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### **Government Guarantees on Pension Fund Returns**

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# **Government Guarantees on Pension Fund Returns**

by

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## **Abstract**

This essay reviews defined contribution pension return guarantees typically made by governments in connection with pension privatizations. Finance theory related to the pricing of options provides a unifying framework for evaluating the cost of these guarantees. The essay considers two types of guarantees on the rate of return earned by an individual pension fund: a guarantee of a fixed minimum rate of return; and a guarantee of a minimum rate of return that is set relative to the performance of other pension funds. A minimum pension benefit guarantee for a participant in a mandatory defined contribution pension plan is also discussed. Costs for each of these guarantees are illustrated using typical parameter values.

\* A detailed technical description of this research is given the working paper "The Value of Guarantees on Pension Fund Returns." A similar description of defined contribution pension guarantees as well as a review of defined benefit pension guarantees is contained in the chapter "Government Guarantees for Old Age Income" to be published by the Pension Research Council of The Wharton School and the University of Pennsylvania Press in the volume *Prospects for Social Security Reform*, Olivia Mitchell, editor.

## Government Guarantees on Pension Fund Returns

Social security reform has been a serious concern to many countries. In Latin America, a number of reforms have been implemented by partially or fully privatizing pension obligations. Most often, these privatization reforms have encouraged or required that individuals switch from a government-run defined benefit pension plan to a privately-run defined contribution system. A potential obstacle exists, however, in gaining political approval for this type of reform. By converting to a defined contribution system, individuals may be exposed to risks not previously faced in a government-sponsored defined benefit plan. Participants in a defined contribution system risk experiencing lower than anticipated investment returns, possibly leaving them with inadequate wealth during their retirement years.<sup>1</sup>

To make reforms involving a conversion to a defined contribution system more attractive to the public, governments have typically provided guarantees that reduce individuals' exposure to investment risks. As a result, guarantees of defined contribution pensions have recently become more common, especially in Latin America which has been at the forefront of pension privatizations.<sup>2</sup> These guarantees have been of two main types. One type insures the periodic rates of return earned by the pension funds in which individuals can invest. Typically, this takes the form of a guarantee that each defined contribution pension fund earns an annual rate of return greater than a pre-specified minimum. The second type of guarantee directly insures each individual's, rather than each pension fund's, return on pension savings. This type of guarantee ensures that participants in a defined contribution system receive a minimum pension payment throughout their retirement years, even if their pension savings are exhausted due to withdrawals during their retirement.

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<sup>1</sup>Bodie, Marcus, and Merton (1988) discuss the relative merits of defined contribution and defined benefit pension plans. Defined contribution plans are increasingly popular throughout the world. For discussions of various countries' pension systems, see Mitchell (forthcoming), Davis (1996), and Turner and Wantanabe (1995).

<sup>2</sup>For descriptions and critical analyses of Latin American pension reforms, see Mitchell and Barreto (1997) and Queisser (1995).

Because governments usually retain an insurance obligation following a pension privatization, estimating the cost of government guarantees is important for gauging the implicit subsidy associated with a particular pension reform. By accounting for the cost of pension guarantees in government budget statistics, an improved, market value-based measure of fiscal spending can be obtained. In addition, these cost estimates could make feasible a system of risk-based insurance premiums that would reduce or eliminate the subsidies from providing guarantees.

This essay presents a number of new results for valuing defined contribution guarantees using a valuation technique known as “contingent claims analysis”(CCA). CCA was first used to value option contracts and corporate liabilities, but it has also been applied to value many different types of government guarantees and insurance contracts, such as loan guarantees, deposit insurance, and defined benefit pension guarantees.<sup>3</sup> It has the attractive feature of requiring relatively few assumptions in order to value claims. Most often, valuation requires only the assumption that equilibrium asset prices do not allow for arbitrage.<sup>4</sup> Assumptions regarding investor preferences or assets' expected rates of return are not needed.

A particular method for calculating contingent claims values, known as “risk-neutral” valuation or “martingale pricing” can be a unifying framework for valuing all types of guarantees. This method, introduced by Cox and Ross (1976) and further developed by Harrison and Kreps (1979), can yield explicit formulas for guarantee values or it can allow for numeric valuation by Monte Carlo simulation. While it is beyond the scope of this essay to provide a detailed analysis of every possible type of pension guarantee, the techniques it discusses can be

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<sup>3</sup>Contingent claims analysis, also known as “option pricing theory”, derives from the seminal work on the pricing of options by Black and Scholes (1973) and Merton (1973). For references to applications of contingent claims analysis, see Merton (1990) chapters 1 and 19. Pennacchi and Lewis (1994) discuss recent research on valuing defined benefit pension guarantees.

