

POVERTY MEASUREMENT

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PLAN OF THE LECTURE

- 1) Review the most popular poverty measures:
 - ▶ Incidence, depth and severity of poverty
 - ▶ Sen's poverty index
 - ▶ Watts index
- 2) Estimation Issues
(time permitting)
- 3) Summary and Recommendations

POVERTY MEASURES

Basic ideas

- o Poverty measures **aggregate** information.
- o A **poverty measure** is a function of individual incomes $\mathbf{x} = (x_1, \dots, x_n)$ and the poverty line z :
 $P: \mathbb{R}^n \rightarrow \mathbb{R}_+$
- o The literature on poverty measures is huge and **technical** in nature. It deals with the choice of the **functional form** of a **suitable** poverty index.
- o In practice **three** indices have taken centre stage:
 - 1) the **headcount ratio**
 - 2) the **poverty gap index**
 - 3) the **poverty gap squared index**

THE HEADCOUNT RATIO

Definition

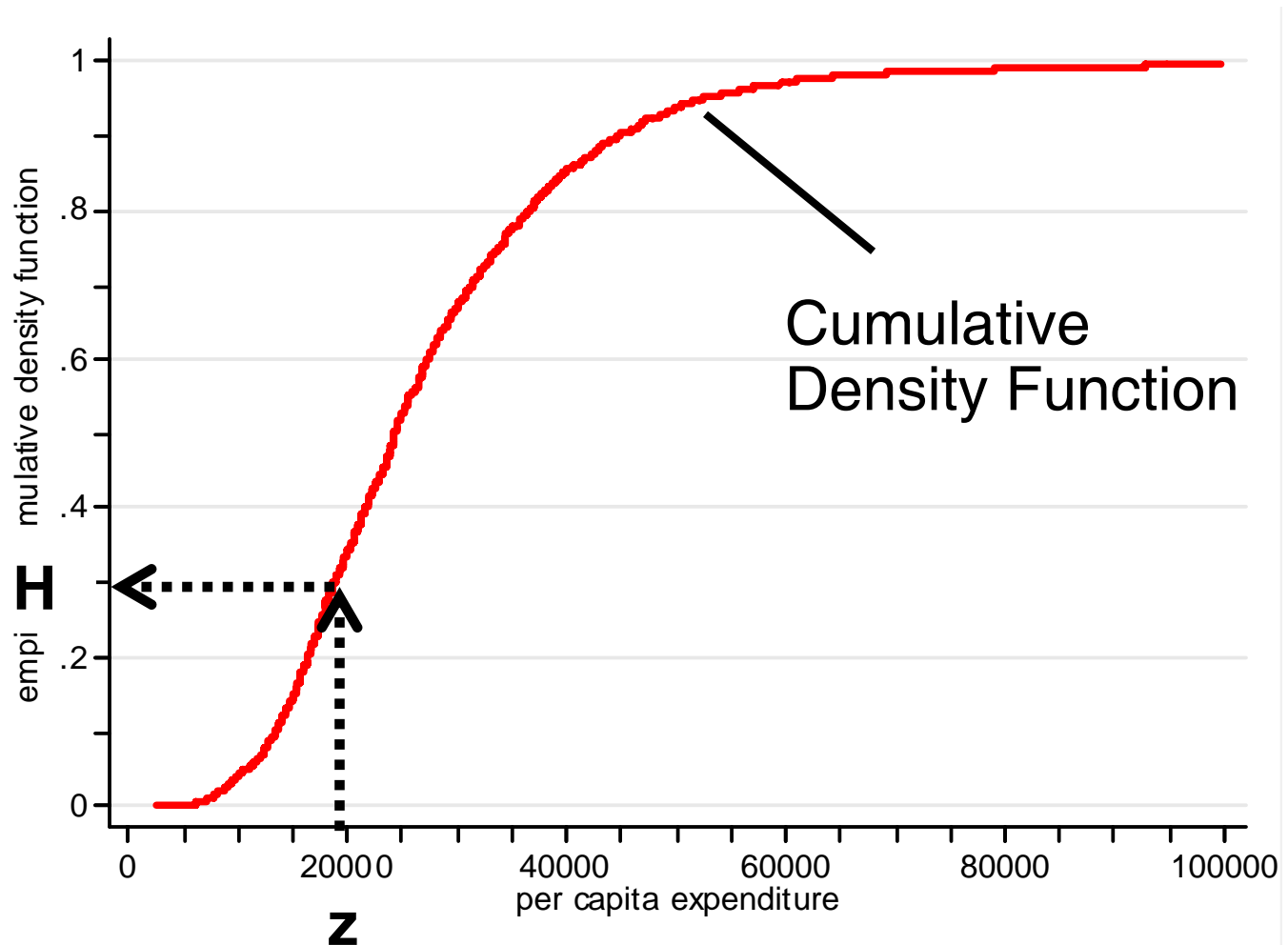
- ▶ The **headcount ratio** is the **proportion** of the population that is classified as poor.
- ▶ Let **q** denote the number of poor households, and **N** the total population.
- ▶ The **headcount ratio H** is defined as:

