A Theory of Contribution Density and Implications for Pension Design

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A theory of contribution density and implications for pension design[1]

by Salvador Valdés-Prieto[2]

The adequacy of contributory pensions for the middle classes depends on density of contribution. Density can be far below 100% because the State is unable or unwilling to impose the mandate to contribute on all jobs, especially on poor workers such as many in self-employment and small firms. The paper presents a model where individuals choose whether to bundle saving for old age in a covered job or to save independently while choosing an uncovered job. The determinants of the effective rate of return offered by the contributory pension plan include the earnings differential. This return is then compared with the returns offered by pure saving in the financial market, to determine the equilibrium density of contribution. The paper also applies the model to assess two standard designs for noncontributory subsidies for the old poor. It finds that these standard designs *crowd out* contributory pensions for the middle classes by reducing density. The paper also considers two second-generation designs for noncontributory subsidies and other approaches to raise density. This model also allows optimization of the combined "multipillar" structure, where participants get noncontributory pensions and also contributory pensions based on both mandates and fiscal incentives.

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1. Introduction

For hundreds of millions of middle-class people around the world, consumption in old age depends on the contributory systems promoted by the State, through either mandates or fiscal incentives. However, if a substantial share of participants have a low density of contribution, the pension replacement rate will be inadequate, even if the benefit formulae are generous for a high density participant.

Low coverage of contributions is prevalent in many countries, despite mandates. Important examples are China - 48% of urban employees contributed in 2005 (Salditt et al 2007), Poland - 68% contributes through ZUS, South Korea - only 58% of the labor force contributes (World Bank, 2000), Brazil –only 49% of the employed contribute, and Mexico - only 38% of employed contributes every two months (Rofman and Lucchetti, 2006). Moreover, average coverage rates like these are misleading when a country exhibits uneven density of contribution, since large groups of rural workers or urban self-employed (most of them poor) are effectively exempt from the mandate, in addition to women engaged in home production.[3] Bucheli et al (2007) use survival analysis to estimate density from individual panel data, starting from incomplete individual histories from Uruguay.

In a situation of uneven density, some middle-income people fall into both absolute and relative poverty in old age, due to inadequate contributory pensions relative to former earnings. This situation may undercut the public's support for contributory pensions. For example, in the debate that led to the 2008 reform in Chile, the argument that inadequate replacement rates are due to uneven density, and that this is a result of incomplete economic development, was criticized as an excuse to preserve an inadequate status quo.

In such situations, it may be tempting to introduce pay-as-you-go finance to grant large supplements to middle class people with inadequate pensions, to be paid by future generations, including the future poor. Since those middle class people did earn enough when active, this outcome is inequitable. It is also inefficient, since introduction of pay-as-

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[3] Density is more important than the coverage of contributions, defined as the ratio of the number of contributions to employment at one point in time. A given coverage rate has different implications depending on whether employment is segmented between individuals that hold covered jobs permanently and others that hold uncovered jobs permanently, or employment is homogenous such that all individuals hold covered jobs intermittenly. The first case exhibits a bimodal distribution of density and the second has a single density.

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you-go finance reduces national saving (Diamond, 1965), reduces the efficiency of the labor market in the future (Abel et al, 1989), induces a decline in fertility (Cigno and Werding, 2007) and may favor populist competition in offering subsidies (Godoy and Valdés-Prieto, 1997). Thus, uneven density of contribution reduces the political stability of funded pensions.[4]

Separately, poverty among the old creates a demand on States to grant progressive subsidies to the old poor. These subsidies are also called "first-pillar pensions", or noncontributory pensions[5]. One possible design is a universal flat subsidy for all the old. However, universal flat pensions are expensive fiscally, leading to benefits that can be too stingy for the poorest among the old. One way to reduce this fiscal cost, and thus to increase redistribution to the poorest, is to reduce benefits for those old that consume above some threshold, as measured by some proxy means test. This requires the proxy means test to have limited error and a modest incremental cost.

A second targeting method is to reduce benefits to those old that receive contributory pensions of sufficient size, which are observable at no cost. This targeting scheme is helpful only if the State-promoted contributory pension achieves an adequate replacement rate, i.e. if it achieve sufficient density of contributions. However, if density is uneven and small for large groups, this method to reduce fiscal cost and to improve subsidies is in jeopardy.

For years, it has been suspected that the density of contribution may be influenced by the design of the non-contributory subsidies themselves. For example, the idea that when a participant already meets the vesting requirement for a Minimum Pension Subsidy, she finds that additional contributions do not increase her total contributory pension, is well established (Diamond and Valdes-Prieto, 1994). However, that intuition was not explicit about job choice and density. Beyer and Valdés-Prieto (2004) were the first to emphasize that insufficient density of contribution can be caused by the design of a first-pillar

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[4]This story contributes to explain the widespread shift from partial funding to pure pay-as-you-go finance in countries where incomes are rising steadily and secularly. Steadily and secularly rising income rules out the intergenerational insurance justification for introducing pay-as-you-go finance.

[5] The design of these subsidies is normally different from the design of subsidies to the poor of other ages. One justification for this separation is to alleviate the consequences of "neglect of old age" (see section 2.1) among formerly middle-class workers that failed to contribute enough.

program, but did not provide a model. A recent survey asserted that "the greatest challenge is to design and manage the link between tax-financed (noncontributory) and contributory social security schemes" (van Ginneken, 2007). No formal model has been available to guide this intuition, and this paper attempts to fill this void.

Related literature includes the one on the marginal link between contributions and benefits in contributory old-age pensions, which affects the supply of hours to the labor market (Auerbach and Kotlikoff, 1987), but was silent on the choice between covered and uncovered jobs. Another literature argues that the design of mandatory old-age pensions affects choice between covered and uncovered jobs, but is silent on the effect of first-pillar design. Hubbard, Skinner and Zeldes (1995) argue that a high rate of withdrawal of subsidies to the old poor reduces the voluntary saving of the poor, but are silent on the impact on density of contribution to second pillars.[6] Valdés-Prieto (2002, p. 241-57) showed how the presence of uncovered jobs allows workers to cap the implicit tax imposed by a second pillar, but is silent on the impact on density. The literature on optimal income tax schedules stresses the disincentive effect on supply of hours of violent withdrawal of subsidies, identifies the optimal two-bracket schedule (Slemrod et al, 1994), and identifies more general optimal tax schedules (Diamond, 1998), but is silent on job choice.

This paper contributes a formal model to determine the density of contribution to State-supported old age pensions, including support through mandates and fiscal incentives. In this model the State is unable or unwilling to impose on the poor a mandate to contribute, specifically on the self-employed and on small firms, but imposes the mandate for jobs in large employers. The model finds that workers choose whether to bundle job selection and saving for old age, on the basis of the effective rate of return offered by the contributory pension plan. This is heavily influenced by the earnings differential relative to the contribution rate. These supply decisions help determine the equilibrium density of contributions, if the demand for labor in the covered sector has some elasticity.

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[6] Although the empirical literature on the impact of tax incentives on voluntary saving finds modest elasticities for the poor (Attanasio and Browning, 1996), violent withdrawal of subsidies create unusually large implicit tax rates, so the effect on voluntary saving can still be sizable.

The paper also applies the model to study whether the standard designs of noncontributory subsidies for the old poor (first-pillar pensions) affect the equilibrium density of contributions. It finds that at given earnings, changes in the design of first-pillar subsidies can have a large impact on density, implying a large elasticity of labor supply to the covered sector to the relative wage paid by that sector. These changes in job choice reduce contribution density in equilibrium. This implies that badly designed noncontributory subsidies *crowd out* contributory pensions and crowd in self-employment, informal jobs and home production among those individuals that value first-pillar subsidies as significant.[7] The policy implication is a new approach to raise density of contribution: to improve the design of the first pillar. This argument changed the scenario in the Chilean reform debate, by pointing towards a constructive method to raise density and to improve the adequacy of contributory pensions.

This paper also explores other benchmark designs for noncontributory pensions. The design called "proportional minimum pension", used by Switzerland, is found to create large horizontal inequities and to have lower efficiency than a program with a small withdrawal rate. Another benchmark is the Swedish system, based on earlier Finnish and Norwegian designs, where a noncontributory pension is withdrawn at a rate of 100% in the first tranche, and subsequently at a rate of 48% in a second tranche (Scherman, 1999). The paper argues that this general shape is supported by some results in optimal income tax theory by Slemrod et al (1994) and Diamond (1998), which recommend that the net marginal tax rate at low income levels be larger than the net marginal tax rate for income levels near the median of the earnings distribution (Valdés-Prieto 2002, p. 67-71). Moreover, simulations by Poblete (2005) for a simplified model found that withdrawal rates at 100% and 48% might be too high and that the optimal rates might be nearer 20%.

The Chilean reform of 2008 adopted this approach, as it replaced two non-contributory subsidies, whose withdrawal rates were 100% and 61%, for a unified new subsidy with a withdrawal rate of about 32% in response to mandatory contributory pensions. That reform has problems, such as cliff withdrawal for other income sources in old age and a lavish size of subsidies relative to earnings when active (Valdes-Prieto 2008, companion paper). Still,

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[7] A similar argument may apply to underreporting of earnings.

the adoption of a first-pillar with a small withdrawal rate can help increase density and reduce its unevenness, and if so it will strengthen the adequacy of contributory pensions. More generally, this approach allows optimization of the combined "multipillar" structure, where each participant can get both noncontributory and contributory pensions in old age.

Section 2 presents a model to determine contribution density, in the absence of noncontributory subsidies for the old poor. Section 3 applies the model to two standard non-contributory subsidy schemes. Section 4 applies the model to analyze two other designs for noncontributory subsidies: a proportional minimum pension, and a subsidy with a small withdrawal rate. Section 5 provides an overview of other policies to raise density of contributions. The final remarks are in section 6.

2. A theory of contribution density in contributory pensions for the middle classes

This section presents a model where each individual chooses between jobs covered by State-supported contributory old-age pension plans (supported with a mandate or with fiscal incentives), and other jobs that are not covered by State-supported contributory pensions. Therefore, coverage of contributions in the contributory plan is endogenous. In this section, noncontributory subsidies are not available in old age.

2.1 The labor market and tax institutions

The State is unable to enforce in all jobs a mandate to contribute based on all labor productivity. This is obvious for home production. Self-employment also facilitates underreporting of work and earnings, for two reasons: (i) there is no employer interested in maximizing reported labor costs in order to minimize corporate income taxes; and (ii) there is no employer interested in minimizing the penalties applied by enforcers of employment protection regulations, which include compliance with social security and labor regulations. Informal firms, defined as those that evade taxes and regulations, also evade the mandate to contribute. The informal employer loses by reporting the earnings of his employee to the State, because this betrays its presence. Thus, there are several reasons why a mandate to participate in contributory old-age pensions is never enforced by the State in all jobs.

