



# The Analytics of the Wage Effect of Immigration

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# 1. The question

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- Do immigrants alter the employment opportunities of native workers?
  - “After World War I, laws were passed severely limiting immigration. Only a trickle of immigrants has been admitted since then. . .By keeping labor supply down, immigration policy tends to keep wages high. Let us underline this basic principle: Limitation of the supply of any grade of labor relative to all other productive factors can be expected to raise its wage rate; an increase in supply will, other things being equal, tend to depress wage rates”
    - Paul Samuelson, *Economics*, 1964.



## 2. The empirical confusion

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- First generation:
  - Friedberg and Hunt (1995): “The effect of immigration on the labor market outcomes of natives is small.” Labor demand curve is perfectly elastic.
- Second generation:
  - Borjas (2003), Mishra (2008). Wage evolution of specific skill groups is correlated with immigration-induced supply shifts. Labor demand curve slopes down, with  $d \log w / d \log L$  around -0.3 to -0.4.
  - Ottaviano-Peri (2005), Card (2009). Imperfect substitution between equally skilled immigrants and natives **and** perfect substitution between unequally skilled workers (high school dropouts and high school graduates) resurrect perfectly elastic demand curve.



## 3. Motivation

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- Take a step back from empirical debate: What does factor demand theory have to say about the potential wage impact of immigration-induced supply shifts?
- Since Marshall's time, economists have understood which factors generate elastic or inelastic labor demand curves, and how the elasticity of labor demand is affected by substitution and scale effects.
- Much of empirical literature on wage impact of immigration (particularly in 1990s) disregarded practically all of these insights, and instead chose data-mining approach.



## 4. Revisiting the theory

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- Goal is to determine the nature of the wage impact of immigration in a model that allows for feedback effects through immigration-induced changes in product demand.
- General equilibrium model explicitly introduces the elasticity of product demand, the rate at which the consumer base expands as immigrants enter the country, the elasticity of supply of capital, and the elasticity of substitution across inputs of production.



## 5. Some implications

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- Factor demand theory constrains the potential sign and numerical value of wage effects. Theoretical results can be used to check plausibility of claims in the literature.
- Under widely used functional restrictions, the impact of immigration on the wage level in the receiving country depends on *completely different* parameters than its impact on the wage distribution.
- If one were in an un-generous mood, the model says that the structural empirical literature has *never* estimated the impact of immigration on the wage level. The “estimated” effects in the literature are instead spewing out restrictions built in by the theory.



## 6. Preliminaries: homogeneous labor, 1-good, closed economy

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- Linear homogeneous aggregate production function,  $Q = f(K, L)$ , with  $f_K$  and  $f_L > 0$ ,  $f_{KK}$  and  $f_{LL} < 0$ , and  $f_{KL} > 0$ .
- Product price is fixed at  $p$ . Assume competitive markets: each input price ( $w$  and  $r$ ) equals its value of marginal product.
- Elasticity of complementarity:  $c_{ij} = f_{ij} f / f_i f_j$
- Short run:  $K$  is fixed
- Long run:  $r$  is fixed.
- Let  $s_L$  be labor's share of income.



## 7. Results from simplest model

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$$\left. \frac{d \log w}{d \log L} \right|_{dK=0} = s_L c_{LL} < 0,$$

$$\left. \frac{d \log K}{d \log L} \right|_{dr=0} = - \frac{s_L c_{KL}}{s_K c_{KK}} = 1,$$

$$\left. \frac{d \log w}{d \log L} \right|_{dr=0} = s_K c_{KL} + s_L c_{LL} = 0,$$



## 8. Results with CES production function

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$$Q = [\alpha K^\delta + (1 - \alpha)L^\delta]^{1/\delta},$$

$$\left. \frac{d \log w}{d \log L} \right|_{dK=0} = -(1 - \delta)s_K.$$

- Suppose production function is Cobb-Douglas (so that  $\delta = 0$ , or equivalently  $\sigma_{KL} = 1$ ). We also know  $s_L = 0.7$ .
- The wage elasticity will be between 0.0 and  $-0.3$ , depending on the extent to which capital has adjusted to the presence of the immigrant influx.