$$H = \frac{q}{N} = \frac{1}{N} \sum_{h=1}^N I(x_h \leq z)$$

$I(\cdot)$ is an **indicator function** that is 1 if its argument is true, 0 otherwise.

THE HEADCOUNT RATIO

CDF for Croatia, 2004



THE HEADCOUNT RATIO

An Assessment

- ▶ **Easy** to understand
- ▶ **Insensitive** to:
 - 1) the **degree of poverty**
 - ▶ *Cut in half every poor's income ...H does not change!*
 - 2) the **distribution of income among the poor**
 - ▶ *transfer from a poor person to a not-so-poor person (still poor after the transfer) ... H does not change!*

THE POVERTY GAP INDEX

Definition

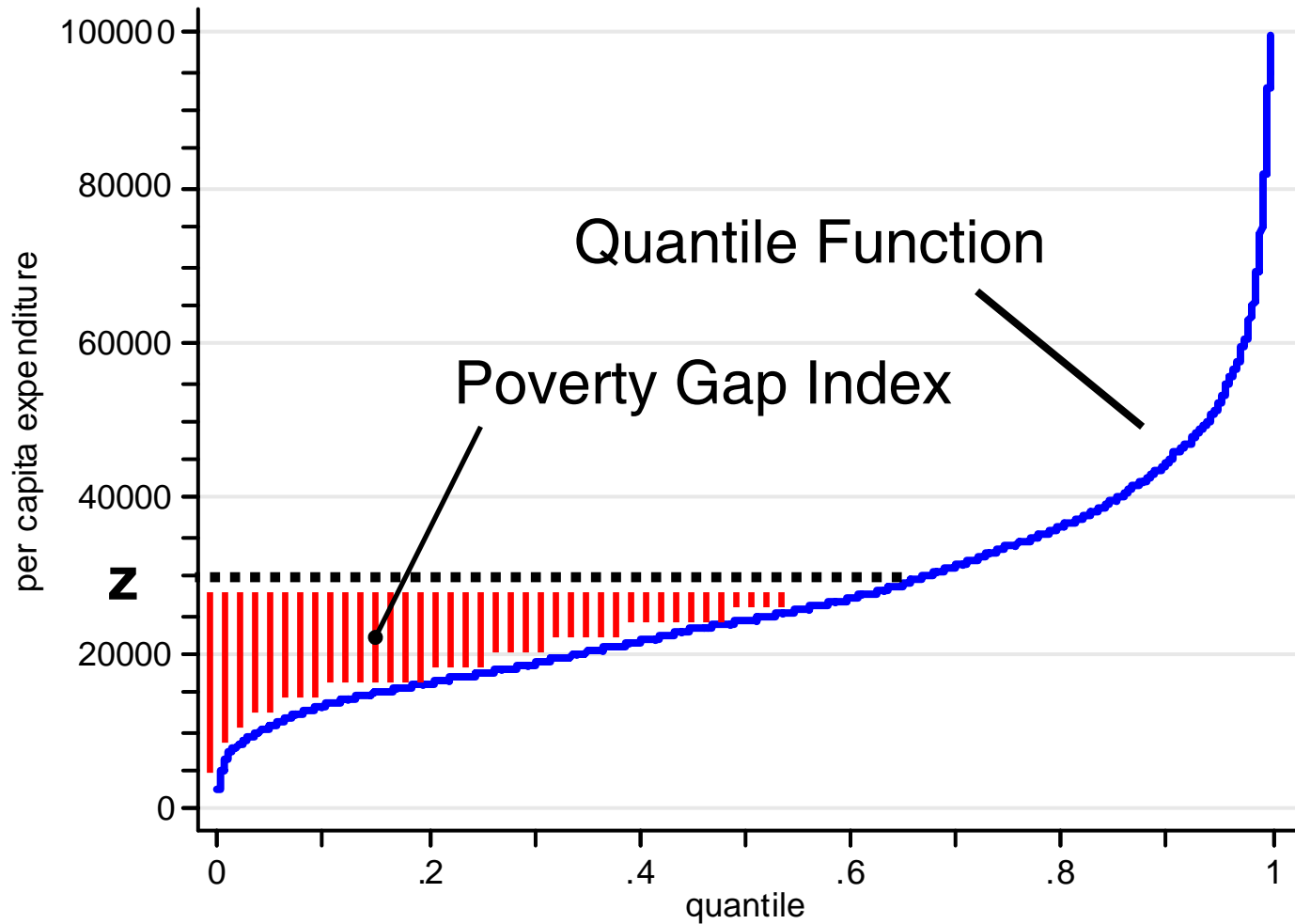
- ▶ The **PG index** is defined as the **average poverty gap** in the population *as a proportion of the poverty line* (where the non-poor have zero gaps):

