The increased interest in weather-based risk management tools in developed and developing nations gives rise to examining a new set of risk-contingent structured financial products. This paper examines a variety of models applicable to agriculture and the sovereign debt of developing agrarian nations including from the corporate side, weather-linked bonds, and from the producer side weather-linked loans and weather linked mortgages. These weather risk management tools are targeted towards mitigating both business and financial risk by reducing the contractual obligation of debt (principal and/or interest) depending on the intrinsic value of an attached weather option (e.g. excess heat or precipitation) which pays off if a specific weather event occurs. The paper also discusses the structure of a famine bond, which provides funds in advance of a high probability famine event brought about by a specific weather event such as drought. The models presented have broad application to agricultural economies that suffer from weather induced volumetric (production) risk.
Weather-Linked Bonds

Introduction

The blending of business and financial risk faced by agricultural producers has recently been discussed in the context of commodity-linked bonds (Turvey 2006, Jin and Turvey 2002). These instruments link the payoff from a derivative such as a forward contract, futures contract, or option to the repayment covenants of a loan or bond (Attah Mensa 1992). It is economically obvious that the relationship between downside commodity price risk and loan default are financially linked. If the two are closely linked at either the firm or project level, then any action designed to mitigate commodity price risk will also reduce the risk of bankruptcy. In turn, bond yields and discounts will fall.

In recent years there has been a significant interest in weather derivatives\(^1\) based on precipitation or heat (Turvey 2001, 2005; Martin et al. 2001; Richards et al. 2004). Some early research included drought insurance pricing or demand with outcomes largely tied to crop yields (Bardsley et al. 1984; Patrick 1988; Quiggen 1986; Sakurai and Reardon 1997). Turvey (2001) identified a number of heat and rainfall products and illustrated how premiums could be calculated using historical probabilities. This was among the first that posited the notion that the weather itself be insured and that risk management can be obtained without measuring loss. Martin et al. (2001) also provide an actuarial model for computing precipitation insurance for agricultural crops while Richards et al. (2004) examine pricing issues related to heat-based derivatives in relation to fruit crop production in California.

However, these papers have examined for the most part the pricing relationship of the option value of weather risk and have not tied this risk to the management of financial risk and debt instruments. A recent exception is Vedenov et al. (2006) who make the case for the use of catastrophe bonds\(^2\) as a mean to reinsure losses from agricultural crop

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1 Geman (1999), Dischel (2002) and Jewson and Brix (2005) offer great review on climate risk, the weather market, weather derivatives and pricing.
2 Using Georgia’s historical state-level cotton yield data, they conclude that the proposed CAT bond based on percent deviations of realized state average yields relative to the long-run average demonstrates potential as risk transfer mechanisms for crop insurance company.
insurance. Also related is the work of Turvey (2006) who makes the case for commodity-linked bonds as a mean to contractually tie the payoff of a bond or other credit instrument to a put or call option on the price of an underlying commodity. Advantages of packaging hedging tools to the debt instruments are widely appreciated in the literatures. The agency relationship is explicit. And is bolstered by O’Hara (1990) who demonstrates that conventional loans can be Pareto dominated by financial contracts explicitly incorporating characteristics of the borrower’s product market, identifies conditions under which commodity-linked debt is desirable and when more complicated revenue-linked loans are optimal, and shows how the type of lending contract can have real effects on production decisions. Miralloc and Smith (2003) have examined how firms jointly determine financing, hedging and investment decisions. They argue that optimum leverage reflects the tradeoff between under and over investment and show that hybrid debt financing (a commodity-linked bond with a linked forward contract) can reduce agency costs and incentives to over and under investment. This can increase firm value relative to standard debt financing. Consequently, one of the advantages to issuing a commodity-linked bond is that it pre-commits the firm to a hedging strategy for the life of the bond in a business and legal environment where covenants to using futures, options or swaps are often prohibited. Although the topic under investigation in this paper is weather-based, the intuitive logic is that by pre-committing to a risk management strategy, within the covenants of a bond, investors can ensure that they are compensated for whatever risks are implied by the weather hedge.

Turvey (2006) also defends the use of commodity-linked bonds in terms of the balance between business risks and financial risks. The relationship between employing risk management strategies and financial leverage is well developed. Turvey and Baker (1989, 1990), Mello and Parsons (2000) show that a firm with no debt gains little from hedging its price risks because the agency costs of debt are reduced or zero. However, a firm that is highly leveraged will find significant economic benefits if the source of business risk such as weather can be managed, and the degree by which benefits accrue is directly related to the degree of leverage. This transcends clearly into the notion of risk balancing (Collins 1985; Featherstone et al. 1988), the leverage effect of beta in the

More generally, managing weather risk involves a transaction that shifts risks from states in which the opportunity costs of liquidity are high to those in which the opportunity costs of liquidity are low (Mello and Parsons 2000). In this sense the purpose of hedging weather risk is to improve liquidity, reduce financial distress and the costs of external financing, and make value-maximizing investments affordable. In addition, maintained liquidity provides the flexibility to undertake and plan future investment opportunities (Mello and Parsons 2000) and higher firm value. It has been argued by Mayers and Smith (1987) and Morrelec and Smith (2003) that risk management policies also allow the firm to control the underinvestment incentives associated with debt financing by increasing the number of states of nature in which shareholders are residual claimants.