Moreover, the State may be *unwilling* to enforce a mandate to contribute on jobs taken primarily by the poor. In many emerging economies, this has originated legal exemptions to

the mandate to contribute, mainly for the self-employed. In some high-income economies, low-wage jobs are exempt from the mandate to contribute (Netherlands, Australia). In the case of State support that consists of exemptions from the income tax, it is clear that it is ineffective for jobs whose tax rate is already zero or negative, including jobs in informal firms and the self-employed poor.

The premise of this model is that uncovered jobs are a significant and massive job option, not a marginal exception. At the same time, it is assumed that some jobs are covered, specifically those at large employers. Empirical work for Chile confirms that uncovered jobs are a realistic option for many workers (Torche and Wagner, 1998). In most emerging economies uncovered jobs are closer to 50%, and in most countries in Africa and South Asia they are closer to 80% (van Ginneken, 1999).

"Contribution density" is defined as the share of (the present value of) earnings in the active phase of life on which the individual contributes to some contributory pension system for old-age. For any given rate of turnover between covered jobs and other uses of time, average density falls when self-employment and informal employment expands and when activity outside the labor force (mainly home production) rises. Density may also change for a different reason: underreporting of earnings, keeping the headcount constant.

This is a model under certainty. It collapses a continuum of dates and ages into just two separate periods, the active and passive phases of life. The "passive" label is inexact because work in old age is allowed. The labor market and tax variables are:

y^c = gross earnings in the covered sector per period, in the active phase.

y^{ex} = earnings per period in jobs that are exempt or uncovered, in the active phase.

z^{ex} = the ratio y^{ex}/y^c.

D = density, ε [0,1]. It is the proportion of (the present value of) earnings in the active phase of life on which the individual contributes to some contributory old-age system.

e_p = earnings in old age, expressed as a proportion of y^c. Exempt from taxes.

t_a = net taxes on earnings applicable only to covered jobs in the active phase.

t_p = net taxes that are applied only to contributory pensions in old age.

Labor supply in active life is assumed to be fixed. Although this is restrictive, it makes transparent that the results of this model do not rely on the income effect on leisure triggered by net taxes or by noncontributory subsidies.

The net tax rates t_a and t_p represent the substantial health insurance and unemployment insurance taxes levied on contributory pensions and covered earnings, but not levied on earnings in exempt jobs nor on the product of voluntary saving. Since health insurance taxes tend to be large and highly redistributive, in the sense that they have almost no compensatory benefits at the margin, the marginal net tax is high. Covered earnings are also subject to the taxes that finance other branches of social insurance, and even to mandatory housing contributions (as in Brazil and Mexico), whose marginal benefits are way below the marginal tax rate. The net tax rate can also include personal income tax rates in cases where the earnings from exempt jobs are effectively free from income tax.

2.2 The contributory old-age pension system for the middle classes

There is a debate about the justification for State promotion of contributory old-age pensions (mandates, fiscal incentives) for the middle classes. On one hand, State promotion is defended as help to those that neglect old age.[8] Many workers have difficulty visualizing the long-term future. Others suffer from excessive optimism and neglect old age and its pessimistic overtones until late in their lives (Bernheim, Skinner and Weinberg, 2001). On the other hand, other authors argue that there is no neglect of old age (Aguiar and Hurst 2005, Scholz et al 2006). A separate justification for State promotion is that workers may choose jobs without giving placing enough value on pension benefits, linked to work in covered jobs, not available in uncovered jobs. These include access to on-the-job learning, to employer-financed training, access to social networks, and access to the future growth of the modern economy driven by general knowledge externalities. Covered jobs also increase consumer-credit limits, since covered earnings can be verified by the lender. Thus, even if there is no neglect of old age, undervaluation of these other benefits justifies some State

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[8] Neglect of old age is different from the hyper rational time-inconsistency in preferences discussed in Laibson (1997), because the latter excludes surprise when the high cost of old age is unveiled as excessive optimism dissipates over the ageing process.

promotion of contributory pensions for the middle classes. Neither neglect of old age nor externalities in job choice are modeled here.

Any contributory system for the middle classes can be described by parameters related to contributions, pension ages, the initial benefit (pension) and the indexation rule for ongoing pensions. Since only two phases of life are considered, several parameters do not apply here. Because the function of progressive redistribution is performed by the noncontributory subsidy, the contributory system is assumed to pay purely proportional benefits or earnings-related i.e. that it leaves intra-generational redistribution to the tax system and social assistance institutions.[9]

The baseline model is simplified further by assuming that the contribution base CB for old age benefits equals covered earnings, i.e. that $CB(y^c) = y^c$. This omits a Maximum on taxable earnings (*MTE*), a positive Least taxable earnings (*LTE*), and a minimum density required to vest second-pillar old-age benefits (D_{2P})[10]. The impact of these parameters is studied in section 5. These assumptions cut down the number of parameters to just two:

θ = effective contribution rate for old age, deducted from gross earnings. This applies only in covered jobs and uniform for all participants. If there are separate contribution rates paid by workers and employers, θ is the equivalent rate that would apply if the total actual contribution were paid by the worker alone, equal to $(\theta_w + \theta_E)/(1 + \theta_E)$.[11]

ρ^c = internal rate of return (real terms) paid by the contributory system to each generation of participants, net of taxes. The financing method used by the contributory pension system and its maturity affects the average ρ^c per generation: full funding should bring ρ^c close to the returns on saving offered by voluntary saving vehicles. Mature pay-as-you-go finance under a rule where at least one parameter is adjusted to insure financial independence from

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[9] This division of labor maximizes transparency and thus minimizes the risk of capture by pressure groups. This division does not impair the ability of the tax system to redistribute on the basis of lifetime wealth, since subsidies and taxes in old-age are conditional on income flows that depend on lifetime earnings, such as contributory pensions and capital income. First pillar subsidies also redistribute based on lifetime wealth.
[10] The individual that does not comply with density D_{2P} loses all contributions (those made in the name of the employer and on the name of the worker), so R^c = -100% if D < D_{2P} . Vesting conditions are standard in traditional contributory systems. In the U.S. Social Security system, D_{2P} = 120 months (10 years). In contrast, notional and fully funded defined contribution systems usually set $D_{2P} = 0$.
[11] This formula assumes also that all contributions to health insurance and other branches of social insurance are paid by the worker, none by the employer. The general case is available in Valdés (2002) chapter 5.3.

the fiscal budget, makes ρ^c equal to the growth rate of the real covered wage bill, which is smaller than the rate of return on physical capital in efficient economies. In funded and partially funded finance, the tax treatment of the returns earned by the pension fund is an important determinant of ρ^c. The replacement rate is $\theta \cdot (1 + \rho^c)$.

In the case of mandates, the net tax rates t_a and t_p also include the implicit taxes associated to *illiquidity*. In the accumulation phase, the stock of accumulated pension rights is not available to cover legitimate emergencies, owing to the widespread use of inalienability rules (a creditor is not allowed to collect a lien on pension rights; Andolfatto 2002). In contrast, in exempt jobs all saving can be held in liquid form, so saving is more valuable on average. The difference in value is an implicit tax on mandatory saving, which can be included in t_a. Similarly, mandatory annuitization creates illiquidity in the passive phase, which is represented by an increase in t_p (Valdés-Prieto 2002, p. 223-229).

In the case where the State supports contributory pensions with fiscal incentives (individual or employer-based), the tax regimes affects t_a, ρ^c and t_p. Each of these rates should be interpreted as net of the tax treatment granted by the State compared to uncovered jobs.

2.3 Income identities and budget constraints for an individual

In each phase of life, the income identities before any voluntary saving are[12]:

(1a) $\quad y_a(D) \equiv (y^c \cdot D) \cdot (1 - \theta - t_a) + (z^{ex} \cdot y^c) \cdot (1 - D)$

(1b) $\quad y_p(D) \equiv (y^c \cdot D) \cdot \theta \cdot (1 + \rho^c) \cdot (1 - t_p) + e_p \cdot y^c$

It is instructive to obtain from (1) the marginal rate of transformation between y_a and y_p:

(1c) $\quad MRT_{CS} \equiv -\dfrac{dy_p}{dy_a} \equiv -\dfrac{\partial y_p / \partial D}{\partial y_a / \partial D} = (1 + \rho^c) \cdot \dfrac{(1 - t_p)}{\left[z^{ex} - (1 - \theta - t_a) \right] / \theta}$

Equation (1c) reveals the critical feature of the model, which can be gleaned by assuming that t_a and t_p are zero. The effective gross rate of return on contributions can be higher than $(1 + \rho^c)$, because the denominator $[((z^{ex} - 1) / \theta) + 1]$ can be smaller than 1. Given that a covered sector exists, the situation $z^{ex} < 1$ is possible if the covered employers compete

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[12] (1a) assumes that health insurance contributions have the same base as old-age pension contributions.

among themselves for a limited aggregate supply of workers with attributes similar to the individual in (1), and make the equilibrium gross earning y^c larger than y^{ex}.

If the salary paid by the large employers is sufficiently above earnings in the uncovered sector, the worker makes only a minor sacrifice when taking a covered job, as compared to the pension benefit. This individual may choose to bundle saving for old age and job choice, despite the inability or unwillingness of the State to mandate bundling of the job selection and the saving decisions in all jobs.

Of course, it is also possible that $z^{ex} > 1$. In this case the denominator is larger than 1 and the effective rate of return on contributions is lower than $(1+ \rho^c)$.

To determine whether saving for old age and job choice are bundled for this individual, it is necessary to compare the MRT offered by the contributory pension system with the gross returns offered by the financial market (see 2.4).

But three myths can be dispelled at this stage: First, a worker that (partially) neglects old age always chooses zero density of contribution. Second, the poor never save for old age because their current needs outweigh any gain from saving for the long term (Titelman and Uthoff 2005, Van Ginneken 2007). Third, illiquidity of pension rights would lead to null valuation of contributory pensions (Diamond and Valdes-Prieto 1994). However, if the effective gross rate of return on contributions is high enough (helped by a small enough $[z^{ex} - (1-\theta -t_a)]/\theta$), she may still find that the covered job is preferable, despite any of these three traits. Note that valuing an old-age pension at less than if old age were fully visualized, is different from valuing it at zero. Thus, the presence of any of these three traits age does not imply that desired saving for old age will be zero.

Remark: a tradeoff between job choice and saving arises *only* if the denominator in (1c) is positive, i.e. if $z^{ex} > (1-\theta -t_a)$. If not, covered jobs are productive enough for the individual, relative to uncovered jobs, to yield more income in *both* phases of life. The extra productivity of covered jobs makes up for contributions θ and for the tax differential t_a, so that the contributory pension comes for free. This leads the individual to choose the

maximum possible density, $D^* = 1$, regardless of her subjective discount rate and of conditions in the financial market for pure saving.