$$PG = \frac{1}{N} \sum_{i=1}^N \left(1 - \frac{x_i}{z}\right) I(x_i \leq z) = \frac{1}{N} \sum_{i=1}^q \left(1 - \frac{x_i}{z}\right)$$

- ▶ The poverty gap index (PG) accounts for the **intensity** (or **depth**) of poverty: how poor the poor are.
- ▶ The contribution of the i -th individual to PG is larger the poorer she is, that is, the larger is her **poverty gap** ($z - x_i$).

THE POVERTY GAP INDEX

Quantile Function for Croatia, 2004



THE POVERTY GAP INDEX

Interpretations

- ▶ Suppose **PG = 0.20**
- ▶ *Interpretation 1:*
“On average, the poor have an expenditure shortfall of 20 percent of the poverty line”
- ▶ Now suppose **z = 1,500 €** (poverty line).
- ▶ *Interpretation 2:*
The **per capita cost of eliminating poverty** is equal to $PG \times z$. In our example: 300 € (= $0.20 \times 1,500$).

THE POVERTY GAP INDEX

Interpretations

- 1) The **minimum** cost of eliminating poverty is achieved when *every* poverty gap is filled up to z :

$$\sum_{i=1}^q (z - x_i) = q(z - \mu_z)$$

PERFECT TARGETING

- 2) The **maximum** cost of eliminating poverty is when z is given to everyone (to be sure the none are poor):

$$z \times N$$

NO TARGETING

- 3) The **ratio** of minimum to maximum cost equals **PG**.
- ▶ PG is an indicator of the **potential saving** for eliminating poverty **from targeting** the poor.