This paper examines the use of weather-linked credit instruments in a variety of models applicable to agriculture in developed and developing agrarian countries, in which weather is among the most important determinants of economic activities and so of their debt repayment capacity. The weather risk management instruments are targeted towards mitigating both business and financial risk by reducing the contractual obligation of debt depending on the intrinsic value of attached weather options. For agribusiness and corporate finance, we propose weather-linked bond that allow firms to hedge part or the entire periodic cash flow requirements to service debt using weather options. The idea can also be applied to agricultural finance in the forms of weather-linked farm mortgage or operating loan, of which the repayments can be contingent upon the performance of weather variable. More interestingly, in development finance, we also consider the application of weather-linked bond as a new form of sovereign debt given to low-income agrarian economies that can provide some form of weather hedging opportunity for the life of the sovereign debt, and the one so called famine bond, which can provide timely and more efficient way for actors in the international aid community to combat famine in the poor and vulnerable countries.

The less developed countries (LDCs) have for years been faced with large outstanding sovereign debt impeding them from economic growth. Recent research by
the World Bank and IMF on an operational framework for debt sustainability in LDCs identified exogenous shocks as one of the three factors strongly affecting the risk of debt distress. Among other shocks—such as terms-of-trade shocks, weather and natural disaster shocks have large impact on growth and fiscal balances in LDCs, of which earnings and subsistent livelihoods of the bulk of the populations depend primarily on agriculture. Statistically, LDCs are more frequently and more severely affected by climate disasters (floods, droughts, extreme temperatures and wind storms) than other developing countries, and such disasters have been growing in frequency and severity. Climate shocks tend to have the strongest impact in LDCs that are more indebted, small and have weaker policies and institutions. These shocks affect LDCs by lowering agricultural output and fiscal revenues while, at the same time, increasing spending needs for delivery of public resources, social programs and safety nets, thus weakening repayment capacity for sovereign debt obligations. With limited capacity to smooth the fiscal balance, the risk of sovereign debt distress can, therefore, increase during and after shocks. These fiscal pressures further force countries to lower long-term investment in physical and human capital, which in turn, impacts growth. Therefore, through their impacts on LDCs, climate shocks also affect official creditors by increasing credit risk thus weakening portfolio quality, and making the environment for new lending more difficult. And as the exogenous weather shocks are significant predictors of debt distress for LDCs, financial instruments that can mitigate sovereign debt risk of these shocks, thus, should benefit LDCs and official creditors alike, and should be in the interest of the international community, whose goal is to promote debt sustainability and economic growth in LDCs.

Theoretical literatures suggest that financial instruments that can effectively help LDCs manage the exogenous shocks can potentially contribute toward increasing economic efficacy in the sovereign lending. With better managed risk exposure, LDCs can concentrate more on investment and productivity growth, which in turn, makes interests of LDCs and official creditors coincide. Many financial instruments are

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3 The other two factors are the debt burden and the quality of policies and institutions. See World Bank and IMF, “Debt sustainability in Low Income Countries: Further Considerations on an Operational Framework and Policy Implications”, September 10, 2004, IDA/SecM2004-0629/1.

considered widely by academics and practitioners. Among others, contractual concessional lending, which smooth the debt service requirements in accordance with debtor’s capacity to pay by indexing the debt repayment to debtor’s major economic indicators such as real GDP growth, major commodity prices or real exchange rates, are intensively scrutinized. Results from many studies have shown that while in principle, there are gains to be realized through risk-sharing between official creditor and LDC debtor, there are, however, practical problems including moral hazard and adverse selection at the country level as well as other operationalization issues for further consideration.

Other hedging and insurance instruments that could potentially be used by creditors to manage shocks in LDCs are also widely considered for the two prominent shocks-- terms-of-trade shocks, and weather and natural disaster shocks. To hedge against terms-of-trade shocks from commodity price fluctuation, official creditor may play facilitating roles in helping LDCs establish and pool multi-commodity hedging instruments; they can attach hedging instruments such as commodity call, put options, future contract, other exotic options or even credit default swaps to the standard concessional credit. The potential to attach hedging instruments to the market-based debt instruments has recently been discussed in the context of commodity-linked bonds (Turvey 2006, Attah-Mensa 2004, Jin and Turvey 2002, among other in the past). These commodity instruments, however, could not insure against other commodity-related shocks that affect a country’s repayment capacity, such as crop yield and climate shock affecting quantity of output.

The recent significant market interest in weather derivatives results in many newly developed hedging and insurance instruments for weather and natural disaster shocks—mostly come in two forms: insurance against natural disaster or the (non-catastrophic) weather insurance. Recent innovations in developing countries in which ‘index’ insurance has been used or proposed (Hess, Richter and Stoppa 2002; Stoppa and

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5 Other exogenous shocks include shocks to the availability of financial resources and shocks caused by conflict and civil unrest in neighboring countries, which are policy oriented and already have specific coping strategies (Varangis et al. 2004).
Hess 2003; Skees et al. 2005, 2006; Hazell and Skees 2006; Gautman et al. 1994) opens up the opportunity to investigate the use of weather-linked bonds in terms of World Bank or International Monetary Fund loans to developing countries. From a conceptual base, the repayment of sovereign debt of developing agrarian economies is largely conditioned on specific weather events such as drought or flood. The risk attached to third world debt is high, default is not infrequent, and for countries that do not default on sovereign debt, the impact on national treasuries can be severe. Loan forgiveness is often tied to political motives or other economic relationships, but forgiveness of third world debt is not taken lightly because of the cost transfer to the taxpayers of donor nations. Krugman (1988, 1989), Sachs (1988), Dornbusch (1988) and Kenen (1990) support the claim that partial forgiveness could conceivably make both debtors and creditors better off. A debt-overhang LDC could increase its economic efficiency and consequently its real income, which would in turn lead to a reduction in its default risk. Despite the apparent merit of this logic, there has not been much debt forgiveness to date. The moral-hazard effect of debt forgiveness is, however, pointed out by Froot, Scharfstein and Stein (1989). They argue that the amount of relief required to induce investment in the LDCs may depend on a variety of factors, some of which may be known only by the borrowing countries. To resolve this problem, they suggest that indexation scheme that links loan repayment to indebted country’s exogenous variables could help distinguish the appropriate amount of relief on a case-by-case basis. O’Hara (1990), as previously discussed, has shown how the type of lending contract can have real effects on production decisions. Atta-Mensah (2004) further argues that in the event of sovereign debt default, substantial bankruptcy, legal and renegotiating costs incurred, and new uncertainties introduced constitute dead-weight losses (as opposed to simple wealth transfer) to the parties involved in the contract. Thus derivative securities may serve to minimize these dead-weight losses, in that state-contingent payments may be tailored to the risk preferences of either borrower or lender, which would avoid these transaction costs and so would minimize the probability of default.

