium models
  - general equilibrium models
All models are wrong, 
but some are useful
TRADE TOOLS
IN A
PARTIAL EQUILIBRIUM SETTING
Partial equilibrium models

- Partial = only looks at part of the economy (sector)
- Limitation is strength:
  - many effects are left out of the model -> limits use
  - results are more rapid to derive and transparent
- Can be used if the effects on rest of the economy are small:
  - small sector (limited income effects)
  - limited links with other parts of economy (limited backward and forward linkages)
Autarky

Price

Quantitate

\[ Q^D = Q^S \]
Free trade for a small country
Free trade for a small country

Diagram showing the relationship between price and quantity for imports. The diagram includes the supply (S) and demand (D) curves, with the equilibrium price (P) and quantity (Q) indicated. The area enclosed by the curves represents the imports.
Free trade: consumers gain

**Diagram:**
- **Price** axis
- **Quantity** axis
- **Imports** shaded area
- **Consumer surplus** shaded area

**Axes:**
- **D** (Demand)
- **S** (Supply)
- **Q^S**
- **Q^D**
- **P_W**
- **P_D**
Free trade: consumers gain

- Consumer surplus
- Imports

Graph showing the relationship between price and quantity with shaded areas representing consumer surplus and imports.
Free trade: producers loose

Price

Producer surplus

Imports

Quantities

Q^S

Q^D
Free trade: producers loose

![Graph showing free trade impact on producers]

- **Price**
- **Quantity**
- Imports

The graph illustrates how free trade affects producers. The shaded area represents the loss of producer surplus (PS) due to competition from imports. The difference between the domestic quantity (Q^S) and the imported quantity (Q^D) is represented as imports.
Free trade: economy as a whole gains

Transferred surplus

Imports
Free trade: economy as a whole gains

[Diagram showing the effects of free trade on the economy, including areas labeled as "Transferred surplus," "Gain," and "Imports." The diagram illustrates the relationship between price and quantity, with demand (D) and supply (S) curves intersecting.]
Import tariff: imports reduce

Imports

Price

Quantit

S

D

P^D

P^{W+T}

P^W
Import tariff: imports reduce

Diagram showing the effect of an import tariff on quantity demanded and quantity supplied. The graph illustrates the change in price due to the tariff, with the imports area highlighted.
Import tariff: consumers loose

In the diagram, the price of imports is denoted as $P^W + T$, where $T$ is the tariff. The domestic price is $P^D$. The quantity supplied by domestic producers is $Q^S$, and the quantity demanded by consumers is $Q^D$. The shaded area represents the consumer surplus loss due to the import tariff.
Import tariff: producers gain

The diagram shows the effect of an import tariff on the market for a good. The supply and demand curves are labeled S and D, respectively. The price paid by domestic consumers is denoted as $P^D$, and the price received by domestic producers is $P^W$. The tariff, denoted as $T$, increases the price paid by consumers to $P^{W+T}$. The shaded area labeled PS represents the gain to producers due to the tariff. The quantity supplied increases from $Q^S$ to $Q^D$, indicating an increase in imports as a result of the tariff.
Import tariff: government gets revenue

**Diagram:**

- **Axes:**
  - Vertical: Price
  - Horizontal: Quantity

- **Lines:**
  - **S** (Supply curve)
  - **D** (Demand curve)

- **Points:**
  - **Q^s**
  - **Q^d**

- **Price Points:**
  - **P^w**
  - **P^w+T**

- **Areas:**
  - **Imports**
  - **Revenue**

**Equations:**

- Imports: \( P^w + T \)
- Revenue: \( P^w \times Q^d - P^w \times Q^s \)
Import tariff: economy has a deadweight loss

\[ \text{Imports} = (P^W + T) - Q^S \]

- PS gain: \( P^W \times Q^S \)
- Revenue: \( P^W + T \times Q^D \)
- Deadweight loss: \( P^W - P^D \times (Q^D - Q^S) \)
Here we have again illustrated an import tax. Import taxes collected are area 1256. Consumer cost is area 12347. Taxes collected amount to area 1265. The welfare gain equals the difference between consumer losses and taxes. Some taxes (area 7456) come from terms of trade gains, as the import price declines relative to domestic price. Therefore the welfare effects depends on the relative size of 243 and 7456.
Here we have illustrated an import quota. In contrast to a tax, the quota creates import QUOTA RENTS, equal to area 1234. These rents are a likely target for lobbying. They are also, potentially, lost to foreigners.