THE POVERTY GAP INDEX

Assessment

▶ Advantages

- 1) reflects depth of poverty
- 2) it does not imply a discontinuity (“jump”) at the poverty line

▶ Drawbacks

- 1) insensitive to distribution of income among the poor

⋮ **poverty line = 4**

	α	β	γ	δ	H	PG	PG2
A	1	2	3	9	0.75	0.375	0.219
B	2	2	2	9	0.75	0.375	0.188

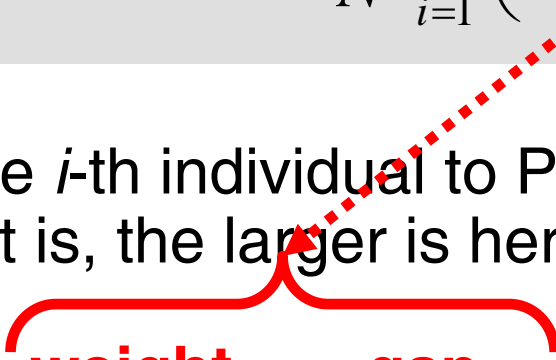
THE POVERTY GAP SQUARED

Definition

- ▶ The **squared poverty gap index** attributes more weight to the poorest among the poor:

$$PG = \frac{1}{N} \sum_{i=1}^N \left(1 - \frac{x_i}{z}\right)^2 I(x_i \leq z) = \frac{1}{N} \sum_{i=1}^q \left(1 - \frac{x_i}{z}\right)^2$$

- ▶ The contribution of the i -th individual to PG is larger the poorer she is, that is, the larger is her **poverty gap** $(z - x_i)/z$:


$$PG = \frac{1}{N} \sum_{i=1}^q \left(1 - \frac{x_i}{z}\right) \times \left(1 - \frac{x_i}{z}\right)$$

THE POVERTY GAP SQUARED

Assessment

▶ **Advantages**

1. sensitive to differences in both **depth** and **severity** of poverty. In other words, takes inequality among the poor into account
2. Hits the point of poverty line smoothly

▶ **Drawbacks**

- 1) difficult to interpret

BACK TO THE HEADCOUNT RATIO

A Must-Read Article

ECONOMETRICA

VOLUME 44

MARCH 1976

NUMBER 2

POVERTY: AN ORDINAL APPROACH TO MEASUREMENT

BY AMARTYA SEN¹

The primary aim of this paper is to propose a new measure of poverty, which should avoid some of the shortcomings of the measures currently in use. An axiomatic approach is used to derive the measure. The conception of welfare in the axiom set is ordinal. The information requirement for the new measure is quite limited, permitting practical use.

THE SEN'S POVERTY INDEX

Pars destruens...

- ▶ The **headcount ratio** is a very crude index.
- ▶ H violates important properties (**axioms**) a desirable poverty measure should satisfy, for instance:
 - ▶ **Monotonicity** – *a reduction in income of poor must increase the poverty measure.*
- ▶ Similarly, the **PG** index violates other axioms.

THE SEN'S POVERTY INDEX

Pars construens

THEOREM 1: *For large numbers of the poor, the only poverty index satisfying Axioms R, M, and N is given by:*

$$(8) \quad P = H[I + (1 - I)G],$$

where G is the Gini coefficient of the income distribution of the poor.

$$P_{SEN} = H \times G_z + PG \times (1 - G_z)$$

P_{SEN} is a weighted average of H and PG.

THE SEN'S POVERTY INDEX

Assessment

▶ Advantages

- ▶ Derived from a set of **axioms**
- ▶ **sensitive** to distribution among the poor (as PG2)

▶ Drawbacks

- ▶ Because Sen's measure depends on the Gini coefficient, it shares its main **inconvenience**:
 - 1) The Gini-and thus the Sen index **cannot be used to decompose poverty** into contributions from different subgroups.

ANOTHER INFLUENTIAL ARTICLE

Econometrica, Vol. 52, No. 3 (May, 1984)

NOTES AND COMMENTS

A CLASS OF DECOMPOSABLE POVERTY MEASURES

BY JAMES FOSTER, JOEL GREER, AND ERIK THORBECKE¹

The **headcount ratio**, the **PG** and **PG2** all belong to the **Foster-Greer-Thorbecke class** of poverty measures.