To identify the impact of net taxes, assume $z^{ex} = 1$. In this case the effective rate of return shown in equation (1c) is $(1 + \rho^c) \cdot (1 - t_p)/[1 + (t_a/\theta)]$, where the tax regimes affects t_a, ρ^c and t_p. Since the earnings of covered jobs are always more taxed than in uncovered jobs, and since benefits from a formal pension system are more easily taxed than liquidation of voluntary savings, differential taxes on earnings and benefits are usually a reason *against* bundling job choice with saving for old age. However, differential subsidies to the rate of return of the pension fund ρ^c could make the combined MRT$_{CS}$ more attractive than saving in the financial market.

The MRT$_{CS}$ offered by the contributory pension system must be compared with those offered by the financial market, which is open for all individuals. Let us endow the individual with the option of making pure voluntary saving or dissaving in amount S. The rate of return in pure saving is r (in real terms, net of taxes), and the level of this rate is allowed to depend on the sign of S. Since it is an empirical fact that the rate of return is larger when the individual is indebted with consumer loans, than when he is a net saver, it is assumed that $r(-) > r(+)$, where $r(-)$ is the rate of return on consumer loans ($S < 0$) and $r(+)$ is the interest rate on saving ($S > 0$). The period budget constraints are:

(2a) $c_a = y_a(D) - S$

(2b) $c_p = y_p(D) + S \cdot [1 + r(sign(S))]$

The next subsection shows how the job selection and the saving decisions interact and may become bundled, as governed by the relative sizes of the MRT$_{CS}$ offered by contributory pensions and the gross returns offered by the financial market, $1 + r(+)$ and $1 + r(-)$.

2.4 Individual optimization

The individual maximizes lifetime utility. As usual, utility is assumed to be additive separable across phases of life. Labor supply is assumed to be inelastic in the active phase to avoid interaction with the allocative effects of income effects, such as those introduced by the non-contributory subsidies considered in section 3. To achieve this, the utility

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function must be quasi-linear in consumption (Diamond, 1998). To simplify further, the utility of leisure in old age is assumed to be additive separable from the utility of consumption in old age ($U_{cp,lp} = 0$). The individual solves the following program:

(P1)
$$Max_{\{D,S,l_p\}} \ U \equiv c_a + u(\hat{l}_a) + v(c_p) + n(l_p)$$
$$subject \ to \ (2), \ to \ D \ \varepsilon \ [0,1] \ and \ to \ l_p \ \varepsilon \ [0,1].$$

where $\hat{l}_a$ is fixed and $l_p \equiv 1 - (e_p y^c / w_p)$ are the proportions of hours taken as leisure in the corresponding phase of life, and w_p is the net wage per hour available for work in old age. As usual, u', v' and n' are positive, while second derivatives are negative. The utility discount factor for old age is incorporated into the functions v and n.

Since in this model the budget constraint is the result of competition between linear options, many corner solutions are possible. Rather than going through all possible combinations as in the Kuhn-Tucker conditions, these corners are ordered using the following identity, obtained from (P1):

(3a)
$$\frac{\partial U}{\partial D} \equiv \left\{ \frac{\partial U}{\partial S} \cdot \left(z^{ex} - (1 - \theta - t_a) \right) + v' \cdot \left[\theta(1 + \rho^c)(1 - t_p) - \left(z^{ex} - (1 - \theta - t_a) \right) \cdot (1 + r(signS)) \right] \right\} \cdot y^c$$

This can be reordered in a more revealing way as:

(3b)
$$\frac{\partial U}{\partial D} \equiv \left\{ \frac{\partial U}{\partial S} + v' \cdot [MRT_{CS} - (1 + r(signS))] \right\} \cdot \left(z^{ex} - (1 - \theta - t_a) \right) \cdot y^c$$

Equations (3) show that the relationship between pure saving and job choice (selection of density D) depends on the sign of two terms: the term $\left(z^{ex} - (1 - \theta - t_a) \right)$, which is the relative productivity of the uncovered sector as seen by the individual in the active phase, and the term in the square bracket of (3b).

If the expression $\left(z^{ex} - (1 - \theta - t_a) \right)$ is negative, meaning that covered jobs dominate uncovered jobs, version (3a) must be used, where it can be seen that both terms in the square brackets are positive. If in addition pure saving S is an interior solution, so that $\partial U / \partial S$ is zero, the positive square bracket in (3a) makes $\partial U / \partial D$ positive always. Thus,

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$D*$ must be in the corner with $D* = 1$. This is intuitive, because a negative $\left(z^{ex} - (1 - \theta - t_a)\right)$ makes choosing covered jobs dominant.

When expression $\left(z^{ex} - (1 - \theta - t_a)\right)$ is positive, version (3b) can be used. If voluntary saving S is in an interior solution (so that $\partial U/\partial S$ is zero), the sign of $\partial U/\partial D$ is governed by the sign of the term in square brackets of (3b). This term is the difference between the return of saving through the contributory system (eq. 1c) and the return on saving through the financial market. Either return may dominate. If the term in square brackets is negative, saving through the mandatory system is inferior to voluntary saving and the labor optimum is at the corner with the lowest possible density ($D* = 0$, case F1 below). If the term in the square bracket is positive, the labor optimum is with $D* = 1$ (case F3 below). An interior solution for $D*$ applies to a range of intermediate cases ($D* \varepsilon$ [0,1], case F2 below).

PROPOSITION 1: The individual's optimum can be in one of only four situations, labeled F1 to F4 and characterized as follows. Job choice and saving are bundled in cases F2, F3 and F4:

    a)   F1: Pure saving for old age has a *higher* net effective return than saving through the contributory system, allowing uncovered jobs to dominate. The individual prefers zero density ($D* = 0$) and channels all saving through pure saving vehicles. $S*$ has any sign.

    b)   F2: The rate of return on saving in the contributory system is *intermediate* between the return on pure saving $r(+)$ and interest on consumer credit $r(-)$. There are two subcases: In case F2a, $D*$ is *interior* and $S* = 0$ (the individual avoids the financial market). In case F2b, $D* = 1$ and $S* > 0$ (the individual is a saver).

    c)   F3: covered jobs dominate only because the return on contributory-system saving is higher than the interest in consumer credit $r(-)$. In this case $D* = 1$ and $S*$ has any sign.

    d)   F4: covered jobs dominate uncovered jobs regardless of returns. Thus $D* = 1$ and $S*$ has any sign.

Proof: A combination of eq. (3) and the global analysis provided in Figure 1 below.

Figure 1 shows the consumption opportunity sets. The heavy line from A to F represents incomes achievable in the *absence* of voluntary saving ($S = 0$). Extreme A corresponds to the case in which the worker chooses *zero* density (absence of contributions to the second pillar), while extreme F corresponds to full contribution density, i.e. to $D = 100$ percent. The equation for AF is obtained by eliminating D from equations (1a) and (1b). The slope of AF is the net return of mandatory saving, $(1+\rho^c)(1-t_p)/([z^{ex}-(1-\theta-t_a)]/\theta)$, which is the MRT_{CS} that appears in equations (1c) and (3b). Recall that this return can be higher than $(1+\rho^c)(1-t_p)$, because the denominator $[z^{ex}-(1-\theta-t_a)]/\theta$ can be smaller than $(1-t_p)$. The height of AF in the vertical dimension is $\theta\cdot(1+\rho^c)(1-t_p)\cdot y^c$, so AF is flat if $\theta = 0$, which represents the case when the State does not promote any pension system.

In Figure 1, opportunities for financial saving are represented by the dashed lines that start at AF and go Northwest. The individual may also choose to save less than θ by issuing consumer debt, an option represented by the dotted lines that starts from AF and go Southeast. The higher slope of the lines going Southeast represents a higher interest rate on consumer debt than on saving.
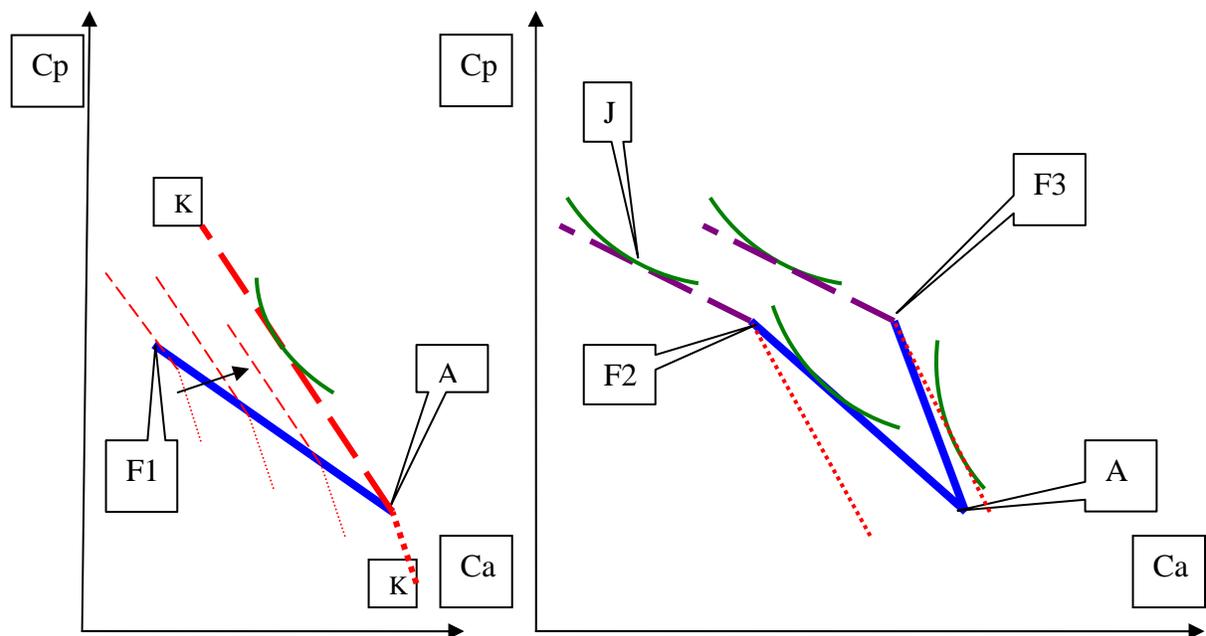

Figure 1: Budget constraints in a State-supported contributory pension plan (left panel: case F1; right panel: cases F2 and F3)

The left panel shows the case F1, where the slope of line AF1 is smaller than $(1+r(+))$ in absolute value (and also smaller than $1+r(-)$, since $r(-) > r(+)$). This means that voluntary saving for old age is more attractive than the contributory saving associated to covered jobs. The implication for contribution density is dramatic: the individual is better off by moving to the corner with zero density ($D^* = 0$) and making any desired saving through voluntary vehicles. This is of direct policy interest: if the contributory systems offer a relatively low net rate of return, more workers prefer zero density and State support for contributory plans fails.