Note: this graph is in terms of IMPORT demand and supply.
Note that, given a tariff $t_1$, the welfare effect of an additional import duty is not only the triangle $635$, but also the rectangle $3542$. This captures the geometric nature of the welfare effect of distortions. To handle this, the model directly calculates $123$ and $146$. 
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  - Assignment 2: graphical policy analysis
- General equilibrium model: mathematical
  - Assignment 3: quantitative policy analysis
A simple PE model without trade: supply, demand and market clearing
ALGEBRA -- effects of tariffs/quotas
A simple PE model with trade

6 equations and 6 unknowns: Qd, Qs, Md, Ms, P, P*
Some manipulations to solve the model:

- We can derive an import demand equation from eq. (3):
  $$\text{Md}(P) = \text{Qd}(P) - \text{Qs}(P) = k_{md} P^{\varepsilon_{md}}$$

- The constant $k_{md}$ can be calibrated from initial data
- The import demand elasticity can then be retrieved as:
Some manipulations to solve the model - II

- To solve the model ‘by hand’ we linearize the equations
- Example:
Some manipulations to solve the model - III

- After some manipulations we can solve the linearized model for the domestic equilibrium price P:

\[ \ln(\tau) + \ln(\epsilon) - \omega ms - \ln(\epsilon) + \ln(\tau) = m d ms + ms dm k k P \]

(Of course we need to calculate \( \exp(\ln(P)) \) to get the level of P.)
Linear or non-linear?

- The log-linear version holds as a local approximation for more general functional forms.
- We can also solve the non-linear version directly through linear approximation methods (EULER, GRAGG)
- We can also solve the non-linear version in a spreadsheet....
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Nonlinear Spreadsheet Models

• The model is specified as a system of implicit equations:
  \[ g(x) - f(x) = 0 \]
  \[ h(x) - k(x) = 0 \]
  etc...

• The model is solved by solving for one of these equations, treating the others as constraints.
A view of the model

Perfect substitutes model

note: use "solver", under the "Tools" menu. Solver settings have been set.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Inputs</th>
<th>Benchmark sales of the domestic industry</th>
</tr>
</thead>
<tbody>
<tr>
<td>Qs0</td>
<td>4722</td>
<td>Benchmark sales of the domestic industry</td>
</tr>
<tr>
<td>Qd0</td>
<td>7936</td>
<td>Benchmark total sales (domestic origin and imported)</td>
</tr>
</tbody>
</table>

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<thead>
<tr>
<th>Variable</th>
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</tr>
</thead>
<tbody>
<tr>
<td>Es</td>
<td>0.5</td>
<td>Es: Elasticity of domestic supply</td>
</tr>
<tr>
<td>Ed</td>
<td>-2</td>
<td>Ed: Elasticity of demand</td>
</tr>
<tr>
<td>Ems</td>
<td>10</td>
<td>Ems: Elasticity of import supply</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Variable</th>
<th>Inputs</th>
<th>Benchmark sales of the domestic industry</th>
</tr>
</thead>
<tbody>
<tr>
<td>t0</td>
<td>18.00%</td>
<td>Initial tariff</td>
</tr>
<tr>
<td>t1</td>
<td>10.00%</td>
<td>New tariff</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Variable</th>
<th>Inputs</th>
<th>Benchmark sales of the domestic industry</th>
</tr>
</thead>
<tbody>
<tr>
<td>w0</td>
<td>0.00%</td>
<td>Initial foreign-held quota price wedge</td>
</tr>
<tr>
<td>w1</td>
<td>0.00%</td>
<td>Final foreign-held quota price wedge</td>
</tr>
</tbody>
</table>