FGT (1984)

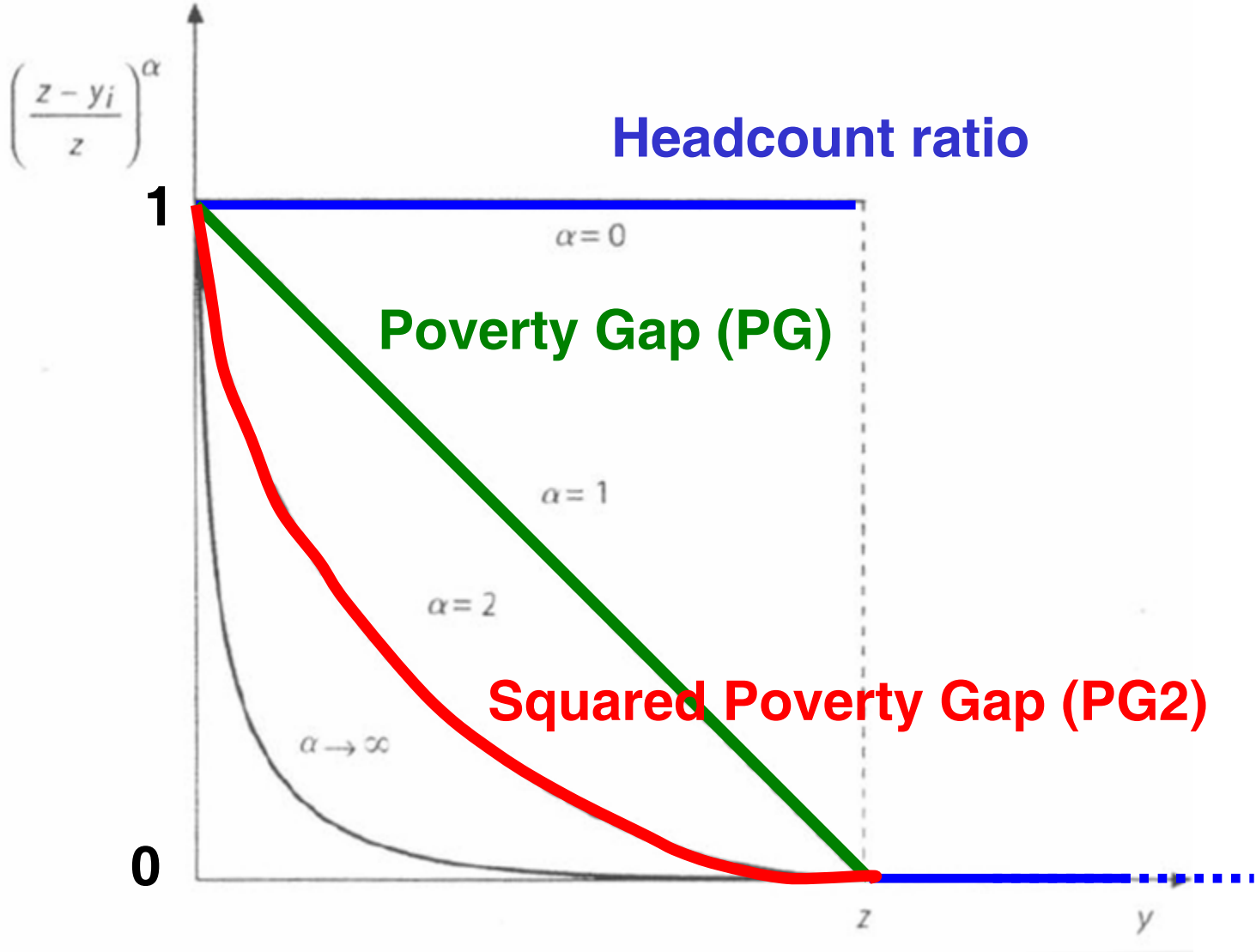
Definition

The **FGT class** of poverty measures has an additive structure:

$$P_{\alpha} = \frac{1}{N} \sum_{h=1}^N \left(\frac{z - x_h}{z} \right)^{\alpha} I(x_h \leq z), \quad \alpha \geq 0$$