The case labeled as F2 in the right panel shows that when the return offered by the second pillar (the slope of line AF2) is intermediate between the return offered by voluntary saving vehicles $(1+r(+))$ and the cost of consumer credit $(1+r(-))$, there are two subcases. In one, the individual chooses an *interior density* of contribution ($0 < D^* < 1$), rather than a corner. This subcase happens when the contribution rate θ is larger than the desired saving rate: full density is selected to take advantage of the higher rate of return of the contributory pension plan, but excess saving is undone by a limited reduction in density rather than by incurring expensive consumer debt. The first order conditions for this case do not depend on the financial market, because it is being bypassed. Therefore, an increase in the rate of return in the financial market has no effect on any individual's labor supply in case F2a.

In subcase F2b the desired saving rate is higher than θ, so point J is preferred, where $D^* = 1$ and $S^* > 0$ to make up for the desired extra saving.

In case F3, also in the right panel of figure 1, an interior contribution density is never optimal. If the contribution rate θ were larger than the desired saving rate, a limited reduction in density would be more expensive than consumer debt. And if the contribution rate θ is smaller than desired, $S^* > 0$ to complete the difference. Thus $D^* = 1$ in all cases.

The case labeled F4 is not shown for simplicity. Its graph is similar to case F3, with one difference: the line AF has a positive slope, because covered jobs dominate.

The model also yields comparative statics results.

PROPOSITION 2: A sequence of increases in the rate of return (the MRT_{CS}) paid by the contributory system creates two discontinuous increases in the density chosen by an individual, at given earnings per period, as follows:

a)   Initially, there is no effect on labor decisions by any individual in cases F1, F2b, F3 and F4, because she still prefers the same densities of contribution. Only the individual in case F2a -the subcase with an interior solution for D – responds, and gradually to the increase in MRTCS.

b)   When the MRTCS rises above a threshold, equal to $1+r(+)$, the individual in case F1 switches discontinuously to case F2a and raises her density from $D^* = 0$ to $D^* \varepsilon (0,1]$.

c)   Further increases in MRTCS attract small and gradual increases in the density of contribution, and only for the individual that chooses an interior solution (F2a).

d)   When additional increases in MRTCS equalizes $1+ r(-)$, the individual in case F2a jumps discontinuously to case F3, where $D^* = 1$.

Proof: Graphically, the slope of AF rises, when the contributory system pays a higher return. Note that the height of point F does not change, because it is given by (1) alone. The result that density rises in response to a higher MRT_{CS} requires that take-home earnings in the active phase are held constant.

Proposition 2 may be relevant for policy, since it shows that contribution density can be quite elastic to the net rate of return paid by mandatory saving in some ranges. To test this proposition, individual data would be needed. For sufficiently smooth distributions of workers' attributes, density of contributions should be observed to vary faster in relatively narrow ranges, given by stages (b) and (d).

Up to here all the results refer to labor supply. Overall equilibrium depends also on the elasticity of labor demand in the covered sector. If the demand of uncovered jobs is elastic,

so that wages do not change much in response to these changes in job choice, equilibrium contribution density changes as well.[13]

This section shows that despite the inability or unwillingness of the State to impose a mandate to participate on all workers, large subsets of workers bundle the job selection and the saving decisions. The next sections consider applications of this model where the net taxes cannot be described by a fixed rate, as in noncontributory subsidies for old age.

3. Standard first-pillar pensions in the presence of a contributory system

This section applies the model to noncontributory subsidies for the old poor ("first pillar pensions"). The standard rationale for them is progressive redistribution. This encompasses the rationales of avoiding horizontal inequity and risk aversion from behind a veil of ignorance.

This section demonstrates the proposition that non-contributory subsidies for the old (first pillars) crowd out contributory pensions (second pillar if mandated, third pillars if fiscally favored). This proposition is valid only for those workers that have *low labor productivity*. This is because the amounts of first pillar pensions must be attractive for their presence to make any difference. This is certainly the case for the lifetime poor, but it can also be the case for the lower middle classes.

Two prevalent designs for first-pillar pensions are assistance pensions and minimum pension subsidies. They are analyzed separately.

3.1 An Assistance Pension in the presence of a contributory system

A standard assistance pension is defined here to be a flat subsidy with cliff withdrawal:

(4) $NCS_{std}^{A}(D) = \begin{cases} A & if \ (y^{c}D) \cdot \theta(1+\rho^{c}) < T \\ 0 & if \ not \end{cases}$

where T is a threshold for the size of the contributory pension, measured in \$/month. An assistance pension may also have analogous thresholds, referred to the size of labor

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[13] The effects on individual saving do not have an impact on the aggregate capital stock when the affected workers receive a modest fraction of national income.

earnings in old age, to the size of per capita household income (which is indicative of intra-household transfers) and to the size of capital consumption. If all thresholds are raised without bounds, a universal flat pension obtains.

A higher density of contribution D raises the contributory pension, and this may trigger the full loss of the subsidy, of size A. The budget constraint for old age must be modified to:

(2b') $c_p = y_p(D) + S \cdot [1 + r(sign(S))] + NCS_{std}^A(D,T)$

The results of going through the new budget equations are presented in detail for case F2, and to save space the results are merely summarized for cases F1, F3 and F4.

The introduction of the Assistance Pension changes the budget constraint from (2b) to (2b') and this twists the thick solid line AF2 in figure 2, by adding the thick dotted line AA'DEF2. This line falls in point D, to register the fact that when density rises sufficiently to make the contributory pension exceed the statutory threshold T, the worker *loses the whole* subsidy A at once. Thus, at E his old age income is limited to income from work and the contributory pension.

The thick budget lines do not consider pure saving S. Pure saving and dissaving opportunities at point F2 create, respectively, the dashed line that goes from F2 to the Northwest (net saver status), and the narrow dotted line that goes from F2 to the Southeast (net debtor status). Similarly, saving opportunities around point D create the dashed line that starts from D to the Northwest, and opportunities for consumer credit create the narrow dotted line that starts from D to the Southeast. Although point F2 is not dominated, it now has serious competitors along line A'D.

*Figure 2: Contribution density falls when an Assistance Pension is introduced
(case F2 with S*= 0 in the absence of noncontributory pensions)*

If the original optimal decision for density was at point B, the introduction of the assistance pension can change the preferred density to a point like C. The new density of contribution is definitely below the original one. This is shown by the thick horizontal arrow, which measures the increase in consumption in the active phase due to the choice of longer spells in uncovered jobs, brought about by the introduction of the assistance pension. The optimal contribution density falls discontinuously due to the introduction of the assistance pension.

The abrupt withdrawal of the assistance pension at point E explains the large incentive to reduce contribution density, that is, to reject covered jobs and give preference to self-employment and home production once D has been reached. If the demand of covered jobs is elastic, so that wages do not change much in response to these changes in job choice, equilibrium contribution density will change. The conclusion is that the *design* of the assistance pension drives a reduction in equilibrium contribution density.

In case F1 uncovered jobs are more desirable and all saving for old age is made through pure saving vehicles. Since the assistance pension is offered to individuals with zero density, its presence is a further reason to keep density at zero. The introduction of the assistance pension does not affect the optimal density, which remains at zero in case F1. (*D*=0*).

In cases F3 and F4, the introduction of an assistance pension can affect the choice of density ($D^*=1$). This happens if the size A of the subsidy is large enough to be similar in size to the contributory pension. In this subcase, there is a tradeoff between reducing density below 100% in order to gain access to the subsidy A, and losing income from a covered job. The introduction of an assistance pension can reduce density discontinuously.

3.2 A Minimum Pension Subsidy in the presence of a contributory system

A "minimum pension subsidy" is defined as:

(5) $\qquad NCS^{MP}(D) = \begin{cases} 0 & if \ D < D_T \\ (1-t_p) \cdot \left[MPGoal - (y^c D) \cdot \theta(1+\rho^c) \right] & if \ D \ is \ in-between \\ 0 & if \ (y^c D) \cdot \theta(1+\rho^c) > MPGoal \end{cases}$

Where D_T is a threshold of contribution density required to have access to (vest) the subsidy. *MPGoal* is the statutory level of the minimum pension, which may be adjusted annually by ad-hoc legislation to follow the evolution of median or average earnings. This is the target that must be reached when adding the noncontributory subsidy to the self-financed contributory pension. The maximum subsidy is [$MPGoal - y^c D_T \theta(1+\rho^c)$].

This design has a double rationale. On the one hand, a higher contributory pension is taken as an indication of less need, so the noncontributory subsidy is reduced. On the other hand, stimulating the attainment of a positive level of contributions (D_T) seems to help alleviate neglect of old age. This rationale is discussed further in section 5.

Because of the subsidy, the budget constraint in old age is modified to:

(2b'') $\quad c_p = y_p(D) + S \cdot [1 + r(sign(S))] + NCS^{MP}(D, D_T, MPGoal)$

For the "in-between" range of D, when voluntary saving is held constant:

(6) $\qquad \left. \dfrac{\partial c_p}{\partial D} \right|_{D \ in-between}^{S=cons \tan t} = \dfrac{\partial y_p}{\partial D} + 0 + \dfrac{\partial NCS^{MP}}{\partial D} = y^c \theta(1+\rho^c)(1-t_p) + \left[-y^c \theta(1+\rho^c)(1-t_p) \right] = 0$

Equation (6) shows that consumption in old age does not increase at all when density rises! Each $1 of additional contributory pension is used by the state to withdraw its own subsidy by $1, leaving nothing extra to the worker. The "withdrawal rate" of the minimum pension subsidy is 100%, in this range. The withdrawal rate is an implicit tax on covered earnings.

Of course, with twisted budget constraints a local analysis such as the one in equation (6) is insufficient. In case F2, the global situation is described by figure 3. Unlike the case of the assistance pension, no subsidy is obtained with zero density, because the contribution requirement of D_T years is not met. Accordingly, there is no point A'. Only when the worker increases contribution density to point B, does he increase his total old age income, including the subsidy, to point B'.

Further contributions after point B' are fully taken away by the 100% withdrawal rate. For example, total income in old age at point C is the same as the one in point B', despite higher density. This is against the worker's interest, because income during his active phase is *less at C than at B'* (he has a lower take-home wage owing to the fact that his contributions are higher). In figure 3, the bold dotted line is flat in the region in which the State reduces the subsidy. The flatness indicates that the withdrawal rate is 100 percent.