7 They found that indexing to endogenous variables (e.g. country’s output or export) has negative moral-hazard effect on investment.
The growing interests of academics and practitioners in expanding weather innovations and markets to developing countries\(^8\) have provided opportunity to mitigate climate shocks in LDCs. In principle, these weather hedging instruments could be offered by official creditors or development partners and partly or fully reinsured by private insurers or investors in the international markets. Furthermore, pooling these weather insurance policies across countries or regions can promote risk diversification, which can substantially reduce premium costs. These, therefore, could suggest the possibility for official creditors to attach weather insurance or hedging instruments to the standard concessional credit or more interestingly to the market-based debt instruments.

This paper takes an alternative look at the problem and asks under what conditions might loan forgiveness take place and at what cost, not to the donor nation, but to the developing nation. Conceptually, the World Bank or any other donor nation could negotiate a rider to a bond issued to an agrarian economy that will provide part or all of a bond repayment depending upon a single, multiple, or aggregate measure of a weather event (e.g. precipitation or heat). We will focus on non-catastrophic events that damage only the agricultural production of the recipient nation so although the weather-linked bond is related to catastrophe bonds, they are structurally different. In return, the recipient country will pay a higher yield on donor bonds, but the nature of the contingency is such that interest and principle is reduced or eliminated should the specified weather event happen.

The increasing in frequency and severity of the adverse covariate shocks (major droughts, floods, cyclones, earthquakes, tsunamis, etc.) that lead to the incidence of humanitarian catastrophe in the vulnerable LDCs constitutes to the raising demand for emergency aids. As insurers of last resort for those LDCs, actors in the international aid community potentially face two critical concerns. First, their available contingent funds are limited, yet needed to be distributed efficiently and equally across all possible catastrophe events. And second, the timing of humanitarian assistance has more critical consequences in the developing world. Most of the time, the extra aid funds are raised from the donors after the loss has occurred. The traditional famine relief program using

\(^8\) World Bank (2005), Managing Agricultural Production Risk: Innovations in Developing Countries provides excellent review.
food surpluses from developed countries is proved inefficient and very cost-ineffective. The aids can come too late, or tremendous transportation costs must be added to speed up the process, and it can be too difficult and costly to distribute.\textsuperscript{9} Therefore, the challenging goal for the aid agencies and other international organizations is to allocate their limited resources to ensure timely and cost-effective emergency response system that can minimize the short-run suffering of the vulnerable population as well as the long-run effect to their human capital. The growing interest in catastrophe risk-sharing and innovative weather derivatives in developing countries have stimulated policy research on the design of disaster aid finance system (Syroka 2006, Alderman and Haque 2006) to conceptualize the financial efficiency gain through the use of (market) weather-based risk transfer instruments, which may promise several advantages including more timely and predicable aid in times of crisis, risk price information for sound development portfolio decisions and great dignity for the beneficiaries. Moreover, the World Bank has started to think of the international aid financing in term of global system pooling catastrophe risks all around the world to take advantage of global diversification.

The predominant risks faced by vulnerable populations in LDCs especially in sub-Saharan Africa are meteorological in nature—particularly drought and risk of excess rainfall, flooding and tropical cyclones in some coastal regions. A network of weather stations and satellites continually report data detailing the meteorological and agricultural conditions through out the continent, which can be used to track events that impact vulnerable populations—abnormally high rainfall in localized areas, meteorological conditions that impact agricultural production or even the outbreak of malaria epidemics (Syroka 2006, Thomson et al. 2006). These data describing the risk faced by vulnerable populations has been collected for several years. High resolution satellites have been consistently monitoring rainfall and other related indicators in sub-Saharan Africa since 1995, complementing the low-resolution satellites used since 1979. Weather data has been collected by National Meteorological Services in a consistent manner for over thirty years in most African countries under the guidance and control of the UN World Meteorological Organization. The availability of quality history data thus allows for new

\textsuperscript{9} Barrett and Maxwell (2005) offers an excellent review of food aid in supporting strategy to reduce poverty and in responding to food insecurity in the low-income, food deficit countries.
development of efficient methods of emergency need assessments through many forms of early warning systems\textsuperscript{10}, which aim mainly to convert meteorological information into meaningful indicators and/or predictors for agriculture, food insecurity and catastrophic events. Developing market-based risk transfer products based on valuable historical information from such early warning systems may open up a new opportunity and provide possible alternative ways to manage emergency humanitarian aids for actors in the international aid community, enabling financial preparation for catastrophe events that may happen in the future, allowing effective distributional planning and decision-making resource to ensure the ultimate goal of timely and cost-effective emergency response.