<sup>4</sup>Arbitrage is defined as a set of financial transactions that does not require any initial wealth and that can produce a positive profit but never a loss. Informally, it is sometimes referred to as a “free lunch”.

customized to handle other specific cases.

When governments guarantee private contracts, such as pension plans, adverse selection and moral hazard problems may arise. These incentive problems can be alleviated by properly structuring and pricing guarantees, and/or regulating the activities of the parties on whose behalf the guarantee is given. Discussions of these important issues can be found in a number of recent papers and, due to a lack of space, will not be repeated here.<sup>5</sup> Because this essay's focus is on valuing guarantees, it often takes the risk decisions of the participating parties as given. But it should be emphasized that these decisions are frequently linked to the guarantee's structure, pricing, or regulation. In some cases, by estimating the costs of guarantees and then charging appropriate risk-based insurance premiums that cover these costs, adverse selection and moral hazard problems can be alleviated.

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<sup>5</sup>See Bodie and Merton (1993), Pesando (1996), and Smalhout (1996).

The plan of the essay is the following. The next section discusses option contracts and how pension guarantees have option-like features. It also describes CCA and how it can be used to value these contracts. The following section considers two types of pension rate of return guarantees: one being a fixed rate of return guarantee and the other being a rate of return guarantee that is relative to the performance of other pension funds. In the next section a guarantee of a minimum pension benefit for a participant in a mandatory defined contribution pension plan is considered. Values for these rate of return and minimum pension guarantees are illustrated using typical parameter values. A concluding section then follows. Mathematical formulas for the guarantees discussed in the essay are given in the Appendix.

### **I. Options, Guarantees, and Contingent Claims Analysis**

Options are examples of derivative contracts or “contingent claims.” They are financial instruments whose values depend on the prices of other assets. Options can be categorized into two types. A *call option* is a contract that gives its holder the right to *buy* some asset at a pre-specified price at some future date. A *put option* is a contract that gives its holder the right to *sell* some asset at a pre-specified price at some future date. This pre-specified price is referred to as the option's “exercise price” or “strike price.” Of the two types of options, put options are the most similar to guarantees. This is illustrated with the following example.

Suppose a defined contribution pension fund holds a portfolio of U.S. equities which comprise the Standard and Poor's 500 stock index (S&P500).<sup>6</sup> The value of the pension fund, and therefore the value of the participants' retirement benefits, will vary with fluctuations in the S&P500 stock index. The pension fund can insure itself from significant declines in the value of its stock holdings by purchasing a put option on the S&P500. For example, assume that value of the pension fund's investments currently equals  $S_0 = \$10$  million, and it wishes to insure itself against a fall in the value of the portfolio below  $X = \$9$  million during the next year. It could purchase S&P500 put options having a one-year maturity and an effective exercise price of  $X =$

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<sup>6</sup>The S&P500 is a market-value-weighted index of common stocks of 500 of the largest United States corporations.

\$9 million.<sup>7</sup>

If we let  $G_1$  denote the value of the put option contracts (guarantees) when they mature in one year's time, then

$$(1) \quad G_1 = \max(X - S_1, 0)$$

where  $S_1$  is the value of the pension fund's stock portfolio at the end of the year and “max(x,y)” means “select the maximum of x and y.”<sup>8</sup> Hence, the put option, being analogous to a guarantee or insurance contract, will ultimately have a positive value only if the end-of-year value of the pension fund's stocks,  $S_1$ , sink to less than  $X = \$9$  million. By owning this “portfolio insurance,” the pension fund will have a combined end-of-year value given by

$$(2) \quad \begin{aligned} S_1 + G_1 &= S_1 + \max(X - S_1, 0) \\ &= \max(X, S_1) \end{aligned}$$

so that it is guaranteed to be worth no less than  $X$ .