We have not considered pure saving *S*. From any given income point where pure saving is zero, positive saving opportunities create the dashed line to the Northwest, and consumer credit creates the dotted line to the Southeast. Figure 3 presents a subcase where F2 is not yet dominated by B'.

Figure 3: Contribution density moves to D_T when a Minimum Pension is introduced (case F2)

A participant finds optimal to turn down covered jobs once he has reached or expects to reach point B'. If he wants extra consumption at any age, it is always more desirable to attain it through pure saving and dissaving than through contributory pensions. For example, pure saving allows a move from B' to E, which dominates a move from B' to point C, because the return on pure saving is not taxed at the expropriatory implicit rate of 100 percent.

For preferences marked D and E in figure 3, the new optimal density of contribution (at point E) is $D^* = D_T$, well below the one chosen in the absence of the minimum pension subsidy (point D). The thick horizontal arrow in the upper half of figure 3 shows a large discontinuous reduction in density of contribution, from point D to B. Some of its consumption consequences are undone by increasing voluntary saving from B' to E.

However, for the preferences marked A and G in figure 3, the new optimal density (which supports point G) is *higher* than the one chosen in the absence of the minimum pension subsidy (point A). This is due to the density requirement $D_T > 0$: the participant observes that by increasing density up to point B, i.e. by choosing $D^* = D_T$, he attains the maximum

subsidy. This strategy requires sacrifice of some take-home wage, but the consumption loss when active can be fully mitigated by taking consumer debt in amount BG (measured in the horizontal axis), along the dotted line that starts from B' to the Southeast. In figure 3 preferences are chosen so that points G and A yield the same consumption when active. Underlying this constancy in consumption there is a large discontinuous increase in density of contribution, marked by the thick horizontal arrow from A to B. Thus, a minimum pension subsidy *alone* has ambiguous consequences for chosen contribution density.

An unintended consequence of a minimum pension subsidy is that it may favor the consumer credit industry, rather than the old poor. Participants that want point G must pay $[r(-) - r(+)] \cdot (c_p^{B'} - c_p^{G})$ in interest for consumer debt. If the supply of consumer credit is less than perfectly elastic, then some of the incidence of the subsidy reaches the owners of consumer credit operations. This reduces the efficiency of first pillar subsidies.[14]

For completeness, consider cases F1 and F3. In case F1 exempt jobs are preferred and all saving for old age is made through pure saving vehicles. In case F1, the minimum pension subsidy *raises* the density of contributions from $D^* = 0$ to $D^* = D_T$, in those subcases where the size of the maximum subsidy is attractive enough. The figure that proves this is omitted for brevity, but the intuition is straightforward: getting to a point like B', where the full subsidy is obtained, can be worth losing the advantages of reducing density to zero.

In case F3, the introduction of a minimum pension subsidy can *reduce* density from $D^*=1$, but only in a rather extreme subcase. Cutting density to $D^* = D_T$ gives access to the maximum subsidy, and this may be larger than the earnings loss due to the lower labor productivity of uncovered jobs. This subcase is extreme because $D = D_T$ can only be better than $D = 1$ if, at the margin, the present value of the subsidy discounted at rate $r(-)$ rises more than what is lost from a more productive job.

3.3 Simultaneous presence of an Assistance Pension and a Minimum Pension Subsidy

A number of countries offer both subsidies together. One reason is that with a minimum pension subsidy alone, women engaged in home production fail and individuals that neglect

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[14] Since take-up of first pillar subsidies is voluntary, this loss cannot exceed the maximum subsidy. This is not the case for mandatory second pillars, where the loss in interest payments in consumer credit can leave the participant worse off than in the absence of the second pillar (Valdes-Prieto 2002, p. 215-218).

old age choose a density D smaller than D_T and can fall into dire poverty when old in the absence of intrafamily transfers.

When both subsidies are offered simultaneously, an incompatibility rule is standard: both subsidies cannot be received simultaneously (in old age). Provided that the contributory system pays an annuity that is constant over time, incompatibility is resolved at pension age, as follows: given that density D and pure saving S are pre-determined as of pension age, each individual calculates the sum of contributory pensions and each of the subsidies, and chooses the highest sum. Since the options will be the same at any time after pension age, choices will remain constant over time too.

Figure 4 shows the empirically relevant case where the size A of the assistance pension is well below the size of the maximum subsidy offered by the minimum pension schedule.
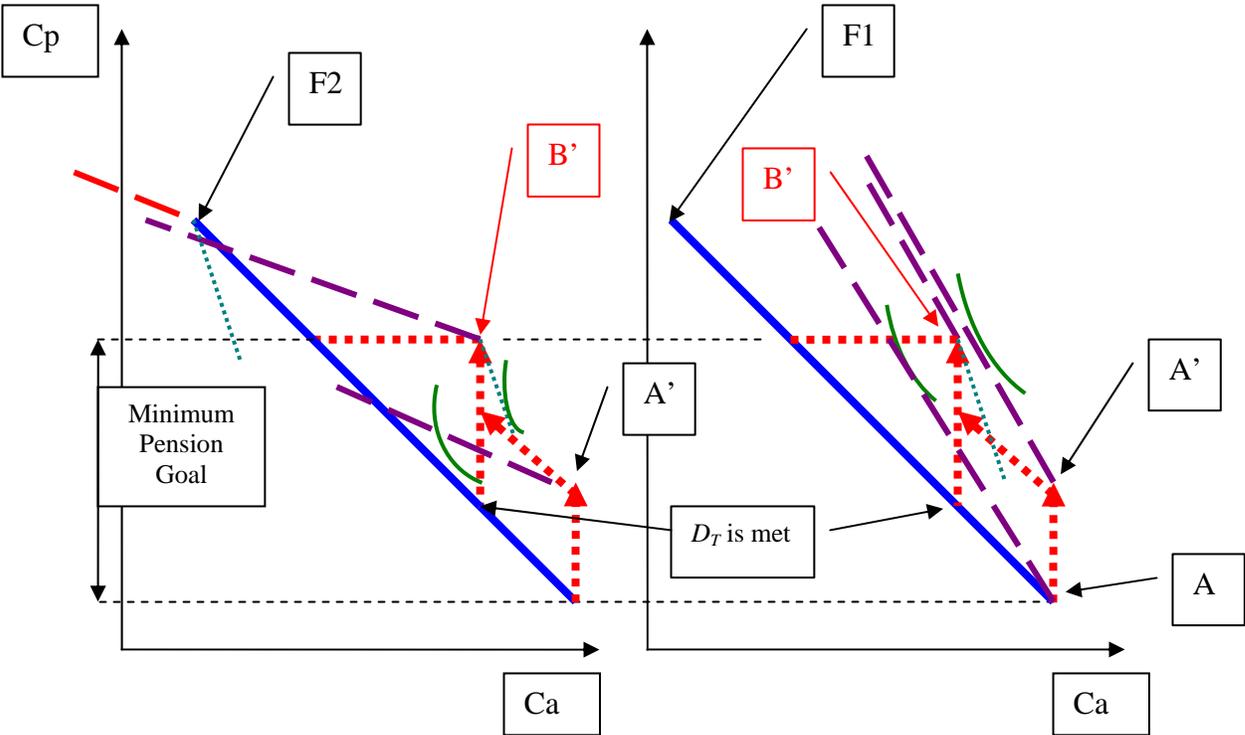
Figure 4: Contribution density falls when both designs are offered simultaneously (case F2 in the left panel and case F1 in the right panel)

The left panel of figure 4 shows the simultaneous offer of both subsidies under the incompatibility rule, for case F2. The presence of the assistance pension makes selection of

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$D^* < D_T$ attractive for the individuals with preferences such that, in its absence, would have chosen $D^* = D_T$ (as in point G in figure 3). Thus, the vesting condition of the minimum pension subsidy is crowded out by an assistance pension with no vesting condition.

Of course, some individuals would still be attracted to $D^* = D_T$, for example those with the indifference curve shown in the left panel of figure 4. However, the figure also shows that in the absence of both subsidies, that individual would have chosen a density *higher* than D_T. Therefore, density always falls (in case F2) when both first-pillar programs are offered.

The right panel of figure 4 shows case F1, where in the absence of subsidies uncovered jobs are preferred and all saving for old age is made through pure saving vehicles. The case shown is one where pure saving that starts from point A' dominates pure saving that starts from point B'. The optimal density D^* remains at zero despite the presence of a minimum pension subsidy that would have attracted an increase in D^* to D_T if present alone. Again, the presence of an assistance pension trumps the vesting requirement of the minimum pension subsidy.[15]

In conclusion, the simultaneous offer of both subsidies eliminates most of the cases where minimum pension subsidies increase the density of contribution.

Summing up, section 3 finds that standard first pillar subsidies reduce the equilibrium density of contribution. The implication is that part of self-employment among the low-productivity workers can be explained by the poor design of these social programs. They create a "low contribution density trap." This trap punishes, through subsidy withdrawal, those who increase their contribution density. At the same time, these programs have succeeded at improving the material well-being of their beneficiaries.

4. Second-generation designs for first-pillar pensions

This section analyzes two second-generation designs: a "proportional" minimum pension, and a subsidy with a single withdrawal rate in proportion to second pillar pensions.

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[15] This may explain why coverage of contributions fell in Chile in the late 1970's as compared to the 1960's: an assistance pension was introduced in 1975.

4.1 A proportional minimum pension

Consider a "proportional" minimum pension, like the one that Switzerland had in its AHV system (Valdés-Prieto 2002, p. 62). In this design, the goal used by the minimum pension subsidy is not a fixed number, but is proportional to the number of years (or months) of contribution attained by each individual participant. This design can be represented by the following expression for the subsidy amount:

$$(8) \quad PMPS^i = Max\Big[\ 0 \ ; \ \big(N^i/N_R\big) \cdot MPGoal_R - CP^i(D(N^i)) \Big] \qquad \text{(\$/month)}$$

where $PMPS^i$ is the amount of the monthly subsidy in old age for participant i, N^i is the number of periods of contributions by participant i (when the period is made smaller, an almost continuous version is obtained), $MPGoal_R$ is the minimum pension goal for a reference number of periods of contribution, namely N_R, and CP^i is the contributory pension built by participant i, which depends on density, which also depends the number of periods of contribution. Note that the minimum density requirement D_T is zero.