## 9. Homogeneous labor, 2-good, open economy

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- We need to introduce a second good to examine how immigration affects aggregate product demand and prices. If there were only one good in the economy, all units of that good are sold regardless of the price.
- But many ways of introducing second good.
- One factor in determining modeling strategy: If immigration and trade were substitutes, as in Mundell's (1957) classic analysis, there would be factor price equalization across countries. Immigration would have no wage effects and would only alter the distribution of outputs as described by the Rybczynski Theorem. In fact, there is no reason for international migration.



## 10. Extension of CES model

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- Two goods: good  $q$  is produced domestically in a large economy; good  $y$  is imported. Assume price of  $y$  is set in global market; price of  $y$  is the numeraire and set to unity. Immigration and trade are complements since there is complete specialization of goods production.
- Product demand for domestic good  $q$  changes because immigration may have changed the price of the good (encouraging native consumers to change their quantity demanded) and because immigrants themselves consume the product.
- Introduction of supply curve of domestic capital.
- Model has much in common with derivations of Marshall's rules of derived demand.



# 11. Demand for domestic good

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- Each consumer  $j$  has quasilinear utility function and budget constraint:

$$U(y, q) = y + g_j^* \frac{q^\xi - 1}{\xi}, \quad Z = y + pq.$$

- A consumer's demand function for domestic good is:

$$q_j = g_j p^{-1/(1-\xi)},$$

- Quasilinear utility implies that consumer's demand for the domestic product does not depend on income.



## 12. Aggregating consumer demand

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- Three types of consumers: domestic workers ( $C_L$ ), domestic capitalists ( $C_K$ ), and consumers in the “rest of the world” ( $C_X$ ). All consumers have same utility function; weighting factor  $g$  differs among 3 groups.
- Total quantity demanded by domestic consumers ( $Q_D$ ) and foreign consumers ( $Q_X$ ) is then given by:

$$Q_D = (g_L C_L + g_K C_K) p^{-1/(1-\xi)},$$

$$Q_X = g_X C_X p^{-1/(1-\xi)}.$$

Also impose the balanced trade restriction.



## 13. Product market neutrality

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$$Q = C p^{-1/(1-\xi)}, \quad C = g_L C_L + g_K C_K + g_X C_X.$$

- How does an immigration-induced increase in the size of the workforce affect the size of the consumer base?
- Let  $C(L)$  be the function relating (weighted) number of consumers to number of workers.
- Let  $\phi = d \log C / d \log L$ . There is *product market neutrality* if  $\phi = 1$ .
- Non-neutrality:  $\phi > 1$ , immigrants are “conspicuous consumers”. Or  $\phi < 1$ , too much money is sent out in remittances.
- We know nothing about the magnitude of  $\phi$ .



## 14. Summary of model

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- Inverse product demand function  $p = C^\eta Q^{-\eta}$ ,
- Aggregate production function:  $Q = [\alpha K^\delta + (1 - \alpha)L^\delta]^{1/\delta}$ ,
- Inverse supply function of  $K$ :  $r = K^\lambda$ ,



## 15. The wage elasticity

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- Labor markets are competitive. Then:

$$\frac{d \log w}{d \log L} = \frac{-\lambda(1 - \delta - \eta)s_K}{(1 + \lambda - \delta) - (1 - \delta - \eta)s_K} - \frac{(1 + \lambda - \delta)\eta(1 - \phi)}{(1 + \lambda - \delta) - (1 - \delta - \eta)s_K}.$$

- Remarks:
  - The denominator is positive; see footnote 12.
  - The equation is the reciprocal of the traditional Hicks-Marshall equation if  $\phi = 0$ .
  - The second term need not vanish in the long run ( $\lambda = 0$ ). So immigration may have permanent wage effects.