Rather than obtaining insurance by purchasing exchange-traded put options, equivalent pension fund insurance might be provided by a government. In this case, a government guarantor would have an end-of-year liability given by  $G_1 = \max(X - S_1, 0)$ . Figure 1 graphs the end-of-year values of the pension fund if it were uninsured ( $=S_1$ ), the guarantee ( $=\max(X - S_1, 0)$ ), and the pension fund if it were insured ( $=\max(X, S_1)$ ). It is obvious that the guarantee puts a lower-bound on the end-of-year value of the insured pension fund.

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<sup>7</sup>S&P500 index options are traded on the Chicago Board Options Exchange. Each contract is written on an underlying stock index value equal to 10 times the index. Hence, if the S&P500 index equals 1,000, each option contract is written on an underlying stock value of \$10,000. In this case, the pension fund would purchase 100 put option contracts having an exercise price of 900.

<sup>8</sup>Technically, since changes in the S&P500 index do not include dividends paid by the stocks in the index, a proper comparison would exclude from  $S_1$  any dividends paid to the pension fund during the previous year.

While the value of the government guarantee is easily determined at the end of the year when  $S_1$  becomes known, the more challenging problem is determining its value (cost) at the beginning of the year prior to the guarantee's maturity. Clearly, if the government guarantee was exactly equivalent to an exchange-traded option contract, such as an S&P500 put option, then its market value could be inferred from the current market price of the S&P500 put. However, it is rarely, if ever, the case that government pension guarantees have an exact private-market counterpart. Thus, we need another method for calculating the guarantee's theoretical market price. This is where CCA can be applied.

CCA is a method for valuing derivative-like contracts. This valuation method has been embraced by both financial economists and financial market practitioners because it is based on only a few, often realistic assumptions. Its main requirement is that prices of derivative contracts and their underlying securities do not allow for arbitrage opportunities. One need not make any particular assumptions regarding investors' preferences, such as their degree of risk aversion, or the expected rates of return on derivatives or their underlying securities. CCA is consistent with most any set of investor preferences or equilibrium asset expected rates of return.

One technique for calculating contingent claims values that has proven to be quite useful is known as the “risk-neutral” valuation technique, or more generally the “martingale pricing” approach. This technique can be illustrated in terms of our previous example. Let  $r$  denote the current risk-free interest rate for borrowing or lending over the next year. Also let  $G_0$  denote the current value (cost) of the government guarantee contract. Then this technique states that the guarantee's value is given by

$$(3) \quad \begin{aligned} G_0 &= \frac{1}{1+r} E^* [ G_1 ] \\ &= \frac{1}{1+r} E^* [ \max( X - S_1, 0 ) ] \end{aligned}$$

where  $E^*[\square]$  is an operator that takes the expected value of its argument subject to the condition that the expected rate of return on all assets equals the risk-free interest rate,  $r$ . For this reason,

$E^*[\square]$  is referred to as the “risk-neutral” expectation, since only in an economy where all investors are risk-neutral would the equilibrium rates of return on all assets equal the risk-free rate, that is, risky assets would bear no risk-premium. Hence, in equation (3) we would compute the expected value of  $\max(X-S_1,0)$  by setting the expected rate of return on the pension fund's stock portfolio equal to the risk rate, that is,  $E^*[S_1] = S_0(1+r)$ .

The risk-neutral expectation is denoted with an asterisk to differentiate it from the true expectation operator,  $E[\square]$ . In general, the true expected rates of return on risky assets will not equal the risk-free rate, and it should be emphasized that this “risk-neutral” valuation or “martingale-pricing” technique *does not* assume that risky asset expected rates of return truly do equal the risk-free rate. As stated earlier, the only assumption being made is that equilibrium values of contingent claims and their underlying securities do not allow for arbitrage. It just turns out that one can compute the arbitrage-free values of contingent claims by calculating their expected payoffs *as if* securities' expected rates of return equaled the risk-free rate and then discounting these expected payoffs by the risk-free rate.<sup>9</sup>

Intuitively, the risk-neutral or martingale approach gives the correct value for the government guarantee,  $G_0$ , because of two erroneous assumptions whose effects cancel each other out, resulting in a correct valuation. One incorrect assumption is that all assets have an average rate of return equal to the risk-free rate, that is, there are no “risk-premia” in asset rates of return. This implies a “risk-neutral” expectation of  $G_1 = \max(X-S_1,0)$  that differs from the “true” expectation of  $G_1$ , leading to the first error. The other incorrect assumption is that this risky payment should be discounted at the risk-free rate,  $r$ , rather than a discount rate that includes a risk-premium, leading to a second error. Because both the first and second “errors” involve a failure to account for risk premia, the first error “overstates” the expected value of  $G_1$  by the risk premia while the second error “understates” the discount factor applied to  $G_1$  by