Equation (8) highlights the fact that the proportion of periods with positive contribution is different from density, which is the proportion of the present value of earnings that is subject to contribution. The distinction is important because N^i affects the contributory pension exponentially: a higher N^i allows the internal rate of return of the second pillar system to operate for a longer time. For the admittedly restrictive case of a flat age-earnings profile with constant per-period density of contribution, the contributory pension is proportional to the future value function: $CP^i = k^i \cdot [(1+\rho^c)^{Ni} - 1]/\rho^c$, where k^i is a proportionality constant that depends on the declared earnings of individual i, the per-period density and life expectancy, and ρ^c is now the per-period internal rate of return paid by the contributory system to each generation.

In contrast, the minimum pension goal grows linearly with N^i. The discrepancy between exponential and linear growth has three implications: [16]

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[16] This discrepancy cannot be captured by the model of sections 2 and 3, where all periods in the active phase collapse into one. This prevents distinction between density D and the number N of periods of contribution.

a)   When the number of years of contribution is small, the minimum pension goal grows faster than the contributory pension. Since the subsidy *grows* with N^i, there exists a marginal subsidy to increase the number of periods of contributions.

b)   For a larger number of years of contribution, the subsidy *falls* with N^i, and thus there is a marginal tax on raising density. The marginal tax is zero when $PMPS^i$ reaches *0*.

c)   The amount of the subsidy has a different interior maximum for each k^i. The N^i that maximizes the subsidy is larger for lower declared salaries.[17]

Valdés-Prieto (2006) argued against a proportional minimum pension on the grounds of inefficiency and horizontal inequity. Inefficiency occurs because the marginal tax/subsidy rate on additional contributions varies in an unintended and very wide range. The value depends on the size of individual earnings (k^i) and on N^i, even for cases with constant per-period density of contribution. According to simulations, the marginal subsidy can be as high as 350% for low values of k^i and falls continuously as N^i rises until very negative rates are reached, according to a curvature that depends on the size of earnings. A 350% subsidy rate is large enough to justify fraud. At the other extreme, the very high implicit tax rates on additional contributions applied to many participants means that one of the less desirable aspects of the minimum pension subsidy is retained.

A withdrawal rate that varies continuously with (k^i, N^i) also has a high level of complexity, relative to a subsidy with a unique withdrawal rate, and this raises the risk of planning errors for individuals. Finally, the proportional minimum pension remains vulnerable to underreporting of earnings.

Moreover, the proportional minimum pension fails the test of horizontal inequity as soon as density of contribution ceases to be constant. The following illustration was offered by Valdés-Prieto (2006).

Case A: A seasonally employed woman works intensively on the fruit harvest for 3 months a year, earning a wage of CLP$ 200,000 a month in exchange for 12-hour workdays sweating in the sun. It is assumed that she works seasonally from the ages of 20 to 39. Then at age 60 she meets the legal age for

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[17] From (8), the marginal subsidy is $\dfrac{\partial PMPS^i}{\partial N^i} \equiv \dfrac{MPGoal_R}{N_R} - \left(\dfrac{k_i}{R^c}\right) \cdot \ln(1+R^c) \cdot (1+R^c)^{Ni}$ if $PMPS^i > 0.$

starting a contributory pension, which may last until she dies at age 82. She postpones consumption of her contributions for 41 years on average.

Case B: Another (middle or upper class) woman works as a part-time secretary for a wage of CLP$ 132,665 a month, 5 hour a day, 12 months a year, between the ages of 40 and 59. Her contributory pension may last from age 60 until she dies at age 82. She postpones consumption of her contributions for only 21 years on average.

In this illustration the numbers are chosen to insure that the account balance at age 60 (pension age) is the same for both women, if the real interest rate is 5 percent a year (the balance is CLP$5.4 million). Therefore, both self-finance a contributory pension of CLP$ 31,105 per month (if annuities yield 4 percent real). Since the accumulation as of pension age is a measure of the overall saving effort, both should get the same subsidy.

The seasonal fruit-picker has only five years (60 months) of contribution. If the minimum pension goal ($MPGoal_R$) is CLP$90,000 for a reference period of $N_R = 20$ years, (close to Chilean reality as of 2006), the proportional minimum pension goal for her is $(5/20)·CLP\$90,000 = CLP\$ 22,500$. As this is smaller than her contributory pension of CLP$31,105/month, her subsidy is zero!

In contrast, the middle class secretary has 20 years of contributions. Her proportional minimum pension goal is $(20/20)·CLP\$ 90,000$. Given her contributory pension of CLP$ 31,105/month, her subsidy is CLP$ 58,895 per month. This subsidy lasts about 22 years on average, counted from age 60. The present value of this subsidy as of age 60, discounted at a real interest rate of 4 percent is CLP$ 10.2 million. Comparing with her accumulation at age 60, the average rate of subsidy for her is 190 percent.

Although the seasonal fruit-picker can collect the assistance pension, the amount of the assistance pension is about half the CLP$90,000. The inequity is obvious.

The general lesson is that a proportional minimum pension suffers from defining effort in terms of the number of years (or months) of contribution, rather than in money terms. Because of the resulting inefficiencies and horizontal inequities, a proportional minimum pension cannot be Pareto efficient, or optimal according to any social welfare function.

4.2 A subsidy with a small withdrawal rate

This section evaluates a family of noncontributory subsidies where the withdrawal rate is set at a much smaller value. Valdés-Prieto (2002, pp. 57 and 70) summarized this design with the following formula:

(9) $NCS(D)^{SW} = \max\left[0 \quad ; \quad BP - \gamma \cdot CP(D)\right]$

where CP(D) is the contributory pension financed by the participant, BP is a basic pension or subsidy given to those that never contributed to the second pillar, and γ is the "withdrawal rate" of the subsidy, with γ ε (0,1). This withdrawal rate can also be defined as γ = BP/MCPWS, where MCPWS is the "maximum contributory pension with subsidy".

Equation (9) includes the minimum pension subsidy of section 3.2 as a polar case, reached by making γ = 1 and $D_T = 0$. In another polar case where γ = 0 (if MCPWS = ∞), (9) generates a flat universal pension (paid to all the old), as in Denmark and New Zealand. In the nonlinear case where γ = 0 when *CP(D) < T* , γ = ∞ at *CP(D) = T*, and γ = 0 for *CP(D) > T,* formula (9) reproduces the standard assistance pension of section 3.1.

Therefore, the "small withdrawal rate" family described by equation (9) is intermediate the two extremes of a standard minimum pension subsidy and a universal flat pension.[18] An imperfect example is the new Swedish "minimum pension supplement", which was legislated in the 1990s and implemented in 2003. This subsidy is nonlinear and has two tranches, identified by γ = 1 when *CP(D) <* 1.26 base amounts, and by γ = 0.48 when CP(D) is between 1.26 and 3.07 base amounts (Scherman, 1999). The basic subsidy is 2.13 base amounts, or 77,532 SKR per year in 1998. According to Palme (2003), the Swedish design is based on older Finnish and Norwegian designs. Another example of "small withdrawal" is the new "solidarity pension" created by the 2008 pension reform in Chile (Valdés-Prieto 2008, companion paper), where the withdrawal rate is γ = 0.32 and there is a single tranche.

In terms of the model of this paper, a small withdrawal rate subsidy is defined by:

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[18] See PowerPoint presentation to the Presidential Advisory Commission on Pension Reform, created by the government of Chile in March 2006 (Valdes-Prieto, 2006).

$$(10) \quad NCS^{SW}(D) = \begin{cases} (1-t_p) \cdot \left[BP - \gamma \cdot (y^c \cdot D \cdot \theta \cdot (1+\rho^c)) \right] & if \ (y^c \cdot D \cdot \theta(1+\rho^c)) < MCPWS \\ 0 & if \ not \end{cases}$$

with $\gamma < 1$. The budget constraint for old age is modified to:

$$(2b''') \qquad c_p = y_p(D) + S \cdot [1 + r(sign(S))] + NCS^{SW}(D, BP, \gamma, MCPWS)$$

For the applicable range of D, when voluntary saving is held constant:

$$(11) \quad \left. \frac{\partial c_p}{\partial D} \right|^{S=cons\tan t} = \frac{\partial y_p}{\partial D} + 0 + \frac{\partial NCS^{SW}}{\partial D} = (1-\gamma) \cdot (1-t_p) \cdot y^c \theta (1+\rho^c)$$

Equation (11) says that consumption in the old age increases in proportion to $(1-\gamma)$ when density rises. Each \$1 of additional contributory pension is used by the state to withdraw its subsidy by \$$\gamma$ cents, leaving \$$(1-\gamma)$ cents to the participant, to increase his total pre-tax pension (which combines subsidies and contributory pensions). In present discounted value terms, the implicit tax rate on covered earnings is $(\gamma + t_p + \gamma' t_p)\theta(1+\rho^c)/(1+r(+))$.

As before, local analysis is insufficient. The optimization program P1 is modified by introducing (2b''') to incorporate the first-pillar pension. The modified program P1' applies to the combined system, and not just to the contributory part. The budget constraint remains the result of competition between linear options. Again, the many corner solutions can be ordered using the following identity obtained from (P1'):

$$(12) \quad \frac{\partial U}{\partial D} \equiv \left\{ \frac{\partial U}{\partial S} + v' \cdot [MRT_{CS} \cdot (1-\gamma) - (1 + r(signS))] \right\} \cdot \left(z^{ex} - (1 - \theta - t_a) \right) \cdot y^c$$

Equation (12) differs from (3b) because the net rate of return offered by the combined contributory-noncontributory system is the MRT_{CS} offered by the contributory part alone, multiplied by $(1-\gamma)$, before comparing with the return of pure saving. This affects the frontier between cases F1 and F2, and it increases the range of parameters in which case F1 applies, where $D^* = 0$. The presence of $(1-\gamma)$ also affects the frontier between case F2 and F3, by reducing the range of parameters where case F3 applies, where $D^* = 1$. The range of cases where F4 applies is not modified by γ.
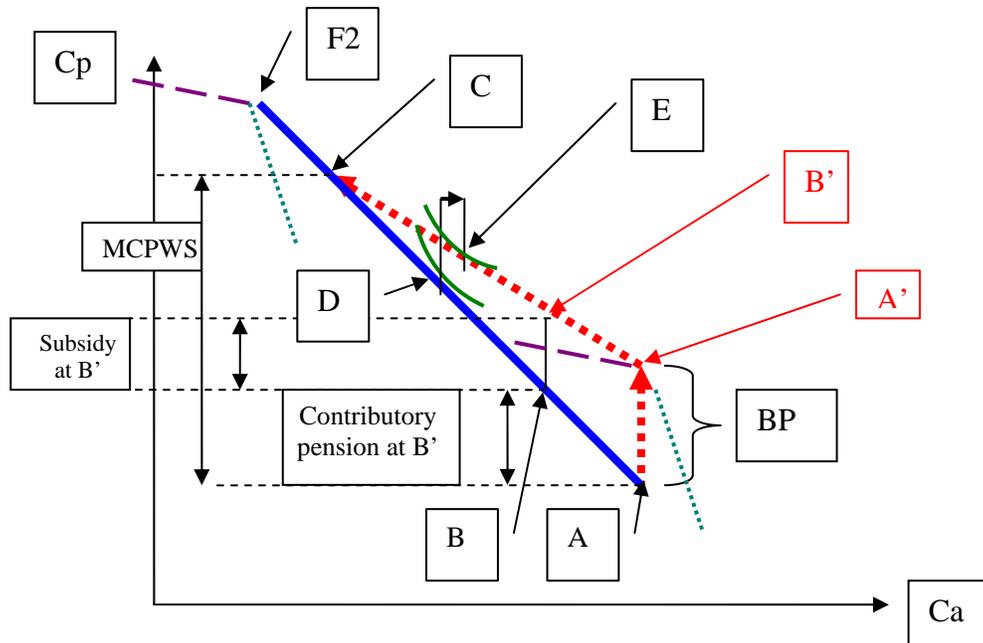
Figure 5: First-pillar subsidy with small withdrawal rate (case F2)