In this paper, we investigate this possibility in the general context of famine bond as a market-based famine relief financing. Conceptually, based on the long historical data used in many early warning systems, if we can develop a powerful empirical model that can (to some extent) accurately forecast the degree of food shortages\textsuperscript{11} and the corresponding expected aid needed by the vulnerable populations, based on a set of climate variables (which may be observed over a specific period of time), we can identify the critical set of these variables that triggers specific famine relief, which can be in a variety of forms. We can consider a bond structure similar to the weather-linked bond that automatically releases the LDCs from repayment of the loan once the threshold set of weather is triggered. Alternatively, the actors in the international aid community can pre-finance the famine emergency relief program by purchasing insurance from the international insurers for LDCs that will provide lump sum cash indemnity directly for

\textsuperscript{10} The USAID’s Famine Early Warning Systems Network (FEWS-NET) can provide expected status of the dominant crop grown by populations in various agricultural areas given the quantity and distribution of rainfall that has been received so far in an agricultural season, using satellite data. The Livestock Early Warning System (LEWS) can provide reliable estimates of the deviation below normal up to 90 days prior to severe food insecurity crisis. Other related early warning systems include the World Food Program’s project on Strengthening Emergency Needs Assessment Capacity (SENEAC), the USAID’s Global Livestock Collaborative Research Support Program (GL-CRSP), and the Livestock Information Network and Knowledge System (LINKS). Another satellite data that is widely used in the EWS generated by LEWS and LINKS is the Normalized Difference Vegetation Index (NDVI), which directly monitors crop production by observing biomass.

\textsuperscript{11} These can be in the form of percentage deviation below the normal level, the degree of malnutrition in children, deviation of domestic food prices and/or productions etc.
the anticipated famine relief whence the threshold is triggered,\textsuperscript{12} and the indemnity can also increase with the severity of the weather event, specified in the insurance contract. We can also imagine a promise or an option sold to the LDCs that whence triggered, it triggers an automatic loan with fixed and pre-established terms from any of the international organizations so that sufficient cash would be available to combat famine in the LDCs. And of course, the amount of loan can also increase with the severity of the weather event.\textsuperscript{13} Many forms of weather derivatives based on famine early warning systems will be considered in the next section.

The Theoretical Framework for Pricing Weather-Linked Bonds

As a starting point we treat the structure of a weather-linked bond (WLB) as a modification of the commodity-linked bond (CLB) discussed and presented in detail by Atta-Mensah (1992), Schwartz (1982, 1987), Carr (1987), Gibson and Schwartz (1990), Miura and Yamauchi (1998), Milterson and Schwartz (1998) and reviewed by Turvey (2006). A general structure of CLB that we believe can be adapted to numerous forms of WLB has been provided following the structure outlined in Milterson and Schwartz (1998), Harrison and Kreps (1979) and Harrison and Pliska (1981). The range of WLBs that we present are applicable to a range of institutions in developing as well as developed economies.

The most general form of a weather insurance product is given by

\textsuperscript{12} The World Food Program has recently entered into the first-ever humanitarian aid weather derivative contract with a leading European reinsurer, AXA Re. This derivative costing US$930,000 is based upon a calibrated index of rainfall data gathered from 26 weather stations across Ethiopia, and provides contingency funding up to US$7.1 million in case of an extreme drought during Ethiopia’s 2006 agricultural season. Specifically, the payout will be triggered when data gathered over a period from March to October indicates that rainfall is significantly below historic averages, pointing to the likelihood of widespread crop failure.

\textsuperscript{13} Noticing that the market-based risk transfer instruments such as the suggested famine bonds can ensure timely and predictable financial resource availability in the time of crisis. How these lump sum cash can effectively be distributed and used to combat famine is another interesting research question and is beyond the interest of this paper. However, as the triggered famine bond make these cash available well-before the occurrence of the crisis, it should allow for timely and effective planning for both aid agencies and local governments of those vulnerable LDCs (either to import food from the neighboring countries or transport the surpluses from countries farther away, etc.).
Note that we are using the notation $W(t)$ on the weather variable. We do this to indicate that the measured weather variable need not be a linear function of the natural weather process $W_t$. Asian and other options on the average as well as dependencies such as heating, cooling or growing degree days can all be considered for a WLB. Note also that we are discounting the terminal value at period $T$ by $e^{-\lambda \sigma t}$, which accounts for the market price of risk often associated with non-tradable or non-hedgable risk. Finally, since the weather variable and the strike value $K$ are measured in physical rather than currency units we need to include the parameter $\psi$ to obtain a currency (e.g. convert degree-days to dollar) denominated option. The scaling parameter $\psi$ may also take into consideration the number of weather options required to cover the face value of the bond ($F$).

Consider for simplicity, a bond embedded with a weather call option, of which the promised payment at maturity is, thus, equivalent to the face value of the bond $F$ for sure plus a call option at strike $K$. If there is a risk of bankruptcy (Schwartz 1982, Carr 1987, Miura and Yamauchi 1998), then bondholders receive the promised payment if the value of the firm $V_T$ is greater than that amount or they will take over the firm in case of default. Therefore, the boundary condition at maturity of this bond can be expressed as:

\[
B(t,T) = \text{Min}[V_T, F \pm \psi \text{Max}(0, W(T) - K)].
\]

Also complicating pricing issues is the fact that the value of bond is equal to the present