Calibrated values

note: initial domestic price index equals 1.0

<table>
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<tr>
<th>Variable</th>
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</tr>
</thead>
<tbody>
<tr>
<td>Ks</td>
<td>4.7E+03 Ks: domestic supply constant term</td>
<td></td>
</tr>
<tr>
<td>Kd</td>
<td>7.9E+03 Kd: total demand constant term</td>
<td></td>
</tr>
<tr>
<td>Kms</td>
<td>1.7E+04 Kms: import supply constant term</td>
<td></td>
</tr>
<tr>
<td>Kmd</td>
<td>3.2E+03 Kmd: import demand constant term</td>
<td></td>
</tr>
<tr>
<td>Md</td>
<td>3.2E+03 Md: Import demand</td>
<td></td>
</tr>
<tr>
<td>Emd</td>
<td>-4.2E+00 Emd: elasticity of import demand</td>
<td></td>
</tr>
</tbody>
</table>

Counterfactual equilibrium price

<table>
<thead>
<tr>
<th>Variable</th>
<th>Inputs</th>
<th>Benchmark sales of the domestic industry</th>
</tr>
</thead>
<tbody>
<tr>
<td>P_In</td>
<td>0.952</td>
<td>Linear domestic price solution:</td>
</tr>
<tr>
<td>P_nln</td>
<td>0.955</td>
<td>Non-linear domestic price solution</td>
</tr>
<tr>
<td></td>
<td>-6.9E-07 non-linear optimization constraint (excess supply), Qd-Qs-M=0</td>
<td></td>
</tr>
<tr>
<td>Pfree_In</td>
<td>0.890</td>
<td>Free trade price (linear)</td>
</tr>
<tr>
<td>Pfree_nln</td>
<td>0.894</td>
<td>Free-trade price (nonlinear)</td>
</tr>
<tr>
<td></td>
<td>0.0E+00 non-linear free trade constraint</td>
<td></td>
</tr>
</tbody>
</table>
We have specified one cell as a problem to be solved given constraints.
Solver options in Excel

- **Max Time:** 100 seconds
- **Iterations:** 100
- **Precision:** 0.000001
- **Tolerance:** 5%
- **Convergence:** 0.001
- **Assume Linear Model**
- **Assume Non-Negative**
- **Use Automatic Scaling**
- **Show Iteration Results**
- **Estimates**:
  - Tangent
  - Quadratic
- **Derivatives**:
  - Forward
  - Central
- **Search**:
  - Newton
  - Conjugate
Over to the computer

- Familiarize yourself with the spreadsheet
- Data for wheat Iran provided in ..\DATA\WheatDataIran.XLS
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Assignments

- Group 1: analyze effect of lower tariff in terms of domestic and import prices, production, trade and welfare
- Group 2: analyze effect of import quota. Provide way to estimate the size of the ‘quota wedge’.
- Group 3: investigate sensitivity of results w.r.t. assumptions on elasticities of demand and supply. How does welfare assessment change if domestic demand is more or less elastic?
Nature of experiment

- Comparative static
- Initial situation $\rightarrow$ new situation

^

shock

Large country model: ToT effects non-zero
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Armingtion preferences

To accommodate two-way trade, we can use the Armington assumption. Under this assumption, imports $x_2$ and domestic goods $x_1$ are used to produce a composite utility good $q$. 

\[
q \leq \rho \alpha x_1 - x_2
\]
Algebra: from first order conditions For CES demands

P is a composite price
P_j is the price of good j
E is total expenditure
k_{si} is a supply function constant term
k_a is a composite demand constant term
\sigma is the elasticity of substitution
\rho = (\sigma - 1)/\sigma
**Inputs**