Figure 5 provides a global analysis for the case where F2 applies before *and* after the introduction of a first pillar with a small withdrawal rate γ. The basic subsidy BP is paid to persons who make no contribution to the second pillars, as indicated by points A and A'. The subsidies are the vertical distance between the solid line, AF2, and the dotted line A'C. The maximum contributory pension that receives a subsidy, the MCPWS, is the contributory pension at point C. The slope of A'C is the net return of the combined two-pillar system, which is the slope of AC, *MRT_{CS}*, times the new factor (1-γ). The slope of CF2 is not affected by (1-γ).

As before, pure saving and dissaving opportunities are indicated in Figure 5 with dashed and dotted lines. The assumption that case F2 applies both before *and* after the introduction of this first pillar is reflected in the slope of the dashed and dotted lines: the dashed lines are flatter than both AC and A'C, while the dotted lines are steeper than both AC and A'C. In this case, figure 5 shows that the introduction of this noncontributory subsidy reduces density from point D to E. Therefore, a small withdrawal rate *also* crowds out contributory pensions.

Figure 5 shows that when case F2 is preserved, the reduction in density is *smaller* than in the standard first pillar designs reviewed in section 3, if γ is smaller than 100%, in the case

of the standard minimum pension. Although section 3 did not show in detail the cases when the introduction of a first pillar changes the case from F2 to F1, and from F3 to F2, a detailed analysis also shows that these case changes apply to a smaller range when the withdrawal rate is smaller, and thus that these drops in density are less likely.

Other configurations are possible: First, if the slope of the dashed line (return $1+r(+)$) is steeper than A'C (but not of AC), then the introduction of this first pillar subsidy changes the situation from case F2 to case F1, and density drops discontinuously from D* ε (0,1) to D* = 0. If the slope of the dotted line (return $1+r(-)$) is flatter than AC (but not of A'C), then the introduction of this first pillar subsidy changes the situation from case F3 to case F3, and density also drops discontinuously from D* = 1 to D* ε (0,1).[19] Despite the approximate preservation of a range where case F2 applies, the introduction of this first pillar subsidy always creates a discontinuous drop in density when a case change occurs.

Summarizing, the gain offered by a small withdrawal rate is that density of contribution to the second pillar is reduced by less than with the standard designs. As the withdrawal rate is reduced towards zero, the crowding out of the contributory system is alleviated by more.

The optimal withdrawal rate

Why not reduce the withdrawal rate to zero, and adopt a universal pension? This appears to eliminate all crowding-out of the contributory pension. The fiscal cost of a universal noncontributory pension is larger than in a scheme with a positive withdrawal rate, and this requires higher tax rates and additional distortions.

Alternatively, why not reduce the withdrawal rate to zero for the poor, by starting withdrawal only after some threshold in second pillar pensions has been reached? It may be argued that the distortions caused by the withdrawal rate would affect only the middle classes, where they matter less, not the low productivity workers. This is indeed the design of the Social Security system in the United States and the age pension in Australia.

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[19] In case F4 covered jobs are more productive than exempt ones and D* = 1 in the absence of a first pillar. The introduction of a subsidy with a small withdrawal rate can reduce density only in an extreme subcase. Cutting density to D* = 0 gives access to the maximum subsidy, and if this subsidy is larger than the earnings loss due to the lower labor productivity of exempt jobs. This subcase is extreme because D = 0 can only be better than D = 1 if, at the margin, the present value of the subsidy, which is affected by (1-γ) and discounted at rate $r(-)$, rises more than what is lost from a more productive job.

The optimal income tax literature introduced by Mirrlees (1971) offers some answers. Although most of the debates spawned by that literature refer to the marginal tax rate at the top of the earnings distribution, the question raised here refers to the median of the earnings distribution. In this region, there is much more consensus: The simulation results of Slemrod et al (1994) for an economy with given factor prices[20], and whose second pillar is a perfect substitute for voluntary saving, show that the optimal two-bracket income tax has two features: (i) all the marginal tax rates are smaller than 100%; (ii) the marginal tax rate for the low-productivity workers should be *larger* than the one for the high-productivity workers. The analytical and empirical work by Diamond (1998) for more general income tax schedules *also* finds that for the distribution of earnings in the U.S., it is optimal to have larger marginal tax rates for low productivity workers than for workers near the median of the earnings distribution.

Thus, the optimal schedule of net taxes is concave near the median of the earnings distribution, rather than convex. This implies that an affluence test (a convex schedule) is less desirable than a "means" test (where means are contributory pensions). Thus, public finance theory recommends that the "low density trap" be mitigated by reducing γ for the poor well bellow 100%, but not to zero, while keeping the withdrawal rate at a lower level such as zero for workers above the median of the earnings distribution (Valdés-Prieto 2002, p. 67-71). However, the level of the contributory pension at which withdrawal should begin has not been dilucidated. It is unknown whether this level should be zero, or should be positive as in the earned income tax credit (Diamond, 1998).

Poblete's 2005 thesis (Universidad Católica de Chile) offers two additional findings, also based on simulations in an economy with given factor prices, where only a noncontributory subsidy is present. A major difference with Slemrod et al (1994) is that exempt jobs (whose saving is not observed by the government in old age) are available to individuals. First, the extreme with $\gamma = 0$ never maximizes social welfare, because the fiscal cost of the program is large and exacerbates tax distortions. Therefore, a universal flat benefit is suboptimal, even if feasible. Second, small reductions in γ from an initial level of 100% are irrelevant,

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[20] This is an appropriate assumption for an open economy, where substitution with foreign saving prevents the impact on domestic saving from affecting capital accumulation and labor productivity.

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because participants remain in corner solutions. This is important for first-pillar policy, since it suggests that the welfare difference between choosing $\gamma = 0.48$ (Swedish second tranche) and $\gamma = 0.32$ (2008 Chilean reform) may be substantial. Poblete (2005) finds that the optimal social value for the withdrawal rate γ is approximately 20 percent, for a substantial range of parameters.

A different way to evaluate a small withdrawal rate scheme is to compare it with a minimum pension subsidy that has the same fiscal cost. To simplify, let us assume also that $D_T = 0$ in the minimum pension subsidy. The condition of equal fiscal cost implies that the Basic Pension is smaller than the Minimum Pension Goal, and that the small withdrawal rate scheme is paid to workers that have a contributory pension *larger* than the Minimum Pension Goal. When the scheme with a small withdrawal rate replaces the minimum pension subsidy, workers with a contributory pension larger than the Minimum Pension Goal, but smaller than the MCPWS, begin to pay an implicit tax at rate γ in covered jobs, which they *would not have paid it* in the absence of this replacement. Thus, this group reduces its density of contribution. On the other hand, the low-productivity participants receive some subsidy under both designs. They always reduce their density of contribution, but do so to a greater extent with the minimum pension subsidy. Since this group is poorer, their utility increases are valued by more. In addition, this group should be smaller in number as compared to the first group. What is the balance of these considerations? the simulations by Slemrod et al (1994) and by Poblete (2005) give an answer: social welfare is higher in a scheme with a small withdrawal rate.

5. Other policies to increase the density of contributions

This section reviews policies to increase density of contribution that do not rely on improving the design of noncontributory subsidies.

One indirect method to raise density is to allow each participant to share contributions with the spouse (Kotlikoff, 1997).[21] This method raises density among home workers and second

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[21] Kotlikoff and Sachs (1997) propose to mandate a 50-50 split of contributions. To see why such a division should *not* be mandatory, consider a couple where she is a working nurse and he is a medical student with little earnings. These problems may be avoided and density may still rise if the State offers mutual division of contributions to all couples, subject to written approval by both members.

earners in a household. This is different from allowing judges in divorce cases to divide pension rights between spouses, because contribution sharing raises density and reduces first-pillar subsidies even in the absence of divorce. Contribution sharing is also different because it is operative even if divorce occurs after pension age.

Another option to increase density is a subsidy *to contributions*, not to benefits, as in noncontributory pensions. Since subsidies to contributions are delivered immediately, they are likely to be more effective than noncontributory subsidies, which are delivered in old age. This is because individuals discount the expected value of those subsidies for illiquidity and inalienability. In addition, persons that neglect old age –who tend to be excessively optimistic- are less likely to give credence to the pessimistic event of poverty in old age.

Of course, contribution subsidies have fiscal costs and thus raise tax distortions. In addition, such subsidies create incentives to raise density to socially *excessive* levels among those that visualize old age correctly (no neglect of old age). Another problem with subsidies to contributions is that some of them would be given to individuals who subsequently accumulate above-average wealth, either because of luck or effort. This reduces the expected social value of the subsidy.

Interactions with employment protection regulations, such as a relatively high minimum salary, should be taken into account. When a contribution subsidy attracts more individuals to covered jobs, they should not find enough jobs in the presence of a minimum salary. In this case a subsidy to contributions merely increases the unemployment rate, and the fiscal expense is dissipated in more search effort. This consequence can be avoided if the law stipulates that some share of the subsidy must be paid to the employer. This provision reduces the effective minimum salary. This portion of the subsidy may increase the employment rate of the target group or may be passed along to workers as a higher contractual wage paid immediately. In both cases density improves.[22]

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[22] This innovation is incorporated in the 2008 pension reform in Chile. See articles 82 to 84 of the 2008 pension reform law (Chamber of Deputies, 2008).

Another tool that affects density of contribution is to lighten up the mandate by reducing the contribution rate θ. This reduces $\left(z^{ex} - (1 - \theta - t_a)\right)$ making cases F3 and F4 more prevalent, where D* = 1. Workers that visualize old age correctly still save voluntarily for old age if θ is reduced. However, if θ is reduced, middle-class workers that neglect old age accumulate fewer pension rights, and this raises the chances of inadequacy. The elasticity of density to θ needs to be above 1 in absolute value for this method to make sense.