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15 The symbol $\pm$ in the boundary condition reflects the fact that the bond can be embedded with a long position of a call option (+) or a short position of that call (-).
value of coupons plus the present value of the payoff. Following the structure outlined in Milterson and Schwartz (1998), Harrison and Kreps (1979) and Harrison and Pliska (1981), the idea is to calculate the discounted expected value of payment directly and so

\[
B(t,0) = E \left[ \int_0^T e^{-\int_0^T f(s,s)ds} cdv \right] + E \left[ e^{-\int_0^T f(s,s)ds} \text{Min}\left[V_T, F \pm \psi \text{Max}(0, W(T) - K_W)\right]\right]
\]

where \(\int_0^T f(s,s)ds\), \(v = t, T\) is the adjusted discount rate which can include convenience yield and interest rate risk under a variety of assumptions including the market price of risk of the non-traded weather variable\(^{16}\) and \(c\) is the coupon payment. In the simplest of cases with zero default risk (exclude \(V_T\), no convenience yield, a weather process that is Gaussian, stationary and self-similar over the time domain (e.g. a geometric Brownian motion) and no interest rate risk, the value of a weather-linked bond with constant coupon rate is given by

\[
B(t,0) = \frac{c}{r} (1 - e^{-rT}) + Fe^{-rT} \pm \text{Min}(F, V_w^c)
\]

where

\[
V_w^c = \psi W(0) N \left\{ \frac{\ln\left( -\frac{W(0)}{K_w} \right) + \left( r + \frac{1}{2} \sigma_w^2 \right) T}{\sigma_w \sqrt{T}} \right\} - \psi K_w e^{-rT} N \left\{ \frac{\ln\left( -\frac{W(0)}{K_w} \right) + \left( r - \frac{1}{2} \sigma_w^2 \right) T}{\sigma_w \sqrt{T}} \right\},
\]

where \(r\) is the instantaneously riskless rate of interest and \(\sigma_w\) is the measured volatility of the weather process. This expression is exactly the same as the results obtained by Schwartz (1982) and Atta-Mensah (1992), and is similar to the one used in Jin and Turvey (2002) except in this case the payout is based on the evolution of a weather event over time rather than a commodity. To reduce this further, note that if \(F = 0, c = 0\) (no bond), and interest rate \(r\) (constant), then terminal cash flows are discounted by \(e^{-rT}\) and

\(^{16}\) See Richards et al. (2004) and Turvey (2005).
(4) reduces to the standard Black (1976) model for pricing options on futures and so the non-tradable commodities. Turvey (2005) has argued that at least in his instance the weather index for degree-days satisfies the assumption of Brownian motion and that the market price of risk is zero because of diversifiable risk. In other words the simplest of structured products is simply the sum of the present value of the cash flow from the bond investment plus the option value of the weather linkage. Finally, note that we retain the $\text{Min}(.)$ operator in (4) to take into account a structure that disallows a payout from the weather option to exceed the face value of the bonds. This is included more as a matter of practicality than economic structure. If the $\text{Min}(.)$ operator is removed then there exists a probability in the event of a severe catastrophe that the payout on the weather option can exceed the face value of the bonds.

**Economics and Scaling**

We turn now to the meaning of the scaling parameter $\psi$ which converts the measure of degree or precipitation to a dollar unit. It can also be broken down into a quantity value as well as the scaling value. The value of the option relative to the bond is an important economic criteria as is the timing and sequencing of the option payoff relative to the cash flow associated with the bond. In the most general instance the option payoff will be tied to the face value of the bond, but it can also be tied to the coupon payments and/or the required sinking fund for disposing of the bond at expiry. Regardless, the nature of the option is to mitigate downside weather related risks that could jeopardize bond or coupon repayment.

There are, of course, any numbers of variations to consider and it is, therefore, worthwhile to consider some variants in order to establish some general rules or guidelines. The broadest categorization establishes whether the bond will be sold at a premium or a discount. A premium bond will provide a weather option as an incentive to the bondholder. In this case the issuer will pay in excess of the bond value if specific weather events are favorable to the firms business. The premium can be in the form of a call or a put, depending on the weather variable of interest. A discount bond is one in
which the bondholder is willing to accept less than par in the event of an adverse weather outcome. Because there is the risk that the bond will not be paid in full, the bond will be issued at a discount. The symbol ± in equations (2)-(5) reflects these structures with ‘+’ indicating the value of a bond sold at a premium and ‘−’ a bond sold at a discount.

As equation (4) is written, it suggests that when the discount (premium) bond matures there is an option effective at the maturity date that will if exercised reduce (increase) the face value or coupon obligation of the issuer. In contrast, we can also consider a simple variant in the following model

$$B(t,0) = \frac{c}{r}(1-e^{-\tau T}) + Fe^{-\tau T} \pm \int_0^T \int_{-\infty}^\infty e^{-t\tau + \sigma \sqrt{t}} \max(0, K_w - W(t))g(W)dw dt,$$

which provides a payout on the option for each year of the bond’s life. What value should be put on the option?

It is typical that a bond issuer will require that a sinking fund be established so that in each year some proportion of the bond’s face value can be retired. Typically the cash required in each year will be $S_t = \frac{F}{T}$, where $T$ is the life of the bond. In addition if the bond pays a periodic coupon then an additional amount of cash flow $c$ of the fixed coupon payment will also have to be paid each year. Thus, the total cash required to pay for a bond on an annual basis is

$$C_t = c + \frac{F}{T}.$$

We now consider the use of the loan principal $F$, which is assumed to have purchased some form of capital $K$. The economic value added (EVA) return from $K$ must, on expectation, generate sufficient cash flow to satisfy (6) and so

$$r_K \geq \frac{c + \frac{F}{T}}{K}.$$
Note that the right hand side is an agency restriction, but the left hand side is a random variable of EVA return on capital $r_k$. We can write the expected EVA return as a weather dependent random variable

(8) \[ E[r_k] = \int_a^b r_k(W_i) g(W_i) dW_i. \]

And by setting (7) as a strict equality and defining $r_k^* = \frac{c + F}{T/K}$, we can consider a triggered weather point $W^* = g^{-1}(r_k^*)$, beyond which the EVA return will meet the agency cash requirement and rewrite (8) as

(9) \[ E[r_k] = \int_a^{W^*} r_k(W_i) g(W_i) dW_i + \int_{W^*}^b r_k(W_i) g(W_i) dW_i, \]

where the first term on the right represents the downside risk, of importance in pricing a discount bond. The second term reflects those weather states in which EVA is sufficiently high to meet all agency cash requirements and is of primary importance to the premium bond.