α	P_{α}	
0	$P_0 = H$	HEADCOUNT RATIO
1	$P_1 = PG$	POVERTY GAP INDEX
2	$P_2 = PG2$	POVERTY GAP SQUARED
∞	P_{∞} weights the poorest person	

The Absolute Contribution to Total Poverty of Individuals at Different Ranks



FGT (1984)

Additivity

Suppose population is divided into m mutually exclusive sub-groups.

$$P = \sum_{j=1}^m \frac{n_j}{n} P_j$$

Aggregate poverty P is a **population-weighted mean** of the sub-group poverty measures.

This type of decomposition allows us to identify the relative contribution of each group to total poverty.

FGT (1984)

Additivity: A Numerical Example 1/2

	x	urban	z
1.	2	urban	5
2.	3	urban	5
3.	6	urban	5
4.	9	urban	5
5.	1	rural	4
6.	1	rural	4
7.	2	rural	4
8.	3	rural	4
9.	5	rural	4
10.	7	rural	4

The dataset:

- ▶ 10 households
- ▶ **x** is income
- ▶ **urban** ... self-explanatory
- ▶ **z** is the poverty line

FGT (1984)

Additivity: A Numerical Example 2/2

```
. povdeco x, varp(z) by(urban)
```

Foster-Greer-Thorbecke poverty indices, FGT(a)

All obs	a=0	a=1	a=2
	0.60000	0.32500	0.19575

FGT(0): headcount ratio (proportion poor)
FGT(1): average normalised poverty gap
FGT(2): average squared normalised poverty gap

Decompositions by subgroup

Summary statistics for subgroup k = 1, .., K

urban	Pop. share	Mean
rural	0.60000	3.16667
urban	0.40000	5.00000

Subgroup FGT index estimates, FGT(a)

urban	a=0	a=1	a=2
rural	0.66667	0.37500	0.23958
urban	0.50000	0.25000	0.13000

0.6000 =
0.6667 x 0.6 +
0.5000 x 0.4

Similarly,
additivity holds
for PG and PG2.

POVERTY AXIOMS

A selection

- ▶ Poverty measures – as inequality indices – can be derived following an **axiomatic approach**.
- ▶ **FOCUS** – A poverty measure should be independent of the income distribution of the non-poor.
- ▶ **MONOTONICITY** – Other things equal, a decrease in a poor person's income should increase the overall poverty level.
- ▶ **TRANSFER** – A progressive transfer from a rich person to a poor person should decrease the poverty measure, and vice versa.
- ▶ **DECOMPOSABILITY** – The poverty index should increase when poverty in a subgroup increases, and viceversa.

AXIOMS AND POVERTY MEASURES

An Intentionally Un-readable Slide!