A more attractive way to lighten up the mandate, proposed by Beveridge (1942, p. 139), is to exempt the first portion of earnings from the mandate, i.e. to make the least taxable earnings positive (*LTE* > 0; see definition in section 2.2). This approach has been applied for decades in the Netherlands. Brazil applies a reduced contribution rate to earnings below a LTE. A beneficiary whose earnings subsequently grow faster than average contributes more later in life, and achieves a substantial contributory pension despite having enjoyed an exemption when young. This substantial pension excludes him from noncontributory subsidies, provided that they are targeted. At the same time, a positive LTE implies that a participant that is "lifetime poor" is allowed to consume more in the active phase, which is socially desirable. This scheme provides relief when it is needed, and is self-targeted, reducing fiscal cost. This scheme also prevents the perverse situation where a mandate leaves a lifetime poor participant worse off than in the absence of all pillars (Valdés-Prieto 2002, p. 150-1). However, since a higher *LTE* with a given contribution rate means that the middle class worker that neglects old age saves a smaller amount, old age poverty looms closer, not farther, for the middle class.

A complementary approach to increase density is to review legal exemptions from the mandate to contribute. Exemptions for workers whose earnings are partially observed by the State are not warranted. However, the self-employed who declare honorarium income to the tax authorities (mostly higher earners) are legally exempt from the mandate to contribute in many emerging countries. However, an elimination of this exemption increases the incentives for the formal self-employed to go informal. In response to this concern, Berstein, Reyes and Pino (2005) proposed the following scheme that redefines the default: the formal self-employed would keep the option not to contribute, but the tax authorities would assume that they want to contribute – and would direct the statutory

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portion of declared honoraria towards the old-age individual account, drawing on the retention of honoraria sent by the employer – unless the participant comes forward and actively requests not to contribute.

This section shows that density of contribution can be increased with several policy tools, and not just by improving the design of noncontributory subsidies. More generally, equilibrium density depends on the design of the combined "multipillar" structure.

6. Final remarks

This paper offers a model where a contributory system is not mandatory for all jobs, but the mandate is operative for some jobs. Covered jobs bundle earnings with saving, forcing job choice and saving decisions to interact. An important finding is that the effective return offered by a contributory system can be enhanced very significantly by the productivity premium of covered jobs over uncovered jobs. The validity of this result extends to individuals that suffer from the illiquidity of pension rights, those that suffer from poverty and those that neglect old age. It is also found that some individuals change their contribution density very significantly in response to modest perceived increases in the rate of return of contributory saving, compared to the return of pure saving.

The design of noncontributory subsidies can create those reversals, because the withdrawal rate of the subsidy affects the rate of return of the combined "two-pillar" system that adds contributory pensions and noncontributory subsidies. This shows how this model can be used to guide the design of "multipillar" pension systems (World Bank, 1994).

Two topics for future research are suggested. One is whether the withdrawal rate should be the same in response to different types on income when old. One such income comes from mandatory pensions (second pillar), another is the one coming from tax-favored pensions (either individual or employer-based: third pillar), a third source of income comes from pure saving, a fourth comes from family transfers (spouses, children) and a final type of income comes from labor earnings (when old). The proper order of these withdrawal rates is not addressed in this paper. The second topic concerns the design of mandatory contributory pensions, regarding parameters such as MTE, LTE and vesting requirements (sections 2.2 and 5).

References

Abel, A., G. Mankiw, L. Summers and R. Zeckhauser (1989) "Assessing dynamic efficiency", *The Review of Economics Studies*, Vol. 56 (1) No. 185, January, p. 1-20.

Aguiar, M. and Hurst, E. (2005) "Consumption versus Expenditure", *Journal of Political Economy*, Vol 113, nº 5 (October).

Andolfatto, D. (2002) "A theory of inalienable property rights", *Journal of Political Economy*, 110, 2, April, p. 382-393.

Attanasio, O. and M. Browning (1996) "Consumption over the life cycle and over the business cycle", *American Economic Review* 85 (5), p. 1118-37.

Auerbach, A. and L. Kotlikoff (1987) *Dynamic Fiscal Policy*, Cambridge University Press.

Beveridge, W. (1942) *Social Insurance and Allied Services*, also *Beveridge Report*, HMSO reprint, London.

Bernheim, D., J. Skinner and J. Weinberg (2001) "What accounts for the variation in retirement wealth among U.S. households" *American Economic Review* 91, 4 p. 832-857.

Berstein, S., Reyes, G. and Pino, F. (2005). "Independent Workers: should they be mandated or subsidized to contribute? A third option", *En foco* Document Expansiva Nº 66 (in Spanish), www.expansiva.cl

Beyer, H. and S. Valdés-Prieto (2004) "Proposal to increase the density of contribution", report for national Office for Women (SERNAM), March, Santiago; later version presented at international seminar organized by Centro de Estudios Públicos and Superintendency of AFP, November 11-12, Santiago (in Spanish), available in www.cepchile.cl

Bucheli, M., A. Forteza and I. Rossi (2007) "Work history and the access to contributory pensions: The case of Uruguay", Working Paper, Departamento de Economía, Facultad de Ciencias Sociales, Universidad de la República, Uruguay.

Chamber of Deputies (2008) Pension Reform Law; Oficio Nº 7259 from the Chamber of Deputies to the President of the Republic of Chile, Valparaíso, January 16. Subsequently published as law Nº 20.255.

Cigno, A. and M. Werding (2007) *Children and Pensions*, The MIT Press.

Diamond, P. (1965) "National debt in a neoclassical growth model", *The American Economic Review*, Vol. 55, No. 5, Part 1, Dec. 1126-1150.

Diamond, P. (1998) "Optimal income taxation: An example with a U-shaped pattern of optimal marginal tax rates", *The American Economic Review* Vol. 88, No. 1, March, 83-95.

Diamond, P. and S. Valdes-Prieto (1994) "Social Security Reforms", chapter 6 in Bosworth, B. R. Dornbusch and R. Labán (eds.), *The Chilean Economy: Policy Lessons and Challenges*, Brookings Institution, Washington, D. C. pp. 257-328.

Godoy, Oscar and S. Valdés-Prieto (1997) "Democracy and pensions in Chile: Experience with two systems", chapter 4 in S. Valdés-Prieto (ed.) *The Economics of Pensions*, Cambridge University Press, also in *Cuadernos de Economía*, Vol. 31, N° 93, pp. 135-160.

Hubbard, G., J. Skinner and S. Zeldes (1995), "Precautionary saving and social insurance", *Journal of Political Economy,* 103 (2), April.

Laibson, D. (1997) "Golden Eggs and Hyperbolic Discounting", *Quarterly Journal of Economics*, Vol. 112, No. 2, May , Pages 443-477.

Mirrlees, J. (1971), "An exploration in the theory of optimum income taxation", *Review of Economic Studies*, 38, p. 175-208.

Palme, J. (2005), "Features of the Swedish pension reform", *The Japanese Journal of Social Security Policy*, Vol. 4, No.1 (June), p. 42-53.

Poblete, Dante (2005) "Optimal degree of targeting of noncontributory pensions in the Third World", Thesis in Economics, Instituto de Economía, Universidad Católica de Chile, www.economia.puc.cl

Rofman, R. and Lucchetti, L. (2006), "Social Security in Latin America: Concepts and Measurements of Coverage," Social protection Discuussion paper Nº 0616, The World Bank.

Salditt, F., P. Whiteford and W. Adema (2007) "Pension Reform in China: Progress and Prospects", OECD Social, Employment and Migration Working Papers No. 53, OECD, Paris.

Scholz, J.K., A. Seshadri and S. Khitatrakun (2006) "Are Americans saving optimally for retirement?" *Journal of Political Economy*, Vol 114, nº 4 (August). p. 607-643.

Slemrod, J., S. Yitzhaki, J. Mayshar and M. Lundholm (1994) "The optimal two-bracket linear income tax", *Journal of Public Economics* 53, 2, p. 269-290.

Titelman, D. and Uthoff, A. (2005), "The Role of Insurance in Social Protection in Latin America", *International Social Security Review*, Vol. 58, 2-3/2005, Oxford (UK) and Malden (USA), p. 43-68.

Torche, A. and G. Wagner (1998) "Social Security: individual valuation of a mandated benefit ", *Cuadernos de Economía*, No 103, p. 363-390, (in Spanish) available in www.economia.puc.cl.

Valdés-Prieto, S. (2002) *Pension policies and pensions markets: a university textbook for Latin America*, Ediciones Universidad Católica, 930 pages, Santiago (in Spanish). Available in www.edicionesuc.cl

Valdés-Prieto, S. (2006) "Presentations to the Presidential Advisory Council on Pension Reform", April 11 and 19, Santiago, English version available in www.economia.puc.cl "Faculty" button.

Valdés-Prieto, S. (2007) "Solidarity pension: advantages, defects and proposals", *Temas de la Agenda Pública* series Nº 13, December, Santiago de Chile, (in Spanish), www.puc.cl/agendapublica/

Valdes-Prieto, S. (2008) "The 2008 Chilean reform to first-pillar pensions" (companion paper), mimeo Instituto de Economia, Catholic University of Chile, available in www.economia.puc.cl

van Ginneken, W. (2007) "Extending Social Security Coverage: Concepts, Global Trends and Policy Issues" . *International Social Security Review*, Vol. 60, No. 2-3, April-September, pp. 39-57.

van Ginneken, W. (editor) (1999) *Social Security for the Excluded Majority*, ILO, Geneva.

World Bank (1994) *Averting the old age crisis: Policies to protect the old and promote growth*, Oxford University Press.

World Bank (2000) *The Korean Pension System at a Crossroads,* Report N°2004-KO, Washington D.C.

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The adequacy of contributory pensions for the middle classes depends on density of contribution. Density can be far below 100% because the State is unable or unwilling to impose the mandate to contribute on all jobs, especially on poor workers such as many in self-employment and small firms. The paper presents a model where individuals choose whether to bundle saving for old age in a covered job or to save independently while choosing an uncovered job. The determinants of the effective rate of return offered by the contributory pension plan include the earnings differential. This return is then compared with the returns offered by pure saving in the financial market, to determine the equilibrium density of contribution. The paper also applies the model to assess two standard designs for noncontributory subsidies for the old poor. It finds that these standard designs crowd out contributory pensions for the middle classes by reducing density. The paper also considers two second-generation designs for noncontributory subsidies and other approaches to raise density. This model also allows optimization of the combined "multipillar" structure, where participants get noncontributory pensions and also contributory pensions based on both mandates and fiscal incentives.

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