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<sup>9</sup>While the proof of this result is beyond the scope of this essay, it can be shown mathematically that this valuation technique relies only on the absence of arbitrage. See Koci<sup>®</sup> (1996) and Duffie (1996) for a detailed discussion of risk-neutral valuation and the martingale pricing approach.

the risk premia. Mathematically, these two errors cancel, leading to a correct valuation formula. Importantly, because this computational technique does not require specification of the actual risk premia of the assets in the economy, no assumptions regarding the signs or magnitudes of risk premia are needed.

To actually calculate the guarantee value in equation (3), one must make an assumption regarding the volatility (standard deviation) of the pension fund's stock portfolio (but, of course, no assumption regarding its expected rate of return is needed).

If one assumes that the stock portfolio's rate of return is normally distributed, computing the risk-neutral expectation in (3) results in the well-known Black-Scholes formula for a put option:

$$(4) \quad G_0 = \frac{X}{1+r} N(-d_2) - S_0 N(-d_1)$$

where  $N(\square)$  is the standard normal distribution function,  $d_1 = \ln[S_0(1+r)/X]/\sigma_s + \square\sigma_s$ ,  $d_2 = d_1 - \sigma_s$ ,  $\ln[\square]$  is the natural logarithm function, and  $\sigma_s$  is the annualized standard deviation of the rate of return on the stock portfolio.

In practice, government guarantees are more complicated than the standard put option-type guarantee in our above example. In some cases, one can apply the martingale pricing approach derive different formulas for these more complicated guarantees. When exact formulas cannot be derived, one can still numerically calculate expected payments and guarantee values using a Monte Carlo simulation technique as in Boyle (1977). The next two sections present examples of both types of cases.

## **II. Valuing Guarantees on a Pension Fund's Rate of Return**

This section considers two sorts of pension fund rate of return guarantees made by governments. These guarantees can be valued by recognizing their similarity to various types of “exotic” options, such as “forward start options,” “options to exchange one asset for another,”

and “options on the minimum of two risky assets.”<sup>10</sup> We begin by considering a relatively simple fixed minimum rate of return guarantee, similar to one provided by Uruguay. We then consider a minimum rate of return guarantee that depends on the average rate of return earned by all pension funds, such as that provided by the government of Chile.

**A Minimum Fixed Rate of Return Guarantee.** Uruguay permits both private and public pension funds, known as “Asociaciones de Fondos de Ahorro Previsional”(AFAP).<sup>11</sup> In the case of public AFAPs (but not private AFAPs), the government guarantees to pension fund participants a minimum annual real rate of return, denoted by  $m$ , equal to 2 percent. Thus, a public AFAP which earns less than 2 percent during a given year would require a government transfer to make up the difference. By applying martingale pricing methods, an explicit formula for the value of these annual rate of return guarantees can be obtained. The derivation, given in Pennacchi (1997), makes use of the similarity between these guarantees and forward start options.

A forward start option is an option having a random exercise price when the option is first issued. The exercise price is set equal to the contemporaneous value of the underlying asset at some future date prior to the maturity date of the option. In other words, the option's exercise price is set to make it “at-the-money” at a pre-specified future date. The analogy between a rate of return guarantee and a forward start option is that a (continuously compounded) rate of return on an asset over some future interval, say from date  $t_1$  to time  $t_2$ , needs to be computed based on two future asset values:  $\ln[S(t_2)] - \ln[S(t_1)]$ , where  $S(t)$  is the value of the (pension fund) asset at date  $t$ . Since, in general, the  $t_1$  beginning date of the rate of return is in the future,  $S(t_1)$  is unknown and analogous to the unknown beginning exercise price of the forward start option.

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<sup>10</sup>For a description and analysis of these exotic options, see Hull (1997).

<sup>11</sup>See Mitchell (1996) for a discussion of pension system reform in Uruguay.