Table 3.1. Axioms of poverty measurement and poverty measures

POVERTY MEASURES

Axioms and measures	Class 0			Class 1				Class 2				Class 3				
	H	I	HI	S	K	T	T _α	C ₁	BD	Ch	C _h	C ₂	F	W	H	HD
Focus axiom	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
Symmetry	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
Replication invariance	✓	✓	✓	X	X	X	✓	✓	X	X	✓	✓	✓	✓	✓	✓
Continuity	X	X	✓	X	X	✓	✓	X	X	✓	✓	✓	✓	✓	✓	✓
Increasing poverty line	X	X	X	✓	✓	✓	X	✓	X	✓	✓	✓	✓	✓	✓	✓
Regressive transfer	X	X	X	X	X	✓	X	X	X	✓	✓	✓	✓	✓	✓	✓
Weak transfer sensitivity	X	X	X	X	X	X	X	✓	X	X	✓	✓	✓*	✓	✓	✓
Subgroup consistency	✓	✓	✓	X	X	X	X	X	X	X	✓	✓	✓	✓	✓	✓
Weak monotonicity	X	✓	✓	✓	✓	✓	X	✓	✓	✓	✓	✓	✓	✓	✓	✓
Strong monotonicity	X	X	✓	✓	✓	✓	X	✓	X	✓	✓	✓	✓	✓	✓	✓
Minimal transfer	X	X	X	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
Weak transfer	X	X	X	✓	✓	✓	X	✓	✓	✓	✓	✓	✓	✓	✓	✓
Progressive transfer	X	X	X	X	X	✓	X	X	X	✓	✓	✓	✓	✓	✓	✓
Restricted continuity	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
Decomposability	✓	X	✓	X	X	X	X	X	X	X	✓	X	✓	✓	✓	✓
Nonpoverty growth	✓	X	✓	✓	✓	✓	✓	✓	✓	X	X	✓	✓	✓	✓	✓
Normalization	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓

H: the headcount ratio; I: the income gap ratio; HI: the poverty gap ratio; S: the Sen measure; K: the Kakwani measure; T: the Thon measure; T_α: the Takayama measure; C₁: the Clark *et al.* ethical measure; BD: the Blackorby–Donaldson measure; Ch: the Chakravarty ethical measure; C_h: the Chakravarty measure; C₂: the Clark *et al.* measure; F: the Foster *et al.* measure; W: the Watts measure; H: the Hagenaaers measure; HD: the Hagenaaers–Dalton measure. The Sen–Kakwani measure and the measures defined by Vaughan are not included.

✓: satisfies the axiom; X: does not satisfy the axiom or needs additional restrictions; *: satisfies the axiom for $\alpha > 2$.

Source: Zheng (1997)

THE WATTS INDEX

An Increasingly Popular Poverty Measure

- ▶ The only poverty measure that satisfies all four axioms described above is the **Watts index**:

$$W = \frac{1}{N} \sum_{h=1}^N [\ln z - \ln x_h] I(x_h \leq z)$$

- ▶ The Watts index is **distributionally-sensitive**, by virtue of its use of logarithms (transferring 1€ to a poor person counts as a larger contribution than transferring to a richer person).

ESTIMATION ISSUES

▶ **Standard Errors**

Poverty calculations are based on a **sample** of households, and samples carry a margin of error in representing the population.

- ▶ *Point estimates should always be associated with standard errors (or confidence intervals).*
- ▶ *Poverty changes should not be taken at the face value.*
- ▶ *Probability weighting, clustering, stratification, etc. are survey design features which should be taken into account in estimating standard errors.*

▶ **Robustness**

How do contaminated data affect poverty measures?

SUMMARY AND RECOMMENDATIONS

- 1) The **headcount ratio** is a crude and 'theoretically inferior' poverty index. H is useful, but ***should not be used exclusively***.
- 2) The **Poverty Gap Index** and the **Squared Poverty Gap Index** are complements to H; poverty analysis should ***combine the three measures***.

SUMMARY AND RECOMMENDATIONS

- 3) The **axiomatic approach** does *not* succeed in identifying the “best” poverty measure. Yet, it is **useful**, as it reveals the principles underlying the poverty measures.
- 4) Allocating anti-poverty resources to minimize the **PG2** or the **Watts index** or the **Sen’s index** would benefit the poorest, a feature that many analysts find appealing
- 5) A **recommended set of poverty indices**: FGT class (H, PG and PG2) + Watts index.

LIST OF PAPERS CITED

- ▶ **Foster**, J.E., J. **Greer** and E. **Thorbecke** (1984), “A class of decomposable poverty measures”, *Econometrica*, 52(3): 761-66.
- ▶ **Sen**, A. (1976), “Poverty: An Ordinal Approach to Measurement”, *Econometrica*, 44(2): 219-31.
- ▶ **Zheng**, B. (1997), “Aggregate Poverty Measures”, *Journal of Economic Surveys*, 11(2): 123-62.