## 16. Special case of $\phi = 1$

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$$\left. \frac{d \log w}{d \log L} \right|_{\phi=1} = \frac{-\lambda(1-\delta-\eta)s_K}{(1+\lambda-\delta) - (1-\delta-\eta)s_K}.$$

- The wage elasticity is 0 in the long run ( $\lambda = 0$ ).
- Let  $\eta^* = 1/\eta$ , and  $\sigma_{KL} = 1/(1-\delta)$ . If there is incomplete capital adjustment, the wage elasticity is negative ONLY IF:

$$\eta^* > \sigma_{KL}.$$

- Same condition that validates Marshall's second rule of derived demand: An increase in labor's share of income leads to more elastic demand "only when the consumer can substitute more easily than the entrepreneur" (Hicks, 1932, p. 246).



## 17. Internalizing the externality

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- The  $w = \text{VMP}$  condition ignores that immigration affects product demand, hence the immigrant's MRP does not equal his VMP. A social planner internalizes this externality and admits the influx that maximizes gross *domestic* product net of any costs.
- $\eta^* > \sigma_{KL}$  is a second-order condition to this problem.
- The wage elasticity must be negative if second-order condition is satisfied (as long as capital does not fully adjust). The scale effect resulting from the immigration-induced expansion in product demand can never be sufficiently strong to lead to a wage increase.



## 18. Non-neutrality

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$$\frac{d \log w}{d \log L} = \frac{-\lambda(1-\delta-\eta)s_K}{(1+\lambda-\delta)-(1-\delta-\eta)s_K} - \frac{(1+\lambda-\delta)\eta(1-\phi)}{(1+\lambda-\delta)-(1-\delta-\eta)s_K}.$$

- The wage elasticity is not zero in the long run if  $\phi \neq 1$ .
- Suppose immigration does not expand the size of the consumer base as rapidly as it expands the size of the workforce (i.e.,  $\phi < 1$ ). The second term is negative and does not vanish as  $\lambda$  goes to zero. There is a permanent wage reduction because there are “too many” workers and “too few” consumers. (Read: remittances).

## 19. Immigration and prices, with product market neutrality

$$\frac{d \log p}{d \log L} = \eta s_K \left( 1 - \frac{d \log K}{d \log L} \right) - \eta(1 - \phi).$$

- Suppose there is product market neutrality. Immigration has no price effect either in the long run, or if product demand is perfectly elastic ( $\eta = 0$ ).
- But immigration must *increase* domestic prices as long as the product demand curve is downward sloping and capital has not fully adjusted
- Intuition: In the absence of full capital adjustment, the immigration-induced increase in domestic product demand cannot be easily met by the existing mix of inputs, raising the price of the domestic product

## 20. The real wage elasticity

- Define the real wage in terms of the domestic price.

$$\begin{aligned} \square \quad \eta^{\square} &= \frac{d \log(w / p)}{d \log L} \\ &= \frac{-\lambda(1-\delta)s_K}{(1+\lambda-\delta) - (1-\delta-\eta)s_K} - \frac{\eta(1-\phi)(1-\delta)s_K}{(1+\lambda-\delta) - (1-\delta-\eta)s_K}. \end{aligned}$$

- Suppose there is product market neutrality. Then:
  - The real wage elasticity is negative.
  - If the production function is Cobb-Douglas, the real wage elasticity is  $-s_K$  in short run ( $\lambda = \infty$ ).



## 21. Marshall's rules redux

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1. The wage effect of immigration is weaker (i.e., less negative) the easier it is to substitute labor and capital.
2. The wage effect of immigration is weaker the more "important" labor is in the production process.
3. The wage effect of immigration is weaker the more elastic the supply of capital.
4. The wage effect of immigration is stronger the more elastic product demand.

And a new rule:

5. The wage effect of immigration is weaker the greater the impact of immigration on the size of the consumer base relative to its impact on the size of the workforce.