The formula for the minimum rate of return guarantee, listed in the Appendix, equals a weighted annual series of “at-the-money” Black and Scholes (1973) type put options, where the weights are proportional to the assumed growth in net new contributions to the pension fund. The value of this guarantee depends on the difference between the risk-free rate and the guaranteed rate of return,  $r-m$ , as well as the rate of return standard deviation of the pension fund,  $\sigma_s$ . Figure 2 plots the annual percentage cost of the guarantee per value of pension fund assets,  $100xG_0/S_0$ , as a function of  $r-m$  for four different values of  $\sigma_s$ . The first value,  $\sigma_s = 0$ , reflects the case in which the AFAP invests entirely in risk-free real assets, earning a certain rate of return equal to  $r$ . The next three cases reflect risky AFAP investments. Because social security reform was enacted in Uruguay only in 1995, and data on AFAP returns is not yet available, the non-zero values of  $\sigma_s$  that we use reflect a parameter estimate taken from Chilean pension fund returns, namely  $\sigma_s = .038$ ,  $.077$ , and  $.154$ , which represents one half, once, and twice the average of all Chilean pension funds. This was estimated from annual data on Chilean pension fund returns for the period 1981 to 1992, as reported in Diamond and Valdés-Prieto (1994, p.300).

As would be expected, Figure 2 shows that the cost of the guarantee rises with the volatility of the AFAP's investments,  $\sigma_s$ , and decreases as the difference between the real interest rate and the minimum guarantee,  $r-m$ , widens. Note that the function becomes more convex as volatility decreases, and for the case of  $\sigma_s = 0$ , the relation is kinked at  $r-m = 0$ . This limiting case reflects the common sense result that if the AFAP invests in risk free assets earning  $r$ , then the value of the guarantee equals zero for  $m \leq r$ , but the value of the guarantee is nonrandom and equal to  $100x(m-r)$  for  $m > r$ .

Another important insight from the guarantee formula is that if a single annual guarantee has any value and the real growth rate of the pension fund is non-negative, the value of the annual series of guarantees will grow without bound as the number of future years for which this guarantee is made increases. A policy implication is that governments should be quite cautious in providing such a guarantee to funds that are expected to grow substantially.

**A Minimum Relative Rate of Return Guarantee.** In Chile, private pension funds, known as “Administradora de Fondos de Pensiones”(AFPs), are required to earn an annual real rate of return that is linked to the average annual real rate of return of all private pension funds. If  $R_a$  is the (ex-post) average annual rate of return earned by all AFPs, then each AFP must earn at least  $\min(R_a - \alpha, \beta R_a)$  where  $\alpha = .02$ ,  $\beta = 1$ , and “ $\min(x,y)$ ” means select the minimum of  $x$  and  $y$ . Thus, if  $R_a$  turns out to be greater than 4 percent, each AFP must earn at least  $1 R_a$ , while if  $R_a$  turns out to be less than 4 percent, each AFP must earn at least  $R_a - 2$  percent. All AFPs are required to hold capital (a guarantee fund) of at least 1 percent of the value of its pension portfolio, invested in the same security portfolio as that of its pension fund. If the fund's return is less than  $\min(R_a - \alpha, \beta R_a)$ , it must make up the difference from its capital and replenish its capital within 15 days. The AFP's license would be revoked if it fails to do so. Thus, given an AFP capital ratio of  $c = .01$ , the government would be exposed to loss following an AFP that earns less than  $\min(R_a - \alpha, \beta R_a) - c = \min(R_a - \alpha - c, \beta R_a - c)$ .

This government guarantee for an individual AFP is similar to an annual series of options to exchange the individual AFP's pension assets for the minimum of two other risky assets. The formula for the cost of this guarantee is listed in the Appendix and is derived by Pennacchi using results in Margrabe (1978), Stulz (1982), and Johnson (1987). As one might expect, the value of this relative rate of return guarantee is sensitive to the standard deviation of the individual AFP's rate of return as well as the correlation between the individual AFP's return and the average return of all AFPs.

Figure 3 plots the annual cost of this Chilean guarantee as a percentage of the current value of the pension fund assets,  $100 \times G_0 / S_0$ . This is done for different assumed correlations between individual AFP and average AFP returns. The guarantee value is shown for three cases: when the individual AFP standard deviation equals, is twice, or is one-half that of the average of AFPs. As would be expected, the value of the guarantee falls as the correlation rises. Interestingly, when the standard deviation of the individual AFP's return exceeds that of the

average of AFPs (which should be the case for the typical AFP since individual risk is diversified by averaging), then even when the correlation is perfect, the guarantee will have positive value.