We are, in principle, concerned with adversity, so the major focus herein is on the value of a discount bond, in which the bondholder accepts the risk of an adverse weather outcome. To avoid these agency costs associated with adversity, we consider a contingent claim with the following indemnity structure, $\psi \text{Max}(0, W^* - W(t))$, so that in the event of an adverse weather where $W(t) < W^*$, a quantity of weather equivalent to $W^*$ is returned to the investment. Hence, by substituting $W^* = W(t)$ into (9), we have the new expected EVA return obtained with this contingent claim as

(10) \[ E[r_{K,W}] = r_k(W^*) \int_a^{W^*} g(W_i) dW_i + \int_{W^*}^b r_k(W_i) g(W_i) dW_i. \]
Subtracting (9) from (10), and multiplying by $K$ provides the value to the firm of the weather contingent claim:

$$
(11) \quad (E[r_{k,W}] - E[r_k])K = \left( r^*_k(W') \int_a^{w^*} g(W_d) dW_d - \int_a^{w^*} r^*_k(W_d) g(W_d) dW_d \right)K \geq 0 .
$$

Therefore, we can now establish the proxy for a particular period $t$,

$$
(12) \quad \psi^p_w = (E[r_{k,W}] - E[r_k])K = \psi \int_{-\infty}^{K_w} (W^* - W(t)) g(W_d) dW_d .
$$

And from this, we can recover the scaling parameter $\psi$ for each and every period\textsuperscript{17}

$$
(13) \quad \psi = \frac{(E[r_{k,W}] - E[r_k])K}{\int_{-\infty}^{K_w} (K_w - W(t)) g(W_d) dW_d} ,
$$

for the weather contingent claim at a general strike $K_w$. The numerator in (13) is currency denominated (e.g. $\$) while the denominator is measured in weather units (e.g. inches of rain, growing degree days etc.). Thus, if the bond is a precipitation bond, to protect against low rainfall measured in inches, the ‘tick’ price or payout per inch below $K_w$ is $\$\psi/\text{inch}$. Finally, the scaling parameter $\psi$ is scalable. Because (13) provides the tick for the entire bond issued, it is more likely that $n$ bonds will be issued to numerous investors. In this case, rather than $\psi$ applying to one bond, $\psi_n = \frac{\psi}{n}$ can be applied to $n$ bonds.

\textsuperscript{17} This result holds if the intertemporal distribution of the underlying weather condition does not change on expectation.
Agricultural Business and Corporate Finance

We examine the instance of an agribusiness (or any other) whose cash flow is affected by adverse weather events. The investment in capital requires an amount \( F \) at \( t=0 \) financed through a bond issued with coupon payment \( c \). The ability to finance coupons and establish a sinking fund for the retirement of the bond is also affected by weather. In the absence of a weather option the present value of the bond is given by

\[
B(.,0) = \frac{c}{r}(1 - e^{-rT}) + Fe^{-rT}.
\]

In each period, the cash flow removed from retained earnings is equal to the coupon payment plus the sinking fund allotment.

\[
C = c + \omega \frac{F}{T}.
\]

If net cash flow from the investment falls below \( C \), then the firm will have to use funds from non-invested projects to make up the shortfall. Thus, one might consider \( C \) as an apt level for the strike price on the weather option. There are several possibilities. First, the firm can hedge the entire cash flow requirement using a weather option with the proceeds going towards any cash flow shortfall in coupons and sinking fund obligations. In other words, \( C = \psi W_K \) or \( \psi = \frac{C}{K_w} \). Second, the firm can ask bondholders to forego coupon payments in order to make up at least part of the shortfall and \( \psi = \frac{c}{K_w} \). A third option is that the option applies only to the weather risk in the year that the bond matures. The risk of this alternative is that all of the risk is put in a single basket and probably does not, at least on an accrual basis, establish a hedge versus a speculative position. Nonetheless a careful look at the literature on commodity-linked bonds, from which much of the above is based, has the risk occurring only at maturity, and it is for this reason that there is so much variability in the structure of such bonds.
We consider the first bond structure, of which firm can hedge the entire periodic cash flow requirement (both coupon payment and sinking fund obligations) using a weather option. This bond value is, therefore, represented by

\[
B(\cdot,0) = \frac{c}{r^*}(1-e^{-r^*T}) + F e^{-r^*T} \left( \frac{c + \omega \frac{F}{K_w}}{r^*} \right) E \left[ \max\left(0, K_w - W(t)\right) \right] \left(1-e^{-r^*T}\right). 
\]

If bond yields \( r^* \) equal to the market rate of interest \( r \), then one can see immediately that the bond will be sold at a discount equal to the present value of the annualized expected payout from the option (assuming that payouts from one year to the next are independent, and intertemporal volatility in the underlying weather condition does not change on expectation). For example, suppose an at-the-money weather call option based on cooling degree days above 60°F has mean CDD\(^{18}\) of 489.50. A 10-year, $1,000,000 bond is issued with 8% coupon rate and sinking fund of $100,000/year. Then, \( \psi = \frac{180,000}{489.50} = 367.72 \).

The expected value of degree-days above the strike is 16.767, so the annual expenditure on weather protection is \( (367.722 \times 16.767) = 6,165.59 \). Assuming an annual discount rate of 6%, the present value of this expected payout is $46,364.04. Consequently, the bond will be discounted by a further $46,364.04 to compensate for the risk. Suppose that in any year the actual degree-days is 600. The option part is in the money by 101.50 degrees. Times this by 367.72, the coupon payments to bondholders would fall by $37,323.58 to $42,676.42.