## 22. Heterogeneous labor, 2-good, open economy

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- Presence of heterogeneous labor implies that the impact of immigration on the wage of any single group of workers depends on how immigration affects the supply of *every* group of workers. The need arises to reduce the dimensionality of the problem, typically by limiting the types of permissible cross-effects across inputs.
- The need for tractability becomes more pronounced if one wishes to allow for heterogeneous labor in the model of a 2-good, open economy.



## 23. Nested CES simplification

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- Specification has become the “preferred” approach in the literature since its introduction in Borjas (2003). Think of  $L$  as a labor aggregate—an agglomeration of workers belonging to different skill groups.

$$L = [\theta_1 L_1^\beta + \theta_2 L_2^\beta]^{1/\beta}, \quad m_i = \frac{dL_i}{L_i}.$$

- An interesting property:  $d \log L = \frac{s_1}{s_L} m_1 + \frac{s_2}{s_L} m_2 = \bar{m}$ .
- No need to know value of the elasticity of substitution across skill groups to calculate by how much immigration increased the number of efficiency units.

## 24. Heterogeneous labor and aggregate demand

- In the simpler model with homogeneous labor, the inverse product demand function was  $p = C^\eta Q^{-\eta}$ , where the effective number of consumers for the domestic good is  $C = g_L C_L + g_K C_K + g_X C_X$ .
- Use the same market demand function in the heterogeneous labor model, with  $C = C(L)$ . Hence the shifter in the inverse product demand function depends on the efficiency units-adjusted number of workers. Those workers who are more productive and have higher wages “count” proportionately more in the aggregation.



## 25. Summary of model

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- Inverse product demand function  $p = C^\eta Q^{-\eta}$ ,
- Aggregate production function:  $Q = [\alpha K^\delta + (1 - \alpha)L^\delta]^{1/\delta}$ ,
- Inverse supply function of  $K$ :  $r = K^\lambda$ ,
- Armington aggregator of  $L$ :  $L = [\theta_1 L_1^\beta + \theta_2 L_2^\beta]^{1/\beta}$ ,



## 26. Marginal productivity condition

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$$w_i = \left[ (1 - \alpha) C^\eta Q^{1-\delta-\eta} L^{\delta-1} \right] \theta_i L^{1-\beta} L_i^{\beta-1}.$$

- The bracketed term is *identical* to the VMP of labor in the homogeneous labor case. The fact that there are now two different skill groups simply adds the multiplicative term to the right of the bracket.
- Let the bracketed term equal  $w$ .

$$\begin{aligned} d \log w_i &= d \log w + (1 - \beta) d \log L + (\beta - 1) d \log L_i, \\ &= d \log w + (1 - \beta)(\bar{m} - m_i). \end{aligned}$$



## 27. Distributional wage effect

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$$d \log w_i = d \log w + (1 - \beta)(\bar{m} - m_i).$$

$$d \log w_1 - d \log w_2 = -\frac{1}{\sigma_{12}}(m_1 - m_2).$$

- The distributional effect does not depend on any of the parameters that determine the impact of immigration on the wage *level*.
- Proportionality between relative wage effects and supply shifts suggests that one should be skeptical when evaluating evidence reported in the literature. It is easy to manipulate results by defining skill groups in ways that either accentuate the relative supply shift or that mask it. See Borjas, Freeman, and Katz (1997).



## 28. Mean wage effect

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$$d \log \bar{w} = \frac{s_1 d \log w_1 + s_2 d \log w_2}{s_L} = d \log w.$$

- The impact of immigration on the average wage in a model with heterogeneous labor is *identical* to the impact of immigration on the wage in a model with homogeneous labor. In the simplest Cobb-Douglas world, it must still be the case that the wage elasticity of immigration is between  $-0.3$  and  $0.0$ .
- The mean wage effect is “pre-determined” (by the values of  $\lambda$ ,  $\delta$ ,  $\eta$ ,  $\phi$ , and  $s_L$ ), and is independent of whatever complementarities may or may not exist among labor inputs in the production process.