### **III. Valuing Minimum Pension Guarantees for Defined Contribution Plans**

This section considers the value of a minimum pension guarantee for a participant in a mandatory defined contribution pension system, where a fixed proportion of a worker's wage is assumed to be contributed to a pension fund that earns risky returns. Two previous studies, both estimating the value of this guarantee for the case of Chile, should be noted. Wagner (1991), whose results are summarized in Diamond and Valdés-Prieto (1994), values this guarantee by simulating its annual cost when the demographics and maturity of the pension system are at their steady state values. The model calculates this cost under different assumptions regarding the real rate of return on pension fund assets and the level of the minimum pension guarantee. Another study by Zarita (1994) applies contingent claims techniques to value Chile's minimum pension guarantee. His model explicitly allows for a stochastic rate of return on pension fund assets, so that a worker's accumulated pension savings at retirement is random. If the worker's savings at retirement is less than the cost of an annuity providing the minimum pension, the government is assumed to make a payment to cover the difference. The risk-neutral expected value of this government payment is calculated using a Monte Carlo simulation of the worker's risky pension investment assuming a deterministic level of wage contributions each period and a constant real interest rate.

The approach taken in this section is similar to that of Zarita (1994) but includes a number of extensions. First, in addition to allowing pension returns to be stochastic, we also allow a worker's real wage, and thus his monthly pension contribution, to follow a random process. The evolution of real wages is also assumed to influence the minimum pension set by the government when the worker retires. Second, real interest rates are assumed to follow a stochastic process. This is potentially important since retirement annuity values are a function of real interest rates. In addition, valuing the government's guarantee requires that real interest rates discount the government's guarantee payments and, in general, these payments are systematically related to not only asset returns and wage levels, but also the real interest rate.

Third, we model the government's payments for a minimum pension in a different, arguably more realistic manner. Upon reaching retirement, a retiree may have a choice regarding his benefit payments. If he has sufficient pension savings, he may choose to close his pension account and use his savings to purchase a lifetime annuity that provides a benefit at or above the minimum pension. Alternatively, he can maintain his pension account and receive benefits by a scheduled withdrawal of funds from his account. For a retiree with an account balance insufficient to purchase a minimum pension annuity, a scheduled withdrawal of funds is required. The maximum amount that a retiree can withdraw each year is determined by a government schedule that depends on the retiree's current pension balance and the value of a lifetime annuity, where this annuity is calculated using the government's "technical" interest rate. If and when a retiree's pension account balance is exhausted, the government guarantees that it will pay him the minimum monthly pension for the remainder of his life.

As discussed in Turner and Wantanabe (1995), and Smalhout (1996), a worker that reaches retirement with a pension balance that is slightly above or at the price of a minimum pension annuity will have an incentive to not purchase an annuity but to choose the scheduled withdrawal option. By choosing this scheduled withdrawal, he will receive free longevity insurance at the government's expense. Should he live longer than expected, the government provides him with a minimum pension. If, instead, he lives less than expected, his heirs will inherit the balance of his pension account. Thus, in some states of the world, he receives a government subsidy that would not occur if he had immediately purchased a lifetime annuity. Hence, for someone reaching retirement with moderate to small pension savings, which is the individual most likely to require minimum pension assistance, it is more realistic to assume a scheduled withdrawal of pension funds. Unlike Zarita (1994), the results reported in this section explicitly consider this scheduled withdrawal.

The following is a brief summary of the model which is detailed in Pennacchi (1997). It is based on three random processes: the rate of return on pension fund assets, the growth in real

wages, and the change in the short term real interest rate. These three processes may be correlated. This short term real interest rate determines the term structure of real yields based on the Vasicek (1977) model. An additional minor source of uncertainty is the individual's mortality. The probability of death at each age is assumed to be uncorrelated with economic variables and is taken from Chile's official life table. A hypothetical male worker is assumed to begin making pension contributions at age 20 and, should he live until the retirement age of 65, begin a scheduled withdrawal of his pension savings at the maximum level allowed by law. The worker's mandatory monthly contribution equals 10 percent of his randomly evolving wage and is invested in his pension fund earning a random rate of return.

At retirement, the maximum that can be withdrawn each month is calculated following the actual Chilean government formula, which is described in Diamond and Valdés-Prieto (1994, p. 290). This formula's "maximum" withdrawal is, however, truly the maximum only if it exceeds the government's minimum pension level. If not, the amount withdrawn is equal to the minimum pension. This occurs until the retiree's pension account is exhausted, should he live that long. After the account balance is exhausted, the government pays the minimum pension until the end of the retiree's life.