Non-linear Armington model

| Benchmark sales of X1 (i.e. domestic) industry | 200 |
| Benchmark sales of country 2 imports (targeted by policy change) | 1000 |
| Benchmark sales of country 3 imports | 200 |
| Benchmark total sales (domestic origin and imported) | 1400 |
| NA: Composite elasticity of demand | 1.5 |
| Elasticity of domestic supply | 3 |
| Elasticity of country 2 import supply | 5 |
| Elasticity of country 3 import supply | 5 |
| Elasticity of substitution | \( \sigma \) |
| Initial tariff on country 2 imports | t2,0: 25.00% |
| Initial tariff on country 3 imports | t3,0: 25.00% |
| Final tariff on country 2 imports | t2,1: 25.00% |
| Final tariff on country 3 imports | t3,1: 25.00% |
| Initial export tax/quota price wedge on country 2 imports | w2,0: 0.00% |
| Initial export tax/quota price wedge on country 3 imports | w3,0: 0.00% |
| Final export tax/quota price wedge on country 2 imports | w2,1: 25.00% |
| Final export tax/quota price wedge on country 3 imports | w3,1: 0.00% |

**Calibrated values**

| Ks1: domestic supply constant term | 2.00E+02 |
| Ks2: country 2 import supply constant term | 3.05E+02 |
| Ks3: country 3 import supply constant term | 6.10E+02 |
| KD: composite demand constant term | 1.40E+03 |
| CES weight for domestic good | 0.68 |
| CES weight for country 2 import | 0.93 |
| CES weight for country 3 import | 0.68 |
| P1: calibrated domestic product market | 1.00 |
| P2: calibrated internal price for good 2 | 1.00 |
| P3: calibrated internal price for good 3 | 1.00 |
| P2*: calibrated country 2 border price | 0.80 |
| P3*: calibrated country 3 border price | 0.80 |
| PA: calibrated composite good price | 1.00 |

**Counterfactual solution values**

<table>
<thead>
<tr>
<th>Excess demands</th>
<th>free-trade in X2</th>
<th>internal prices and Y</th>
</tr>
</thead>
<tbody>
<tr>
<td>counterfactual</td>
<td>free trade in X2</td>
<td>counterfactual</td>
</tr>
<tr>
<td>excess demands</td>
<td></td>
<td></td>
</tr>
<tr>
<td>-9.2E-08</td>
<td>0.0E+00</td>
<td>P1: domestic product market</td>
</tr>
<tr>
<td>-8.4E-07</td>
<td>2.3E-13</td>
<td>P2: good 2 market</td>
</tr>
<tr>
<td>-1.6E-07</td>
<td>-5.7E-14</td>
<td>P3: good 3 market</td>
</tr>
<tr>
<td>-2.6E-11</td>
<td>0.0E+00</td>
<td>PA: composite good price</td>
</tr>
<tr>
<td>8.6E-08</td>
<td>0.0E+00</td>
<td>Y: total expenditure</td>
</tr>
</tbody>
</table>
end
Supplementary material

- Theoretical exercises with PE model
Assignment 1

Form 3 groups that each analyze a scenario:
- Import tariff
- Import quota
- Increased foreign capital inflow

Give a justification of your policy:
- What are the benefits for the country?
- What are the costs for the country?
Impact of an import tariff/tax – Group 1

- Assume world price is lower than domestic price
- Objective is to compensate producers for losses

- Benefits:
  - Producers gain in surplus
  - Government gains tariff revenue

- Costs
  - Consumer surplus decreases
  - Efficiency loss (deadweight losses – triangles)
Impact of an import quota – Group 2

- Assume world price is lower than domestic price
- Objective is to compensate producers for losses

- Benefits:
  - Producers gain in surplus

- Costs
  - Consumer surplus decreases
  - Efficiency loss (deadweight losses – triangles)
  - Administration cost
Effects of increased foreign capital inflow –
Assumed: investment in manufacturing sector
Gr.3

- Direct benefits: Producer Surplus UP; Cons Surplus UP: Total Econ Gain UP
- Indirect benefits:
  - Employment opprtunity UP
  - Foreign Excha. Stock UP
  - Backward linkage: domestic input demand UP
  - Forward linkage: other downstream industry Benefits
- In sum, GNP rise

- Costs:
  - maintaining political-economic stability through:
  - fiscal policy; promotion; law enforcement; etc.