In the alternative, the issuer may alter the coupon rate to reflect the risk to the bondholders. In this scenario, the intent is to offer a coupon rate \( c^* \) that will make the value of the weather-linked bond equivalent to the value of the bond without weather risk. Therefore, \( c^* \) solves

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\(^{18}\) This example is taken from Turvey 2006, Table 6, Toronto 1936-1996, 90 days in summer.
\[
\frac{c}{r}(1-e^{-rt})+Fe^{-rt} = \frac{c^*}{r}(1-e^{-rt})+Fe^{-rt} - \left( c^* + \omega T \right) \frac{E\left(\max(0, K_w - W(t))\right)}{K_w} \left(1-e^{-rt}\right) \frac{r}{r}.
\]

Solving yields

\[
c^* = \frac{c + \omega T E\left(\max(0, K_w - W(t))\right)}{K_w} \left(1 - \frac{E\left(\max(0, K_w - W(t))\right)}{K_w}\right).
\]

And therefore,

\[
c^* = \frac{80,000 + 1.0 \cdot 1,000,000}{10} \cdot \frac{16.767}{489.40} = \frac{86,389}{1}.
\]

In other words, to compensate for the weather risk, the coupon would increase from 8% to 8.6389%. The coupon rate on a WLB will always be larger than that of a straight bond if the bond yields are to be equivalent. If the option is designed to compensate the sinking fund then the coupon will increase, i.e. \( \frac{\partial c^*}{\partial \omega} > 0 \).

**Agricultural Finance**

There are two obvious applications to agricultural finance. The first is a very simple structure in which the repayment of a non-revolving operating loan is contingent on the performance of a weather variable. The second is with loan repayment on a farm mortgage. The most likely weather risk would be precipitation to protect crop yield losses from drought.
The value of the non-revolving operating loan embedded with a weather option to the lender’s portfolio is

\[
B(.,0) = e^{-rT} \left[ F - \psi E[\text{Max}(0, K_W - W(t))] \right]
\]

where \( F = fe^{r^*T} \) and \( f \) is the initial amount borrowed for operating costs, and \( r^* \) is the interest rate charged on the operating loan as opposed to \( r \), which reflects the lender’s cost of capital. This rate will reflect the risk that the embedded weather option will be exercised and will differ from a rate \( r^{**} \) that would be charged on operating loans without the option (i.e. \( F = fe^{r^{**}T} \)). The interest rate \( r^* \), therefore, is the rate that would make the lender indifferent towards an operating loan with the linked weather option and one without.

\[
r^* = -\frac{\ln \left[ \frac{\psi E[\text{Max}(0, K_W - W(t))]}{f} + e^{(r^*)T} \right]}{T} \quad \text{and} \quad r^* > r^{**} > r.
\]

Using the values above for a $100,000 operating loan with a degree-day call option of 1-year duration, and defining \( \psi = \frac{f}{K_W} = \frac{100,000}{489.50} = \$204.29 \),

then \( r^* = \ln \left[ \frac{204.29 \times 16.767}{100,000} + e^{0.08} \right] = \ln(1.1175) = 0.1111 \).

And so the risk adjusted interest rate is 11.11% or a risk premium of 3.11%. Thus, if the weather event had degree-days of 600, then the payout on the option part would be \([600 - 489.50] \times $204.29 = $22,574.045\), and the loan value would be reduced from $100,000 to $77,425.96. The effective premium on the option part was the incremental increase in the interest expense of $3,110.

For a weather-linked mortgage, we note that the annuity on a T-year mortgage of value \( F \) is given by
(21) \( A(i) = F \left( \frac{1 - (1 + i)^{-T}}{i} \right)^{-1} \),

where \( i \) is the interest rate on the mortgage. Applying \( \psi = \frac{A(i)}{K_w} \), the value of this mortgage with an attached weather option is

\[
B(.0) = \frac{A(i^*)}{r}(1 - e^{-iT}) - \left( \frac{A(i)}{K_w} \right) E\left[ Max(0, K_w - W(t)) \right] \frac{(1 - e^{-iT})}{r}.
\]

As before, we assume the lender will offer this mortgage at a higher interest rate so that the present value of the mortgage with the weather option is equivalent to that without. Thus, the interest rate on the mortgage with the weather option \( (i^*) \) solves

\[
\frac{A(i)}{r}(1 - e^{-iT}) = \frac{A(i^*)}{r}(1 - e^{-iT}) - \left( \frac{A(i)}{K_w} \right) E\left[ Max(0, K_w - W(t)) \right] \frac{(1 - e^{-iT})}{r}.
\]

The solution cannot be solved in closed form but will be the numerical solution to \( i^* \) in

\[
\left[ \frac{1 - (1 + i^*)^{-T}}{i^*} \right] = \left[ \frac{1 - (1 + i)^{-T}}{i} \right] \left( 1 + \frac{E\left[ Max(0, K_w - W(t)) \right]}{K_w} \right)^{-1}.
\]

The amortization per $100,000 of 10-year mortgage at 8% interest is $14,902.95/year. Assuming as above a linked weather 60f growing degree-day call option with a strike of 489.50 degrees, and expected loss at the money of 16.767 degrees. Solving (24) for \( i^* \) gives an interest rate on this weather-linked mortgage of 8.7546%, which when applied to the mortgage, yields an amortization of $15,414.13 /year, an increase of about 3.4% or $511.18/year. Suppose that in a particular year, the option part expires in the money with
\( W(t) = 600 \). Then, with \( \psi = \frac{A(i)}{K_w} = \frac{14,902.95}{489.50} = 30.445 \), the payout against the mortgage is \( 30.445 \times (600 - 489.5) = 3,364.20 \). The mortgage remittance would therefore be $12,049.93 rather than the $15,414.13 that would be paid had the weather event not occurred.