The minimum pension is set at the discretion of the government, and it depends on a number of political and economic factors. For simplicity, the model assumes that the minimum pension at the beginning of an individual's retirement follows the formula: minimum pension =  $\alpha$ \*(average wage at start of individual's working life)\*(growth in the individual's real wages over his working life)\* $\beta$ . This assumed formula reflects the likelihood that the government will tend to raise the minimum pension should real wages (and the standard of living) rise. Since Turner and Wantanabe (1995, p. 210) report that the minimum pension is approximately 25 percent of the average wage and because our model assumes that the individual's real wage will almost double over his 45 years of work (1.5 percent average annual growth), the formula should maintain a minimum pension-average wage ratio of approximately 25 percent.<sup>12</sup>

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<sup>12</sup>One component of the lifetime growth of an individual's real wage is likely to reflect

Once again, the martingale approach is used to value this guarantee. The three random process are transformed into “risk-neutral” counterparts so that the guarantee's value can be computed as the expectation of the government's discounted minimum pension payments. This expectation is calculated using a Monte Carlo simulation, where contributions or withdrawals from the individual's pension fund account occur each month. Parameter values typical of Chile were used and are given in Pennacchi (1997).

Guarantee values are calculated for the case of a 20 year-old male beginning wage earner starting with a zero pension fund balance. Mortality is based on the Chilean life tables for male annuitants. Assuming an average Chilean monthly real wage of 100 at the time this individual begins work, we find that the average level of the minimum pension set by the government (according to the formula discussed above) at the worker's retirement date was 44.7.

Figure 4 graphs the present values of the minimum pension guarantee for this 20 year-old worker for different initial monthly wages ranging between 10 and 100. The value of this guarantee ranges from 251.8 for an individual with an initial monthly wage of 10 to 5.8 for an individual with an initial monthly wage of 100. The shape of the relationship is convex as one might expect given the put option-like nature of this guarantee. Also plotted in Figure 4 is the individual's age at which his pension fund account would be depleted, should he live that long. This ranges from age 72.1 for an initial wage of 10 to age 91.8 for an initial wage of 100. Note that this age profile has a concave shape. Higher initial wages increase the time before the pension account is depleted, but less than proportionally. While higher initial wages tend to result in proportionally higher accumulated pension savings at retirement, the government's scheduled withdrawal formula allows greater pension withdrawals for individuals with higher

(.continued)

increased (economy-wide) average productivity, while another component should reflect the individual's increased productivity due greater experience and seniority. Thus, it is reasonable to expect that an individual's lifetime real wage growth will exceed the economy-wide average. For this reason, the formula includes a final factor of one-half. The result is that our simulations give an average minimum pension at the individual's retirement date equal to 44.7 percent of the initial average real wage, implying that, on average, there is a slightly less than doubling (from 25 percent) of the minimum pension.

savings. Thus the withdrawal schedule tends to subdue the effect that greater retirement savings have on the age at which pension funds are depleted.

#### **IV. Conclusions**

Pension privatizations frequently require that individuals contribute to defined contribution pension plans. However, when contributions to these pension plans are mandatory, individuals will be subject to investment risks that they did not previously face in a government-sponsored defined benefit plan. To make privatization reforms politically attractive to the public, governments have typically offered guarantees that reduce individuals' exposure to investment risks.

Recent advances in contingent claims analysis have provided important insights for valuing pension guarantees. This essay illustrates how the martingale pricing approach, also known as the risk-neutral valuation method, can be applied to value a variety of guarantees on pension fund returns. Perhaps the most attractive feature of this approach is the relatively few assumptions needed to calculate guarantee values. The main restriction imposed by this approach is that equilibrium asset prices do not allow for arbitrage opportunities.

This paper analyzed guarantees at a microeconomic level. It considered the values of defined contribution rate of return guarantees for individual pension funds and the value of a minimum pension guarantee for an individual worker in a defined contribution pension system. A potential benefit of this ability to value individual guarantees is that a system of risk-based insurance (guarantee) premiums might be established. Requiring that riskier pension funds, and possibly riskier individuals, pay higher insurance premiums could help control adverse selection and moral hazard behavior. It would reduce the subsidies and the economic distortions associated with government guarantees. The potential for reducing such distortions through risk-based premiums may ultimately change the type of pension system that a government chooses to adopt.