## 29. Some more implications

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- Complementarities among skill groups simply “place” the wage effect for each of the groups around this pre-determined mean wage effect.
- Simulations in literature claim that estimation of *structural* factor demand models implies that the impact of immigration on the wage of “average” worker is  $x$  percent. These simulations typically use a Cobb-Douglas. This assumption builds in *numerically* what the mean wage effect must be. The “estimated” wage effects have nothing to do with the data; they are simply “spewing out” the constraints imposed by factor demand theory.

## 30. Within-group imperfect substitution (Ottaviano-Peri)

- Inverse product demand function  $p = C^\eta Q^{-\eta}$ ,
- Aggregate production function:  $Q = [\alpha K^\delta + (1 - \alpha)L^\delta]^{1/\delta}$ ,
- Inverse supply function of  $K$ :  $r = K^\lambda$ ,
- Armington aggregator of  $L$ :  $L = [\theta_1 L_1^\beta + \theta_2 L_2^\beta]^{1/\beta}$ ,
- Armington aggregator of  $L_i$ :  $L_i = [\rho_N N_i^\gamma + \rho_F F_i^\gamma]^{1/\gamma}$ ,



## 31. Marginal productivity conditions

$$w_i^N = \left[ (1 - \alpha) C^\eta Q^{1-\delta-\eta} L^{\delta-1} \right] (\theta_i L^{1-\beta} L_i^{\beta-1}) (\rho_N L_i^{1-\gamma} N_i^{\gamma-1}),$$

$$w_i^F = \left[ (1 - \alpha) C^\eta Q^{1-\delta-\eta} L^{\delta-1} \right] (\theta_i L^{1-\beta} L_i^{\beta-1}) (\rho_F L_i^{1-\gamma} F_i^{\gamma-1}),$$

- Within-group imperfect substitution adds another multiplicative term to VMP conditions. The bracketed term still represents the “average” wage in the economy, aggregated across all skill groups. This is the wage level determined by the factor demand theory parameters. The product of this bracketed term and the first multiplicative term gives the mean wage for group  $i$ .



## 32. Implications

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- Within-group inequality depends on elasticity  $\sigma_{NF}$ , but nothing else does.

$$d \log w_i^M - d \log w_i^N = -\frac{1}{\sigma_{NF}} f_i.$$

- Impact of immigration on average wage of group  $i$ :

$$d \log \bar{w}_i = d \log w + (1 - \beta)(\bar{m} - m_i),$$

- Impact of immigration on average wage in labor market:

$$d \log \bar{w} = d \log w.$$



## 33. A lesson from the nested CES

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- Wage impact at a particular level of nesting depends only on the elasticities that enter the model at or above that level, and does not depend on any of the elasticities that enter the model below that level.
- So the impact of immigration on the aggregate wage does not depend on the value of the elasticity of substitution across skill groups or on the presence or absence of within-group complementarities.
- The impact of immigration on the wage of a particular skill group is unaffected by within-group complementarities between immigrants and natives.



## 34. Conclusion

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- Framework reveals that the effect of immigration on mean wage depends on parameters Marshall first identified in his rules of derived demand. *Plus* one new parameter: the impact of immigration on the size of the consumer base relative to its impact on the size of the workforce.
- Short-run wage effect of immigration is negative in a wide array of possible scenarios. Even long run effect may be negative if immigration increases size of consumer base by less than it increases size of workforce.
- A lot of unfinished business: variety of domestic goods (labor-intensive, non-tradeable); how does immigration affect the size of the consumer base?



## 35. One last thought

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- A disconnect between factor demand theory and some of the empirical literature. The theory predicts that the short-run wage effect of immigration will be negative and numerically sizable in many plausible scenarios, even after accounting for feedback and scale effects.
- But some claim that immigration wage effects are negligible *even in the short run*. If claim is correct, theoretical implications of factor demand theory need to be dismissed. We are left without framework for understanding or predicting how immigration influences labor markets in sending and receiving countries