**Development Finance**

As indicated in the introduction, there is a great interest in weather risk management in developing countries, especially those where the agricultural economy is a significant portion of GNP. To these countries many NGO’s and formal organizations such as the World Bank or Asia Development Bank, provide funds with fixed repayment terms. However, these countries are susceptible to weather risks that make repayment of sovereign debt difficult. Many of the loans made to LDCs can be converted to weather based loans or bonds using the principles discussed above. However, there are a number of bonds that are very interesting, especially in the context of famine relief. These so-called famine bonds establish a trigger based on a forecast of future famine event based on observable weather conditions used in many early warning systems. There are two types of bonds to consider. The first releases the LDC from repayment of the loan so that government funds can be put to use to address the future famine. The second is an option, similar to a look-back option that whence triggered either provides cash directly for anticipated famine relief, or triggers an automatic loan with fixed and pre-established terms so that sufficient cash would be available to combat famine.

For the first, suppose there exists an empirical model \( h = h(W^*) \) that predicts famine based on an observable set of weather events \( W^* \). There is a critical value \( \bar{h} \)
which triggers famine relief. This trigger can be tied to a specific weather event \( \bar{W}^* \) that is obtained from the solution to\(^{19}\)

\[
(25) \quad \bar{W}^* = h^{-1}(\bar{h}).
\]

The option value (an insurance) equivalent to equation (1) for a put option, is therefore, given by

\[
(26) \quad V_{\bar{W}} = e^{-(r+\Delta \bar{\sigma})T} \psi \int_{0}^{\bar{W}^*}(\bar{W}^* - W^*(T))g(W^*_T)dW^*_T.
\]

The variant on this is an option that does not provide cash per se, but rather the right to receive a loan or a bond from the World Bank or other actors in the international aid community. When the weather event is triggered, a pre-negotiated bond amount is triggered. The value of the bond may increase as the weather signal indicates the intensity and duration of the potential famine. The probability of the adverse weather event that can indicate the onset of famine is given by

\[
(27) \quad P(W^*_T \leq \bar{W}^*) = \int_{0}^{\bar{W}^*}g(W^*_T)dW^*_T.
\]

And so the expectation that the bank would issue a bond and how large the bond issued would be determined, a priori by

\[
(28) \quad E[B(.)] = \int_{0}^{\bar{W}^*}B(\bar{W}^*_T, t)g(W^*_T)dW^*_T.
\]

\(^{19}\) Noticing that \( W^* \) can be a set of weather events observable for a specific period of time. A simple example can be the cumulative rainfalls during the agricultural season relative to the historical data, combination of heat and rainfall etc.
This is similar to a look-back option. Once the weather trigger is observed, then immediately an amount \( F \) is forwarded to the famine-prone region. The bond itself, once exercised can have an option attached that increases the bond value depending on the severity of the weather event, i.e.

\[
(29) \quad B(W^*, t) = F + \psi \text{Max}(0, K_{W^*} - W^*(t))\]^{n}.
\]

Note that the payoff can be raised to a power to take into account the possibility in cases of famine that the rate of nutritional deterioration declines non-linearly with the extent of the weather event (e.g. drought). Consequently, the monies required to avert the famine will not simply increase linearly with the extent, but non-linearly (i.e. \( n \geq 1 \)). There are two mechanisms for this to work. First, because a specific event is required for the bond to be released, (29) can be provided in the form of a promise should the event triggered with probability as defined in (27). The second involves a prepayment of the obligation in the form

\[
(30) \quad E[B(.)] = \int_{0}^{\pi} F + \psi \text{Max}(0, K_{W^*} - W^*(t))\]^{n} g(W^*)dW^*.
\]

In other words, the LDC or actors in the international aid community can purchase an option (basically an insurance) for the LDC that will provide a base level of famine release \( F \) that will increase with the severity of the weather event. Since the funding is prepaid in the option value, it need not be repayed. This is another form of (26).

**Conclusions**

The interest in weather derivatives has not formally been presented in relation to bonds, loans, mortgages and other credit instruments as outlined in this paper. The presentation in this paper follows closely the logic, economics, and model formulation of
the more popular commodity-linked loans, and from a mathematical point of view the logic, benefits and costs are interchangeable. The usual problems that hamper weather derivatives or weather insurance, however, are no more resolved when attached to a bond as when used to manage volumetric risk in isolation of credit. These include the nature of the stochastic process and probability distribution determining the risk and payoffs to the option, the location of measurement and basis, the comprehension by end users and so on. Nonetheless, there are many specific weather events that are highly correlated with production variability. These could be localized events, but in some cases such as has been discussed in terms of famine, widespread with catastrophic consequences.

The models provided in this paper can be used in many applications, and examples have been provided. Perhaps one of the more innovative components, dealing mostly with developing countries, comes in the form of weather-linked bonds provided for famine relief.

The successful innovation of these weather-based debt instruments and risk management instruments in both developed and developing agrarian economies depends largely on the quality of the weather data used to derive underlying weather indices. Some of the strict quality requirements include reliable and trustworthy on-going daily collection and reporting procedures; daily quality control and cleaning; an independent source of data for verification purposes; and a long, clean and consistent historical record to allow for proper actuarial analysis of the risks involved as the premium charged by the players in the international weather market will reflect the probability and severity of the linked weather events. More importantly, in the international weather risk market, at least thirty years of daily data are ideally required.
References


at the workshop Innovations in Agricultural Production Risk Management in Central America: Challenges and Opportunities to Reach the Rural Poor, Antigua, Guatemala, May 2005.