The ability to price guarantees can also allow government budgets to be measured on a market-value basis. A government's total liability from providing guarantees can be calculated by aggregating the values of individual guarantees.<sup>13</sup> This aggregation requires detailed data on the economy's individual pension funds, and/or worker demographics. While such an exercise was beyond the scope of this essay, the analysis presented here provides a foundation for obtaining a more accurate indicator of government fiscal policy.

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<sup>13</sup>A similar aggregation is performed in Cooperstein, Pennacchi, and Redburn (1995), where the aggregate value of deposit insurance is calculated by aggregating the values of deposit insurance provided to individual banks.

## Appendix

This appendix lists guarantee formulas for the (Uruguayan) minimum fixed rate of return guarantee and the (Chilean) minimum relative rate of return guarantee. Derivation of these formulas is given in Pennacchi (1997).

### Minimum Fixed Rate of Return Guarantee

Define  $h(\tau)$  as

$$(A.1) \quad h(\tau) = e^{(m-r)\tau} N(-d_2) - N(-d_1)$$

where  $d_1 = (r - m + \frac{1}{2}\sigma_s^2)\tau / (\sigma_s \sqrt{\tau})$  and  $d_2 = d_1 - \sigma_s \sqrt{\tau}$ . Also assume that the pension fund is growing due to net new contributions at a proportional real growth rate of  $g$ . Then if a government makes this guarantee on an annual basis ( $\tau=1$ ) for  $n$  consecutive years, the total value of the guarantee,  $H_n$ , is

$$(A.2) \quad H_n = S_0 h(1) \sum_{y=0}^{n-1} e^{gy}$$

### Minimum Relative Rate of Return Guarantee

Let  $\sigma_s$  and  $\sigma_a$  be the standard deviations of the rate of return on the individual guaranteed AFP and on the average of all AFPs, respectively, and let  $\rho$  be the instantaneous correlation between the individual AFP's portfolio return and that of the average of all AFPs. Then define  $h(\tau)$  as

$$\begin{aligned}
(A.3) \quad h(\tau) = & e^{-q_x \tau} N_2 \left( \frac{-q_x}{\sigma_1 \sqrt{\tau}} + \%_0 \sigma_1 \sqrt{\tau}, \frac{q_x - q_v}{\sigma \sqrt{\tau}} - \%_0 \sigma \sqrt{\tau}, \frac{\rho_{12} \sigma_2 - \sigma_1}{\sigma} \right) \\
& + e^{-q_v \tau} N_2 \left( \frac{-q_v}{\sigma_2 \sqrt{\tau}} + \%_0 \sigma_2 \sqrt{\tau}, \frac{q_v - q_x}{\sigma \sqrt{\tau}} - \%_0 \sigma \sqrt{\tau}, \frac{\rho_{12} \sigma_1 - \sigma_2}{\sigma} \right) \\
& - N_2 \left( \frac{-q_x}{\sigma_1 \sqrt{\tau}} - \%_0 \sigma_1 \sqrt{\tau}, \frac{-q_v}{\sigma_2 \sqrt{\tau}} - \%_0 \sigma_2 \sqrt{\tau}, \rho_{12} \right)
\end{aligned}$$

where  $N_2(\square, \square, \square)$  is the bivariate normal distribution functions,  $q_x = \alpha + c$ ,  $q_v = (1 - \beta)r + c$ ,  $\sigma_1^2 \equiv \sigma_a^2 + \sigma_s^2 - 2\rho\sigma_a\sigma_s$  3,  $\sigma_2^2 \equiv \beta^2\sigma_a^2 + \sigma_s^2 - 2\rho\beta\sigma_a\sigma_s$  4, and  $\rho_{12} = (\beta\sigma_a^2 - \sigma_a\sigma_s\rho(1 + \beta) + \sigma_s^2)/(\sigma_1\sigma_2)$  5. Assume that the pension fund is growing due to net new contributions at a proportional real growth rate of  $g$ . Then if a government makes this guarantee on an annual basis ( $\tau=1$ ) for  $n$  consecutive years, the total value of the guarantee,  $H_n$ , is

$$(A.4) \quad H_n = S_0 h(1) \sum_{y=0}^{n-1} e^{gy}$$

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